Advanced Integrate with Ansys optiSLang and Mechanical Software

Powering Innovation That Drives Human Advancement

Robust Design Optimization – Theoretical Background

Release 2023 R1

Please note:
- These training materials were developed and tested in Ansys Release 2024 R1. Although they are expected to behave similarly in later releases, this has not been tested and is not guaranteed.
- The screen images included with these training materials may vary from the visual appearance of a local software session.

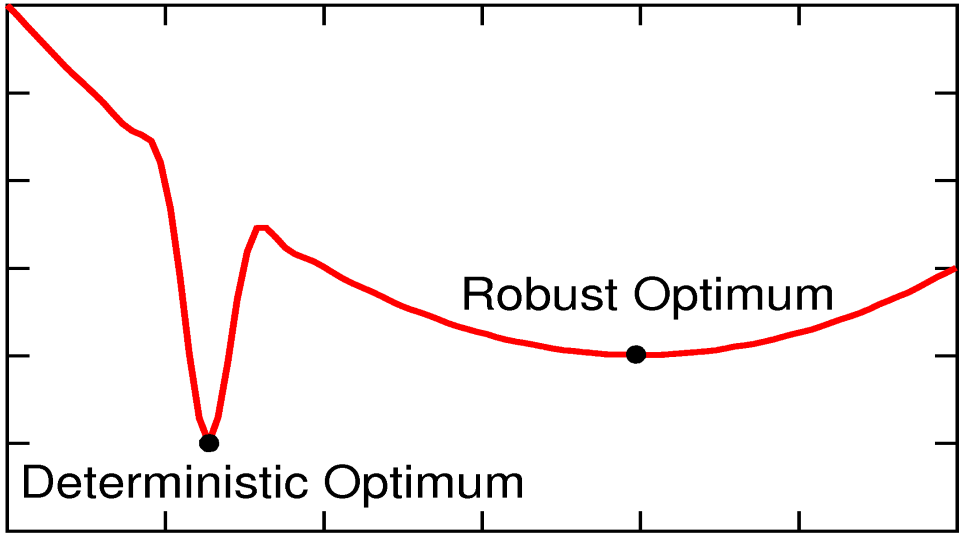
Agenda

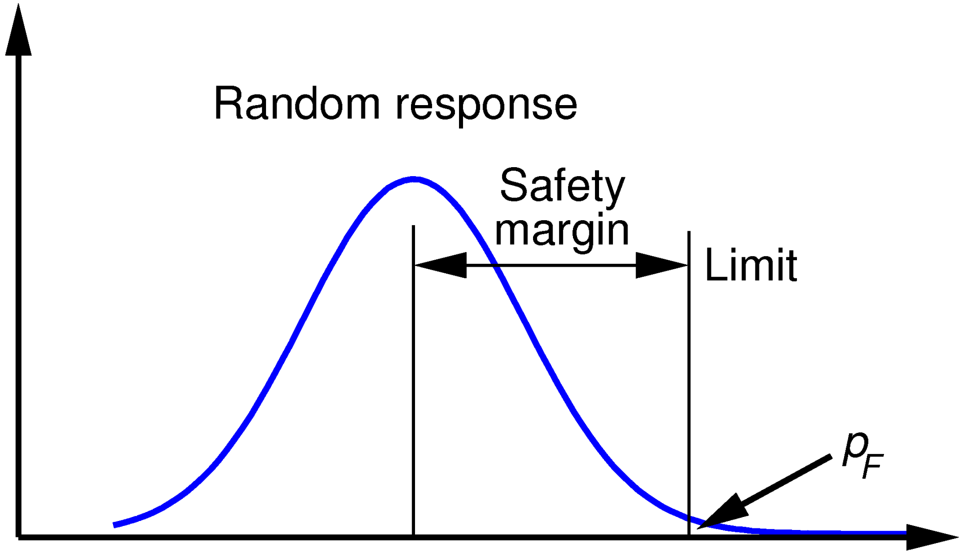
Session	Slide Set	Time	Topic
1	0	5’	Agenda
	1	25’	Introduction to Ansys optiSLang
		10’	Ansys optiSLang in the Ansys Learning Hub – Find your Examples
		15’	Q/A
	2	30’	Sensitivity Study and Optimization – Theoretical Background
2	3	75’	Hands-on – Process Integration, Sensitivity Study and Postprocessing Steel Hook – optiSLang inside Workbench
		15’	Q/A
3	4	40’	Hands-on – Optimization Steel Hook – optiSLang inside Workbench
	5	**20’**	**Robust Design Optimization – Theoretical Background**
	6	40’	Hands-on – Robustness Evaluation Steel Hook – optiSLang inside Workbench
		15’	Q/A

How to Define the Robustness of a Design?

- **Intuitively:** The performance of a robust design is largely unaffected by random perturbations
- **Variance indicator:** The coefficient of variation (CoV) of the objective function and/or constraint values is not greater than the CoV of the input variables
- **Sigma level:** Keep an undesired performance outside an interval of mean +/- sigma level (e.g. design for six-sigma)
- **Probability indicator:** The probability of reaching undesired performance is smaller than an acceptable value

How to Define the Robustness of a Design?





- Performance (objective) of robust optimum is less sensitive to input uncertainties
- Minimization of statistical evaluation of objective function *f* (e.g. minimize mean and/or standard deviation):

$$\bar{f} \rightarrow min \text{ or } \bar{f} + \sigma_f \rightarrow min$$

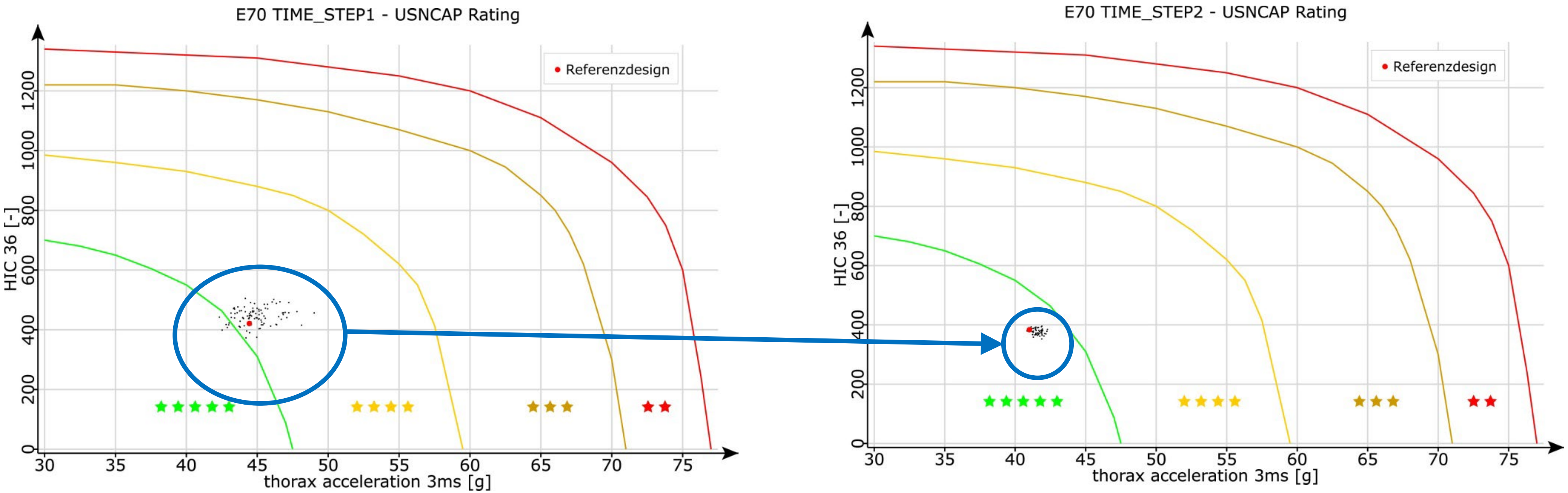
- Safety margin (sigma level) of one or more responses y: $(y_{limit} - \mu_Y)/\sigma_Y \geq a$
- Reliability (failure probability) with respect to given limit state:

$$p_F \leq p_F^{target}$$

Performance of Passive Safety Systems

- Initial configuration does not meet USNCAP 5-star requirements
- Goal is not only a shift into 5-star performance
- But also a low probability slipping into 4-star region

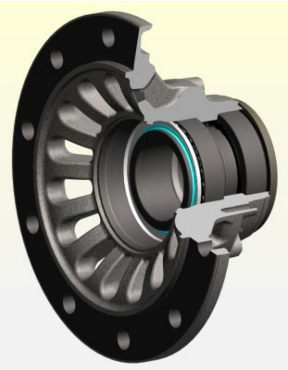
→ Shift mean values and reduce scatter

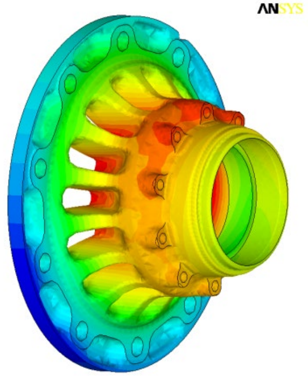


[Will, J., Baldauf, H.: Robustheitsbewertungen bezüglich der virtuellen Auslegung passiver Fahrzeugsicherheit. WOST 2.0, 2005]

Robustness Evaluation as Early as Possible

by courtesy of TIMKEN
Where You Turn





Goal: Tolerance check before any hardware exist!

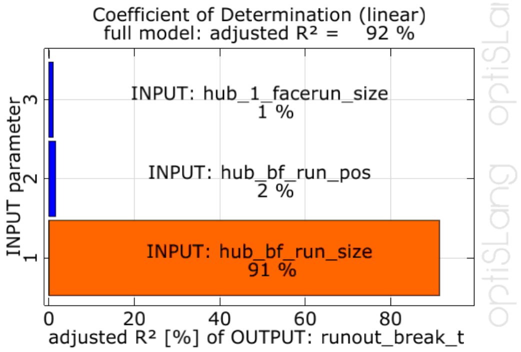
Classical tolerance analysis tend to be very conservative

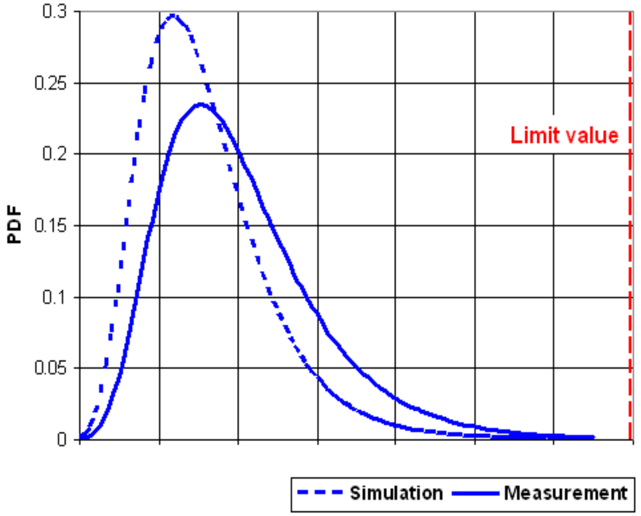
Robustness evaluation against production tolerances an material scatter (43 scattering parameter) shows:

- Press fit scatter is o.k.
- only single tolerances are important (high cost saving potentials)

Production shows good agreement

Design Evaluations: 150
solver: ANSYS/optiSLang





[Suchanek, J.; Will, J.: Stochastik analysis as a method to evaluate the robustness of light truck wheel pack. WOST 6.0, 2009.]

Robustness Analysis

Statistical Characterization of Random Variables

- Mean value and standard deviation

$$\mu_X = E\left[X\right] \qquad \sigma_X = \sqrt{E\left[(X-\mu_X)^2\right]}$$

- Coefficient of variation of one variable

$$CV(X) = \frac{\sigma_X}{\mu_X}$$

- Coefficient of correlation between two variables

$$\rho_{12} = \frac{E\left[(X_1-\mu_{X_1})(X_2-\mu_{X_2})\right]}{\sigma_{X_1}\sigma_{X_1}}$$

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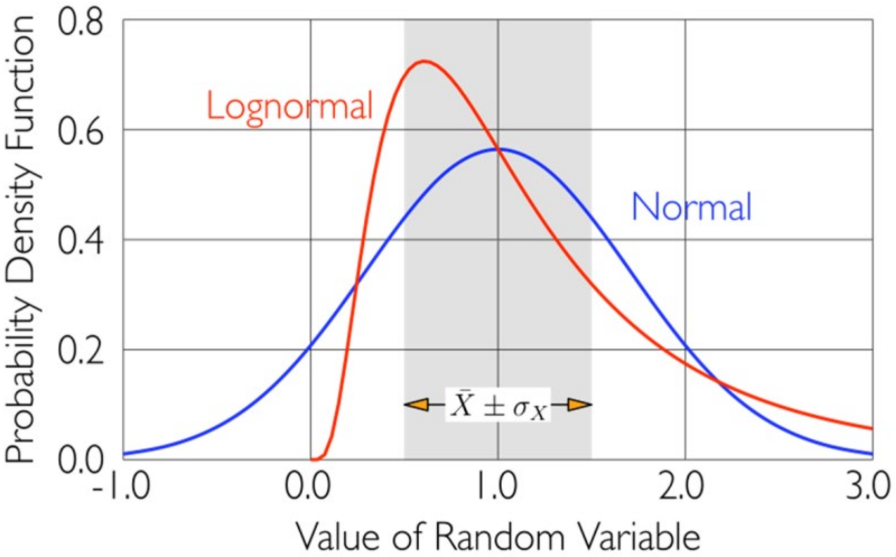
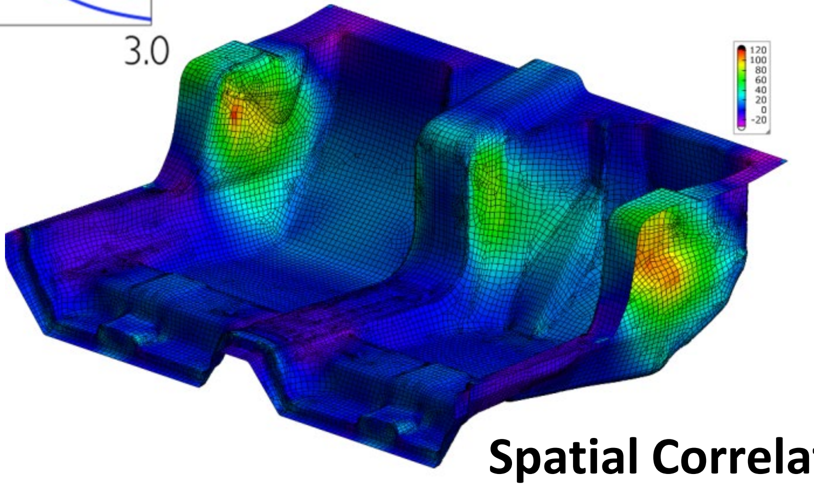
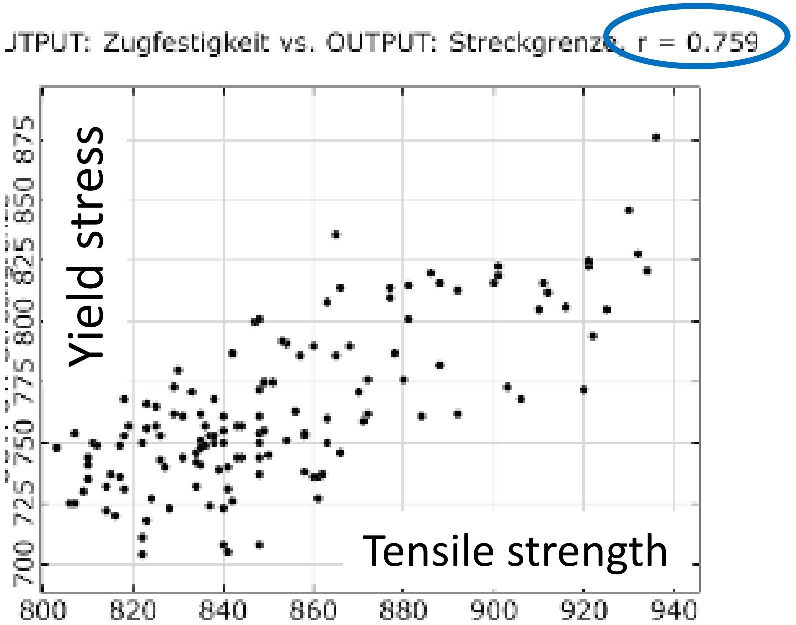
Definition of Uncertainties

- Translate know-how about uncertainties into proper scatter definition



Distribution types
define variable scatter





Correlation is an important characteristic of stochastic variables

Spatial Correlation = random fields

Definition of Input Scatter in optiSLang

- The definition of the random variable properties is implemented in table format by automatically taken the reference values as mean
- Probability density functions for all random variables are plotted corresponding to the defined variable properties
- Either standard deviation, or Coefficient of Variation (CoV) can be specified
- Reference design can be imported from arbitrary flows or result files

	Name	Parameter type	Reference value	PDF	Type	Mean	Std. Dev.	CoV	Distribution parameter
1	m	Opt.+Stoch.	1		NORMAL	1	0.02	2 %	1; 0.02
2	k	Opt.+Stoch.	20		NORMAL	20	1	5 %	20; 1
3	D	Stochastic	0.02		NORMAL	0.02	0.002	10 %	0.02; 0.002
4	Ekin	Stochastic	10		NORMAL	10	1	10 %	10; 1

Definition of Input Correlations in optiSLang

- The definition of linear (Pearson's) input correlations is possible
- Definition of all correlation coefficients or a submatrix for selected parameters

	Name	Parameter type	Reference value	PDF	Type	Mean	Std. Dev.	CoV	Distribution parameter
1	m	Opt.+Stoch.	1		NORMAL	1	0.02	2 %	1; 0.02
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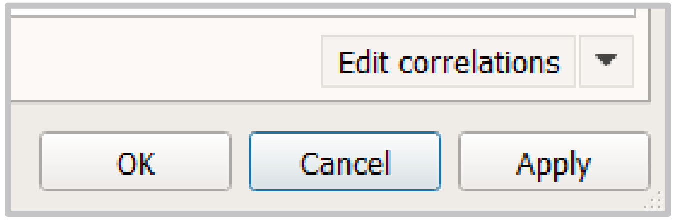
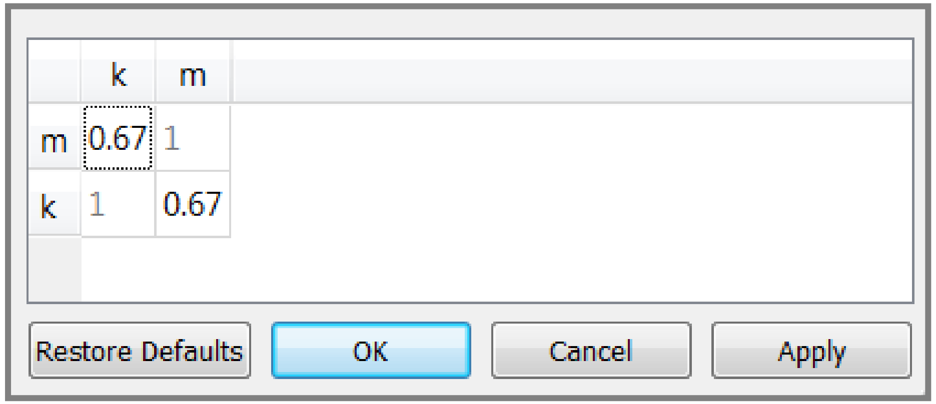
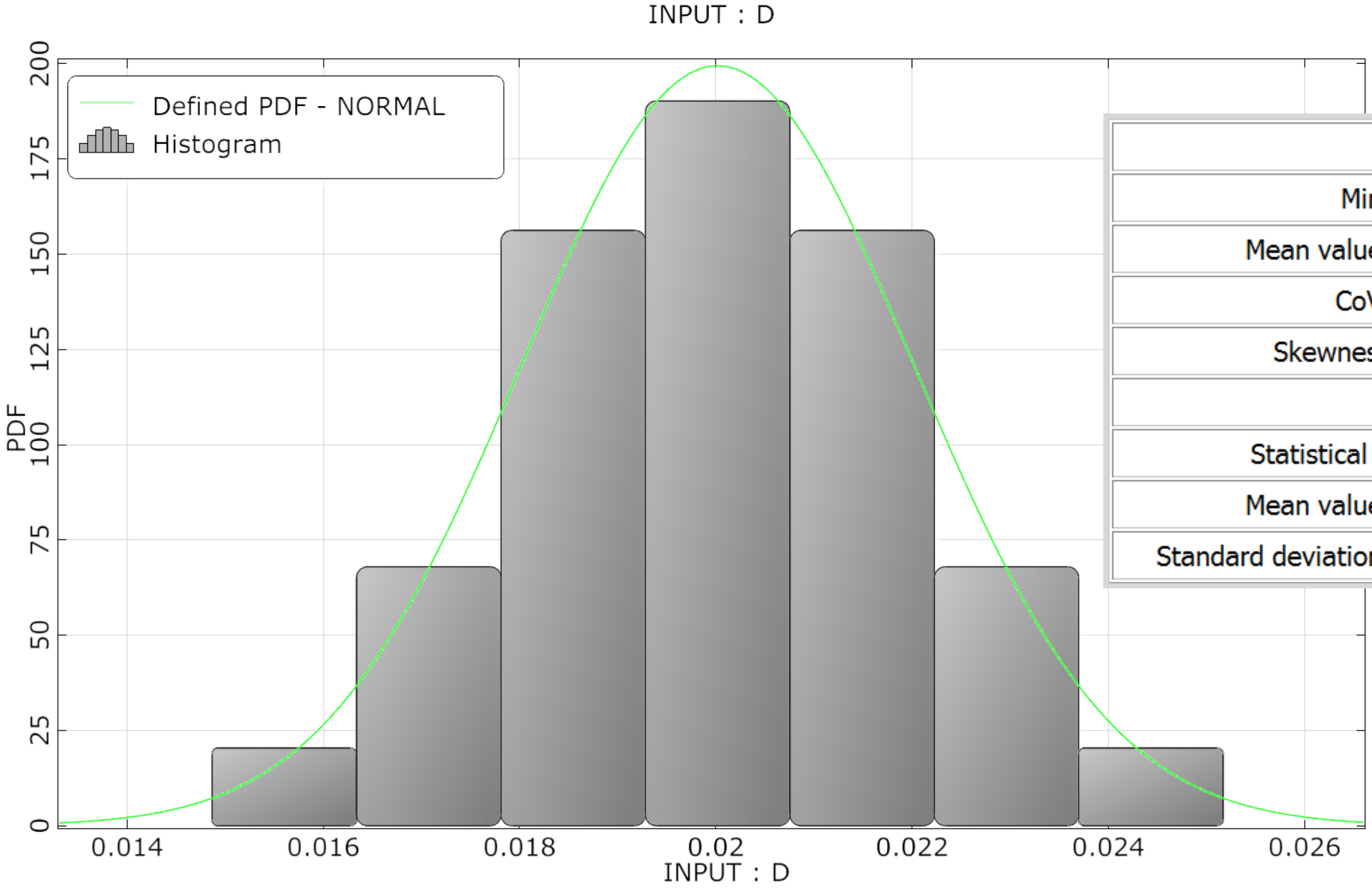




Robustness Post-Processing

Defined Parameter Distributions

- Check Histogram and compare with defined PDF of inputs
- Sample mean value and standard deviation should agree with defined



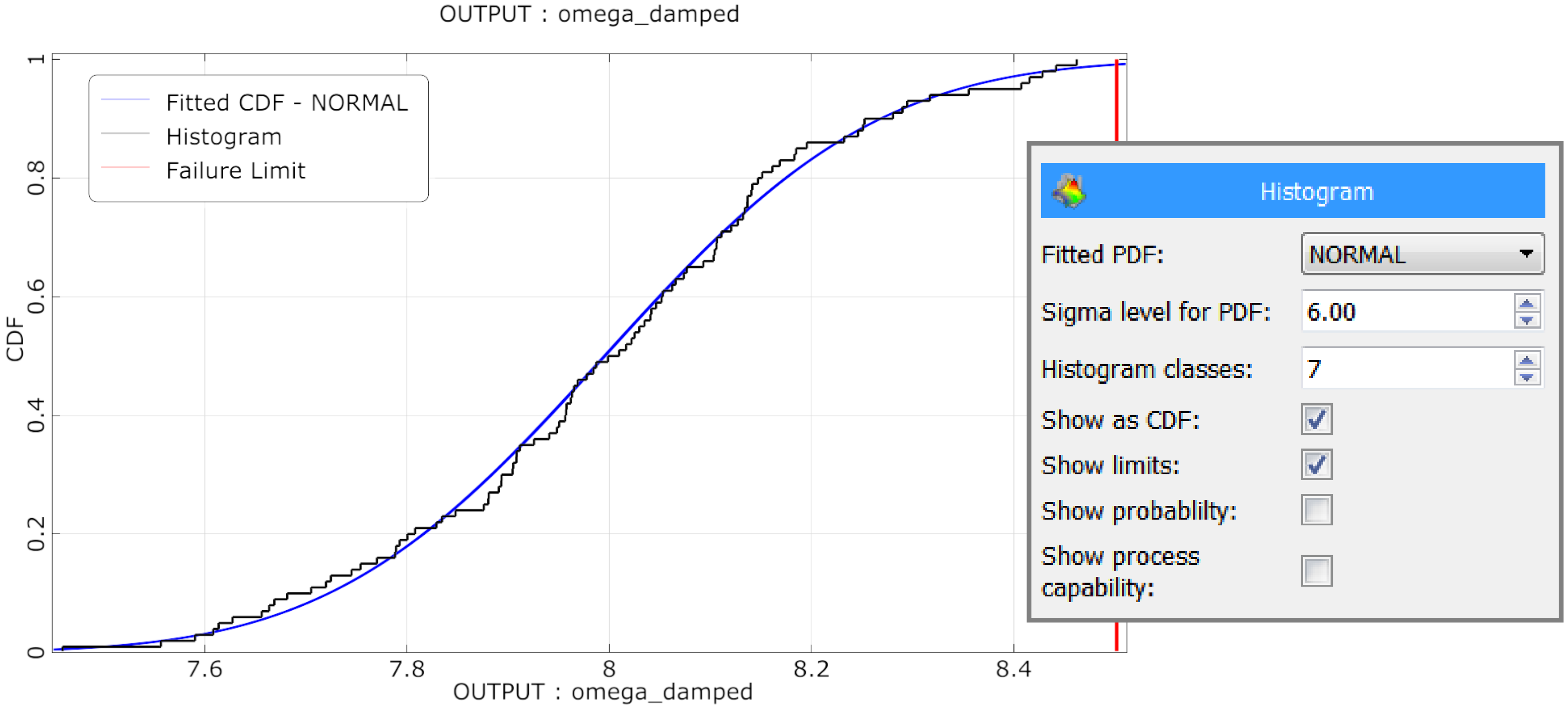
Statistical data			
Min:	0.0148483	Max:	0.0251517
Mean value:	0.02	Standard deviation:	0.00199728
CoV:	0.099864		
Skewness:	-4.37401e-16	Excess kurtosis:	-0.111953
Defined PDF : NORMAL			
Statistical Moments		Parameter	
Mean value:	0.02	Mean value mu:	0.02
Standard deviation:	0.002	Standard deviation s:	0.002

Distribution Fit

- Automatic fit compares deviation of empirical (sample) distribution function with analytical CDF of candidate distribution types

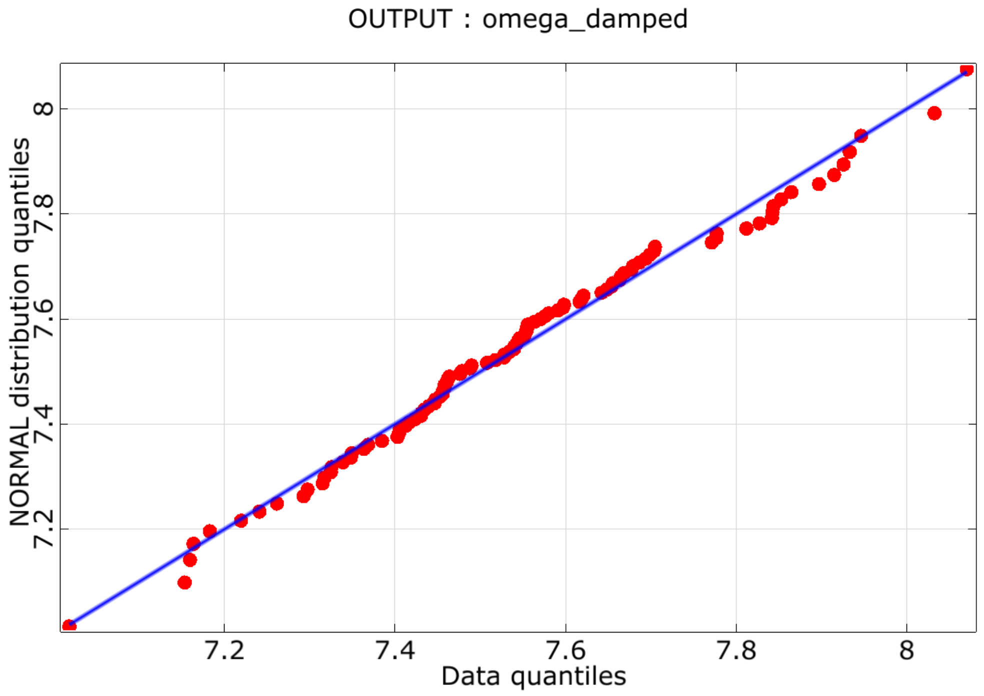
→ Recommended distribution type has minimum sum of squared errors

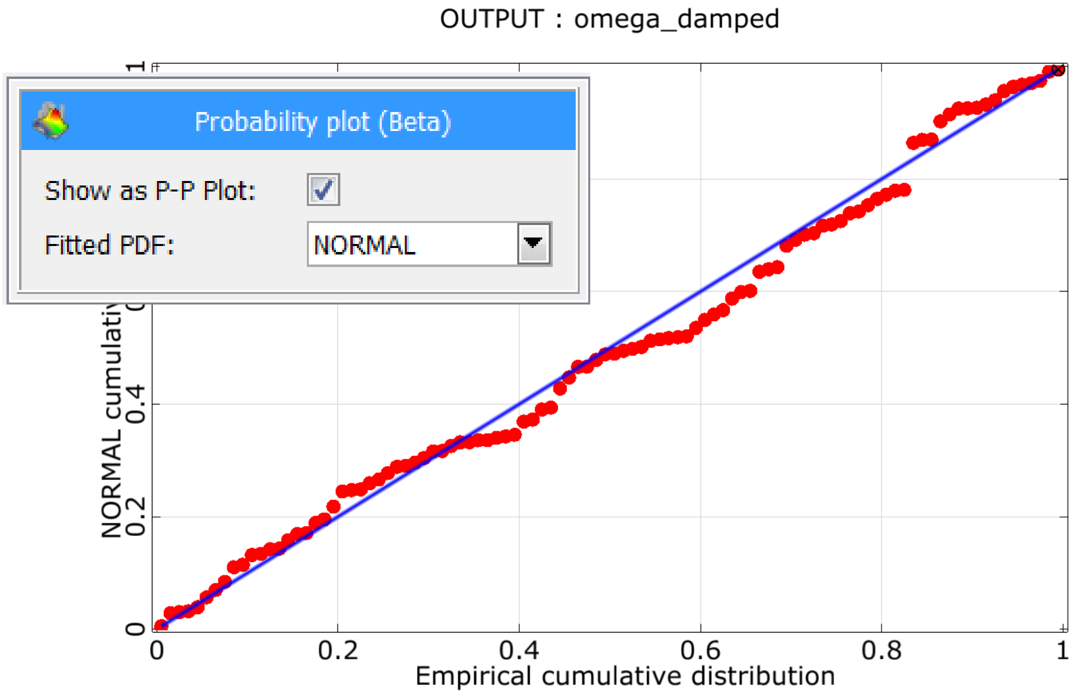
- Single distribution type is fitted via moments to data points



Probability Plot (beta)

- The quantile or Q–Q plot (default) compares quantile values (i.e., *X*-values for given probabilities) of the ranked data to a specified theoretical distribution
- The probability or P–P plot compares the probabilities assigned to the *X*-values of ranked data to a theoretical distribution





Limits

- Define lower and/or upper safety and/or failure limits

→ Limits are indicated in the histogram, box-whisker and traffic light plots

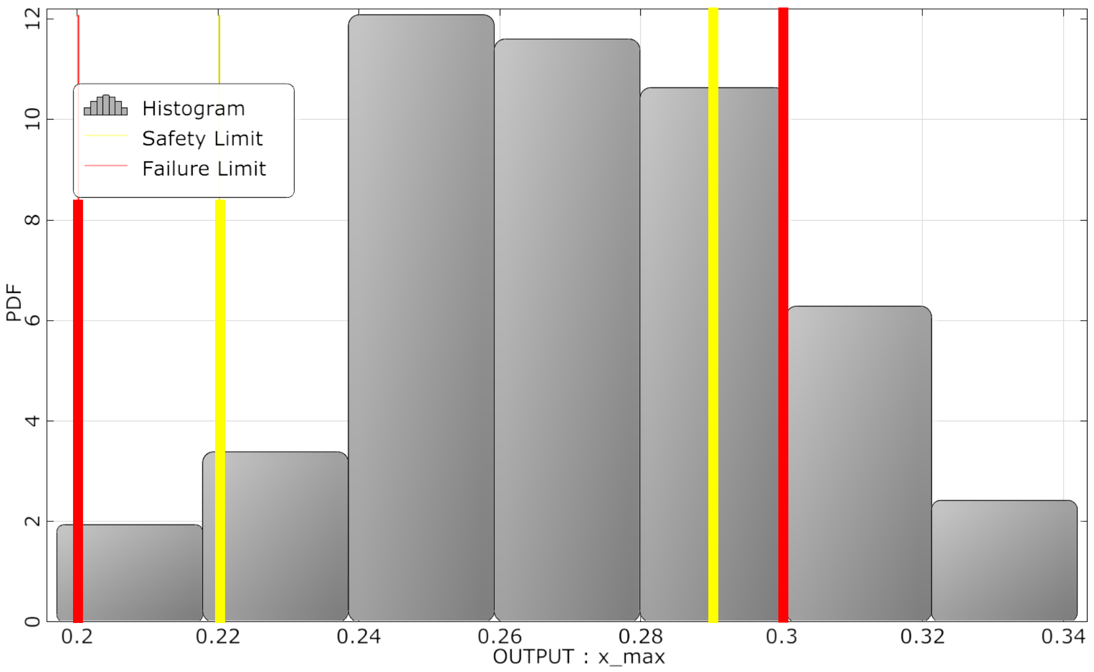
→ Probabilities of violating the limits are calculated

Edit View Windows MOP Help

External files and processes...

Edit limits...

	Dimension	Safety Limit		Failure Limit		
1	omega_damped		8.3	Lower	8.5	Target
2	x_max	0.22	0.29	0.2	0.3	

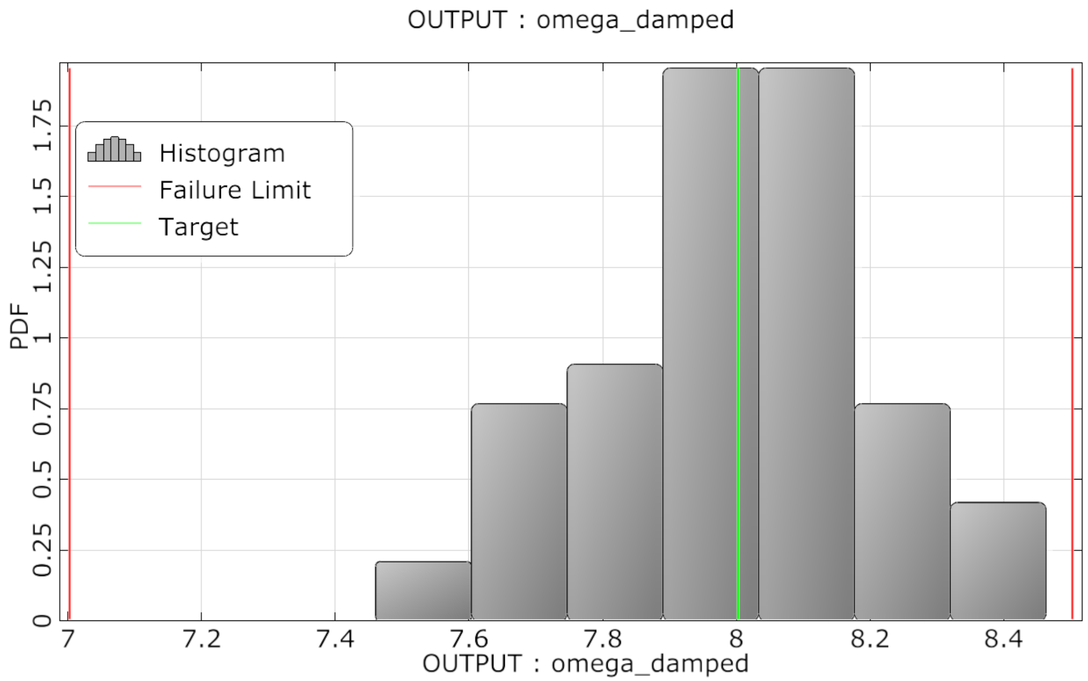


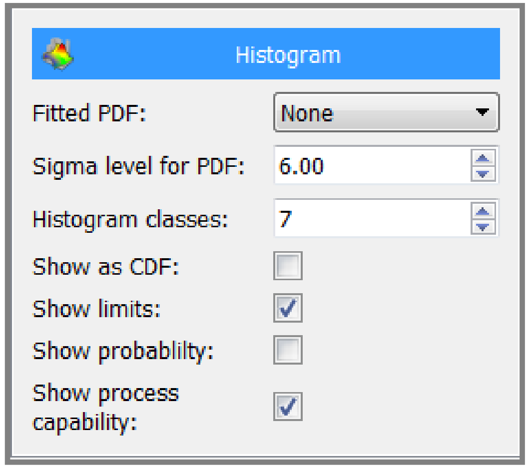
Limit : Safety Limit			
	Lower value = 0.22	Upper value = 0.29	Total
P_rel:	0.05	0.32	0.37
Sigma-Level:	1.74967	0.616846	
Limit : Failure Limit			
	Lower value = 0.2	Upper value = 0.3	Total
P_rel:	0.01	0.18	0.19
Sigma-Level:	2.42582	0.954919	

Process Capability Indices

- Considers lower and upper failure limits and possible target
- Considers estimated sample mean and standard deviation
- Assumes normal distribution
- **Six Sigma** requires **process capability ≥ 2.0**

$$\hat{C}_{pk} = \min\left[\frac{USL - \hat{\mu}}{3\hat{\sigma}}, \frac{\hat{\mu} - LSL}{3\hat{\sigma}}\right]$$

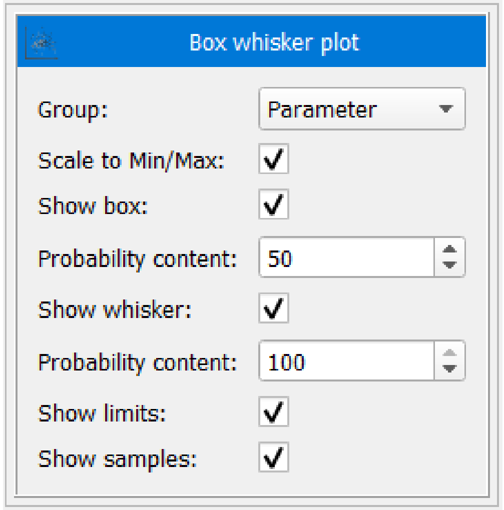


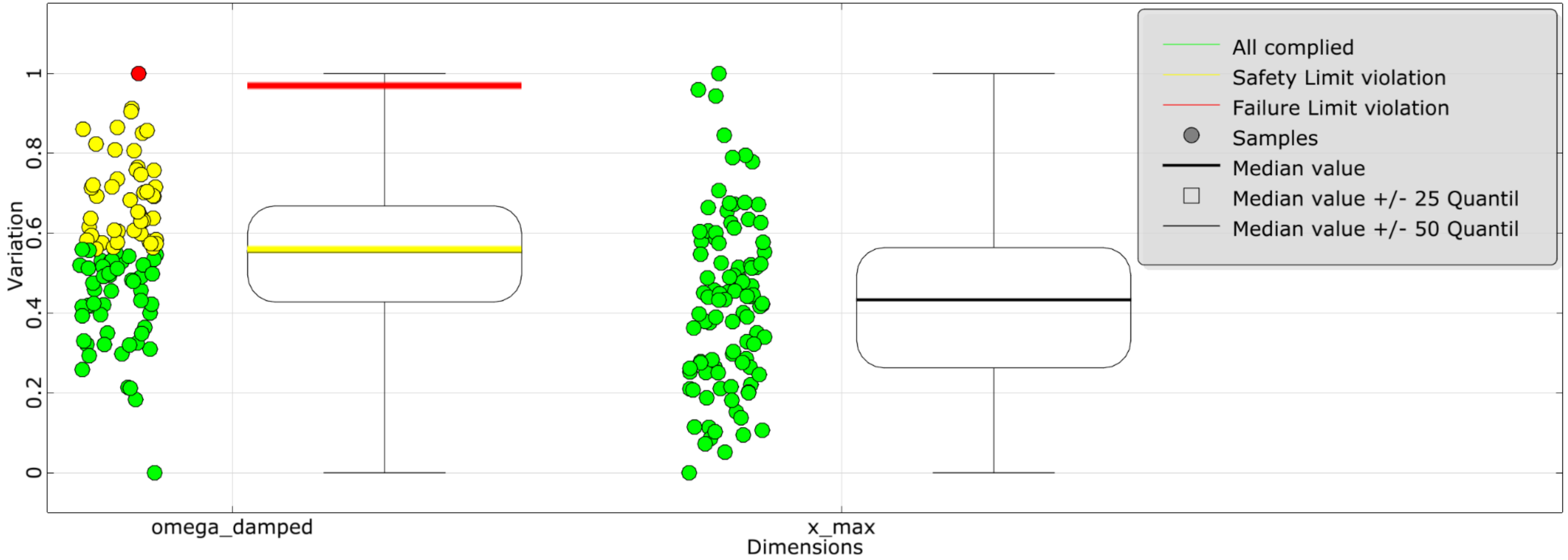


Process capability index			
LSL = 7 | USL = 8.5 | Target = 8			
C_p,lower:	1.55597	C_p,upper:	0.792532
C_p:	1.17425	C_pk:	0.792532
C_pm:	1.17375	C_pkm:	0.792197

Box-Whisker Plot

- Box and whiskers show different, settable probability contents above and below the median value
- Different scales (absolute or min. to max.) are possible
- Limits are shown as colored lines
- and as colored samples besides the box/whisker
- Horizontal spread of samples is for visualization only





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