

Given a simplified aircraft dynamic model, complete the following tasks:

- Build a nonlinear simulation model using MATLAB/Simulink;
- Select appropriate values of altitude and speed, find the corresponding trimmed point;
- Find the linearized model at the trimmed point;
- Set the trimmed point as the initial condition, assume appropriate control surface inputs, conduct linear and nonlinear simulations, and compare the simulation results.

Please submit a written report including but not limited to the following information:

- Linear and nonlinear simulation results and discussions;
- MATLAB codes, including m-files and Simulink block diagrams.

### Parameters of the aircraft:

#### Mass Properties

Weight (lbs):	$W = 20,500$
Moments of Inertia (slug-ft <sup>2</sup> ):	$J_{xx} = 9,496$
	$J_{yy} = 55,814$
	$J_{zz} = 63,100$
	$J_{xz} = 982$

#### Wing Dimensions

Span =	30 ft
Area =	300 ft <sup>2</sup>
m.a.c =	11.32 ft

#### Reference CG Location

$$X_{cg} = 0.35\bar{c}$$

#### Control Surface Actuator Models

deflection limit

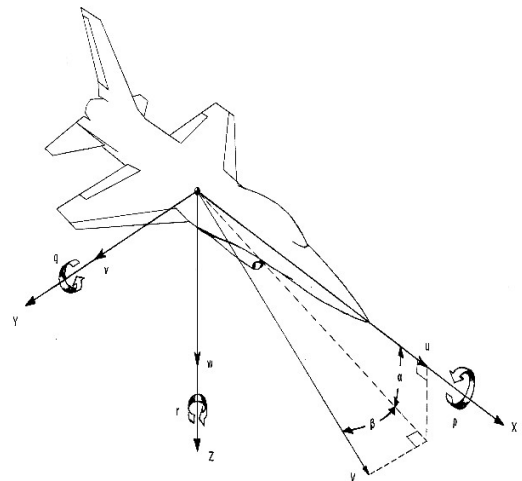
Elevator	$\pm 25.0^\circ$ ,
Ailerons	$\pm 21.5^\circ$ ,
Rudder	$\pm 30.0^\circ$ ,

#### Aerodynamic forces and moments:

$$\begin{aligned} F_{xa} &= C_{xa} \bar{q} S & F_{ya} &= C_{ya} \bar{q} S & F_{za} &= C_{za} \bar{q} S \\ L_a &= C_{L_{Roll}a} \bar{q} S b & M_a &= C_m \bar{q} S \bar{c} & N_a &= C_N \bar{q} S b \end{aligned}$$

#### Engine forces and moments:

$$\begin{aligned} F_{XT} &= T & F_{YT} &= 0 & F_{ZT} &= 0 \\ L_T &= 0 & M_T &= 0 & N_T &= 0 \end{aligned}$$



**Total force and moment coefficients:**

$$C_{Xa} = C_X(\alpha, \delta_e) + \frac{\bar{c}Q}{2V_\infty} C_{X_Q}(\alpha)$$

$$\begin{aligned} C_{Za} &= C_Z(\alpha, \beta, \delta_e) + \frac{\bar{c}Q}{2V_\infty} C_{Z_Q}(\alpha) \\ &= C_Z(\alpha) \left[ 1 - \left( \frac{\beta}{57.3} \right)^2 \right] - 0.19 \cdot \frac{\delta_e}{25} + \frac{\bar{c}Q}{2V_\infty} C_{Z_Q}(\alpha) \end{aligned}$$

$$\begin{aligned} C_{Ya} &= C_Y(\beta, \delta_a, \delta_r) + \frac{b}{2V_\infty} [C_{Y_R}(\alpha)R + C_{Y_P}(\alpha)P] \\ &= \left( -0.02\beta + 0.021 \frac{\delta_a}{20} + 0.86 \frac{\delta_r}{30} \right) + \frac{b}{2V_\infty} [C_{Y_R}(\alpha)R + C_{Y_P}(\alpha)P] \end{aligned}$$

$$C_{Ma} = C_M(\alpha, \delta_e) + \frac{\bar{c}Q}{2V_\infty} C_{M_Q}(\alpha) + C_{Za} \cdot (x_{cg,ref} - x_{cg})$$

$$\begin{aligned} C_{L_{Roll}a} &= C_{L_{Roll}}(\alpha, \beta) + \Delta C_{L_{Roll}, \delta_a=20^\circ}(\alpha, \beta) \cdot \frac{\delta_a}{20} + \Delta C_{L_{Roll}, \delta_r=30^\circ}(\alpha, \beta) \cdot \frac{\delta_r}{30} \\ &\quad + \frac{b}{2V_\infty} [C_{L_{Roll}R}(\alpha)R + C_{L_{Roll}P}(\alpha)P] \end{aligned}$$

$$\begin{aligned} C_{Na} &= C_N(\alpha, \beta) + \Delta C_{N, \delta_a=20^\circ}(\alpha, \beta) \cdot \frac{\delta_a}{20} + \Delta C_{N, \delta_r=30^\circ}(\alpha, \beta) \cdot \frac{\delta_r}{30} \\ &\quad + \frac{b}{2V_\infty} [C_{N_R}(\alpha)R + C_{N_P}(\alpha)P] - C_{Ya} (x_{cg,ref} - x_{cg}) \frac{\bar{c}}{b} \end{aligned}$$

**Lookup tables:**

$\alpha$	$C_{X_Q}$	$C_{Y_R}$	$C_{Y_P}$	$C_{Z_Q}$	$C_{L_{Roll_R}}$	$C_{L_{Roll_P}}$	$C_{M_Q}$	$C_{N_R}$	$C_{N_P}$
-10	-0.2670	0.8820	-0.1080	-8.8000	-0.1260	-0.3600	-7.2100	-0.3800	0.0610
-5	-0.1100	0.8520	-0.1080	-25.8000	-0.0260	-0.3590	-5.4000	-0.3630	0.0520
0	0.3080	0.8760	-0.1880	-28.9000	0.0630	-0.4430	-5.2300	-0.3780	0.0520
5	1.3400	0.9580	0.1100	-31.4000	0.1130	-0.4200	-5.2600	-0.3860	-0.0120
10	2.0800	0.9620	0.2580	-31.2000	0.2080	-0.3830	-6.1100	-0.3700	-0.0130
15	2.9100	0.9740	0.2260	-30.7000	0.2300	-0.3750	-6.6400	-0.4530	-0.0240
20	2.7600	0.8190	0.3440	-27.7000	0.3190	-0.3290	-5.6900	-0.5500	0.0500
25	2.0500	0.4830	0.3620	-28.2000	0.4370	-0.2940	-6.0000	-0.5820	0.1500
30	1.5000	0.5900	0.6110	-29.0000	0.6800	-0.2300	-6.2000	-0.5950	0.1300
35	1.4900	1.2100	0.5290	-29.8000	0.1000	-0.2100	-6.4000	-0.6370	0.1580
40	1.8300	-0.4930	0.2980	-38.3000	0.4470	-0.1200	-6.6000	-1.0200	0.2400
45	1.2100	-1.0400	-0.2270	-35.3000	-0.3300	-0.1000	-6.0000	-0.8400	0.1500

$C_X(\alpha, \delta_e)$

$\delta_e^\circ \backslash \alpha^\circ$	-24	-12	0	12	24
-10	-0.0990	-0.0480	-0.0220	-0.0400	-0.0830
-5	-0.0810	-0.0380	-0.0200	-0.0380	-0.0730
0	-0.0810	-0.0400	-0.0210	-0.0390	-0.0760
5	-0.0630	-0.0210	-0.0040	-0.0250	-0.0720
10	-0.0250	0.0160	0.0320	0.0060	-0.0460
15	0.0440	0.0830	0.0940	0.0620	0.0120
20	0.0970	0.1270	0.1280	0.0870	0.0240
25	0.1130	0.1370	0.1300	0.0850	0.0250
30	0.1450	0.1620	0.1540	0.1000	0.0430
35	0.1670	0.1770	0.1610	0.1100	0.0530
40	0.1740	0.1790	0.1550	0.1040	0.0470
45	0.1660	0.1670	0.1380	0.0910	0.0400

$C_Z(\alpha)$

$\alpha^\circ$	$C_Z(\alpha)$
-10	0.7700
-5	0.2410
0	-0.1000
5	-0.4160
10	-0.7310
15	-1.0530
20	-1.3660
25	-1.6460
30	-1.9170
35	-2.1200
40	-2.2480
45	-2.2290

$$C_M(\alpha, \delta_e)$$

$\delta_e^\circ \backslash \alpha^\circ$	-24	-12	0	12	24
-10	0.2050	0.0810	-0.0460	-0.1740	-0.2590
-5	0.1680	0.0770	-0.0200	-0.1450	-0.2020
0	0.1860	0.1070	-0.0090	-0.1210	-0.1840
5	0.1960	0.1100	-0.0050	-0.1270	-0.1930
10	0.2130	0.1100	-0.0060	-0.1290	-0.1990
15	0.2510	0.1410	0.0100	-0.1020	-0.1500
20	0.2450	0.1270	0.0060	-0.0970	-0.1600
25	0.2380	0.1190	-0.0010	-0.1130	-0.1670
30	0.2520	0.1330	0.0140	-0.0870	-0.1040
35	0.2310	0.1080	0	-0.0840	-0.0760
40	0.1980	0.0810	-0.0130	-0.0690	-0.0410
45	0.1920	0.0930	0.0320	-0.0060	-0.0050

$$C_{L_{Roll}}(\alpha, \beta)$$

$$C_{L_{Roll}}(\alpha, -\beta) = -C_{L_{Roll}}(\alpha, \beta)$$

$\beta^\circ \backslash \alpha^\circ$	0	5	10	15	20	25	30
-10	0	-0.0010	-0.0030	-0.0010	0	0.0070	0.0090
-5	0	-0.0040	-0.0090	-0.0100	-0.0100	-0.0100	-0.0110
0	0	-0.0080	-0.0170	-0.0200	-0.0220	-0.0230	-0.0230
5	0	-0.0120	-0.0240	-0.0300	-0.0340	-0.0340	-0.0370
10	0	-0.0160	-0.0300	-0.0390	-0.0470	-0.0490	-0.0500
15	0	-0.0220	-0.0410	-0.0540	-0.0600	-0.0630	-0.0680
20	0	-0.0220	-0.0450	-0.0570	-0.0690	-0.0810	-0.0890
25	0	-0.0210	-0.0400	-0.0540	-0.0670	-0.0790	-0.0880
30	0	-0.0150	-0.0160	-0.0230	-0.0330	-0.0600	-0.0910
35	0	-0.0080	-0.0020	-0.0060	-0.0360	-0.0580	-0.0760
40	0	-0.0130	-0.0100	-0.0140	-0.0350	-0.0620	-0.0770
45	0	-0.0150	-0.0190	-0.0270	-0.0350	-0.0590	-0.0760

$$C_N(\alpha, \beta)$$

$$C_N(\alpha, -\beta) = -C_N(\alpha, \beta)$$

$\beta^\circ \backslash \alpha^\circ$	0	5	10	15	20	25	30
-10	0	0.0180	0.0380	0.0560	0.0640	0.0740	0.0790
-5	0	0.0190	0.0420	0.0570	0.0770	0.0860	0.0900
0	0	0.0180	0.0420	0.0590	0.0760	0.0930	0.1060
5	0	0.0190	0.0420	0.0580	0.0740	0.0890	0.1060
10	0	0.0190	0.0430	0.0580	0.0730	0.0800	0.0960
15	0	0.0180	0.0390	0.0530	0.0570	0.0620	0.0800
20	0	0.0130	0.0300	0.0320	0.0290	0.0490	0.0680
25	0	0.0070	0.0170	0.0120	0.0070	0.0220	0.0300
30	0	0.0040	0.0040	0.0020	0.0120	0.0280	0.0640
35	0	-0.0140	-0.0350	-0.0460	-0.0340	-0.0120	0.0150
40	0	-0.0170	-0.0470	-0.0710	-0.0650	-0.0020	0.0110
45	0	-0.0330	-0.0570	-0.0730	-0.0410	-0.0130	-0.0010

$$\Delta C_{L_{Roll}, \delta_a=20^\circ}(\alpha, \beta)$$

$\beta^\circ \backslash \alpha^\circ$	-30	-20	-10	0	10	20	30
-10	-0.0410	-0.0410	-0.0420	-0.0400	-0.0430	-0.0440	-0.0430
-5	-0.0520	-0.0530	-0.0530	-0.0520	-0.0490	-0.0480	-0.0490
0	-0.0530	-0.0530	-0.0520	-0.0510	-0.0480	-0.0480	-0.0470
5	-0.0560	-0.0530	-0.0510	-0.0520	-0.0490	-0.0470	-0.0450
10	-0.0500	-0.0500	-0.0490	-0.0480	-0.0430	-0.0420	-0.0420
15	-0.0560	-0.0510	-0.0490	-0.0480	-0.0420	-0.0410	-0.0370
20	-0.0820	-0.0660	-0.0430	-0.0420	-0.0420	-0.0200	-0.0030
25	-0.0590	-0.0430	-0.0350	-0.0370	-0.0360	-0.0280	-0.0130
30	-0.0420	-0.0380	-0.0260	-0.0310	-0.0250	-0.0130	-0.0100
35	-0.0380	-0.0270	-0.0160	-0.0260	-0.0210	-0.0140	-0.0030
40	-0.0270	-0.0230	-0.0180	-0.0170	-0.0160	-0.0110	-0.0070
45	-0.0170	-0.0160	-0.0140	-0.0120	-0.0110	-0.0100	-0.0080

$$\Delta C_{L_{Roll}, \delta_r=30^\circ}(\alpha, \beta)$$

$\beta^\circ \backslash \alpha^\circ$	-30	-20	-10	0	10	20	30
-10	0.0050	0.0070	0.0130	0.0130	0.0150	0.0210	0.0230
-5	0.0170	0.0160	0.0130	0.0130	0.0140	0.0110	0.0100
0	0.0140	0.0140	0.0110	0.0110	0.0130	0.0100	0.0110
5	0.0100	0.0140	0.0120	0.0120	0.0130	0.0110	0.0110
10	-0.0050	0.0130	0.0110	0.0110	0.0120	0.0100	0.0110
15	0.0090	0.0090	0.0090	0.0090	0.0110	0.0090	0.0100
20	0.0190	0.0120	0.0080	0.0080	0.0110	0.0080	0.0080
25	0.0050	0.0050	0.0050	0.0050	0.0100	0.0100	0.0100
30	0	0	0	0	0.0080	0.0060	0.0060
35	-0.0050	0.0040	0.0050	0.0050	0.0080	0.0050	0.0140
40	-0.0110	0.0090	0.0030	0.0030	0.0070	0	0.0200
45	0.0080	0.0070	0.0050	0.0050	0.0030	0.0010	0

$$\Delta C_{N, \delta_a=20^\circ}(\alpha, \beta)$$

$\beta^\circ \backslash \alpha^\circ$	-30	-20	-10	0	10	20	30
-10	0.0010	0.0020	-0.0060	-0.0110	-0.0150	-0.0240	-0.0220
-5	-0.0270	-0.0140	-0.0080	-0.0110	-0.0150	-0.0100	0.0020
0	-0.0170	-0.0160	-0.0060	-0.0100	-0.0140	-0.0040	-0.0030
5	-0.0130	-0.0160	-0.0060	-0.0090	-0.0120	-0.0020	-0.0050
10	-0.0120	-0.0140	-0.0050	-0.0080	-0.0110	-0.0010	-0.0030
15	-0.0160	-0.0190	-0.0080	-0.0060	-0.0080	0.0030	-0.0010
20	0.0010	-0.0210	-0.0050	0	-0.0020	0.0140	-0.0090
25	0.0170	0.0020	0.0070	0.0040	0.0020	0.0060	-0.0090
30	0.0110	0.0120	0.0040	0.0070	0.0060	-0.0010	-0.0010
35	0.0170	0.0160	0.0070	0.0100	0.0120	0.0040	0.0030
40	0.0080	0.0150	0.0060	0.0040	0.0110	0.0040	-0.0020
45	0.0160	0.0110	0.0060	0.0100	0.0110	0.0060	0.0010

$$\Delta C_{N, \delta_r=30^\circ}(\alpha, \beta)$$

$\beta^\circ \backslash \alpha^\circ$	-30	-20	-10	0	10	20	30
-10	-0.0180	-0.0280	-0.0370	-0.0480	-0.0430	-0.0520	-0.0620
-5	-0.0520	-0.0510	-0.0410	-0.0450	-0.0440	-0.0340	-0.0340
0	-0.0520	-0.0430	-0.0380	-0.0450	-0.0410	-0.0360	-0.0270
5	-0.0520	-0.0460	-0.0400	-0.0450	-0.0410	-0.0360	-0.0280
10	-0.0540	-0.0450	-0.0400	-0.0440	-0.0400	-0.0350	-0.0270
15	-0.0490	-0.0490	-0.0380	-0.0450	-0.0380	-0.0280	-0.0270
20	-0.0590	-0.0570	-0.0370	-0.0470	-0.0340	-0.0240	-0.0230
25	-0.0510	-0.0520	-0.0300	-0.0480	-0.0350	-0.0230	-0.0230
30	-0.0300	-0.0300	-0.0270	-0.0490	-0.0350	-0.0200	-0.0190
35	-0.0370	-0.0330	-0.0240	-0.0450	-0.0290	-0.0160	-0.0090
40	-0.0260	-0.0300	-0.0190	-0.0330	-0.0220	-0.0100	-0.0250
45	-0.0130	-0.0080	-0.0130	-0.0160	-0.0090	-0.0140	-0.0100

**Engine model** (Engine thrust force  $T(M_a, h, \delta_T)$ )

The relationship between the power and throttle:

$$\text{If } \delta_T < 0.77$$

$$P = 64.94\delta_T$$

$$\text{If } \delta_T \geq 0.77$$

$$P = 217.38\delta_T - 117.38$$

The relationship between the thrust force and power:

$$\text{If } P < 50$$

$$T = T_{idle} + (T_{mil} - T_{idle})(P / 50)$$

$$\text{If } P \geq 50$$

$$T = T_{mil} + (T_{max} - T_{mil})(P - 50) / 50$$

Idle thrust levels  $T_{idle}(M_a, h)$

h Ma	0	10000	20000	30000	40000	50000
0	1060	670	880	1140	1500	1860
0.2	635	425	690	1010	1330	1700
0.4	60	25	345	755	1130	1525
0.6	-1020	-710	-300	350	910	1360
0.8	-2700	-1900	-1300	-247	600	1100
1.0	-3600	-1400	-595	-342	-200	700

Military thrust levels  $T_{mil}(M_a, h)$

h Ma	0	10000	20000	30000	40000	50000
0	12680	9150	6200	3950	2450	1400
0.2	12680	9150	6313	4040	2470	1400
0.4	12610	9312	6610	4290	2600	1560
0.6	12640	9839	7090	4660	2840	1660
0.8	12390	10176	7750	5320	3250	1930
1.0	11680	9848	8050	6100	3800	2310

Maximum thrust levels  $T_{max}(M_a, h)$

h Ma	0	10000	20000	30000	40000	50000
0	20000	15000	10800	7000	4000	2500
0.2	21420	15700	11225	7323	4435	2600
0.4	22700	16860	12250	8154	5000	2835
0.6	24240	18910	13760	9285	5700	3215
0.8	26070	21075	15975	11115	6860	3950
1.0	28886	23319	18300	13484	8642	5057