Gradient Descent-Momentum

In [1]:

```
from __future__ import division, print_function, unicode_literals
import numpy as np
import matplotlib
import matplotlib.pyplot as plt
# import matplotlib.cm as cm
# import matplotlib.mlab as mlab
np.random.seed(2)

X = np.random.rand(1000, 1)
y = 4 + 3 * X + .2*np.random.randn(1000, 1)

# Building Xbar
one = np.ones((X.shape[0],1))
Xbar = np.concatenate((one, X), axis = 1)

A = np.dot(Xbar.T, Xbar)
b = np.dot(Xbar.T, y)
w_exact = np.dot(np.linalg.pinv(A), b)


def cost(w):
    return .5/Xbar.shape[0]*np.linalg.norm(y - Xbar.dot(w), 2)**2;

def grad(w):
    return 1/Xbar.shape[0] * Xbar.T.dot(Xbar.dot(w) - y)


def numerical_grad(w, cost):
    eps = 1e-4
    g = np.zeros_like(w)
    for i in range(len(w)):
        w_p = w.copy()
        w_n = w.copy()
        w_p[i] += eps
        w_n[i] -= eps
        g[i] = (cost(w_p) - cost(w_n))/(2*eps)
    return g

def check_grad(w, cost, grad):
    w = np.random.rand(w.shape[0], w.shape[1])
    grad1 = grad(w)
    grad2 = numerical_grad(w, cost)
    return True if np.linalg.norm(grad1 - grad2) < 1e-6 else False


print( 'Checking gradient...', check_grad(np.random.rand(2, 1), cost, grad))
```

```
Checking gradient... True
```

In [2]:

```
def GD_momentum(w_init, grad, eta, gamma):
    w = [w_init]
    v = [np.zeros_like(w_init)]
    for it in range(100):
        v_new = gamma*v[-1] + eta*grad(w[-1])
        w_new = w[-1] - v_new
#         print(np.linalg.norm(grad(w_new))/len(w_new))
        if np.linalg.norm(grad(w_new))/len(w_new) < 1e-3:
            break
        w.append(w_new)
        v.append(v_new)
    return (w, it)
w_init = np.array([[2], [1]])
(w_mm, it_mm) = GD_momentum(w_init, grad, .5, 0.9)
# print(it_mm, w_mm)
```

In [3]:

```
N = X.shape[0]
a1 = np.linalg.norm(y, 2)**2/N
b1 = 2*np.sum(X)/N
c1 = np.linalg.norm(X, 2)**2/N
d1 = -2*np.sum(y)/N
e1 = -2*X.T.dot(y)/N

matplotlib.rcParams['xtick.direction'] = 'out'
matplotlib.rcParams['ytick.direction'] = 'out'

delta = 0.025
xg = np.arange(1.5, 7.0, delta)
yg = np.arange(0.5, 4.5, delta)
Xg, Yg = np.meshgrid(xg, yg)
Z = a1 + Xg**2 +b1*Xg*Yg + c1*Yg**2 + d1*Xg + e1*Yg
```

In [4]:

```
import matplotlib.animation as animation
```

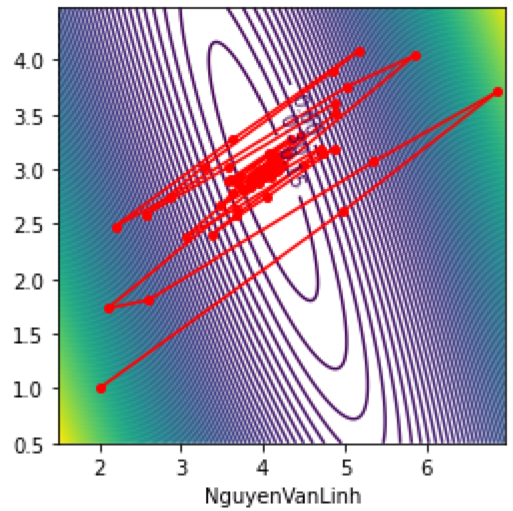
```
from matplotlib.animation import FuncAnimation
def save_gif2(eta, gamma):
    (w, it) = GD_momentum(w_init, grad, eta, gamma)
    fig, ax = plt.subplots(figsize=(4,4))
    plt.cla()
    plt.axis([1.5, 7, 0.5, 4.5])
#     x0 = np.linspace(0, 1, 2, endpoint=True)

    def update(ii):
        if ii == 0:
            plt.cla()
            CS = plt.contour(Xg, Yg, Z, 100)
            manual_locations = [(4.5, 3.5), (4.2, 3), (4.3, 3.3)]
            animlist = plt.clabel(CS, inline=.1, fontsize=10, manual=manual_locations)
#             animlist = plt.title('labels at selected locations')
            plt.plot(w_exact[0], w_exact[1], 'go')
        else:
            animlist = plt.plot([w[ii-1][0], w[ii][0]], [w[ii-1][1], w[ii][1]], 'r-')
        animlist = plt.plot(w[ii][0], w[ii][1], 'ro', markersize = 4)
        xlabel = "NguyenVanLinh"
        ax.set_xlabel(xlabel)
        return animlist, ax

    anim1 = FuncAnimation(fig, update, frames=np.arange(0, it), interval=200)
#     fn = 'img2_' + str(eta) + '.gif'
    fn = 'LR_momentum_contours.gif'
    anim1.save(fn, dpi=100, writer='imagemagick')

eta = 1
gamma = .9
save_gif2(eta, gamma)
# save_gif2(.1)
# save_gif2(2)
```

```
MovieWriter imagemagick unavailable; using Pillow instead.
```



Nesterov accelerated gradient (NAG)

In [5]:

```
from __future__ import division, print_function, unicode_literals
import numpy as np
import matplotlib
import matplotlib.pyplot as plt
from matplotlib.animation import FuncAnimation
# import matplotlib.cm as cm
# import matplotlib.mlab as mlab
np.random.seed(2)
```

In [6]:

```
X = np.random.rand(1000, 1)
y = 4 + 3 * X + .2*np.random.randn(1000, 1)

# Building Xbar
one = np.ones((X.shape[0],1))
Xbar = np.concatenate((one, X), axis = 1)

A = np.dot(Xbar.T, Xbar)
b = np.dot(Xbar.T, y)
w_exact = np.dot(np.linalg.pinv(A), b)
```

In [7]:

```
def cost(w):
    return .5/Xbar.shape[0]*np.linalg.norm(y - Xbar.dot(w), 2)**2;

def grad(w):
    return 1/Xbar.shape[0] * Xbar.T.dot(Xbar.dot(w) - y)
```

In [8]:

```
def numerical_grad(w, cost):
    eps = 1e-4
    g = np.zeros_like(w)
    for i in range(len(w)):
        w_p = w.copy()
        w_n = w.copy()
        w_p[i] += eps
        w_n[i] -= eps
        g[i] = (cost(w_p) - cost(w_n))/(2*eps)
    return g

def check_grad(w, cost, grad):
    w = np.random.rand(w.shape[0], w.shape[1])
    grad1 = grad(w)
    grad2 = numerical_grad(w, cost)
    return True if np.linalg.norm(grad1 - grad2) < 1e-6 else False


print( 'Checking gradient...', check_grad(np.random.rand(2, 1), cost, grad))
print(grad(np.random.rand(2, 1)))
print(numerical_grad(np.random.rand(2, 1), cost))
```

```
Checking gradient... True
[[-5.11424113]
 [-2.71307444]]
[[-4.33336451]
 [-2.27599051]]
```

In [9]:

```
def GD_momentum(w_init, grad, eta, gamma):
    w = [w_init]
    v = [np.zeros_like(w_init)]
    for it in range(100):
        v_new = gamma*v[-1] + eta*grad(w[-1])
        w_new = w[-1] - v_new
#         print(np.linalg.norm(grad(w_new))/len(w_new))
        if np.linalg.norm(grad(w_new))/len(w_new) < 1e-3:
            break
        w.append(w_new)
        v.append(v_new)
    return (w, it)
w_init = np.array([[2], [1]])
(w_mm, it_mm) = GD_momentum(w_init, grad, .5, 0.9)
print(it_mm, w_mm)
```

```
87 [array([[2],
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[3.03548975]]), array([[4.18294457],
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[2.9970889 ]]), array([[4.02179181],
[2.98686539]]), array([[3.99108174],
[2.97336854]]), array([[3.97365425],
       [2.96655669]]), array([[3.9785549 ],
       [2.97109199]]), array([[3.99999528],
       [2.98392403]]), array([[4.02247402],
       [2.99695692]]), array([[4.03147176],
       [3.00261938]]), array([[4.02245776],
       [2.99855499]])]
```

In [10]:

```
def myGD(w_init, grad, eta):
    w = [w_init]
    for it in range(100):
        w_new = w[-1] - eta*grad(w[-1])
        if np.linalg.norm(grad(w_new))/len(w_new) < 1e-3:
            break
        w.append(w_new)
        # print('iter %d: ' % it, w[-1].T)
    return (w, it)

w_init = np.array([[2], [1]])
(w1, it1) = myGD(w_init, grad, 0.1)
(w2, it2) = myGD(w_init, grad, 1)
(w3, it3) = myGD(w_init, grad, 2)

print(it1, it2, it3)
```

```
99 49 99
```

In [11]:

```
N = X.shape[0]
a1 = np.linalg.norm(y, 2)**2/N
b1 = 2*np.sum(X)/N
c1 = np.linalg.norm(X, 2)**2/N
d1 = -2*np.sum(y)/N
e1 = -2*X.T.dot(y)/N

matplotlib.rcParams['xtick.direction'] = 'out'
matplotlib.rcParams['ytick.direction'] = 'out'

delta = 0.025
xg = np.arange(1.5, 6.0, delta)
yg = np.arange(0.5, 4.5, delta)
Xg, Yg = np.meshgrid(xg, yg)
Z = a1 + Xg**2 +b1*Xg*Yg + c1*Yg**2 + d1*Xg + e1*Yg
```

In [12]:

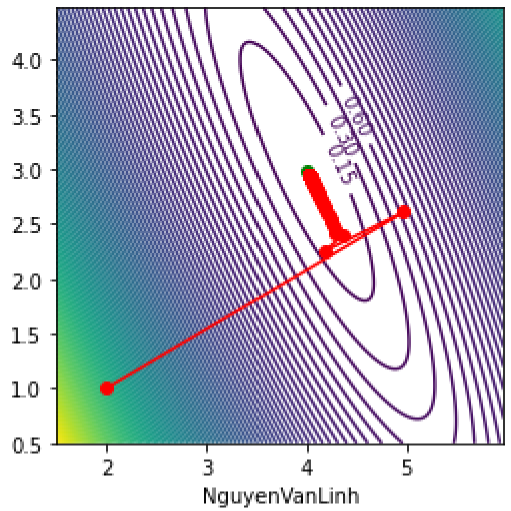
```
def save_gif2(eta):
    (w, it) = myGD(w_init, grad, eta)
    fig, ax = plt.subplots(figsize=(4,4))
    plt.cla()
    plt.axis([1.5, 6, 0.5, 4.5])
#     x0 = np.linspace(0, 1, 2, endpoint=True)

    def update(ii):
        if ii == 0:
            plt.cla()
            CS = plt.contour(Xg, Yg, Z, 100)
            manual_locations = [(4.5, 3.5), (4.2, 3), (4.3, 3.3)]
            animlist = plt.clabel(CS, inline=.1, fontsize=10, manual=manual_locations)
#             animlist = plt.title('labels at selected locations')
            plt.plot(w_exact[0], w_exact[1], 'go')
        else:
            animlist = plt.plot([w[ii-1][0], w[ii][0]], [w[ii-1][1], w[ii][1]], 'r-')
        animlist = plt.plot(w[ii][0], w[ii][1], 'ro')
        xlabel = "NguyenVanLinh"
        ax.set_xlabel(xlabel)
        return animlist, ax

    anim1 = FuncAnimation(fig, update, frames=np.arange(0, it), interval=200)
    fn = 'img2_' + str(eta) + '.gif'
    anim1.save(fn, dpi=100, writer='imagemagick')

save_gif2(1)
# save_gif2(.1)
# save_gif2(2)
```

```
MovieWriter imagemagick unavailable; using Pillow instead.
```



In [ ]: