Summer 2023 Writeup

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1 Introduction

This writeup focuses on the experimental creation and verification of the states outlined in Eritas' interesting states to test pdf in the Quantum Optics Group Sakai in the Quantum Optics Group Resources/HMC QO Group Work/spring 2023 folder in the Resources section. Specifically, the states of the form $|\Psi\rangle = \cos\alpha\,|\psi^+\rangle + e^{i\beta}\sin\alpha\,|\psi^-\rangle$. The state creation procedure followed Professor Lynn's given procedure in her notes document titled "Extended correct version of measuring elliptical pol". The states were verified by determining their experimental density matrices through a full tomography and comparing them to their theoretical counterparts. The code for running this algorithm can be found in the Lynn-Quantum-Optics GitHub at Summer-2023/framework folder/interesting_state_sweep.py.

2 Methods

In order to convert the states listed above into a form which we can derive a procedure for our experimental setup, we rewrite them in the $|H\rangle, |V\rangle$ basis. From here, we can use the procedure outlined in Topic II of Professor Lynn's notes to create the desired entangled state.

2.1 State Creation

Starting with $|\Psi\rangle = \cos\alpha\,|\psi^+\rangle + e^{i\beta}\sin\alpha\,|\psi^-\rangle$, since $|\psi^+\rangle = \frac{|H\rangle|V\rangle+|V\rangle|H\rangle}{\sqrt{2}}$ and $|\psi^-\rangle = \frac{|H\rangle|V\rangle-|V\rangle|H\rangle}{\sqrt{2}}$, the following substitutions and simplifications can be performed:

$$\begin{aligned}|\Psi\rangle &= \cos\alpha\left(\frac{|H\rangle|V\rangle+|V\rangle|H\rangle}{\sqrt{2}}\right) + e^{i\beta}\sin\alpha\left(\frac{|H\rangle|V\rangle-|V\rangle|H\rangle}{\sqrt{2}}\right)\\ &= \frac{1}{\sqrt{2}}\left((\cos\alpha + e^{i\beta}\sin\alpha)\,|H\rangle|V\rangle + (\cos\alpha - e^{i\beta}\sin\alpha)\,|V\rangle|H\rangle\right)\end{aligned}$$

The desired form we want as per the notes is $|\Psi\rangle = \cos\theta\,|H\rangle\,|V\rangle + e^{i\phi}\sin\theta\,|V\rangle\,|H\rangle$. In this case, the $|\alpha\rangle$ in Topic II is equivalent to $|H\rangle$. We can now write the equivalences

$$\begin{aligned}
\cos\theta &= \frac{1}{\sqrt{2}}(\cos\alpha + e^{i\beta}\sin\alpha) \\
e^{i\phi}\sin\theta &= \frac{1}{\sqrt{2}}(\cos\alpha - e^{i\beta}\sin\alpha)
\end{aligned}$$

By squaring the first equation, we can find an expression for θ.

$$\begin{aligned}
\cos^2\theta &= \left|\frac{\cos\alpha + e^{i\beta}\sin\alpha}{\sqrt{2}}\right|^2 \\
&= \frac{\cos^2\alpha + \sin^2\alpha + 2\cos\beta\sin\alpha\cos\alpha}{2} \\
&= \frac{1 + \cos\beta\sin 2\alpha}{2} \\
\theta &= \cos^{-1}\sqrt{\frac{1 + \cos\beta\sin 2\alpha}{2}}
\end{aligned}$$

In order to find ϕ, we can use a geometric interpretation of the coefficients in the complex plane. The complex numbers $z_1 = \cos\theta$ and $z_2 = e^{i\phi}\sin\theta$ have magnitudes $\cos\theta$ and $\sin\theta$ respectively with the phase difference between the two being ϕ. Using the above equivalences, we can write

$$\begin{aligned}
z_1 &= \frac{1}{\sqrt{2}}(\cos\alpha + e^{i\beta}\sin\alpha) \\
z_2 &= \frac{1}{\sqrt{2}}(\cos\alpha - e^{i\beta}\sin\alpha) \\
z_1 &= \frac{\cos\alpha}{\sqrt{2}} + e^{i\beta}\frac{\sin\alpha}{\sqrt{2}} \\
z_2 &= \frac{\cos\alpha}{\sqrt{2}} - e^{i\beta}\frac{\sin\alpha}{\sqrt{2}}
\end{aligned}$$

Drawing out z_1 and z_2 yields the following diagram:

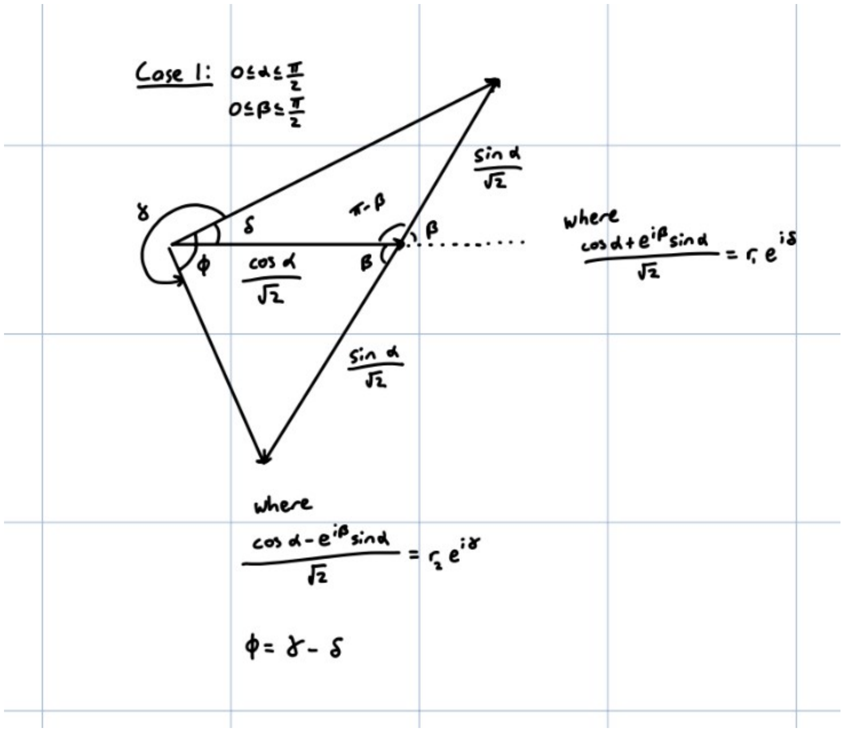

Figure 1: Diagram of z_1 and z_2 for $0 \le \alpha, \beta \le \frac{\pi}{2}$

This shows the case where both α and β are between 0 and $\frac{\pi}{2}$. From here we can use the Law of Sines to determine the angle ϕ since $\phi = \gamma - \delta$. Using Law of Sines,

$$\frac{r_1}{\sin\beta} = \frac{\frac{\sin\alpha}{\sqrt{2}}}{\sin\delta}$$
$$\frac{r_2}{\sin\beta} = \frac{\frac{\sin\alpha}{\sqrt{2}}}{\sin(2\pi - \gamma)}$$

where r_1 and r_2 are the magnitudes of z_1 and z_2 respectively. Now we can find expressions for γ and δ in terms of α and β.

$$\sin\delta = \frac{\sin\alpha\sin\beta}{\sqrt{2}r_1}$$
$$\sin\gamma = -\frac{\sin\alpha\sin\beta}{\sqrt{2}r_2}$$

where

$$
\begin{aligned}
r_1 &= (\frac{\cos\alpha}{\sqrt{2}} + \frac{\sin\alpha\cos\beta}{\sqrt{2}})^2 + (\frac{\sin\alpha}{\sqrt{2}}\sin\beta)^2 \\
&= \frac{1+\sin 2\alpha\cos\beta}{2} \\
r_2 &= (\frac{\cos\alpha}{\sqrt{2}} - \frac{\sin\alpha\cos\beta}{\sqrt{2}})^2 + (\frac{\sin\alpha}{\sqrt{2}}\sin\beta)^2 \\
&= \frac{1-\sin 2\alpha\cos\beta}{2}
\end{aligned}
$$

Using the above equations in combination, the value of ϕ can be found in terms of α and β. However, since Bob's Creation HWP introduces a negative sign by changing H into H and V into $-V$, we need to add a phase shift of π to ϕ in order to account for this. Otherwise, the initial created state will be $|\Phi^-\rangle$ instead of the desired $|\Phi^+\rangle$.

Having found θ and ϕ, we can now use the procedure outlined in the notes to move the quartz plate and UV HWP to their desired positions. Per the notes, we define $b = \frac{\pi}{4}$ and $\mu = \frac{\phi}{2}$. Now in order to set the measurement wave plates to measure any state of the form $|\Psi\rangle = \cos\alpha\,|H\rangle + e^{i\beta}\sin\alpha\,|V\rangle$, we move the HWP to an angle of $\frac{b+\mu}{2}$ and the QWP to an angle of $\mu + \frac{\pi}{2}$. The get_params function does this in the file interesting_state_sweep.py.

The final procedure is as follows:

1. Set the creation state to $|\Phi^+\rangle$.
2. Move Alice's measurement waves plates to measure the state $\frac{1}{\sqrt{2}}|H\rangle + \frac{1}{\sqrt{2}}|V\rangle$ by turning the HWP to $\frac{\pi}{8}$ and the QWP to $\frac{\pi}{2}$
3. Move Bob's measurement wave plates to measure the state $\frac{1}{\sqrt{2}}|H\rangle - e^{i\phi}\frac{1}{\sqrt{2}}|V\rangle$ by turning the HWP to $\frac{b+\mu}{2}$ and the QWP $\mu + \frac{\pi}{2}$ with $b = \frac{\pi}{4}$ and $\mu = \frac{\phi}{2}$.
4. Sweep the QP's entire range and find the angle that minimizes the counts for the predetermined measurement basis.
5. Move the QP to the angle found in the previous step and sweep the UVHWP through its entire range to find the angle that gives a VV to HH count ratio of $\tan^2\theta$ (or a $\cos^2\theta$ fraction of HH counts). First, do a sweep in HH, then do a sweep in VV and use fit functions to find the desired ratio. The angle at this ratio is the UVHWP's desired angle.
6. To set the lab components to create the desired state, move the QP and UVHWP to the angles found from the sweeps, Bob's Creation HWP to 45 degrees, and the PCC to it's home (which is currently 4.005 degrees).

2.2 State Verification

The creation of each state was verified by performing a full tomography and comparing the experimental denstiy matrix with its theoretical counterpart. The fidelities and purities of each of the created states were then calculated to verify that the correct state had been created. The full tomography script used was full_tomo.py in the framework folder in the Lynn-Quantum-Optics GitHub.

3 Results

12 different states were created and verified using this algorithm. α was set at $45°$ or $30°$ while β ranged from 0 to $90°$ in increments of $18°$. This graph was generated using Oscar's code in the GitHub in the Summer-2023/oscar/machine_learning folder using the process_expt.py file. Here AT denotes "Adjusted Theory". The dashed Adjusted Theory curves refer to the theoretical values of the fidelity and minimum witness value that the experiment should fit. The closer the solid experimental fit curves are to the AT curves, the more accurate the experimental data is.

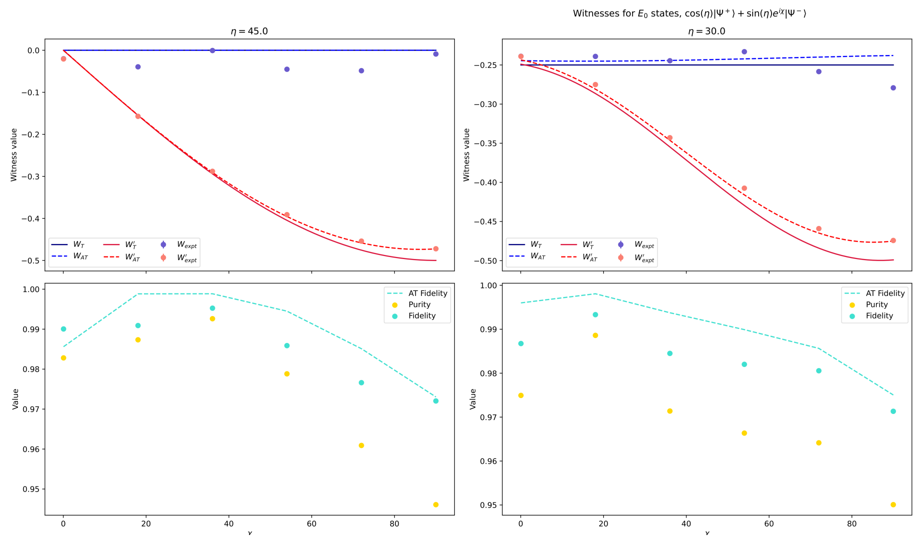

Figure 2: Witness Values, Fidelities, and Purities for Created States

The purities and fidelities for $\alpha = 45$ and $\alpha = 30°$ were consistently high above 95 percent and closely matched the theoretical witness values. For $\alpha = 60°$, there was a marked drop in fidelity starting from $\beta = 18°$ since the minimization algorithm scipy.opt.minimize was incorrectly guessing the minimum. The code was changed to use scipy.opt.brute instead, but the data for $\alpha = 60°$ has not been taken yet.

4 Conclusion

This method seems to be an accurate way of creating and determining states in the form outlined in Eritas' interesting states to test.pdf file. Each state takes around 27 minutes to create and verify. One of the main flaws in this method however is that the sweeps of the QP and UV HWP must go almost exactly through their entire range of angles to be as accurate as possible. On the other hand, if the angles are slightly off due to drift because of the laser not being warmed up or other reasons, the algorithm will adjust the angle measurements accordingly. In the future, the algorithm could either be improved by accounting for drifts in the angle ranges of the QP and UV HWP, and it could be generalized to more general state creation.