Drift Identification and Analysis

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1 Some Helpful Relationships

Basis states:

$$|\mathsf{H}\rangle = \frac{|\mathsf{D}\rangle + |\mathsf{A}\rangle}{\sqrt{2}} = \frac{|\mathsf{R}\rangle + |\mathsf{L}\rangle}{\sqrt{2}} \tag{1}$$

$$|\mathsf{V}\rangle = \frac{|\mathsf{D}\rangle - |\mathsf{A}\rangle}{\sqrt{2}} = \frac{|\mathsf{R}\rangle - |\mathsf{L}\rangle}{i\sqrt{2}} \tag{2}$$

$$|\mathsf{D}\rangle = \frac{|\mathsf{H}\rangle + |\mathsf{V}\rangle}{\sqrt{2}} = \frac{1-i}{2}|\mathsf{R}\rangle + \frac{1+i}{2}|\mathsf{L}\rangle \tag{3}$$

$$|\mathsf{A}\rangle = \frac{|\mathsf{H}\rangle - |\mathsf{V}\rangle}{\sqrt{2}} = \frac{1+i}{2}|\mathsf{R}\rangle + \frac{1-i}{2}|\mathsf{L}\rangle \tag{4}$$

$$|\mathsf{R}\rangle = \frac{|\mathsf{H}\rangle + i|\mathsf{V}\rangle}{\sqrt{2}} = \frac{1+i}{2}|\mathsf{D}\rangle + \frac{1-i}{2}|\mathsf{A}\rangle \tag{5}$$

$$|\mathsf{L}\rangle = \frac{|\mathsf{H}\rangle - i|\mathsf{V}\rangle}{\sqrt{2}} = \frac{1-i}{2}|\mathsf{D}\rangle + \frac{1+i}{2}|\mathsf{A}\rangle \tag{6}$$

(Select) two qubit basis states:

$$|\mathsf{HH}\rangle = \frac{|\mathsf{DD}\rangle + |\mathsf{DA}\rangle + |\mathsf{AD}\rangle + |\mathsf{AA}\rangle}{2} \tag{7}$$

$$|\mathsf{VV}\rangle = \frac{|\mathsf{DD}\rangle - |\mathsf{DA}\rangle - |\mathsf{AD}\rangle + |\mathsf{AA}\rangle}{2} \tag{8}$$

$$\tag{9}$$

Bell states:

$$\left|\Phi^{\pm}\right\rangle = \frac{1}{\sqrt{2}}\left(|\mathsf{HH}\rangle \pm |\mathsf{VV}\rangle\right) \tag{10}$$

$$\left|\Psi^{\pm}\right\rangle = \frac{1}{\sqrt{2}}\left(|\mathsf{HV}\rangle \pm |\mathsf{VH}\rangle\right) \tag{11}$$

2 Theory of Birefringent Optical Components

This section is a little theoretical background. Most of our optical components are (or could be) made of quartz. Quartz has this great property that light with a linear polarization along it's fast axis experiences an index of refraction n_f whereas light along the slow axis (orthogonal to the fast axis) experiences an index of refraction n_s where $n_f \neq n_s$. Let's call the fast axis $\hat{\mathbf{x}}$, the slow axis $\hat{\mathbf{y}}$, and the optical axis $\hat{\mathbf{z}}$. The electric field propagating along the optical axis is given by

$$\mathbf{E} = \mathbf{E}_0^{(+)} e^{i(kz-\omega t)} + \mathbf{E}_0^{(-)} e^{-i(kz-\omega t)} \tag{12}$$

For some complex $\mathbf{E}_0^{(+)} = \left(\mathbf{E}_0^{(-)}\right)^* \in \mathbb{C}^3$. By convention, we focus only on the $\mathbf{E}^{(+)}$ component of this equation. Furthermore we can drop and $\hat{\mathbf{z}}$ components since $\mathbf{E} \cdot \hat{\mathbf{z}} = 0$. Then we can express

$$\mathbf{E}^{(+)} = \hat{\mathbf{x}} E_{0,x}^{(+)} e^{i(kz-\omega t)} + \hat{\mathbf{y}} E_{0,y}^{(+)} e^{i(kz-\omega t)} \tag{13}$$

Note that we will be dropping the common factor of $e^{-i\omega}$ from here on out, since the frequency ω (and by proxy, energy) of the photon will not change in any linear process, and the overall phase does not change the observable quantities. If the index of refraction is different in the two axes we will have a different k_f for the $\hat{\mathbf{x}}$ component and k_s for the $\hat{\mathbf{y}}$ component. After traveling a distance d_0 in the quartz, the electric field will become

$$\mathbf{E}^{(+)} = e^{ik_f d_0} \left(\hat{\mathbf{x}} E_{0,x}^{(+)} + \hat{\mathbf{y}} E_{0,y}^{(+)} e^{i(k_s - k_f) d_0} \right) \tag{14}$$

This is how all of our optical components function. Quarter and half wave plates have their thickness tuned so that $(k_2 - k_1)d_0 = -\pi$ or $-\pi/2$, respectively.

3 Jones Matrices

To describe the linear manipulation of photons by optical components, we use Jones matrices. If an optical component has a Jones matrix $\hat{J}$ then the quantum state vector $|\psi\rangle$ will become $|\psi'\rangle = \hat{J}\,|\psi\rangle$ after passing through it.

3.1 Half Wave Plate

A half wave plate with it's fast axis horizontal has the Jones matrix

$$\mathrm{HWP}_0 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \tag{15}$$

To emulate rotating the plate you can apply the inverse rotation matrix to the light, then rotate the result by the ordinary rotation matrix. Note these rotation matrices are given by

$$R(\theta) = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} \tag{16}$$

$$R^\dagger(\theta) = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \tag{17}$$

And so the Jones matrix for a half wave plate with it's fast axis an angle θ from the horizontal is given by

$$\mathrm{HWP}(\theta) = R\,\mathrm{HWP}_0\,R^\dagger = \begin{pmatrix} \cos 2\theta & \sin 2\theta \\ \sin 2\theta & -\cos 2\theta \end{pmatrix} \tag{18}$$

3.2 Quarter Wave Plate

A quarter wave plate applies a phase shift of $-i$ to the component along it's slow axis, and so if the fast axis is aligned with the horizontal then the Jones matrix is

$$\mathrm{QWP}_0 = \begin{pmatrix} 1 & 0 \\ 0 & i \end{pmatrix} \tag{19}$$

And applying the same rotation trick, if its fast axis lies an angle θ from the horizontal then

$$\mathrm{QWP}(\theta) = R\,\mathrm{QWP}_0\,R^\dagger = \begin{pmatrix} \cos^2\theta + i\sin^2\theta & (1-i)\sin\theta\cos\theta \\ (1-i)\sin\theta\cos\theta & \sin^2\theta + i\cos^2\theta \end{pmatrix} \tag{20}$$

3.3 Quartz Plate

The quartz plate in our setup works by changing the distance that the light travels through it. By rotating the plate to make an angle θ with the horizontal (perpendicular to the optical axis), then light will travel a distance $d = 1 + d_0 \sec\theta$ and the vertical component will incur a relative phase difference of $(k_2 - k_1)(1 + d_0\sec\theta)$. In other words, we can model the quartz plate with the jones matrix

$$\mathrm{QP} = \begin{pmatrix} 1 & 0 \\ 0 & e^{i\phi} \end{pmatrix} \tag{21}$$

Where

$$\phi = A + B\sec\theta \tag{22}$$

for some $A, B \in \mathbb{R}$.

4 UV HWP Problems

Throughout our calibration, we noticed that the angle θ for the UV-HWP that balances $|\mathrm{HH}\rangle$ and $|\mathrm{VV}\rangle$ production (which we might care about if we want to create say a Bell state like $|\Phi^+\rangle$) changes over time. To eliminate motion of the UV-HWP itself as a potential cause of the difference, we setup a series of "drift experiments" to study how the proportion of $|\mathrm{HH}\rangle$ to $|\mathrm{HH}\rangle$ and $|\mathrm{VV}\rangle$ drifted over time.

In these experiments, we essentially just configured the $|\Phi^+\rangle$ state to the best of our ability, and then began measuring $|\mathrm{HH}\rangle$ and $|\mathrm{VV}\rangle$ coincidence counts over extended periods of time. The results of the first experiment are shown in fig. 1 and the second experiment's results can be found in fig. 2.

The UV-HWP is the only component in the setup that changes the proportion of $|\mathrm{HH}\rangle$ and $|\mathrm{VV}\rangle$ being produced, and so we strongly suspect that it is undergoing some change throughout our test. Personally, I believe this to be a temperature change. The

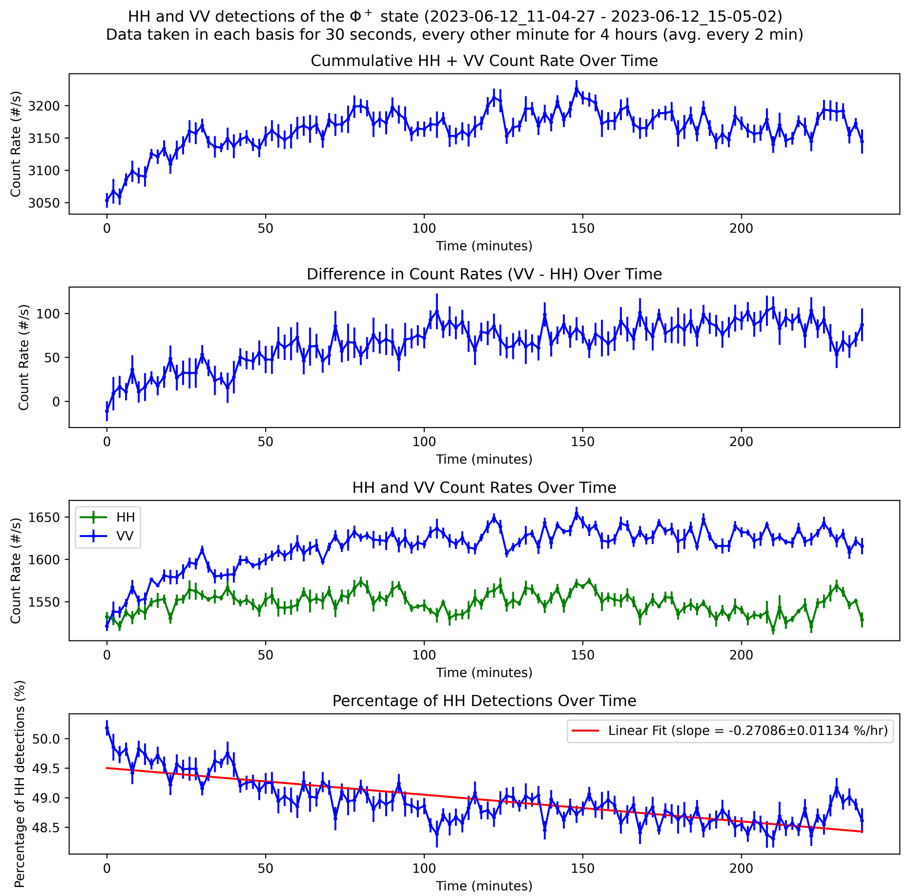

Fig. 1: Results from the first drift experiment. Note the significant increase in overall production rates over the first 90 minutes. It Seems though that this figure could be explained by some overall preference for the $|VV\rangle$ state over time, since the middle plot shows $|HH\rangle$ counts effectively flatline throughout the experiment. To get a better grasp on this this data, we ran another experiment shortly after this one with the exact same setup, but for 5 hours.

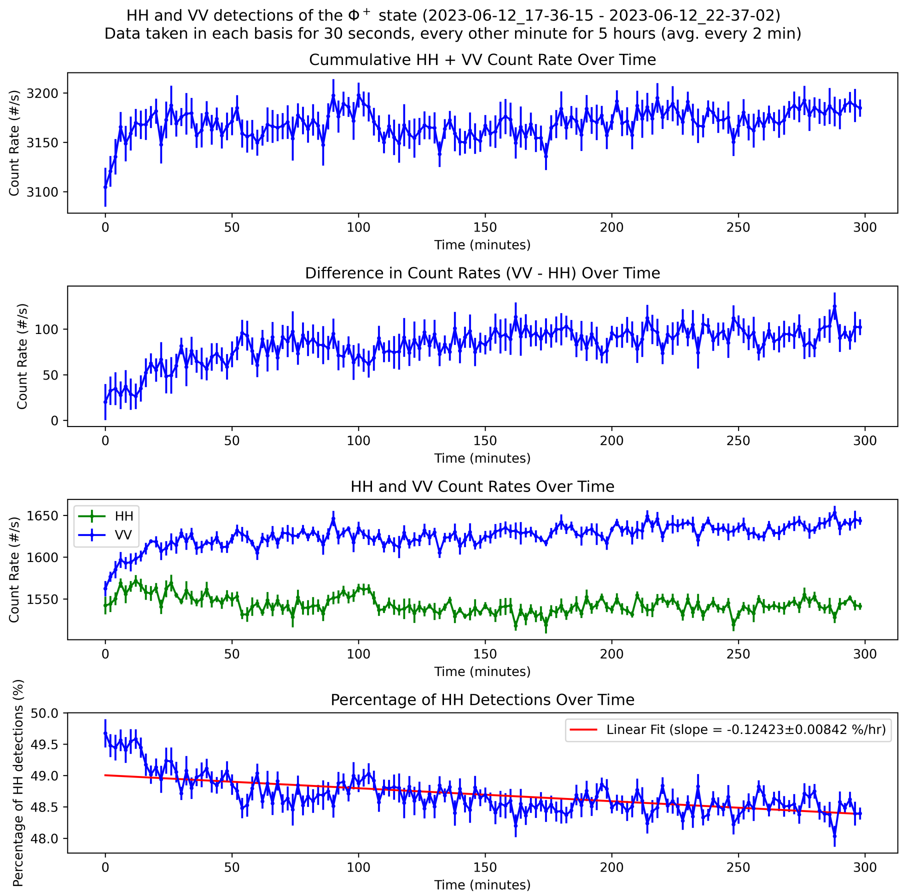

Fig. 2: The results of the second drift experiment we ran. Note that this was shortly after the first, and so if we believe temperature to be a factor, these plots should indicate that the components were already warm. Indeed, we see most of the values in these plots reaching equilibrium much faster than in fig. 1. The most provocative feature of this plot is that we see the relative ratio of $|HH\rangle$ to $|VV\rangle$ production changing *after* the overall production rate has stabilized. This indicates that it likely is some (perhaps temperature-related) change in the UV-HWP causing the bias towards $|VV\rangle$ production, rather than just some kind of overall preference for excess $|VV\rangle$ production over time.

4.1 Modeling the Issue

If we believe that temperature affects the path length of the light in the crystal, then we can model the extra retardance applied to the vertical component with a *small* phase error term $\gamma > 0$, so that the UV HWP will have a jones matrix

$$\mathrm{UVHWP} = \begin{pmatrix} 1 & 0 \\ 0 & -e^{-i\gamma} \end{pmatrix} \tag{23}$$

Then the expanded expression when this plate is rotated to an angle θ from the horizontal becomes

$$\mathrm{UVHWP}(\theta) = R\ \mathrm{UVHWP}\ R^\dagger \tag{24}$$

$$= \begin{pmatrix} \cos^2\theta - e^{-i\gamma}\sin^2\theta & (1+e^{-i\gamma})\cos\theta\sin\theta \\ (1+e^{-i\gamma})\cos\theta\sin\theta & \sin^2\theta - e^{-i\gamma}\cos^2\theta \end{pmatrix} \tag{25}$$

And so the light from our laser becomes the state

$$\mathrm{UVHWP}\,|\mathsf{H}\rangle = \begin{pmatrix} \cos^2\theta - e^{-i\gamma}\sin^2\theta \\ (1+e^{-i\gamma})\cos\theta\sin\theta \end{pmatrix} \tag{26}$$

And the probability of measuring this state to be in the state $|\mathsf{H}\rangle$ or $|\mathsf{V}\rangle$ (equal to the probability of measuring $|\mathsf{VV}\rangle$ or $|\mathsf{HH}\rangle$, respectively) is

$$P(|\mathsf{H}\rangle) = P(|\mathsf{VV}\rangle) = 1 - \frac{1}{2}\sin^2(2\theta)(1+\cos\gamma) \tag{27}$$

$$P(|\mathsf{V}\rangle) = P(|\mathsf{HH}\rangle) = \frac{1}{2}\sin^2(2\theta)(1+\cos\gamma) \tag{28}$$

You'll notice that both these probabilities will have extrema at $\theta = n \cdot 45^\circ$ for $n \in \mathbb{Z}$ no matter what the value of γ is. However, if we are doing something a bit more complex like say fine-tuning the balance of $|\mathsf{H}\rangle$ and $|\mathsf{V}\rangle$ states, we end up caring much more about γ.

Ignoring the fact that the quartz plate will reflect vertically and horizontally polarized light at different rates, we can pretend that the ratio of $|\mathsf{H}\rangle$ to $|\mathsf{V}\rangle$ will be perfectly balanced at the angle $\theta = \pi/8$ and $\sin^2(2\theta) = \frac{1}{2}$. One interesting prediction from the above equations is that as $\gamma \neq 0$, since $\cos$ is even and decreasing around the origin, we will see $P(|\mathsf{V}\rangle) = P(|\mathsf{HH}\rangle)$ decrease, and $P(|\mathsf{H}\rangle) = P(|\mathsf{VV}\rangle)$ increase, which is exactly what fig. 1 and fig. 2 show us experimentally.

In fact, using our experimental data we can even make a back-of-the envelope calculation for what γ in our setup must be. Making the same assumption as above, that $\theta = \pi/8$ balances $|\mathsf{H}\rangle$ and $|\mathsf{V}\rangle$, then we can set $P(|\mathsf{V}\rangle) = \frac{1}{2} - \delta$ for some real percentage error $\delta > 0$ and solve for the phase error γ. This leaves us with

$$P(|\mathsf{V}\rangle) = \frac{1}{4}(1+\cos\gamma) = \frac{1}{2} - \delta \tag{29}$$

$$\gamma = \arccos(1 - 4\delta) \tag{30}$$

In our experiment, we saw the $|\mathsf{HH}\rangle$ counts drop by about $\delta \approx 1.5\%$ throughout the test, indicating a phase error of $\gamma \approx 0.348 \approx 20^\circ$ which can also be expressed as $\frac{\gamma}{2\pi} = 0.055\lambda$ which is really quite a significant difference!

4.2 Temperature Changes

One hypothesis we explored was that possibility that the heating of the quartz plate by our laser was causing the plate to expand, changing the path length of light through the plate and thus the retardance. However, since the phase shift is linear with the path length and the thermal expansion coefficient of quartz is like[1,2] $\epsilon \approx 5.5 \times 10^{-7}$ K^{-1} then the phase shift with respect to a change in temperature will look like $\phi = \phi_0(1 + \epsilon\Delta T)$ or $\gamma = \Delta\phi = \phi_0\epsilon\Delta T$. For our HWP where $\phi_0 = \pi$ this means that to achieve a phase difference γ we require a temperature change $\Delta T = \gamma/\pi\epsilon$ and perhaps you can see the problem when noting the order of magnitude of ϵ. To achieve $\gamma = 0.348$ rad we would require a temperature change of $\Delta T \approx 200,000$ K.

But that's a bit of a silly approximation, since wave plates employ all kinds of tricks and gimmicks and so it's a bit disingenuous to model them as slabs of quartz. The other (probably better) way to go about this approximation is by noting a typical thermal dependence for zero order wave plates[3] being something like $\epsilon = 0.0001\ \lambda$ C^{-1} and then we can directly compare with the observed retardance error of $0.055\ \lambda$ to get a required temperature change of about 550 K, which is still not very realistic for our circumstances.

These calculations suggest that temperature changes in the quartz will not be enough to appreciably affect the drift of the $|\mathrm{H}\rangle : |\mathrm{V}\rangle$ balance that we are measuring.

4.3 Laser Drift

Another factor that affects the retardance of a wave plate is the wave length of light passing through it. The wave length of our laser should be centered around 405 nm, but nothing is ever perfect. Luckily, ThorLabs provides great data sheets on their HWPs[4], a plot of which is shown in fig. 3. I've zoomed in on the most relevant part of the plot here, where it is clear a deviation of ± 5 nm from the central wavelength of 405 nm could easily cause a retardance difference of $0.055\ \lambda$, explaining the percentage difference that we see. However, after discussing these ideas with Prof. Lynn, she believes this would be well outside of the drift of a laser with these specifications. Measurements on the grating spectrometer from fifteen years ago suggest temperature related drift was $< 0.1°$A, however their relevance is of suspicion as the measurements are fifteen years old.

5 Identification of the Cause of Drift

At this point we think it necessary to perform a simple experiment in order to determine if the drift is being caused by the laser or by a crystal. We still somewhat believe the change to be temperature related, as few other measurements follow such clear stabilization curves as shown in fig. 1 and fig. 2. This experiment works

[1] https://www.heliosquartz.com/prodotti/proprieta-del-quarzo/?lang=en
[2] http://www.mt-berlin.com/frames_cryst/descriptions/quartz%20.htm
[3] https://www.newport.com/n/introduction-to-waveplates
[4] https://www.thorlabs.com/newgrouppage9.cfm?objectgroup_id=711

exactly as the previous two, however prior to running the test we will allow for the laser to warm up while being blocked for three hours. The results of this test are shown in fig. 4.

This test shows that the majority of the drift that we've seen in earlier experiments is in fact due to some effect of the laser. However, the experiment also shows that even once the laser is 'stabilized' the percentage of $|HH\rangle$ detections can still move around on the order of 0.1% every hour. This is bound to cause issues in our experiments, especially with the preset state calibrations, but we will just have to work around it for now.

We suspect that the laser drift is either due to or proportional to modulation laser's te

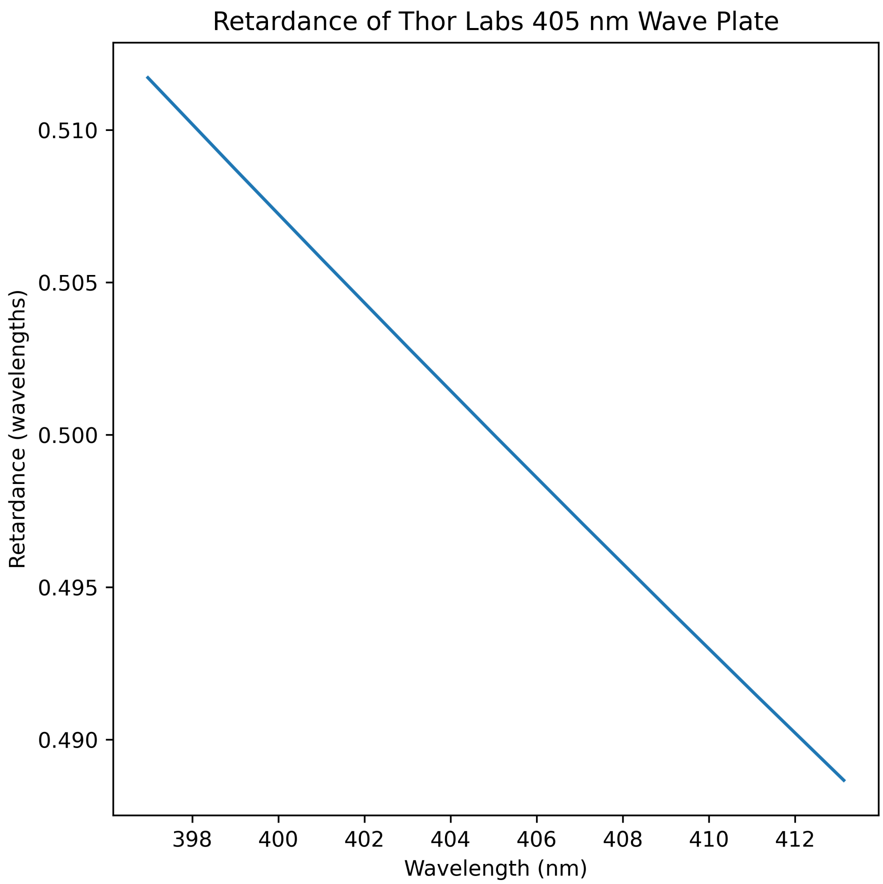

Fig. 3: Plotted data from ThorLabs showing the retardance in of the ThorLabs zero-order quartz crystal 405 nm wave plate (measured in wavelengths) versus the wavelength of light.

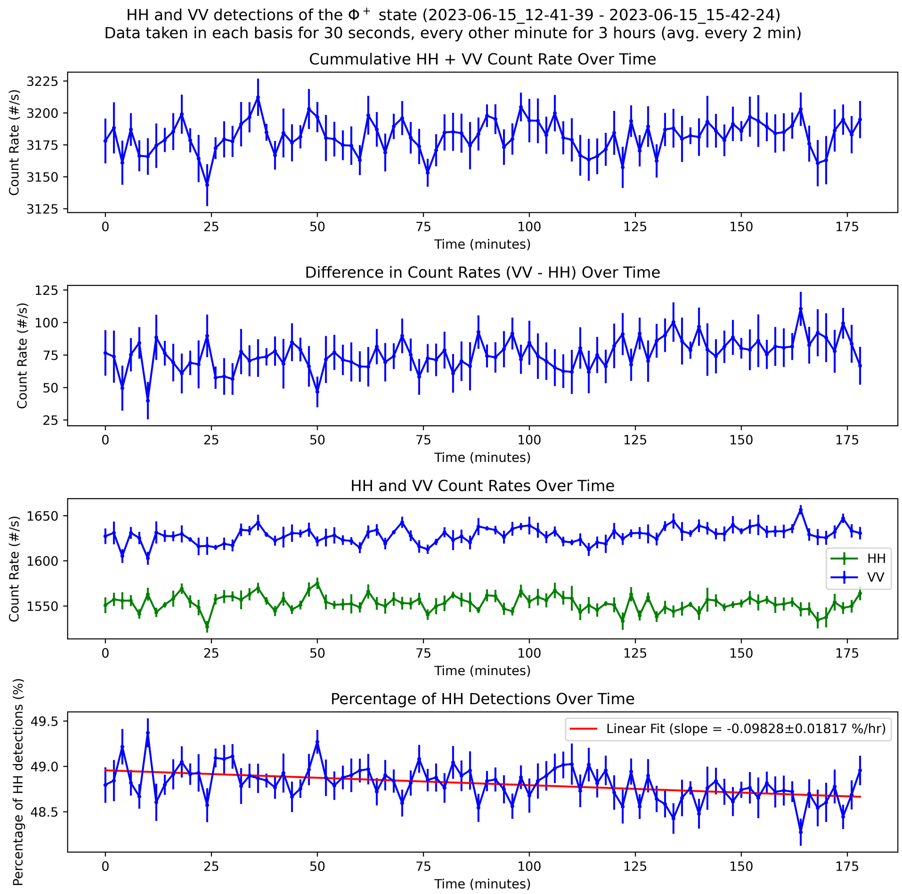

Fig. 4: In this experiment, the laser was allowed to warm up for three hours prior to data collection. However, as you can see from the linear fit that is plotted, we still are seeing movement of the $|HH\rangle$ percentage count away from 50:50. This experiment was a bit discouraging, however when compared to fig. 1 and fig. 2 there is certainly a drastic difference in terms of the initial drift. The linear fit on this plot actually seems justified, whereas the others began with a clear and sharp drop.