

$$\varphi^*(x) = \frac{Bx}{x^2+b^2} + i \frac{bB}{x^2+b^2} = \langle \varphi(x) |$$

$$\langle \varphi | \varphi \rangle = \int_{-\infty}^{+\infty} \varphi^*(x) \varphi(x) dx = \int_{-\infty}^{+\infty} \left(\frac{(Bx)^2}{(x^2+b^2)^2} + \frac{(Bb)^2}{(x^2+b^2)^2} \right) dx$$

$$1 = B^2 \int_{-\infty}^{+\infty} \left(\frac{x^2}{(x^2+b^2)^2} + \frac{b^2}{(x^2+b^2)^2} \right) dx = B^2 \int_{-\infty}^{+\infty} \frac{(x^2+b^2)}{(x^2+b^2)^2} dx = 1$$

$$\frac{1}{B^2} = \int_{-\infty}^{+\infty} \frac{dx}{x^2+b^2} = \frac{1}{b} \arctan \frac{x}{b} \Big|_{-\infty}^{+\infty} = \left[\frac{\pi}{2} - \left(-\frac{\pi}{2} \right) \right] \frac{1}{b} = \Rightarrow$$

$$= \frac{1}{B^2} = \frac{\pi}{b} \Rightarrow B^2 = \frac{b}{\pi} \Rightarrow B = \pm \sqrt{\frac{b}{\pi}}$$

Задача 3 (1)

$$\langle \varphi | \varphi \rangle = ? \quad \text{Решение: } A = \pm a \sqrt{\frac{2a}{\pi}}$$

$$\int_{-\infty}^{+\infty} \left(\frac{B}{x+ib} \right)^* \frac{A}{x^2+a^2} dx = \dots$$

$$B = \pm \sqrt{\frac{b}{\pi}}$$

$$\frac{B}{x+ib} = \frac{B(x-ib)}{x^2+b^2} \Rightarrow \left(\frac{B}{x+ib} \right)^* = \frac{B(x+ib)}{x^2+b^2}$$

$$\langle \varphi | \varphi \rangle = \int_{-\infty}^{+\infty} \frac{AB(x+ib)}{(x^2+a^2)(x^2+b^2)} dx = AB \int_{-\infty}^{+\infty} \frac{(x+ib) dx}{(x^2+a^2)(x^2+b^2)} = ABibI$$

$$\text{где } I = \int_{-\infty}^{+\infty} \frac{dx}{(x^2+a^2)(x^2+b^2)} = \frac{1}{a^2-b^2} \int_{-\infty}^{+\infty} \frac{(a^2-b^2) dx}{(x^2+a^2)(x^2+b^2)} =$$

$$= \frac{1}{a^2-b^2} \int_{-\infty}^{+\infty} \frac{(a^2+x^2-b^2-x^2) dx}{(x^2+b^2)(x^2+a^2)} = \frac{1}{a^2-b^2} \int_{-\infty}^{+\infty} \frac{dx}{x^2+b^2} - \frac{1}{a^2-b^2} \int_{-\infty}^{+\infty} \frac{dx}{x^2+a^2} =$$

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HOMEWORK - 1

QUANTUM MECHANICS

Задача 1 (1)

$$\psi(x) = \frac{A}{x^2 + a^2}$$

$$A = ?$$

Решение: $\int_{-\infty}^{+\infty} |\psi(x)|^2 dx = \int_{-\infty}^{+\infty} \psi^*(x) \psi(x) dx = 1$

$$\int_{-\infty}^{+\infty} \frac{A^2 dx}{(x^2 + a^2)^2} \Rightarrow \frac{1}{A^2} = \left[\frac{1}{2a^3} \arctg \frac{x}{a} + \frac{x}{2a^2(x^2 + a^2)} \right] \Bigg|_{-\infty}^{+\infty} = (\dots)$$

$$(\dots) = \frac{1}{2a^2} \left[\frac{1}{a} \arctg \frac{x}{a} + \frac{x}{x(x + \frac{a^2}{x})} \right] \Bigg|_{-\infty}^{+\infty} = \frac{1}{2a^2} \left[\dots \right] \Bigg|_{-\infty}^{+\infty} = (\dots)$$

$$(\dots) = \frac{1}{2a^2} \left[\frac{1}{a} \left(\frac{\pi}{2} - 0 \right) - \frac{1}{a} \left(-\frac{\pi}{2} + 0 \right) \right] = \frac{1}{2a^2} \cdot \frac{1}{a} \cdot \left(\frac{\pi}{2} + \frac{\pi}{2} \right) = \frac{1}{2a^2} \cdot \pi$$

$$\frac{1}{A^2} = \frac{\pi}{2a^2} \Rightarrow A^2 = \frac{2a^3}{\pi} \Rightarrow A = \sqrt{\frac{2a^3}{\pi}} = \sqrt{\frac{2a}{\pi}} \cdot a$$

Ответ: $\psi(x) = \pm \sqrt{\frac{2a}{\pi}} \cdot \frac{a}{(x^2 + a^2)}$

Задача 2 (1)

$$\psi(x) = \frac{B}{x + ib}$$

$$B = ?$$

$$\int_{-\infty}^{+\infty} \psi^*(x) \psi(x) dx = 1 = \langle \psi | \psi \rangle$$

$$|\psi(x)| = \frac{B}{x + ib} \cdot \frac{(x - ib)}{(x - ib)} = \frac{Bx}{x^2 + b^2} - i \frac{bB}{x^2 + b^2} \Rightarrow \langle \psi(x) | = \frac{Bx}{x^2 + b^2} + i \frac{bB}{x^2 + b^2}$$

$$= \frac{1}{a^2 - b^2} \cdot \frac{1}{b} \operatorname{arctg} \frac{x}{b} \Big|_{-\infty}^{+\infty} - \frac{1}{a^2 - b^2} \cdot \frac{1}{a} \operatorname{arctg} \frac{x}{a} \Big|_{-\infty}^{+\infty} = (\dots)$$

$$(\dots) = \frac{1}{a^2 - b^2} \left(\frac{\pi}{1} \cdot \frac{1}{b} - \frac{\pi}{1} \cdot \frac{1}{a} \right) = \frac{1}{a^2 - b^2} \left(\frac{\pi}{b} - \frac{\pi}{a} \right) =$$

$$\Rightarrow I = \left(\frac{\pi}{b} - \frac{\pi}{a} \right) \frac{1}{a^2 - b^2} \Rightarrow \langle \varphi | \psi \rangle = ABibI =$$

$$ABib \left(\frac{\pi}{b} - \frac{\pi}{a} \right) \frac{1}{a^2 - b^2} = a \frac{\sqrt{2ab}}{\pi} ib \left(\frac{\pi}{b} - \frac{\pi}{a} \right) \cdot \frac{1}{a^2 - b^2}$$

$$\text{Отсюда: } \langle \varphi | \psi \rangle = a \frac{\sqrt{2ab}}{\pi} ib \left(\frac{\pi}{b} - \frac{\pi}{a} \right) \cdot \frac{1}{a^2 - b^2}$$

Задача 4 (1) Дана $\mathcal{L}(f(x)) = \sum_{x_i} \frac{1}{|f'(x_i)|} \delta(x - x_i)$
 где x_i — нули первого порядка
 функции $f(x)$

$$f(x) = f(x_0) + f'(x_0)(x - x_0) + \dots$$

допускаю $f(x_0) = 0$, откуда $f(x) = f'(x_0)(x - x_0)$

$$\mathcal{L}(f(x)) = \mathcal{L}(f'(x_0)(x - x_0)) = \frac{1}{|f'(x_0)|} \delta(x - x_0)$$

Отсюда, зная из x_i , что мы имеем n корней...

$$\mathcal{L}(f(x)) = \sum_{x_i} \frac{\delta(x - x_i)}{|f'(x_i)|}$$

Задача 5 (3)

$$\psi(x) = \sqrt{\frac{2}{a}} \sin \frac{\pi x}{a}$$

$$f_1(x) = d_1 \exp(i\pi x/a)$$

$$f_2(x) = d_2 \exp(-i\pi x/a)$$

1. проверить $\langle f_1 | f_2 \rangle = 0$

2. $d_1 = ?$ $d_2 = ?$

3. разложить $|\psi\rangle$ по базису $|f_{1,2}\rangle$, т.е. найти

$c_1 = ?$, $c_2 = ?$ $|\psi\rangle = c_1 |f_1\rangle + c_2 |f_2\rangle$

Решение: (1)

$$\langle f_1 | f_2 \rangle = \int_0^a d_1 d_2 \sqrt{\frac{2}{a}} \sqrt{\frac{2}{a}} dx = \int_0^a d_1 d_2 e^{\frac{-i\pi x}{a}} e^{\frac{i\pi x}{a}} dx = \int_0^a d_1 d_2 e^{\frac{-2i\pi x}{a}}$$

$$= \left[\frac{e^{Axi}}{A} = \cos Ax + i \sin Ax \right] = \int_0^a d_1 d_2 \left(\cos \frac{2\pi x}{a} - i \sin \frac{2\pi x}{a} \right) dx =$$

$$= d_1 d_2 \frac{a}{2\pi} \sin \frac{2\pi x}{a} \Big|_0^a + d_1 d_2 \frac{a}{2\pi} \cos \frac{2\pi x}{a} \Big|_0^a = d_1 d_2 \frac{a}{2\pi} (\sin 2\pi - \sin 0)$$

$$+ d_1 d_2 \frac{a}{2\pi} (\cos 2\pi - \cos 0) = d_1 d_2 \frac{a}{2\pi} (1 - 1) = \underline{\underline{0}}$$

Решение: (2)

$$e^{\frac{-i\pi x}{a}} e^{\frac{i\pi x}{a}} = e^0 = 1$$

$$f_1: \langle f_1 | f_1 \rangle = \int_0^a d_1^2 dx = 1 = \int_0^a d_1^2 dx = 1 \Rightarrow d_1^2 = x \Big|_0^a$$

$$d_1^2 = \frac{1}{a} \Rightarrow d_1 = \frac{1}{\sqrt{a}}$$

$$f_2: \langle f_2 | f_2 \rangle = \int_0^a d_2^2 dx = 1 \Rightarrow d_2^2 = a \Rightarrow d_2 = d_1 = \frac{1}{\sqrt{a}}$$

Peut-être : (3) $|\psi\rangle = c_1 |1\rangle + c_2 |2\rangle$

$$\frac{\sqrt{2}}{\sqrt{a}} \sin \frac{\pi x}{a} = \alpha_1 c_1 e^{\frac{\pi x i}{a}} + \alpha_2 c_2 e^{-\frac{\pi x i}{a}}$$

$$\frac{\sqrt{2}}{\sqrt{a}} \sin \frac{\pi x}{a} = \frac{c_1}{\sqrt{a}} \left(\cos \frac{\pi x}{a} + i \sin \frac{\pi x}{a} \right) + \frac{c_2}{\sqrt{a}} \left(\cos \frac{\pi x}{a} - i \sin \frac{\pi x}{a} \right)$$

$$\frac{\sqrt{2}}{\sqrt{a}} \sin \frac{\pi x}{a} = (c_1 + c_2) \frac{1}{\sqrt{a}} \cos \frac{\pi x}{a} + (c_1 - c_2) \frac{i}{\sqrt{a}} \sin \frac{\pi x}{a}$$

$$\frac{\sqrt{2}}{a} \sin \frac{\pi x}{a} = \frac{(c_1 + c_2)}{\sqrt{a}} \cos \frac{\pi x}{a} + \frac{i(c_1 - c_2)}{\sqrt{a}} \sin \frac{\pi x}{a}$$

$$\begin{cases} \frac{\sqrt{2}}{\sqrt{a}} = \frac{i(c_1 - c_2)}{\sqrt{a}} \\ 0 = \frac{(c_1 + c_2)}{\sqrt{a}} \end{cases} \Rightarrow \begin{cases} \sqrt{2} = i(c_1 - c_2) \\ 0 = c_1 + c_2 \end{cases} \Rightarrow \begin{cases} \frac{\sqrt{2}}{i} = c_1 - c_2 \\ -c_2 = c_1 \end{cases}$$

$$\begin{cases} c_1 = \frac{\sqrt{2}}{i} + c_2 \\ c_2 = -c_1 \end{cases} \Rightarrow \begin{cases} c_1 = \frac{\sqrt{2}}{i} - c_1 \\ c_1 = \frac{\sqrt{2}}{2} i \Rightarrow c_2 = -\frac{\sqrt{2}}{2} i \end{cases}$$

Donc on a : $|\psi\rangle = c_1 |1\rangle + c_2 |2\rangle$

$$|\psi\rangle = \frac{\sqrt{2}}{2} i |1\rangle - \frac{\sqrt{2}}{2} i |2\rangle$$