Homework 4 Answers

1. a) $Smoke \Rightarrow Smoke$ is valid.

b) $Smoke \Rightarrow Fire$ is true if $Smoke$ is false and False if $Smoke$ is true and $Fire$ is false. So it is neither valid nor unsatisfiable.

c) $(Smoke \Rightarrow Fire) \Rightarrow (\neg Smoke \Rightarrow \neg Fire)$ is neither. It is true if $Smoke$ is true and $Fire$ is false since it makes the premise of the main implication false. It is false if $Smoke$ is false and $Fire$ is true. So neither.

d) $Smoke \lor Fire \neg Fire$ is equivalent to $Smoke \lor True$ which is always true. So it is valid.

e) $((Smoke \land Heat) \Rightarrow Fire) \Leftrightarrow ((Smoke \Rightarrow Fire) \lor (Heat \Rightarrow Fire))$

This is unintuitive, but is valid. The only case the left hand side is false is if $Smoke$ and $Heat$ are true and $Fire$ is false. In all other cases it is true. On the right hand side, you have exactly the same situation. The disjunction is false if and only if both the implications are false, which is the same condition as on the l.h.s.

f) $(Smoke \Rightarrow Fire) \Rightarrow ((Smoke \land Heat) \Rightarrow Fire)$

Valid. If the claim on the l.h.s. is true, the r.h.s. must be, since $Smoke$ itself should imply $Fire$.

g) $Big \lor Dumb \lor (Big \Rightarrow Dumb)$

Valid. Expanding the implication, we have $Big \lor Dumb \lor \neg Big \lor Dumb$ which is always True.

2. We can state the following facts from what is given.

$Mythical \Rightarrow \neg Mortal$

$\neg Mythical \Rightarrow Mortal \land Mammal$

$(\neg Mortal \lor Mammal) \Rightarrow Horned$

$Horned \Rightarrow Magical$

We can prove both $Horned$ and $Magical$ but not $Mythical$.

We can start from a valid statement $Mythical \lor \neg Mythical$. Since $Mythical \Rightarrow \neg Mortal$ and $\neg Mythical \Rightarrow Mortal \land Mammal$,
we can infer $(\neg Mortal \lor Mammal)$, which in turn implies $Horned$ according to the third rule, and $Magical$ according to the last rule.

Setting $Mythical$ to true and $Mortal$ to false satisfies all the clauses, as does setting $Mythical$ to false, and $Mortal$ and $Mammal$ to true. Hence we cannot show $Mythical$.

3. a) Let S be the sentence: $((Food \Rightarrow Party) \lor (Drinks \Rightarrow Party)) \Rightarrow ((Food \land Drinks) \Rightarrow Party)$

no	Food	Party	Drinks	Food $\Rightarrow$ Party	Drinks $\Rightarrow$ Party	Food $\land$ Drinks $\Rightarrow$ Party	S
1	F	F	F	T	T	T	T
2	F	F	T	T	T	T	T
3	F	T	F	T	F	T	T
4	F	T	T	T	T	T	T
5	T	F	F	F	T	T	T
6	T	T	F	F	F	F	T
7	T	F	T	T	T	T	T
8	T	T	T	T	T	T	T

The sentence is valid since it is true in all interpretations.

b) Let us start with the left hand side of S: (Food $\Rightarrow$ Party) $\vee$ (Drinks $\Rightarrow$ Party)

Eliminating the two implications, we get:

($\neg$ Food $\vee$ Party) $\vee$ ($\neg$ Drinks $\vee$ Party)

Since this repeats Party twice and by idempotence, one of the copies can be removed, giving us:

$\neg$ Food $\vee$ Party $\vee$ $\neg$ Drinks

Let us now consider the right hand side of S: $((Food \vee Drinks) \Rightarrow Party)$.

By eliminating the implication, we get:

$\neg$ (Food $\wedge$ Drinks) $\vee$ Party

By taking the negation inside and applying Demorgan's law, this further reduces to:

$\neg$ Food $\vee$ $\neg$ Drinks $\vee$ Party

Which is the same as the left hand side. So each implies the other.

c) To apply resolution, we start with a negation of the original sentence and derive a contradiction. Since the original sentence is of the form $P \Rightarrow Q$, its negation is of the form $\neg(P \Rightarrow Q)$, which is equivalent to $\neg(\neg P \vee Q)$ which is by Demorgan's law equivalent to $P \wedge \neg Q$.

So we get

((Food $\Rightarrow$ Party) $\vee$ (Drinks $\Rightarrow$ Party)) $\wedge$ $\neg$ ((Food $\wedge$ Drinks) $\Rightarrow$ Party)

Applying similar transformation to (Food $\wedge$ Drinks) $\Rightarrow$ Party), we get

((Food $\Rightarrow$ Party) $\vee$ (Drinks $\Rightarrow$ Party)) $\wedge$ Food $\wedge$ Drinks $\wedge$ $\neg$ Party

Eliminating the first two implications, we get:

($\neg$ Food $\vee$ Party $\vee$ $\neg$ Drinks $\vee$ Party) $\wedge$ Food $\wedge$ Drinks $\wedge$ $\neg$ Party

So we have the following 4 clauses as we start:

1. $\neg$ Food $\vee$ Party $\vee$ $\neg$ Drinks;
2. Food
3. Drinks
4. $\neg$ Party

With resolution, we get the following:

5. Party $\vee$ $\neg$ Drinks; by resolution between clauses 1 and 2.
6. Party; by resolution between clauses 5 and 3.
7. Empty; by resolution between clauses 4 and 6.