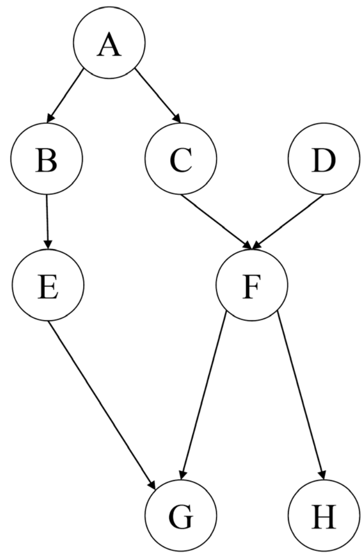
CS331: Introduction to Artificial Intelligence
Homework #5

1. Consider the Bayesian network below. Answer true or false for the following questions on d-separation. Show the blocked paths for partial credit.



a) I (D, G | { })
No. There are 2 paths between D and G. DFG is not blocked by evidence because F is not observed. Although one unblocked path is enough to conclude no, it is worth noting that DFCABEG is blocked by evidence because F is a collider and neither F nor its descendants (G and H) are observed.

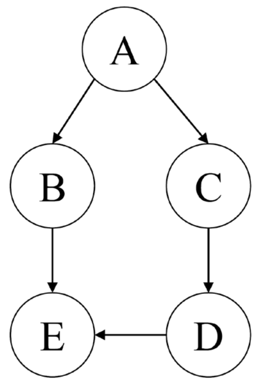
b) I (B, C | { })
No. Path BAC is not blocked by evidence. The other path BEGFC is blocked at G.

c) I (B, C | {A})
Yes. Path BAC is blocked by evidence at A. The other path BEGFC is blocked at G since it is collider and not part of the evidence.

d) I (B, C | {A, G})
No. Path BAC is blocked by evidence at A. The other path BEGFC is not blocked at G since it is collider and is now part of the evidence.

e) I (B, C | {A, C, G})
C occurs in both evidence nodes and the target pair. You can say B is independent of C, given C regardless of everything else, but the question is incoherent. Everybody gets credit for this.

2. Calculate the following probabilities using the Bayesian network below. The CPTs for each node are shown below the network. You may need to use the various probability formulas such as marginalization, the chain rule, conditional independence, Bayes rule, etc.



Conditional probability tables are given below:

A	P(A)
true	0.4
false	0.6

A	B	P(B|A)
true	True	0.9
true	False	0.1
false	True	0.25
false	False	0.75

C	D	P(D|C)
true	true	0.75
true	false	0.25
false	true	0.9
false	false	0.1

B	D	E	P(E|B,D)
true	true	true	0.1
true	true	false	0.9
true	false	true	0.2
true	false	false	0.8
false	true	true	0.3
false	true	false	0.7
false	false	true	0.4
false	false	false	0.6

A	C	P(C|A)
true	True	0.25
true	False	0.75
false	True	0.8
false	False	0.2

a) P(A=true, B=false, C=true, D=false, E=false)
Notation: I use a for "A=true" and -a for "A=false" when not in summation. If inside the summation each letter ranges over both true and false.
= P(a) P(-b|a)P(c|a)P(-d|c)P(-e|-b,-d)=0.4*0.1*0.25*0.25*0.6=0.0015

b) P(B=false, C=false)
=\sum_{a,e,d} P(a) P(-b|a)P(-c|a)P(d|-c)P(e|-b,d) //This is a long way to do it.
=\sum_{a} \sum_{e,d} P(a) P(-b|a)P(-c|a)P(d|-c)P(e|-b,d)
=\sum_{a} P(a) P(-b|a) P(-c|a) \sum_{d} P(d|-c) \sum_e P(e|-b,d)
=\ sum_{a} P(a) P(-b|a) P(-c|a) \sum_{d} P(d|-c)

= \sum_{a} P(a) P(-b|a) P(-c|a)
= P(a) P(-b|a) P(-c|a) + P(-a)P(-b|-a) P(-c|-a)
= 0.4*0.1*0.75+0.6*0.75*0.2
=0.03+0.09=0.12

c) P(A=true | B=false, C=false)
= P(a,-b,-c)/P(-b,-c)
=P(a)*P(-b|a)*P(-c|a)/P(-b,-c) // Short way to compute P(a,-b,-c)
=0.4*0.1*0.75/0.12 // P(-b,-c) is borrowed from (b)
=0.03/0.12= 0.25