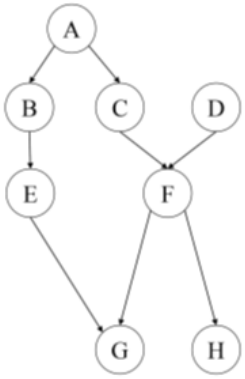
1 Consider the Bayesian network below. Answer true or false for the following questions on d-separation. Show the blocked paths for partial credit.



1.1 I (D, G |{ }) [2 points]

$D \to F \to G$ (False)
$\therefore$ False, the path $D \to F \to G$ is not blocked

1.2 I (B, C |{ }) [2 points]

$B \leftarrow A \to C$ (False)
$B \to E \to G \leftarrow F \leftarrow C$ (True, case 3 by node G)
$\therefore$ False, the path $B \to E \to G \leftarrow F \leftarrow C$ is not blocked

1.3 I (B, C |{A}) [2 points]

$B \to A \to C$ (True, Case 1 by node A)
$B \to E \to G \leftarrow F \leftarrow C$ (True, case 3 by node G)
$\therefore$ True, all paths blocked

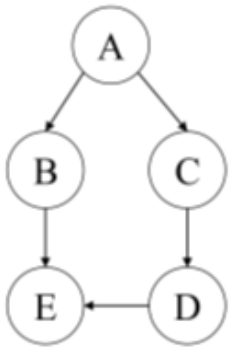
1.4 I (B, C |{A, G}) [2 points]

$B \to A \to C$ (True, Case 1 by node A)
$B \to E \to G \leftarrow F \leftarrow C$ (False, case 3 by node G)
$\therefore$ False, the path $B \to E \to G \leftarrow F \leftarrow C$ is not blocked

1.5 I (B, C |{A, C, G}) [2 points]

$B \to A \to C$ (True, Case 1 by node A)
$B \to E \to G \leftarrow F \leftarrow C$ (False, case 3 by node G)
$\therefore$ False, the path $B \to E \to G \leftarrow F \leftarrow C$ is not blocked

2 Calculate the following probabilities using the Bayesian network below. The CPTs for each node are shown below the network. You may need to use the various probability formulas such as marginalization, the chain rule, conditional independence, Bayes rule, etc.



2.1 P(A=true, B=false, C=true, D=false, E=false) [3 points]

$$
\begin{aligned}
&P(A = true, B = false, C = true, D = false, E = false) \\
&\quad = P(A = true) \cdot P(B = false|A = true) \cdot P(C = true|A = true) \\
&\qquad \cdot P(D = false|C = true) \cdot P(E = false|B = false, D = false) \\
&\quad = (0.4) \cdot (0.1) \cdot (0.25) \cdot (0.25) \cdot (0.6) \\
&\quad = 0.0015
\end{aligned}
$$

2.2 P(B=false, C=false) [4 points]

$$
\begin{aligned}
&P(B = false, C = false) \\
&\quad = P(A = a, B = false, C = false, D = d, E = e) \\
&\quad = \sum_a P(A = a) \cdot P(B = false|A = a) \cdot P(C = false|A = a) \\
&\qquad \cdot P(D = d|C = false) \cdot P(E = e|B = false, D = d) \\
&\quad = \sum_a P(A = a) \cdot P(B = false|A = a) \cdot P(C = false|A = a) \\
&\quad = P(A = True) \cdot P(B = false|A = True) \cdot P(C = false|A = True) \\
&\qquad + P(A = false) \cdot P(B = false|A = false) \cdot P(C = false|A = false) \\
&\quad = (0.4) \cdot (0.1) \cdot (0.75) + (0.6) \cdot (0.75) \cdot (0.2) \\
&\quad = 0.12
\end{aligned}
$$

2.3 P(A=true | B=false, C=false) [3 points]

$$
\begin{aligned}
&P(A = true|B = false, C = false) \\
&\quad = \frac{P(A = true, B = false, C = false)}{P(B = false, C = false)} \\
&\quad = \frac{\sum_d \sum_e P(A = true, B = false, C = false, D = d, E = e)}{P(B = false, C = false)} \\
&\quad = \frac{\sum_d \sum_e P(A = true)P(B = false|A = true)P(C = false|A = true)}{\sum_a \sum_d \sum_e P(A = a, B = false, C = false, D = d, E = e)} \\
&\quad = \frac{(0.4)(0.1)(0.75)}{0.12} \\
&\quad = 0.25
\end{aligned}
$$