Machine Learning
Decision Tree Induction

Thanks to: Dan Weld, University of Washington

Learning from Training Experience

- Credit assignment problem:
  - **Direct** training examples:
    - E.g. individual checker boards + correct move for each
    - Supervised learning
  - **Indirect** training examples :
    - E.g. complete sequence of moves and final result
    - Reinforcement learning
- Which examples:
  - Random, teacher chooses, learner chooses
- Unsupervised Learning

Machine Learning Outline

- Machine learning:
  - √ Function approximation
  - √ Bias
- Supervised learning
  - √ Classifiers & concept learning
  - Decision-trees induction (pref bias)
- Overfitting
- Ensembles of classifiers
- Co-training

Need for Bias

- Example space: 4 Boolean attributes
- How many ML hypotheses?

Two Strategies for ML

- **Restriction bias**: use prior knowledge to specify a restricted hypothesis space.
  - Version space algorithm over conjunctions.
- **Preference bias**: use a broad hypothesis space, but impose an ordering on the hypotheses.
  - Decision trees.

Decision Trees

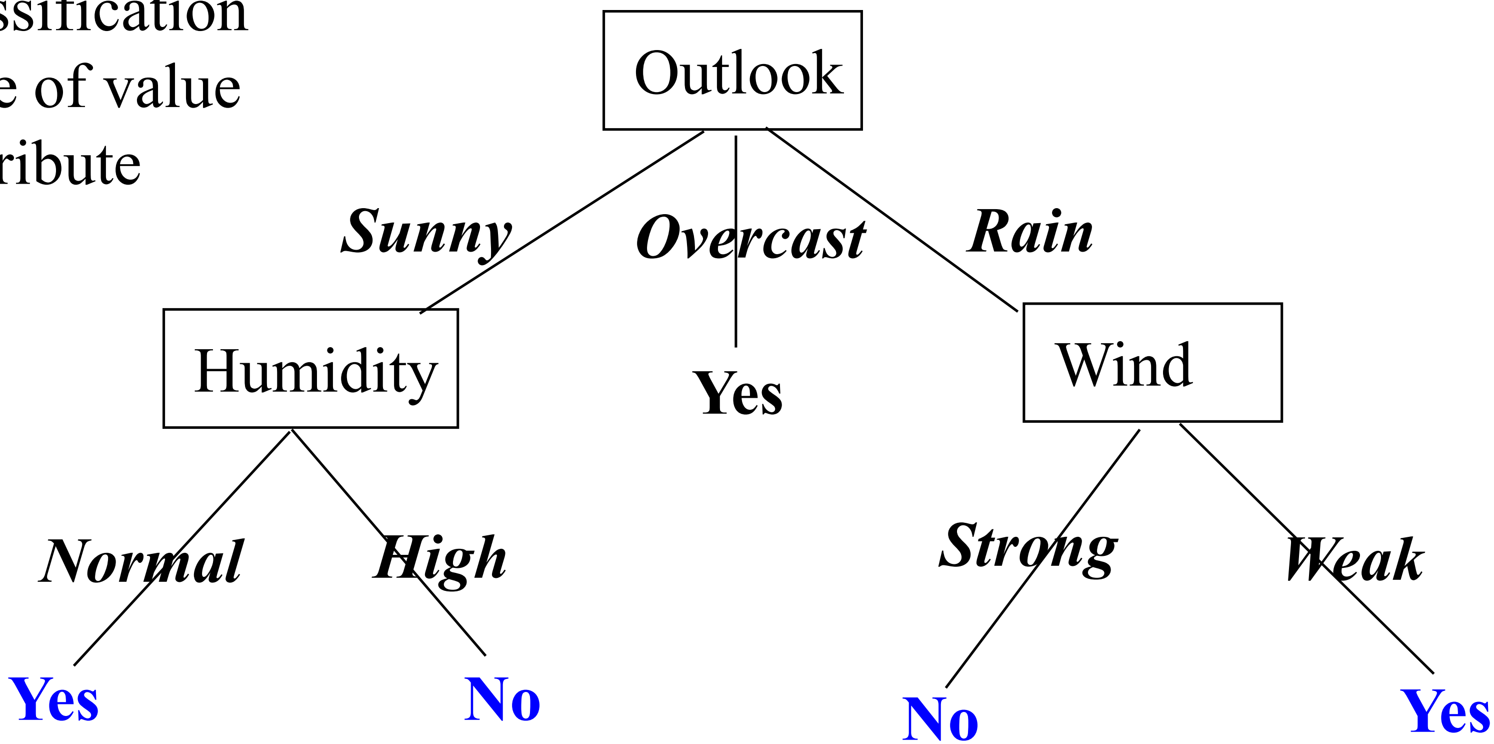
- Convenient Representation
  - Developed with learning in mind
  - Deterministic
- Expressive
  - Equivalent to propositional DNF
  - Handles discrete and continuous parameters
- Simple learning algorithm
  - Handles noise well
  - Classify as follows
    - **Constructive (build DT by adding nodes)**
    - **Eager**
    - **Batch (but incremental versions exist)**

Decision Tree Representation

Good day for tennis?

Leaves = classification

Arcs = choice of value for parent attribute



Decision tree is equivalent to logic in disjunctive normal form

G-Day $\Leftrightarrow$ (Sunny $\wedge$ Normal) $\vee$ Overcast $\vee$ (Rain $\wedge$ Weak)

DT Learning as Search

- Nodes

  Decision Trees

- Operators

  Tree Refinement: Sprouting the tree

- Initial node

  Smallest tree possible: a single leaf

- Heuristic?

  Information Gain

- Goal?

  Best tree possible (???)

- Type of Search?

  Hill climbing

Decision Tree Algorithm

```
BuildTree(TraingData)
    Split(TrainingData)

Split(D)
    If (all points in D are of the same class)
        Then Return
    For each attribute A
        Evaluate splits on attribute A
    Use best split to partition D into D1, D2
    Split (D1)
    Split (D2)
```

Key Questions

- How to choose best attribute?
  - Mutual Information (Information gain)
    - Entropy (disorder)
- When to stop growing tree?
- Non-Boolean attributes
- Missing data

A Better Heuristic From Information Theory

Let V be a random variable with the following probability distribution:

$P(V = 0)$	$P(V = 1)$
0.2	0.8

The ***surprise***, $S(V = v)$ of each value of V is defined to be

$$S(V = v) = -\lg P(V = v).$$

An event with probability 1 gives us zero surprise.

An event with probability 0 gives us infinite surprise!

It turns out that the surprise is equal to the number of bits of information that need to be transmitted to a recipient who knows the probabilities of the results.

This is also called the ***description length*** of $V = v$.

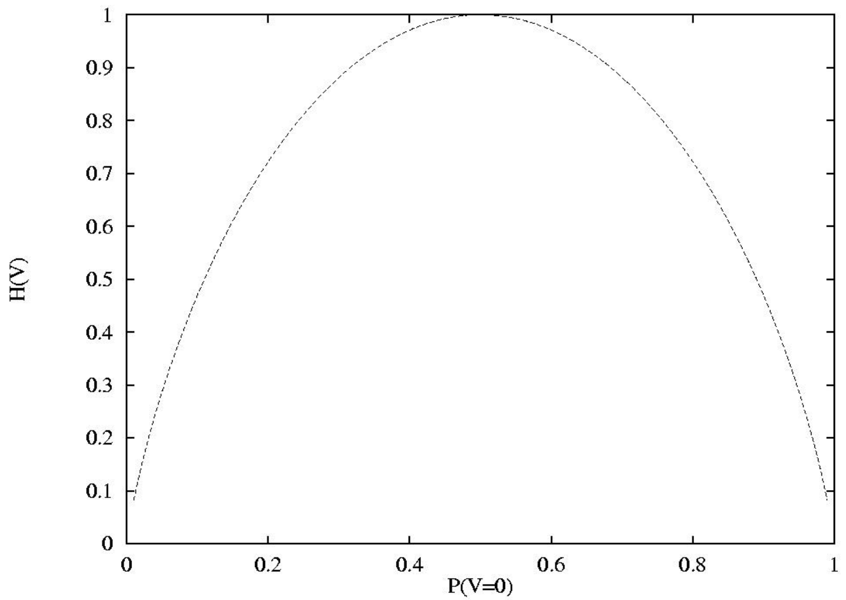
Fractional bits only make sense if they are part of a longer message (e.g., describe a whole sequence of coin tosses).

Entropy

The *entropy* of V, denoted $H(V)$ is defined as follows:

$$H(V) = \sum_{v=0}^{1} -P(H = v) \lg P(H = v).$$

This is the average surprise of describing the result of one "trial" of V (one coin toss).



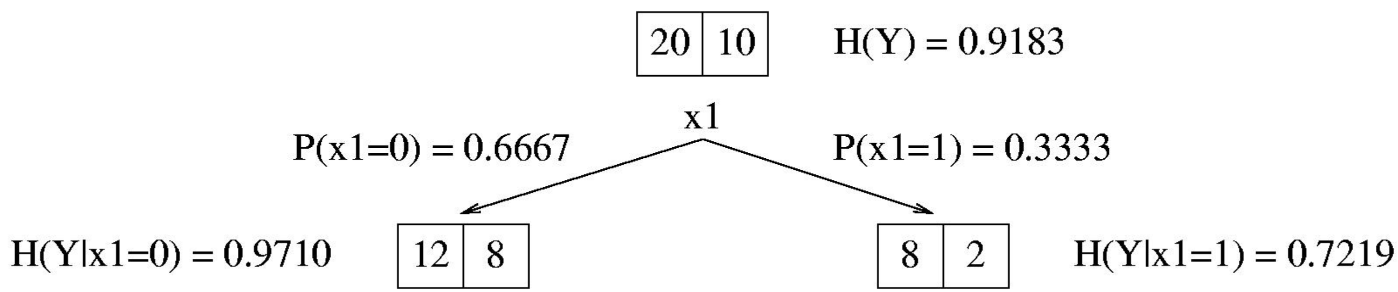
Entropy can be viewed as a measure of uncertainty.

Mutual Information

Now consider two random variables A and B that are not necessarily independent. The *mutual information* between A and B is the amount of information we learn about B by knowning the value of A (and vice versa—it is symmetric). It is computed as follows:

$$I(A;B) = H(B) - \sum_b P(B=b) \cdot H(A|B=b)$$

In particular, consider the class Y of each training example and the value of feature x_1 to be random variables. Then the mutual information quantifies how much x_1 tells us about the value of the class Y.



I(Y;x1) = 0.0304

Issues

- Non-Boolean Attributes
- Missing Data
- Scaling up

Missing Data 1

Day	Temp	Humid	Wind	Tennis?
d1	h	h	weak	n
d2	h	h	s	n
d8	m	h	weak	n
d9	c		weak	yes
d11	m	n	s	yes

- Don't use this instance for learning?
- Assign attribute …

  most common value at node, or

  most common value, … given classification

Fractional Values

Day	Temp	Humid	Wind	Tennis?
d1	h	h	weak	n
d2	h	h	s	n
d8	m	h	weak	n
d9	c		weak	yes
d11	m	n	s	yes

[0.75+, 3-]

[1.25+, 0-]

- 75% h   and 25% n
- Use in information gain calculations
- Further subdivide if other missing attributes
- Same approach to classify test ex with missing attr

  Classification is most probable classification

  Summing over leaves where it got divided

Non-Boolean Features

- Features with multiple discrete values
  - Construct a multi-way split
  - Test for one value *vs.* all of the others?
  - Group values into two disjoint subsets?

- Real-valued Features
  - Discretize?
  - Consider a threshold split using observed values?

Attributes with many values

Problem:

- If attribute has many values, *Gain* will select it
- Imagine using $Date = Jun\_3\_1996$ as attribute

- So many values that it
  - Divides examples into tiny sets
  - Each set is likely ***uniform*** → high info gain
  - But poor predictor...
- Need to penalize these attributes

One approach: Gain ratio

One approach: use $GainRatio$ instead

$$GainRatio(S, A) \equiv \frac{Gain(S, A)}{SplitInformation(S, A)}$$

$$SplitInformation(S, A) \equiv - \sum_{i=1}^{c} \frac{|S_i|}{|S|} \log_2 \frac{|S_i|}{|S|}$$

where S_i is subset of S for which A has value v_i

SplitInfo ≅ entropy of S *wrt* values of *A*

(Contrast with entropy of S *wrt* **target** value)

⇓ attribs with many uniformly distrib values

e.g. if A splits S uniformly into n sets

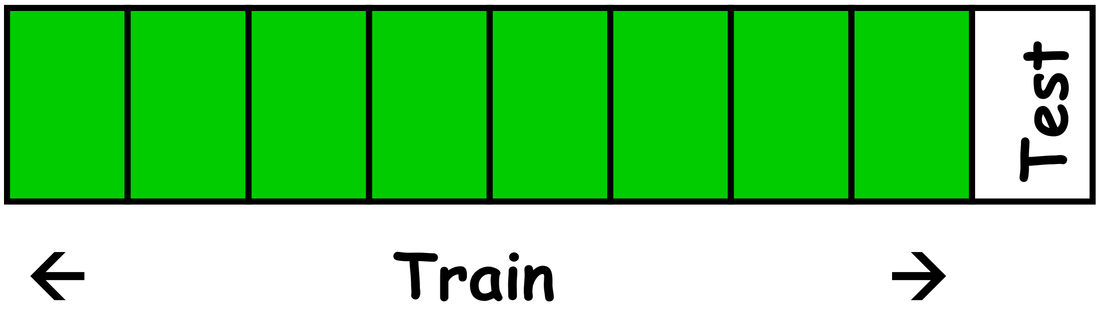
SplitInformation = $\log_2(n)$... = 1 for Boolean

Cross validation

- Partition examples into *k* disjoint equiv classes
- Now create *k* training sets

  Each set is union of all equiv classes ***except one***

  So each set has (k-1)/k of the original training data



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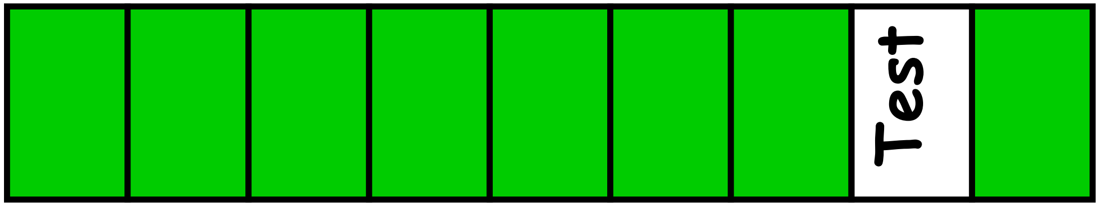

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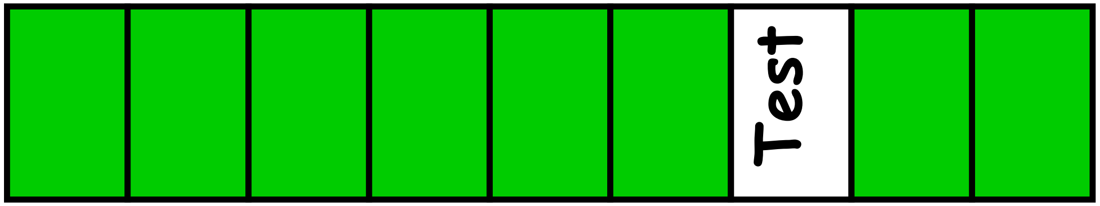
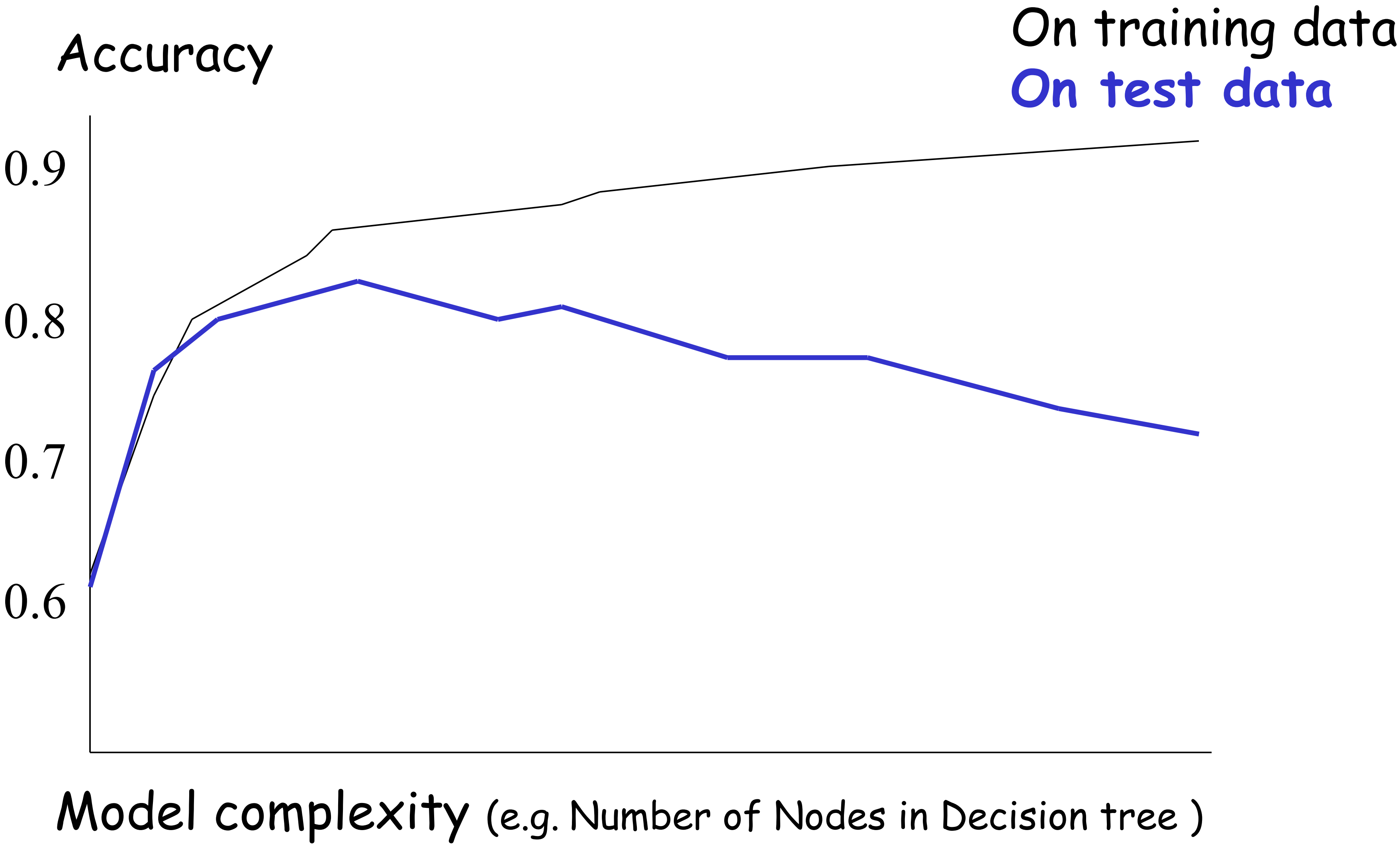


Machine Learning Outline

- Machine learning:
- Supervised learning
- Overfitting
  - What is the problem?
  - Reduced error pruning
- Ensembles of classifiers
- Co-training

Overfitting



Overfitting…

- DT is *overfit* when exists another DT' and
  - DT has ***smaller*** error on training examples, but
  - DT has ***bigger*** error on test examples
- Causes of overfitting
  - Noisy data, or
  - Training set is too small

Avoiding Overfitting

How can we avoid overfitting?

- Stop growing when data split not statistically significant
- Grow full tree, then post-prune

How to select "best" tree:

- Measure performance over training data
- Measure performance over separate validation data set
- Add complexity penalty to performance measure

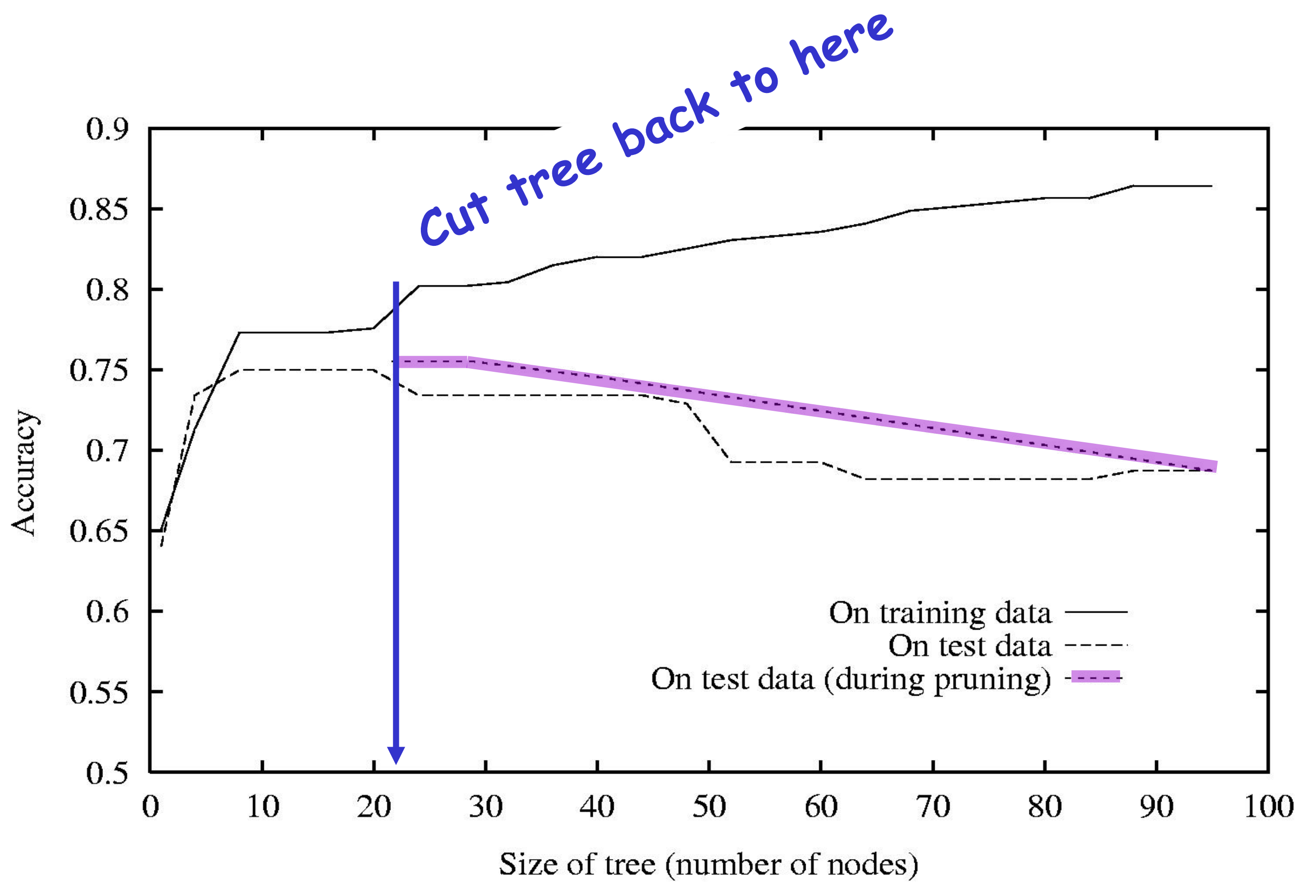
Reduced-Error Pruning

Split data into *training* and *validation* set

Do until further pruning is harmful:

1. Evaluate impact on *validation* set of pruning each possible node (plus those below it)
2. Greedily remove the one that most improves *validation* set accuracy

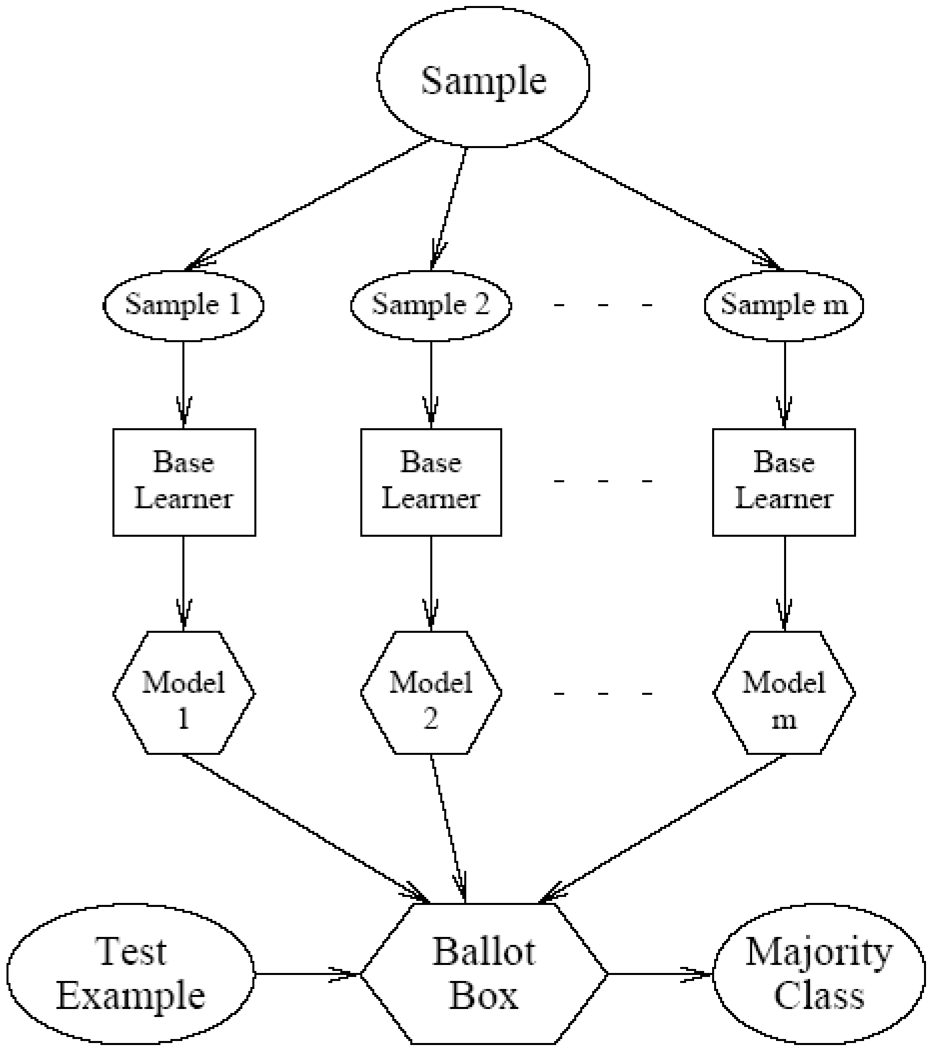
Effect of Reduced-Error Pruning



Machine Learning Outline

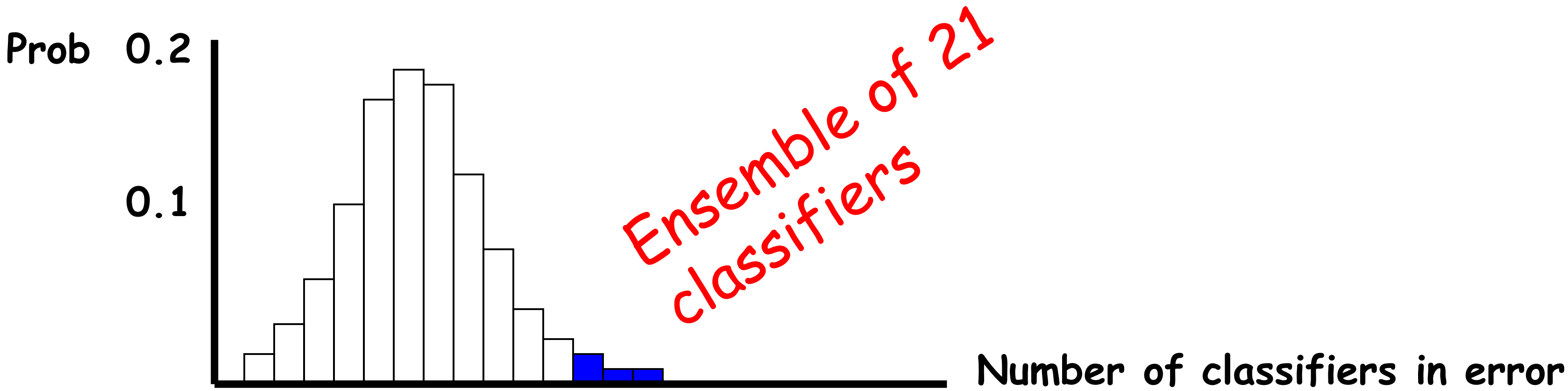
- Machine learning:
- Supervised learning
- Overfitting
- Ensembles of classifiers
  - Bagging
  - Cross-validated committees
  - Boosting
  - Stacking

Voting


Sample
Sample 1
Sample 2
- - -
Sample m
Base Learner
Base Learner
- - -
Base Learner
Model 1
Model 2
- - -
Model m
Test Example
Ballot Box
Majority Class

Ensembles of Classifiers

- Assume
  - Errors are independent (suppose 30% error)
  - Majority vote
- Probability that majority is wrong…
  - = area under binomial distribution

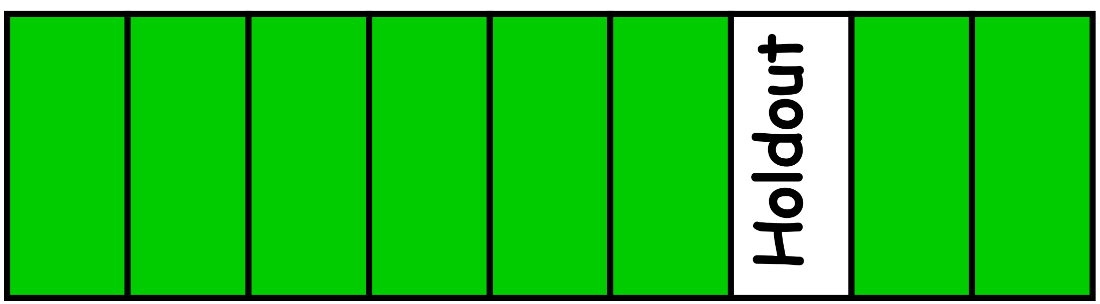


- If individual area is 0.3
- **Area under curve for ≥11 wrong is 0.026**
- Order of magnitude improvement!

Constructing Ensembles

Cross-validated committees

- Partition examples into *k* disjoint equiv classes
- Now create *k* training sets
  - Each set is union of all equiv classes ***except one***
  - So each set has (k-1)/k of the original training data
- Now train a classifier on each set



Ensemble Construction II

Bagging

- Generate k sets of training examples
- For each set

    Draw m examples randomly (with replacement)
    From the original set of m examples
- Each training set corresponds to

    63.2% of original
    (+ duplicates)

- Now train classifier on each set

Ensemble Creation III

Boosting

- Maintain prob distribution over set of training ex
- Create k sets of training data iteratively:
- On iteration *i*

  Draw m examples randomly (like bagging)

  But use probability distribution to bias selection

  Train classifier number *i* on this training set

  Test partial ensemble (of *i* classifiers) on all training exs

  Modify distribution: increase P of each error ex

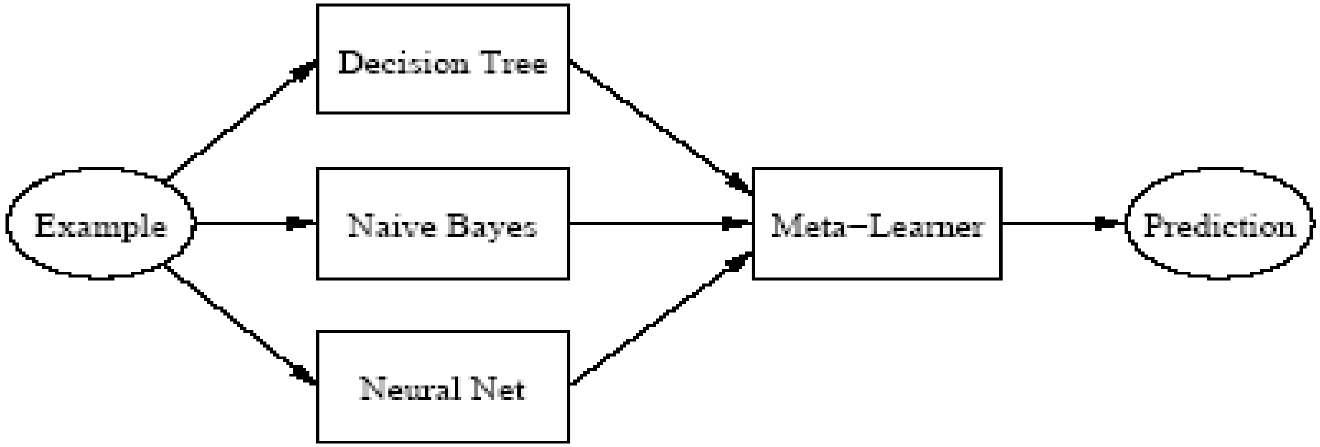
- Create harder and harder learning problems...
- "Bagging with ***optimized*** choice of examples"

Ensemble Creation IV
Stacking

- Train several base learners
- Next train meta-learner

    Learns when base learners are right / wrong

    Now meta learner arbitrates



Train using cross validated committees

- Meta-L inputs = base learner predictions
- Training examples = 'test set' from cross validation