Homework 2 (Due April 29, 2020, 23:59)

BLM 2502: Theory of Computation — Spring 2020

Print family (or last) name: ———————————————————————————

Print given (or first) name: ———————————————————————————

Print given student number: ———————————————————————————

I see that this homework has 10 questions in total 5 pages.

I agree that I have to submit my homework solution before the deadline (April 29, 2020, 23:59) otherwise my homework solution will not be accepted and will not be graded. I accept that ***I will add the signed version of this instruction page as a first page into my homework solution***; otherwise my homework solution will not be graded. I know that ***I have to give my solutions written on white A4-sized pages***; otherwise my homework solution will not be graded. I will take care of the readability of my solutions, from which I may lose 10 points. For any proofs, I am sure to provide a step-by-step argument, with justifications for every step. I understand that, during solving this homework, it is prohibited to exchange information about solutions with any other person in any way, including by talking or ex-changing solutions / papers.

I know that the course book is "Introduction to the Theory of Computation, 2nd Ed., Massachusetts Institute of Technology, by Micheal Sipser."

I have read, understand and accept all of the instructions above. On my honor, I pledge that I have not violated the provisions of the Academic Integrity Code of Yıldız Technical University.

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Signature and Date

1	2	3	4	5	6	7	8	9	10
20 pts	**20 pts**	**20 pts**	**20 pts**	**20 pts**	**20 pts**	**20 pts**	**20 pts**	**20 pts**	**20 pts**

Total
200 pts

1. **[20 points]** For each of the following languages, state the class of the language (whether it is regular, context-free (but not regular) or neither). Prove your answer. Make sure, if you claim that a language is context free, that you show that it is not also regular.

(a) $L=\{w \in \{0,1\}^* : \exists k \geq 0$ and w is a binary encoding (leading zeros allowed) of $2k+1\}$.

(b) $L=\{a^*b^*c^* - \{a_n b_n c_n : n \geq 0\}\}$.

(c) L={x ∈ {a, b}* : |x| is even and the first half of x has one more a than the second half}.

2. **[20 points]** Let L = {w ∈ {a, b}* : the first, middle, and last characters of w are identical}.

   a. Show a context-free grammar that generates *L*.

   b. Design a PDA that accepts *L*.

c. Prove that L is not regular.

3. **[20 points]** Consider the following grammar G:

$S \rightarrow 1\ S\ 1 \mid T$

$T \rightarrow 1\ X\ 1 \mid X$

$X \rightarrow 0\ X\ 0 \mid 1$

a. What are the first (shortest) four strings L(G)?

b. Give an example of a string $w \in \{0, 1\}^*$ such that $|w| > 7$ and $w \notin L(G)$.

c. Show that G is ambiguous.

4. Find grammars for the given languages
   a. $L_1 = L(aaa^*b + b)$.

   b. $L_2 = \{a_n c_m b_n : m,n \geq 1\}$.

c. $L_3 = \{a_n b_{n+1} : n \geq 0\}$

5. **[20 points]** Convert the Grammars given below to Chomsky Normal Form of the. Do not forget to give modified 4-Tuple Grammar.

   a. S → a | aA | B |C
      A →aB |ε
      B →Aa
      C→ bCD
      D→ bbb

b. S→ aA|aBB
A →aaA |ε
B →bB|bbC
C→ B

6. **[20 points]** Find a context-free grammar that generates the language accepted by the PDA

M = ({q0,q1}, {a, b}, {A, z},δ, q0, {q1}), with transitions
$\delta(q_0, a, z) = \{(q_0, Az)\}$,
$\delta(q_0, b, A) = \{(q_0, AA)\}$,
$\delta(q_0, a, A) = \{(q_1, \varepsilon)\}$

7. **[20 points]** Determine whether the following languages are context-free or not
   a. $L=\{a_n w w^R a_n : n \geq 0, w \in \{a,b\}^*\}$

   b. $L= \{a_n b_j a_n b_j : n \geq 0, j \geq 0\}$.

   c. $L= \{a_n b_j a_j b_n : n \geq 0, j \geq 0\}$.

   d. $L= \{a_n b_j a_k b_l : n + j \leq k + l\}$.

   e. $L= \{a_n b_j a_k b_l : n \leq k, j \leq l\}$.

f. $L= \{a_n b_n c_j : n \leq j\}$.

g. $L= \{w \in L((a + b + c)^*) : n_a(w)= n_b(w)=2n_c(w)\}$, where $n_x(w)$ denotes the number of symbol X in the string w.

8. **[20 points]** Let L be a context-free language. Prove that there exists an integer $p \geq 1$, such that the following is true:
   For every string s in L with $|s| \geq p$, there exists a string z in L such that $|s|<|z|\leq|s|+p$.

9. **[20 points]** For the context free languages **in problem 7**, find the grammar of the language in Chomsky Normal Form.

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10. **[20 points]** For the grammars **in problem 9**, obtain the PDA using the obtained context free grammar.

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