Çankaya University
Department of Computer Engineering

CENG 491 - Formal Languages and Automata

Name-Surname: 19.10.2015

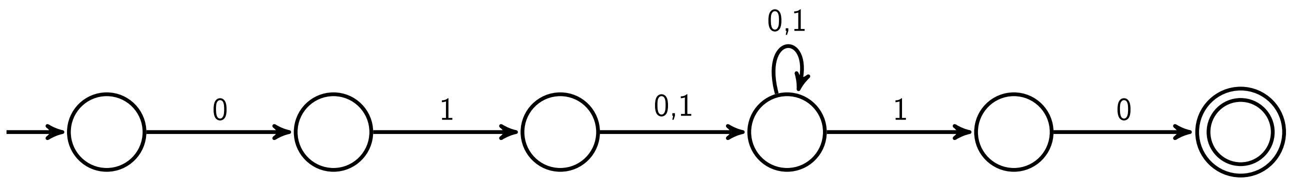
ID Number:

CLASSWORK 1

Find an NFA that recognizes the language A over $\Sigma = \{0, 1\}$ where

$$A = \{w \mid w \text{ starts with 01, ends with 10 and is of length at least 5}\}$$

Answer:



Name-Surname: 20.10.2015

ID Number:

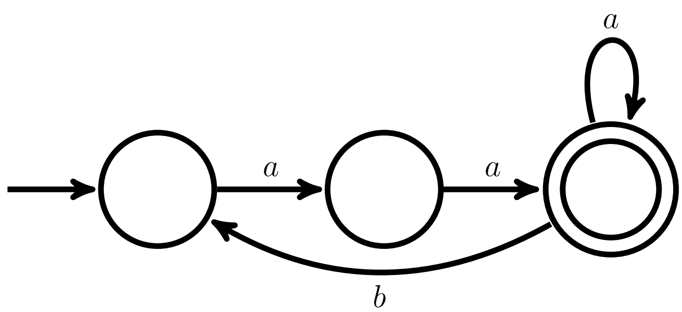
CLASSWORK 1

Find an NFA that recognizes the language A over $\Sigma = \{a, b\}$ consisting of strings that

- Start and ends with a. Length must be at least two.
- Contains a's in sequences of two or more.
- Does not contain bb. (b's are isolated)

Examples:
$aaa, aabaaaa, aaabaaaaabaaaaa$: ACCEPT
$a, b, baa, aababaaa, aabbaaa$: REJECT

Answer:



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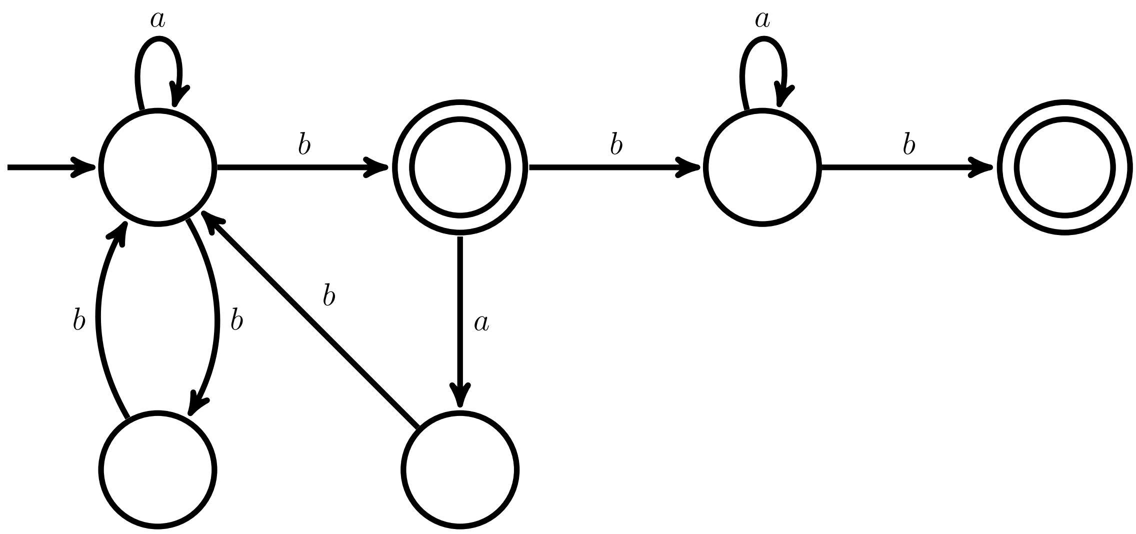
CENG 491 - Formal Languages and Automata

Name-Surname: 26.10.2015

ID Number:

CLASSWORK 2

Find a regular expression equivalent to the language recognized by the following NFA:



Answer:

$$(a \cup bb \cup bab)^* \, b \, (\varepsilon \cup ba^* b)$$

OR

$$b \left(ab \, (a \cup bb)^* \, b\right)^* (\varepsilon \cup ba^* b)$$

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CENG 491 - Formal Languages and Automata

Name-Surname: 27.10.2015

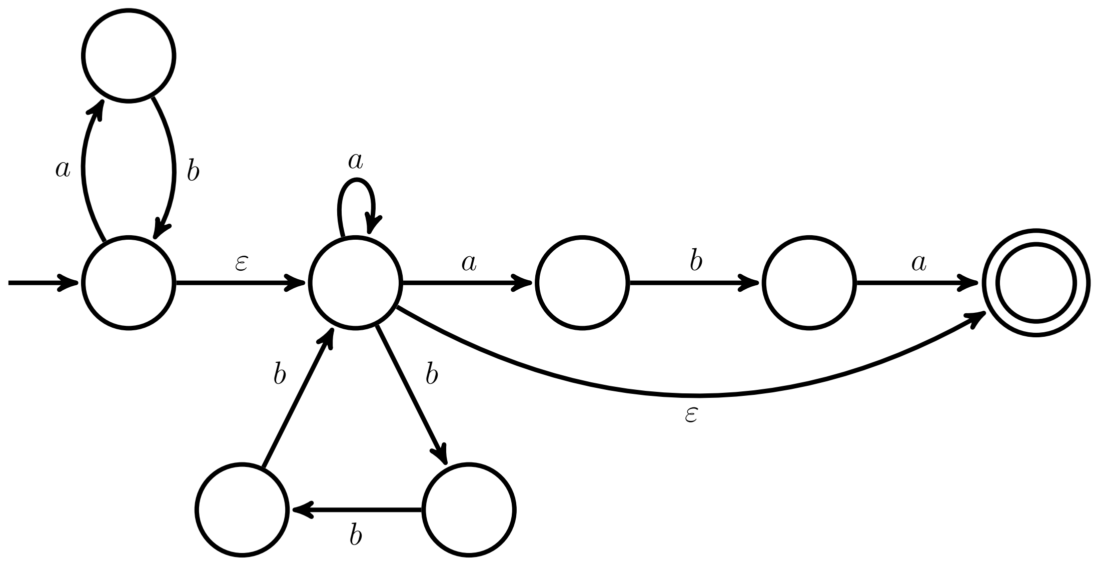
ID Number:

CLASSWORK 2

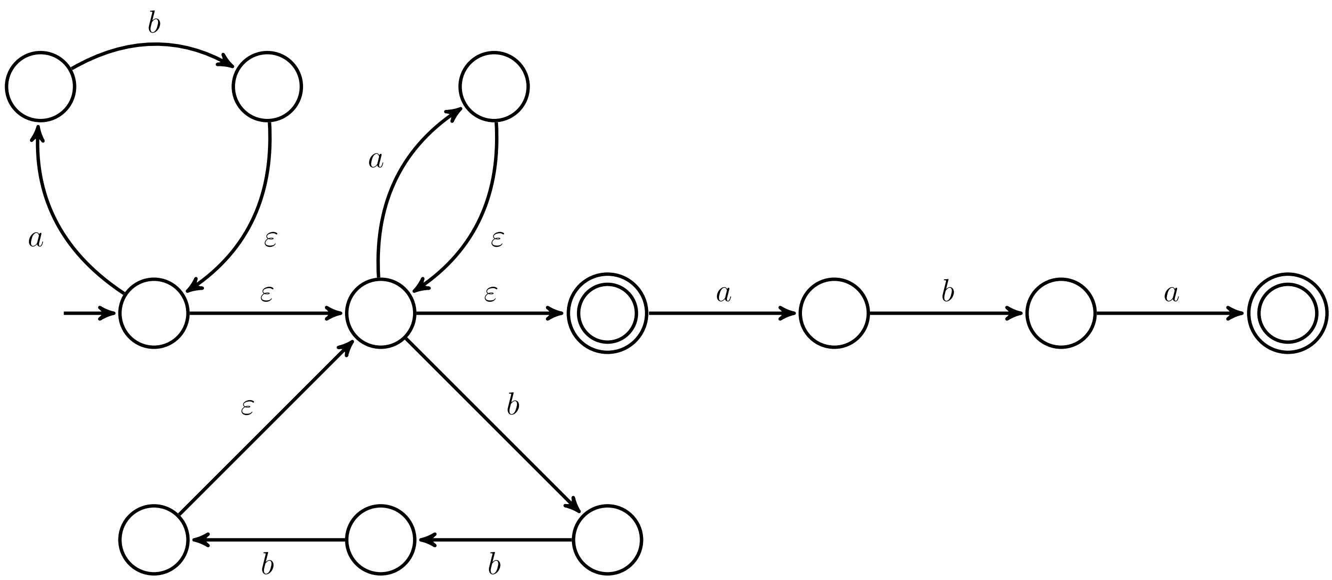
Find an NFA recognizing the regular expression:

$$(ab)^* \left(a^* \cup bbb\right)^* \left(aba \cup \varepsilon\right)$$

Answer:



OR



Name-Surname: 02.11.2015

ID Number:

CLASSWORK 3

Consider the following languages over the alphabet $\Sigma = \{a, b, c\}$

$A = \{a^n\, b^m\, c^{2n+m} \mid n, m \geqslant 0\}$

$B = \{a^{n+3}\, b^{m+2}\, c^q\, a^2 \mid n, m, q \geqslant 0\}$

One of these languages is regular. Find a regular expression for it.
The other is not regular. Prove that it is not regular using pumping lemma.

Answer:

B is regular. We can describe it by the regular expression $aaaa^*\, bbb^*\, c^*\, aa$.

Suppose A is regular, assume the pumping length to be p. Consider the string

$$s = a^p\, b^p c^{3p}$$

Clearly, $|s| > p$ so we should be able to pump it by pumping lemma.

$$s = xyz$$

We also know that $|xy| \leqslant p$. Therefore y consists of a's only. If $|y| = k$, then

$$xyyz = a^{p+k}\, b^p c^{3p} \notin A$$

Therefore our assumption is wrong. s can not be pumped, so A is not regular.

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CENG 491 - Formal Languages and Automata

Name-Surname: 03.11.2015

ID Number:

CLASSWORK 3

Consider the following languages over the alphabet $\Sigma = \{a, b, c\}$

$$A = \{a^n\, b^m\, c^{nm} \mid n, m \geqslant 0\}$$

$$B = \{a^{2n}\, b^m\, c^{q+1}\, a^2 \mid n, m, q \geqslant 0\}$$

One of these languages is regular. Find a regular expression for it.
The other is not regular. Prove that it is not regular using pumping lemma.

Answer:

B is regular. We can describe it by the regular expression $(aa)^*\, b^*\, cc^*\, aa$.

Suppose A is regular, assume the pumping length to be p. Consider the string

$$s = a^p\, b^p c^{p^2}$$

Clearly, $|s| > p$ so we should be able to pump it by pumping lemma.

$$s = xyz$$

We also know that $|xy| \leqslant p$. Therefore y consists of a's only. If $|y| = k$, then

$$xyyz = a^{p+k}\, b^p c^{p^2} \notin A$$

Therefore our assumption is wrong. s can not be pumped, so A is not regular.

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CENG 491 - Formal Languages and Automata

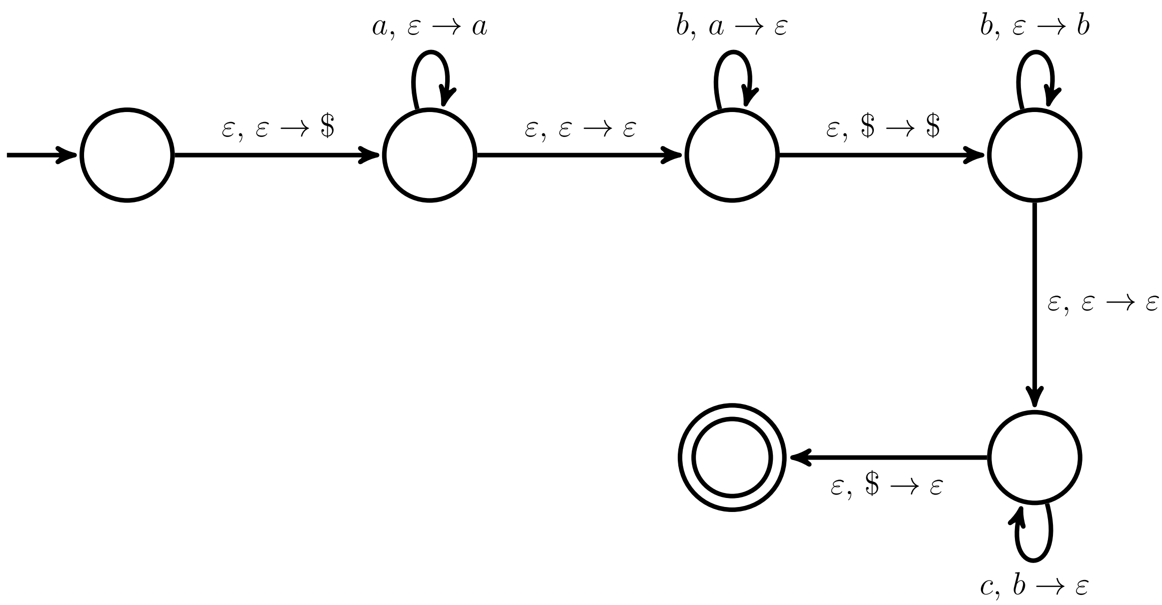
Name-Surname: 16.11.2015

ID Number:

CLASSWORK 4

Find a PDA that recognizes the language $\{a^m\, b^{m+n}\, c^n \mid m, n \geqslant 0\}$.

Answer:



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CENG 491 - Formal Languages and Automata

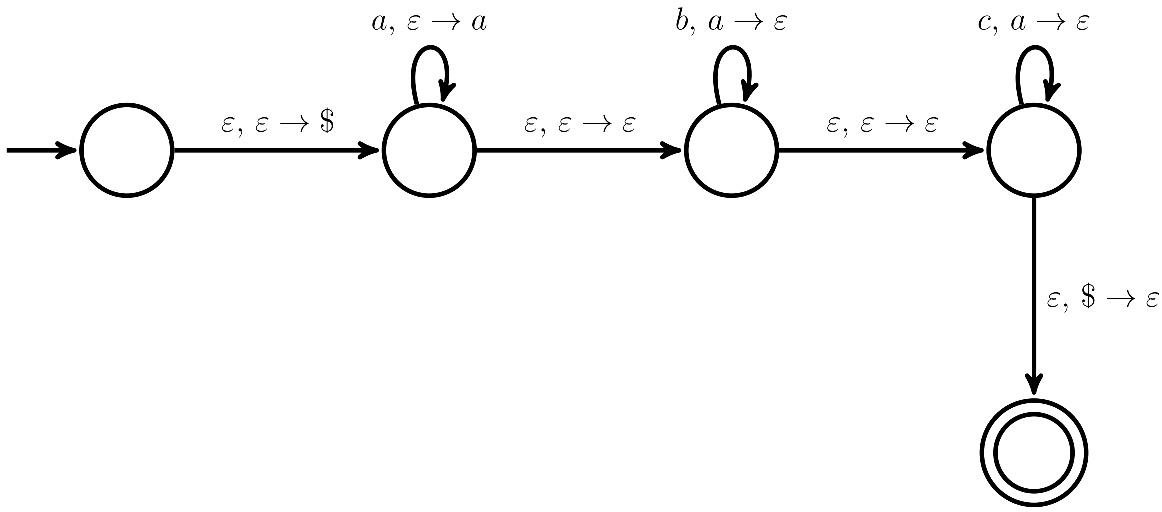
Name-Surname: 17.11.2015

ID Number:

CLASSWORK 4

Find a PDA that recognizes the language $\{a^{m+n}\, b^m\, c^n \mid m, n \geqslant 0\}$.

Answer:



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CENG 491 - Formal Languages and Automata

Name-Surname: 23.11.2015

ID Number:

CLASSWORK 5

Among the following languages over $\Sigma = \{0, 1, 2\}$, choose one that is not context-free. Then, prove that it is not context free using pumping lemma:

1. $\{0^i\, 1^j\, 2^k \mid i > j > k > 0\}$
2. $\{0^i\, 1^j \mid i = j^2\}$
3. $\{0^i\, 1^j\, 2^i \mid i \text{ is odd}, j \text{ is even}\}$
4. $\{0^i\, 1^j\, 2^k \mid k > i \text{ AND } k > j\}$

Answer:

Number 3 is context-free. Therefore it is the wrong answer. The other languages are non-context-free.

1. Suppose A is context free. Let the pumping length be p. Consider

   $$s = 0^{p+2}\, 1^{p+1}\, 2^p$$

   Let $s = uvxyz$. If v (or y) contains more than one type of symbol, $uvvxyyz \notin A$ because symbols are out of order. Therefore v and y must contain one type of symbol only.

   - v and y contain no 0's. Then, v and y contain 1 and/or 2. Pumping up, we have more 1's (or 2's) than 0's. Impossible.
   - v contains 0's, y contain 2's. Pumping up, we have more 2's than 1's. Impossible.
   - v contains 0's, y contain 1's. (BE CAREFUL!) We can pump up, but we cannot pump down, because number of 0's will be equal or less than number of 2's.
   - v contains 0's, $y = \varepsilon$. Similar to previous case.

2. Left as exercise.

3. Pumping length is 4. Consider any s with $|s| \geqslant 4$ in the language. Any such string can be pumped (up or down) as follows:

   - It contains two or more 1's:

$$\underbrace{0\cdots01\cdots1}_{u}\ \underbrace{1}_{v}\ \underbrace{1}_{y}\ \underbrace{1\cdots12\cdots2}_{z}$$

   - It does not contain any 1's:

$$\underbrace{0\cdots0}_{u}\ \underbrace{00}_{v}\ \underbrace{22}_{y}\ \underbrace{2\cdots2}_{z}$$

4. Similar to 1.

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CENG 491 - Formal Languages and Automata

Name-Surname: 30.11.2015

ID Number:

CLASSWORK 6

Let A be the language in $\Sigma = \{0, 1\}$ made of strings where number of zeros is more than twice the number of ones. Describe a Turing Machine recognizing A.
Is it a decider?

Answer:

1. Go to start. Search for 1.
   IF found, cross it. (Replace by $\times$)
   ELSE, Go to 4.
2. Repeat 2 times:
   Go to start. Search for 0.
      IF found, cross it.
      ELSE, REJECT.
3. Go to 1.
4. Go to start. Search for 0.
   IF found, ACCEPT.
   ELSE, REJECT.

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CENG 491 - Formal Languages and Automata

Name-Surname: 01.12.2015

ID Number:

CLASSWORK 6

Let B be the language in $\Sigma = \{0, 1\}$ defined as

$$B = \{w \mid w = 0^n 1^m, m = 5n - 3, n \geqslant 1\}$$

Describe a Turing Machine recognizing B.
Is it a decider?

Answer:

1. Sweep from left to right. IF symbols are out of order, REJECT.
2. Repeat 3 times:
   Go to end. Write 1 to tape.
3. Go to start. Search for 0.
   IF found, cross it. (Replace by $\times$)
   ELSE, Go to 6.
4. Repeat 5 times:
   Go to start. Search for 1.
      IF found, cross it.
      ELSE, REJECT.
5. Go to 3.
6. Go to start. Search for 1.
   IF found, REJECT.
   ELSE, ACCEPT.

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CENG 491 - Formal Languages and Automata

Name-Surname: 07.12.2015

ID Number:

CLASSWORK 7

Let A be the language in $\Sigma = \{a, b\}$ defined as

$$A = \{w \mid w = a^n\, b\, a^{2n-1}, n \geqslant 1\}$$

Describe a Turing Machine recognizing A.

Answer:

1. Sweep from left to right.
   IF there is no b, or more than one b's,
   REJECT.
2. Move head to end. Write a to tape.
3. Move head to start. Search for a until meeting b.
   IF found
   Cross it. (Replace by $\times$)
   ELSE
   Go to 6.
4. Repeat 2 times:
   Move head to start. Search for b.
   Search for a.
   IF found
   Cross it.
   ELSE
   REJECT.
5. Go to 3.
6. Move head to start. Search for b.
   Search for a.
   IF found
   REJECT.
   ELSE
   ACCEPT.

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CENG 491 - Formal Languages and Automata

Name-Surname: 08.12.2015

ID Number:

CLASSWORK 7

Let B be the language in $\Sigma = \{a, b, c\}$ defined as

$B = \{w \mid w = a^i b^j c^k, i < j \text{ AND } i < k\}$

Describe a Turing Machine recognizing B.

Answer:

1. Sweep from left to right. IF symbols out of order, REJECT.
2. Move head to start. Search for a.
   IF found
      Cross it. (Replace by $\times$)
   ELSE
      Go to 6.
3. Move head to start. Search for b.
   IF found
      Cross it.
   ELSE
      REJECT.
4. Move head to start. Search for c.
   IF found
      Cross it.
   ELSE
      REJECT.
5. Go to 2.
6. Move head to start.
   Search for b.
   IF found
      Search for c.
      IF found
         ACCEPT.
      ELSE
         REJECT.
   ELSE
      REJECT.

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Department of Computer Engineering

CENG 491 - Formal Languages and Automata

Name-Surname: 21.12.2015

ID Number:

CLASSWORK 8

Find a 1-1 correspondence (one-to-one, onto function) between these two sets:

$A = \{\text{positive odd integers}\}$

$B = \{\text{positive multiples of } 3\}$

Answer:

$$f : A \to B, \quad f(n) = \frac{3(n+1)}{2}$$

$$g : B \to A, \quad g(n) = \frac{2n}{3} - 1$$

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CENG 491 - Formal Languages and Automata

Name-Surname: 22.12.2015

ID Number:

CLASSWORK 8

Find a 1-1 correspondence (one-to-one, onto function) between these two sets:

$A = \{\text{All integers}\}$

$B = \{\text{Positive even integers}\}$

Answer:

$$f : A \to B, \quad f(n) = \begin{cases} 4n & \text{if} \quad n > 0 \\ -4n + 2 & \text{if} \quad n \leqslant 0 \end{cases}$$

$$g : B \to A, \quad g(n) = \begin{cases} n/4 & \text{if} \quad n/2 \text{ is even} \\ -(n-2)/4 & \text{if} \quad n/2 \text{ is odd} \end{cases}$$

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CENG 491 - Formal Languages and Automata

Name-Surname: 21.12.2015

ID Number:

CLASSWORK 9

What language does the following TM recognize? Describe.

1. Sweep from left to right.
   IF there is any a after b, REJECT.

2. Move head to start.
   Search for a.
   IF found
       Cross it. (Replace by $\times$)
       Search for a.
       IF found
           Cross it.
       ELSE
           REJECT.
   ELSE
       Go to 5.

3. Search for b.
   IF found
       Cross it.
   ELSE
       REJECT.

4. Go to 2.

5. Move head to start.
   Search for b.
   IF found
       REJECT.
   ELSE
       ACCEPT.

Answer:

$\{a^{2n}b^n \mid n \geqslant 0\}$

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CENG 491 - Formal Languages and Automata

Name-Surname: 22.12.2015

ID Number:

CLASSWORK 9

What language does the following TM recognize? Describe.

1. Sweep from left to right.
   IF there is any a after b, REJECT.

2. Move head to start.
   Search for a.
   IF found
   - Cross it. (Replace by $\times$)
   - Search for a.
   - IF found
     - Cross it.
   - ELSE
     - REJECT.

   ELSE
   - Go to 5.

3. Search for b.
   IF found
   - Cross it.

   ELSE
   - REJECT.

4. Go to 2.

5. Move head to start.
   Search for b.
   IF found
   - Search for b.
   - IF found
     - REJECT.
   - ELSE
     - ACCEPT.

   ELSE
   - REJECT.

Answer:

$\{a^{2n}b^{n+1} \mid n \geqslant 0\}$

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Department of Computer Engineering
2015 - 2016 Fall Semester

CENG 491 - Formal Languages and Automata

Final Examination

1) Consider the regular expression $(b \cup aa)^* \, (bb)^* \, (\varepsilon \cup aba)$.

Draw the diagram of an NFA recognizing a language equivalent to this.

2) **a)** Find a CFG over $\Sigma = \{0, 1\}$ that generates strings containing an odd number of symbols, starting and ending with 1.

b) Express the same grammar in Chomsky normal form.

3) Let A be the language in $\Sigma = \{a, b\}$ defined as

$$A = \{w \mid w = a^{n^3} \, b^n, n \geqslant 1\}$$

Describe a Turing Machine recognizing A.

4) Find a 1-1 correspondence (one-to-one, onto function) between these two infinite sets A and B. Denote domain and range of the function. (For example $f : A \to B$ or $g : B \to A$)

$$A = \{\ldots, -3, -1, 1, 3, 5, \ldots\}$$

$$B = \{8, 15, 22, 29, 36, \ldots\}$$

5) Show that the following problem is in NP:

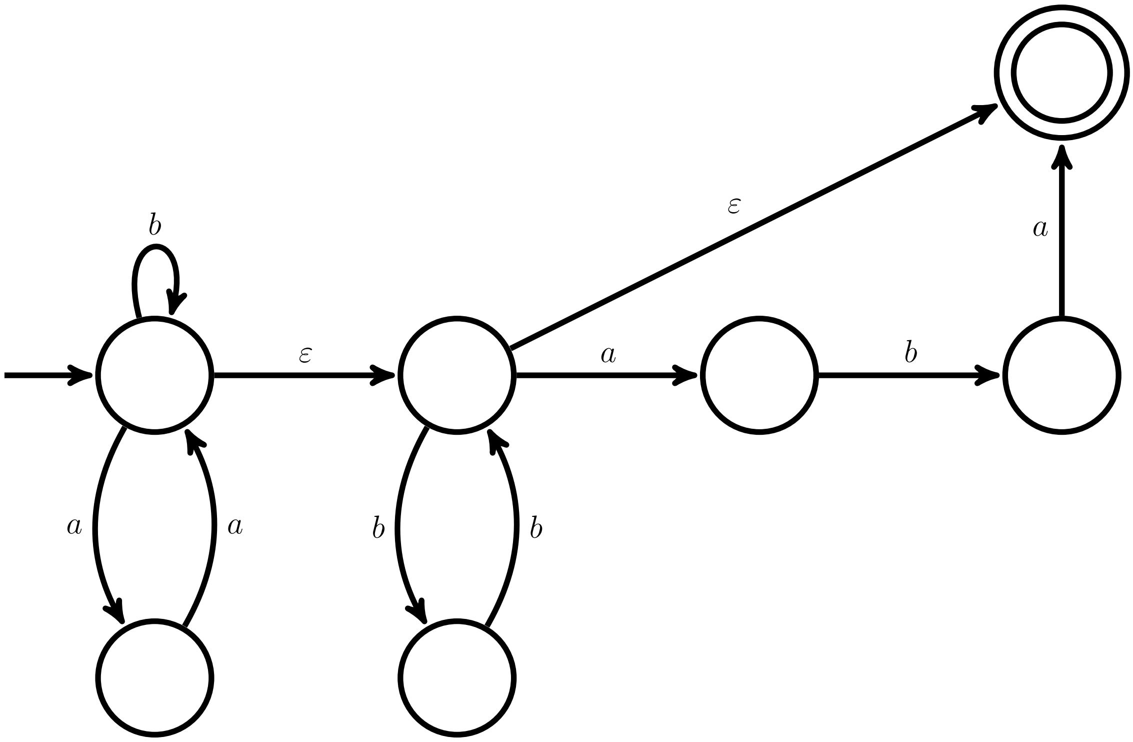
You have n objects. Each object has some weight, w_i, where $i = 1, 2, \ldots n$. You are given m containers. The max weight each container can carry is C. You will accept the containers if your objects fit in them and reject otherwise.

Bonus) Show that the following problem is in P:

There are n senior students in a class. I want to find 3 students A, B, C among them such that the couples A−B, A−C and B−C have been lab or project partners in some course. To make my job easier, each student gives me a list of all students in the class he/she ever worked with.

Answers

1)



2) a)

$$S \to 1A1 \mid 1$$

$$A \to BAB \mid B$$

$$B \to 0 \mid 1$$

b)

$$S \to TZ \mid 1$$

$$T \to ZA$$

$$Z \to 1$$

$$A \to BU \mid 0 \mid 1$$

$$B \to 0 \mid 1$$

$$U \to AB$$

3)

1. Sweep from left to right.
   IF there is no a, or no b, or if they are out of order
   REJECT.

2. Move head to start. Search for b.
   IF found,
   Mark it.
   Move head to end. Write c to tape.
   Go to 2.
   //Write n *c's to tape. (Assuming there were* n *b's in the beginning.*

3. Unmark all b's.

4. Move head to start. Search for b.
   IF found
   Mark it.
   Unmark all marked c's.
   Shuttle between c's and right end. Write one d for each c.
   IF c is not found, Go to 4.
   //Write n^2 d's to tape.

5. Unmark all b's.

6. Move head to start. Search for b.
   IF found
   Mark it.
   Unmark all marked d's.
   Shuttle between d's and right end. Write one e for each d.
   IF d is not found, Go to 6.
   //Write n^3 e's to tape.

7. Move head to start.
   Shuttle between a's and e's, mark each one for one.
   IF they are equal
   ACCEPT.
   ELSE
   REJECT.

3) (Alternative Solution by Oğuz Küçükcanbaz)

1. Sweep from left to right.
   IF there is no a, or no b, or if they are out of order
       REJECT.

2. For each b
       Write c to tape.
       Write d to tape.

3. For each b
       For each c
           For each d
               Move head to start. Search for a.
               IF found,
                   Mark it.
               ELSE
                   REJECT.

4. Move head to start. Search for a.
   IF found,
       REJECT.
   ELSE
       ACCEPT.

4)

$$f : A \to B, \quad f(n) = \begin{cases} 7n+1 & \text{if} \quad n > 0 \\ -7n+8 & \text{if} \quad n < 0 \end{cases}$$

$$g : B \to A, \quad g(n) = \begin{cases} \dfrac{n-1}{7} & \text{if} \quad n \text{ is even} \\ \dfrac{-n+8}{7} & \text{if} \quad n \text{ is odd} \end{cases}$$

5) Given a solution, we have n objects and m containers, together with the list of objects in each container.

To check the solution, we have to do $n - m$ additions and m comparisons.

This is $\Theta(n)$ operations.

We can check a given solution in polynomial time $\quad\Rightarrow\quad$ The problem is in NP.

Bonus) There are $\binom{n}{3} = \Theta(n^3)$ triples. We have to check their connections by going over their lists. This makes $\Theta(n)$ operations for checking each triple, so in total we need $\Theta(n^4)$ operations to find the solution. $\quad\Rightarrow\quad$ The problem is in P.
(Better algorithms are possible but the result is the same.)

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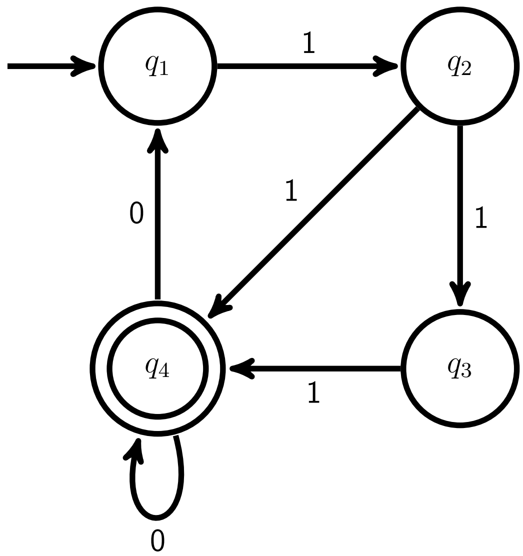
Department of Computer Engineering
2015 - 2016 Fall Semester

CENG 491 - Formal Languages and Automata

First Midterm Examination

1) Find a DFA that accepts all strings over $\Sigma = \{0, 1\}$ that contains one or two 0's in the first three symbols.

2) Find a DFA that recognizes the same language as the following NFA:



3) Consider the following languages over $\Sigma = \{a, b, c\}$:

- $A = \{a^n\, b^m\, c^n \mid m, n \geqslant 0\}$
- $B = \{a^n\, b^m \mid n > m \geqslant 0\}$
- $C = \{a^{3n+2}\, b^{m+1}\, c^k \mid m, n \geqslant 0, k = 5 \text{ or } k = 7\}$

a) One of them is regular. Find a regular expression for it.

b) One of them is not regular. Show that it is not regular using pumping lemma.

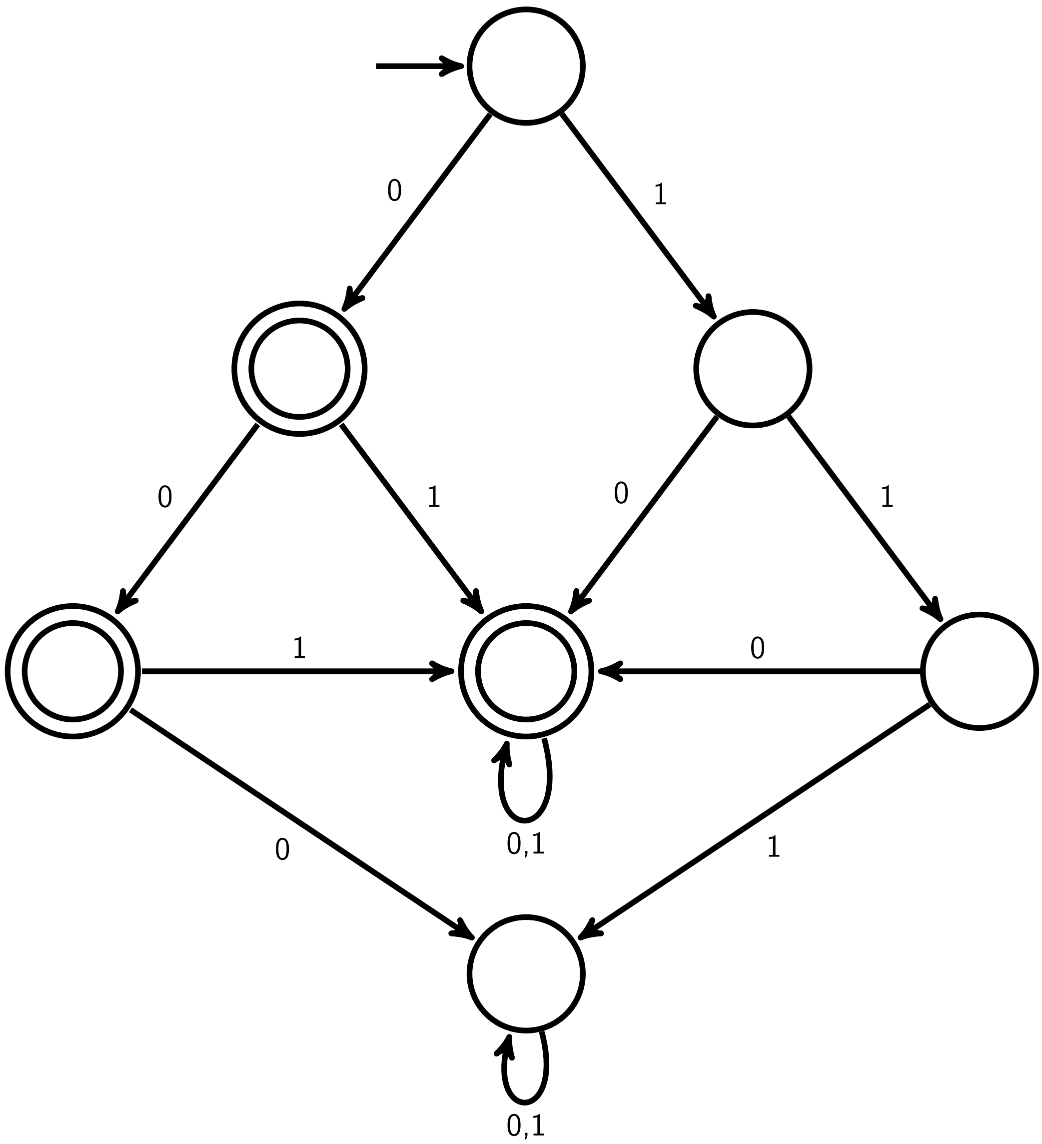
4) Find a CFG that generates the the language $\{a^{2n}\, b^{2m+1}\, c^n \mid m, n \geqslant 0\}$ over $\Sigma = \{a, b, c\}$.

5) Find a PDA that recognizes the language $\{a^n\, b^m \mid n > m\}$ over $\Sigma = \{a, b\}$.

Bonus) Find a CFG in Chomsky normal form that generates strings with a multiple of 8 symbols over $\Sigma = \{0, 1, 2\}$.

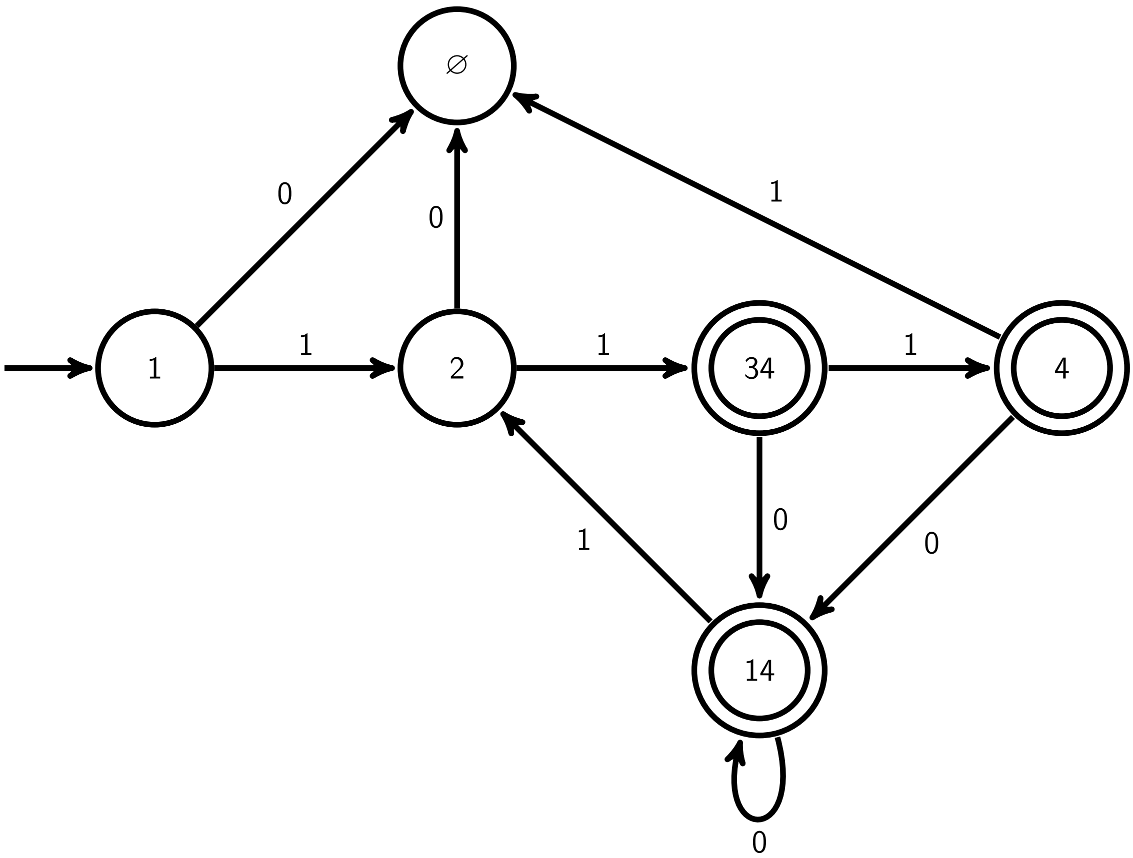
Answers

1)



Note that, once you get 10 or 01 in the beginning, you don't care about the third input.

2)



3) **a)** The language C is regular. Its regular expression is:

$$aa(aaa)^*\, bb^*\, (ccccc \cup ccccccc)$$

b) The languages A and B are not regular. Suppose they are regular, let the pumping length be p.

For A, choose the test string as $s = a^p\, b\, c^p$.
If $xyz = s$ and $|xy| \leqslant p$, then y consists of a's only.
In other words $y = a^k$ therefore $xyyz = a^{p+k}\, b\, c^p \notin A$ because there are more a's than c's.

For B, choose the test string as $s = a^{p+1}\, b^p$.
If $xyz = s$ and $|xy| \leqslant p$, then y consists of a's only.
In other words $y = a^k$ therefore (BE CAREFUL HERE!) $xyyz = a^{p+k+1}\, b^p \in B$ because there are more a's than b's. Similarly for $xyyyz$ and others.

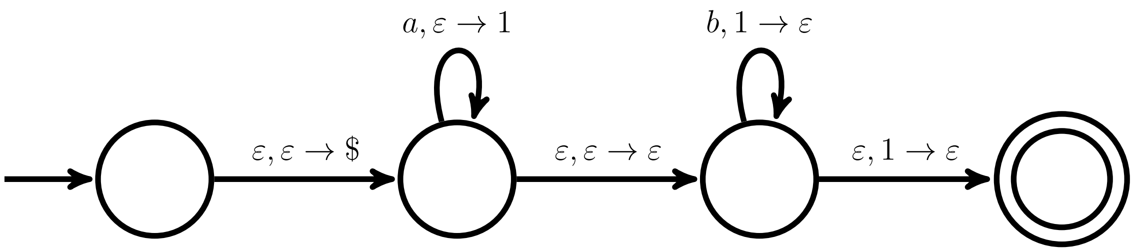
Here, we have to pump down. Use the pumping lemma for $i = 0$.
As usual, y must be a^k, but now, consider $xy^0z = xz = a^{p-k+1}\, b^p \notin B$ because there are less a's than b's.

(*It's a better idea to choose A for the proof!*)

4)

$$
\begin{array}{rcl}
S & \to & aaSc \;\big|\; T \\
T & \to & bbT \;\big|\; b
\end{array}
$$

5)



Bonus) A simple grammar that generates the given language is:

$$
\begin{array}{rcl}
S & \to & T \;\big|\; \varepsilon \\
T & \to & TT \;\big|\; CCCCCCCC \\
C & \to & 0 \;\big|\; 1 \;\big|\; 2
\end{array}
$$

Using the first rule, we can reach zero or one T. Using the second rule repeatedly, we can reach two or more T's. Then, replacing each T by 8 C's and then terminals, we reach our aim.

This grammar is NOT in Chomsky normal form, but we can easily transform it as follows:

$$
\begin{array}{rcl}
S & \to & TT \;\big|\; AA \;\big|\; \varepsilon \\
T & \to & TT \;\big|\; AA \\
A & \to & BB \\
B & \to & CC \\
C & \to & 0 \;\big|\; 1 \;\big|\; 2
\end{array}
$$

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2015 - 2016 Fall Semester

CEng 491 - Formal Languages and Automata

Second Midterm Examination

1) Show that the following language over $\Sigma = \{a, b, c\}$ is not context-free using pumping lemma:

$$A = \{a^{2n}\, b^{n+1}\, c^{n-1} \mid n \geqslant 1\}$$

2) Let B be the language over $\Sigma = \{a, b, c\}$ defined as

$$B = \{w \mid w = a^i\, b^j\, c^k, i \leqslant j \text{ OR } i \leqslant k\}$$

Describe a Turing Machine recognizing B.

3) (See Next Page)

4) There are two computer programs. Each one takes a finite page of text as input, and produces another page of text as output.

A Turing machine can give any input it wants to these two programs. It will ACCEPT if they return the same output for some input, REJECT otherwise.

Is this TM a decider?

5) Consider the alphabet $\Sigma = \{a, b, c, d, e\}$

a) Let A be the set of all words that can be generated with this alphabet. Is A countable? (Words are always of finite length)

b) Let B be the set of all infinite strings that can be generated with this alphabet. Is B countable?

Explain.

Bonus) Consider the infinite two dimensional grid, $G = \{(m, n) \mid m \text{ and } n \text{ are integers}\}$. Every point in G has 4 neighbours, North, South, East and West.

Starting at the origin $(0, 0)$, a string of commands N, S, E, W, generates a path in G. A path is closed if it starts at the origin and ends at the origin.

Let C be the collection of all strings over $\Sigma = \{N, S, E, W\}$ that generate a closed path.

a) Describe C.

b) Prove that C is not context-free.

3) What language does the following TM recognize? Describe.

1. Sweep from left to right.
   IF there are less than or more than two 1's,
       REJECT.
   //There must be exactly two 1's on the tape.

2. Move head to end. Write 000 to tape.

3. Move head to start. Search for 0 until meeting 1.
   IF found
       Cross it. (Replace by $\times$)
   ELSE
       Go to 7.

4. Move head to start. Search for 1.
   Repeat 2 times:
       Search for 0 until meeting 1.
           IF found
               Cross it.
           ELSE
               REJECT.

5. Move head to start. Search for 1. Search for 1.
   //In other words, search for the second 1.
   Repeat 3 times:
       Search for 0.
           IF found
               Cross it.
           ELSE
               REJECT.

6. Go to 3.

7. Move head to start.
   Search for 0.
   IF found
       REJECT.
   ELSE
       ACCEPT.

Answers

1) Suppose A is context free. Let p be the pumping length. Choose s as $s = a^{2p}b^{p+1}c^{p-1}$. How can we choose v and y such that $s = uvxyz$?

1) They contain more than one symbol.

In this case, the pumped string uv^2xy^2z will have symbols out of order. For example, it will contain a's after b's. Therefore $uv^2xy^2z \notin A$.

2) They contain a single symbol.

In that case, we can pump at most two of the symbols $\{a, b, c\}$. The remaining one will have the same power in the pumped string. For example, if we choose $v = aa$ and $y = b$ we obtain $uv^2xy^2z = a^{2n+2}b^{n+2}c^{n-1} \notin A$

Therefore we cannot pump this string. By pumping lemma, A is not context free.

2)

1. Sweep from left to right. IF symbols out of order, REJECT.
2. Move head to start. Search for a.
   IF found
   Cross it. (Replace by $\times$)
   ELSE
   ACCEPT.
3. Move head to start. Search for b.
   IF found
   Cross it.
   ELSE
   Go to 5.
4. Go to 2.
5. Restore all a's.
6. Move head to start. Search for a.
   IF found
   Cross it. (Replace by $\times$)
   ELSE
   ACCEPT.
7. Move head to start. Search for c.
   IF found
   Cross it.
   ELSE
   REJECT.
8. Go to 6.

3) $\{0^n 1 0^{2n} 1 0^{3n-3}, n \geqslant 1\}$

4) Yes, it is a decider. The TM can check all possible inputs, because there are finitely many inputs. Supposing there are k different characters on the keyboard and n characters per page, it needs to check n^k different inputs.

5) **a)** Yes. We can count (list) them as follows:

$$\{a, b, c, d, e, aa, ab, ac, \ldots, ee, aaa, aab, \ldots\}$$

b) No, infinite strings are not countable. They are similar to real numbers on $[0, 1]$. Suppose there is a list. We can easily produce a string not on the list using Cantors diagonal proof.

Bonus) **a)** These strings contain

- Equal number of N and S symbols.
- Equal number of E and W symbols.

(Order does not matter)

b) Assuming pumping length is p, start with $s = N^p E^p S^p W^p$ and use pumping lemma.