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Computation Theory | Homework 1

Şafak B.

(1) Why do we need computation? Why do we need programming language for computation? Why do we need automatons that accept programming language?

Mathematicians such as Kurt Gödel, Alan Turing discovered that some specific problems cannot be solved by computers, one example of this is the problem of determining whether a mathematical statement is true or false. As an example, computer cannot determine that for integers $n>2$, the equation $x^n+y^n=z^n$ cannot be solved with positive integers x, y, z. But some are can be solved by 'computational model'. Computation allows us to solve some computational problems. To solve that mathematical or algorithmic problems, we need power of computation, and to make a computation on computational model, we need some rules to show that computation. This is the programming language. But how this computational model or the computer recognizes this language that is called programming language? We design a model that is called automaton or machine. For example when we say in the language "if", the automaton recognizes that it is a conditional statement. Then the rest will come.

(2) For any $n \in \mathbb{N}$, prove that the following equality is valid. $1^6+2^6+\dots+n^6=\frac{n}{42}\cdot(n+1)(2n+1)(3n^4+6n^3-3n+1)$

Proving by induction.

We take an infinite set, $N=\{1,2,3,\dots\} \in \mathbb{N}$ and say that the property is called P.

For each $i \geq 1$, if $P(i)$ is true, then so is $P(i+1)$.

for $n=1$, $P(1)=1^6=1$

for $n=2$, $P(2)=1^6+2^6=65$

for $n=K$, $P(K)=1^6+2^6+\dots+K^6=\frac{K}{42}(K+1)(2K+1)(3K^4+6K^3-3K+1)$

for $n=K+1$, $P(K+1)=\underbrace{1^6+2^6+\dots+K^6}_{\text{equals } P(K)}+(K+1)^6=\frac{K+1}{42}(K+2)\cdot(2K+3)(3(K+1)^4+6(K+1)^3-3K-2)$

$$(K+1)^6=\frac{K+1}{42}\cdot(K+2)\cdot(2K+3)(3(K+1)^4+6(K+1)^3-3K-2)-\frac{K}{42}(K+1)(2K+1)\cdot(3K^4+6K^3-3K+1)$$

$$\sum_{i=0}^{6}\binom{6}{i}K^i 1^{n-i}=K^6+6K^5+15K^4+20K^3+15K^2+6K+1=\frac{1}{42}\Big[(K+1)(2K+3)(3(K+1)^4+6(K+1)^3-3K-2)-(K+1)(2K+1)(3K^4+6K^3-3K+1)\Big]$$

is equal

3) Let α and β be two positive integer numbers. If $\alpha^2-\beta^2$ is not odd, then prove that $\alpha+\beta \geq 2$

Let say $\alpha+\beta < 2$, $2k \in \mathbb{N}$, then

$$\alpha^2-\beta^2 = \frac{(\alpha+\beta)(\alpha-\beta)}{2} = k \to \alpha+\beta \nmid 2 \text{ and } \alpha-\beta \nmid 2 \leftrightarrow k \in \mathbb{N}$$

$\therefore$

$\alpha+\beta \geq 2$

Q.E.D

4) Let a,b,c,d be integers. If $a>c$ and $b>c$, then prove that $\max(a,b)-c$ is always positive.

Let say that $\max(a,b)-c$ is always positive

then , $\max(a,b)-c<0$ $\longrightarrow$ if $a>b$, then $\max(a,b)=a \to a-c<0$, $a<c$

$\max(a,b)<c$ $\qquad$ if $b>a$, then $\max(a,b)=b \to b-c<0$, $b<c$

but we said $b>c$ and $a>c$.

Q.E.D

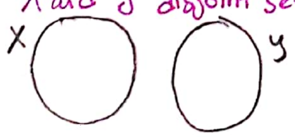
5) X and Y closed sets. $X\times Y$ is cartesian product of X, Y. Write $|X\times Y|$ in terms of $|X|$ and $|Y|$.

$X\times Y = \{(x,y) \mid x\in X,\ y\in Y\}$

Let say $|X|=m$ and $|Y|=n$; where $m,n\in\mathbb{N}$

Then $|X\times Y| = |X|\cdot|Y| = m\cdot n$

6) X and Y disjoint sets. Their joint $S=X\cup Y$. Sum of element in S is $\Sigma(S)$, product is $\Pi(S)$. Write then in terms of $\Sigma(X)$, $\Sigma'(Y)$, $\Pi(X)$, $\Pi(Y)$



$\to$ disjoint two set , $S=X\cup Y$

Let us say $\Sigma'(X)=m_x$, $\Sigma(Y)=m_y$; $m_x, m_y, P_x, P_y \in \mathbb{R}$

$\Pi(X)=P_x$, $\Pi(Y)=P_y$

Due to $X\cap Y=\emptyset$

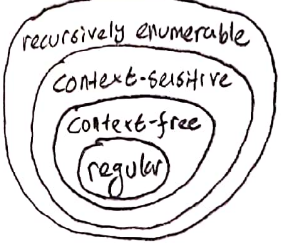
$$\Sigma(S) = \Sigma'(X)+\Sigma'(Y) = m_x+m_y$$

$$\Pi(S) = \Pi(X)\,\Pi(Y) = P_x\cdot P_y$$

7 What is the relation between programming language and the power of machine that accepts that programming language?

The theory of computation classifies languages by the computations they can express.

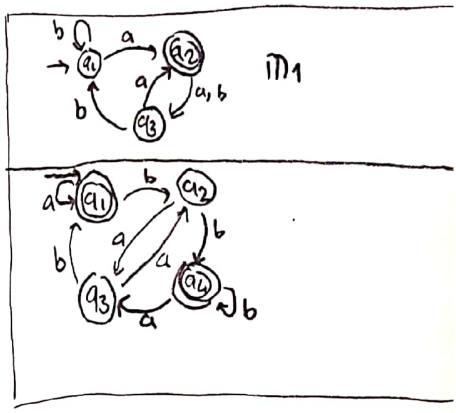
Chomsky Hierarchy



if the power of a machine increases, it can recognizes much more complex programming languages.
if the power of a machine decreases, it can recognizes more simple programming languages.

8 what is start state?

a) Start state of M_1 is q_1, Start state of M_2 is q_1



what is the set of accept states?

b) Set of accept state for M_1 is $\{q_2\}$

Set of accept state for M_2 is $\{q_1, q_4\}$

what sequence of states does the machine go through on input aabb?

c) for M_1, the sequence: $q_1 \rightarrow q_2 \rightarrow q_3 \rightarrow q_1 \rightarrow q_1$

for M_2, the sequence: $q_1 \rightarrow q_1 \rightarrow q_2 \rightarrow q_4$

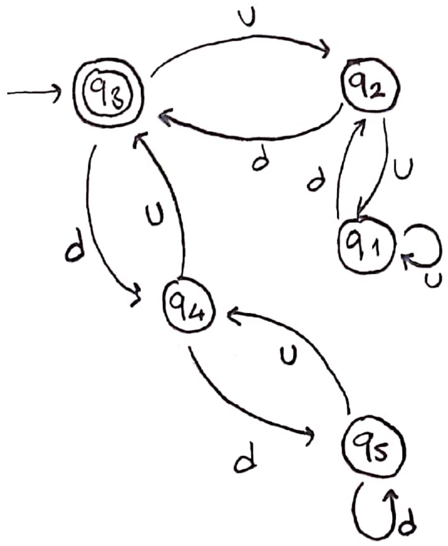
Does the machine accept string aabb?

d) Machine M_2 accepts but machine M_1 does not accepts. (aabb)

Does the machine accept string ϵ?

e) Machine M_2 accepts but machine M_1 does not accepts. (ϵ)

9



The formal 5-tuple definition of DFA M is

$(\{q_1, q_2, q_3, q_4, q_5\}, \{u, d\}, \delta, q_3, \{q_3\})$

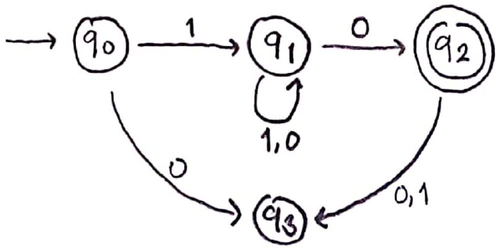
Where δ is:

	u	d
q_1	q_1	q_2
q_2	q_1	q_3
q_3	q_2	q_4
q_4	q_3	q_5
q_5	q_4	q_5

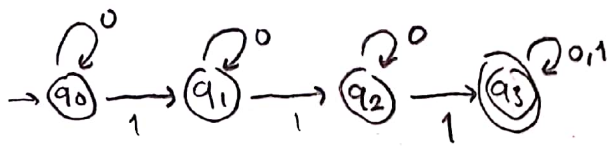
Give the state diagram of M.

10 Give state diagrams of DFAs recognizing the following languages. Alphabet $\Sigma = \{0,1\}$.

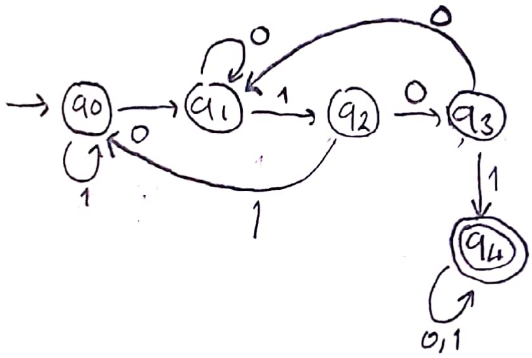
- a) $\{w \mid w = 1v0,\ v \in \Sigma^*\}$



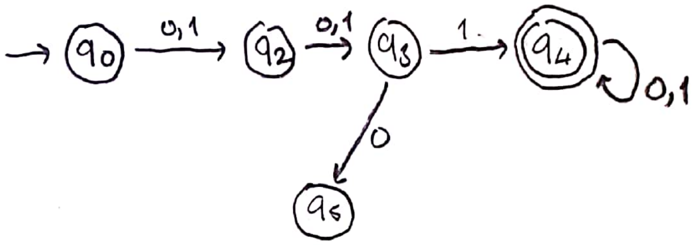
- b) $\{w \mid w \text{ contains at least three 1s}\}$



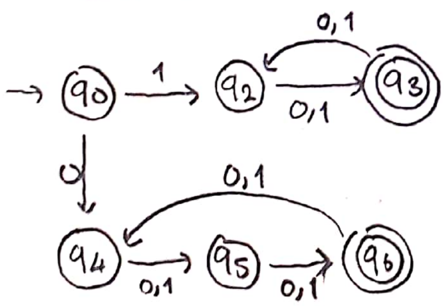
- c) $\{w \mid w = v0101z;\ v,z \in \Sigma^*\}$



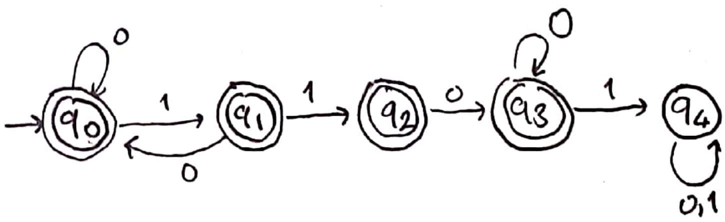
- d) $\{w \mid w = xy1v;\ x,y \in \Sigma;\ v \in \Sigma^*;\ |w| \geq 3\}$



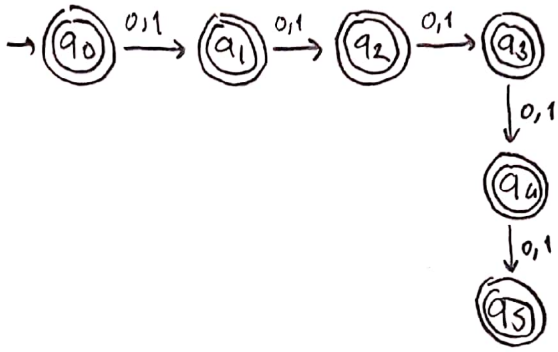
- e) $\{w \mid w$ starts with 0 and $|w| \in 2N+1$
or
w starts with 1 and $|w| \in 2N\}$



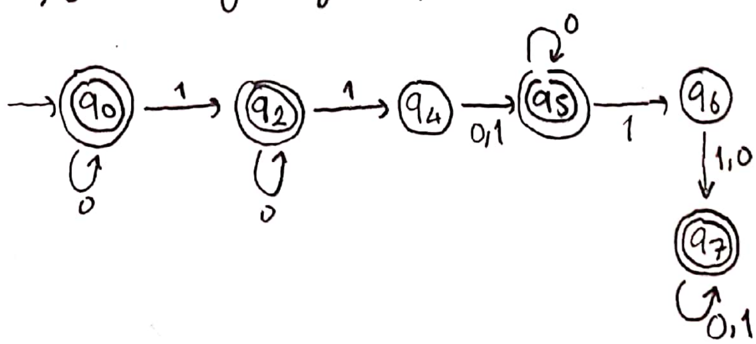
- f) $\{w \mid w$ does not contain the substring $1101\}$



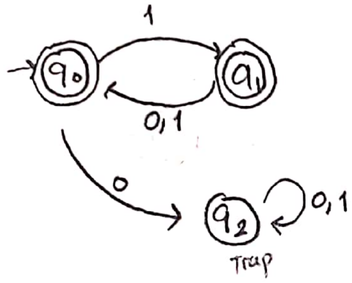
- g) $\{w \mid w$ the length of w is at most $5\}$



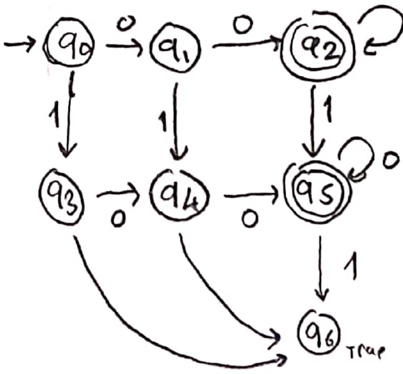
- h) $\{w \mid w$ is any string except 11 and $1111\}$



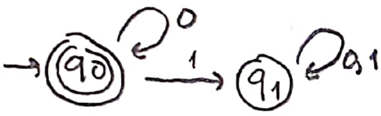
i) $\{w \mid$ every odd position of w is a 1$\}$



j) $\{w \mid w$ contains at least two 0s and at most one 1$\}$

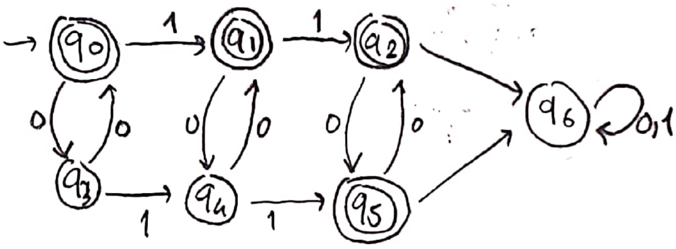


k) $\{\varepsilon, 0\}$

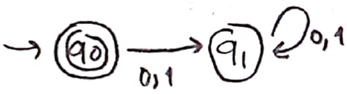


l) $\{w \mid w$ contains an even number of 0s, or contains exactly two 1s$\}$

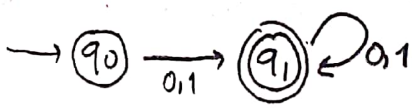
εV



m) The empty set



n) All strings except the empty string

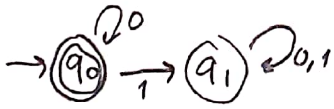


11

$L(M_1) = \{w \mid w = \varepsilon \text{ or } w = 0;\ \Sigma \neq \{0,1\}\}$

, $L(M_1) = C$

M:



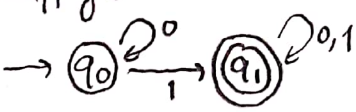
$q_0 = q_0$

$F = \{q_0\}$

non final states $= \{q_1\}$

Swapping:

M_2:



$L(M_2) = \{w \mid w = 1V, V \in \Sigma^*\}$, $L(M_2) = D$

$L(M_2)$ can be everything in Σ^* but ε and 1

$\therefore$ $L(M_2) = \overline{L(M_1)}$

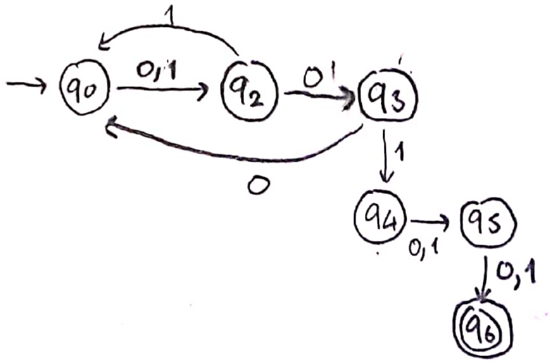
$D = \overline{C}$

Show by giving an example that if M is DFA that recognizes Language C, swapping the final and non-final states in M yields a new DFA that recognizes $\overline{C}$.

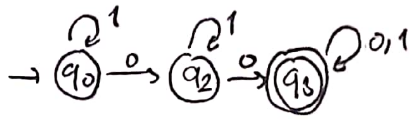
12) Design automata (DFA) to accept the following languages.

- a) $A = \{w \in \{0,1\}^* : w \text{ has a } 1 \text{ in the third position from the right}\}$

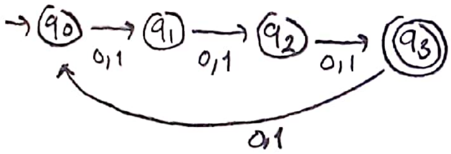
$w = V1xy \,;\, V \in \Sigma^* ; x, y \in \{0,1\}$



- b) $B = \{w \in \{0,1\}^* : w \text{ contains at least two } 0\text{s}\}$

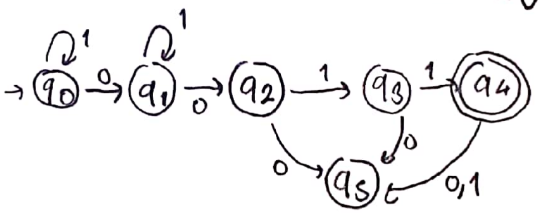


- c) $C = \{w \in \{0,1\}^* : 3 \mid |w|\}$



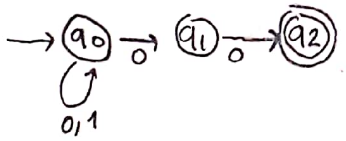
- d) $D = \{w \in \{0,1\}^* : w \text{ contains exactly two } 0\text{s and at least two } 1\text{s}\}$

$\{1\}^+ \cup$

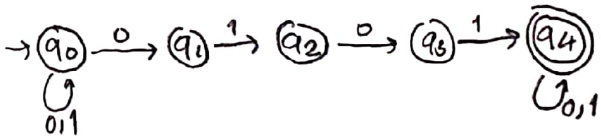


13) Give state diagrams of NFAs with the specified number of states. Alphabet $\Sigma = \{0,1\}$

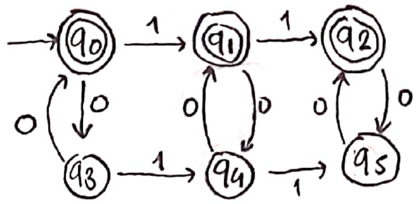
- a) The language $\{w \mid w = v00, v \in \Sigma^*\}$ (3 states)



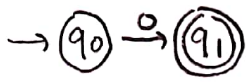
- b) $\{w \mid w = x0101y \,;\, x, y \in \Sigma^*\}$ (5 states)



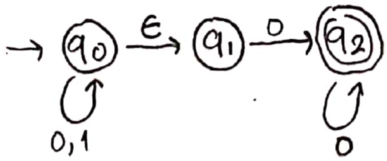
- c) $\{w \mid w$ contains even number 0s, or contains exactly two 1s$\}$ (6 states)



- d) $\{0\}$ (2 states)

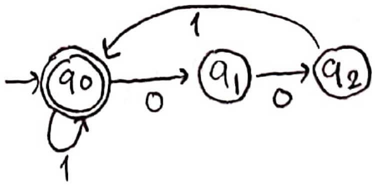


- e) $0^*1^*0^+$ (3 states)

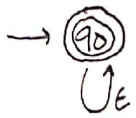


- f) $1^*(001^+)^*$ (3 states)

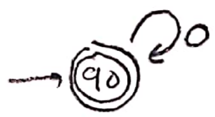
0011



- g) {ε} (1 state)

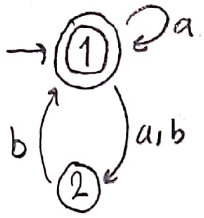


- h) 0^* (1 state)



14 Use the construction given in theorem 1.39 in the book to convert the following two NFA to equivalent DFA.

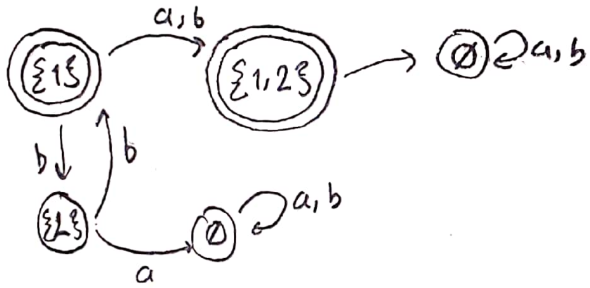
- a)



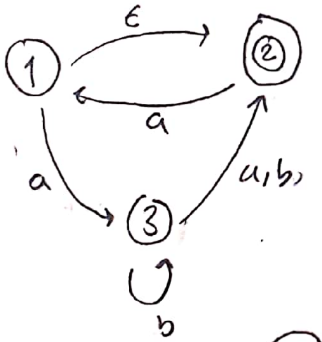
NFA initial step → q
DFA initial step → {1}

$\delta^*(1,a) = \{1,2\}$ $\delta^*(2,b) = \{1\}$
$\delta^*(1,b) = \{2\}$ $\delta^*(2,a) = \emptyset$

$\delta^*(1,a) \cup \delta^*(2,a) = \{1,2\}$
$\delta^*(1,b) \cup \delta^*(2,b) = \{1,2\}$

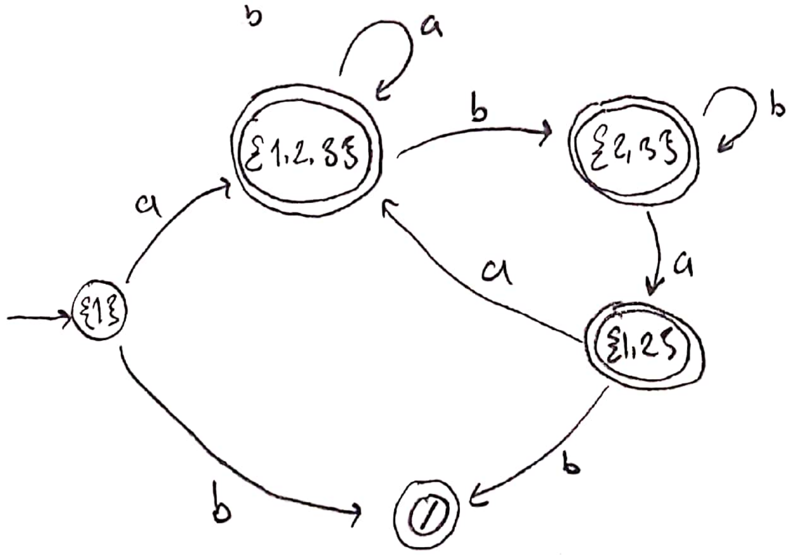


- b)



$\delta^*(1,a) = \{1,2,3\}$ $\delta^*(2,a) = \{1,2\}$ $\delta^*(3,a) = \{2\}$
$\delta^*(1,b) = \emptyset$ $\delta^*(2,b) = \emptyset$ $\delta^*(3,b) = \{2,3\}$

not necessary



$\delta^*(1,a) \cup \delta^*(2,a) \cup \delta^*(3,a) = \{1,2,3\}$
$\delta^*(1,b) \cup \delta^*(2,b) \cup \delta^*(3,b) = \{2,3\}$
$\delta^*(2,a) \cup \delta^*(3,a) = \{1,2\}$
$\delta^*(2,b) \cup \delta^*(3,b) = \{2,3\}$
$\delta^*(1,a) \cup \delta^*(2,a) = \{1,2,3\}$
$\delta^*(1,b) \cup \delta^*(2,b) = \emptyset$

15 is the machine NFA or DFA? why?

a) It is a NFA due to automaton can transition to, and be in, multiple states at once for some given input.

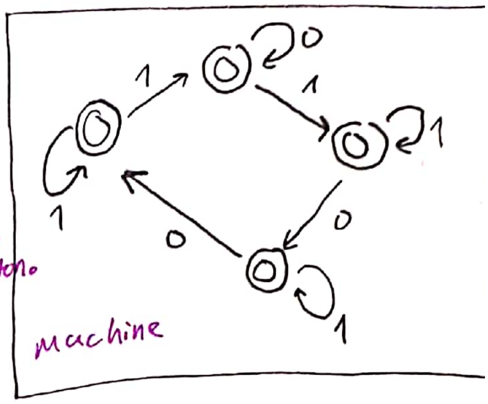
Give its regular expression.

b) $1^*10^*11^*01^*0$ $\{1100, 11100...\}$

$= 1^*0^*1^+01^*0$

write the language which is a set consisting of strings that are recognized by this automaton.

c) $L(M) = \{w \mid w = 1^m 0^n 1^k 0 1^l 0 ;\ m,n,l \geq 0;\ k>0;\ m,n,l,k \in \mathbb{N}\}$



16 Give regular expressions describing the following languages:

- a) $A = \{w \in \{0,1\}^* : w$ contains at least three 1s$\}$

  $w = v1x1y1z$; $v,x,y,z \in \Sigma^*$

  $\{0,1\}^* 1 \{0,1\}^* 1 \{0,1\}^* 1 \{0,1\}^*$

- b) $B = \{w \in \{0,1\}^* :$ w contains at least two 1s and at most one 0$\}$.

  $((\Sigma^* 1 \Sigma^* 1 \Sigma^*) \cap (1^* 0 1^*)) \cup ((\Sigma^* 1 \Sigma^* 1 \Sigma^*) \cap (1^*))$

- c) $C = \{w \in \{0,1\}^* :$ w contains even number of 0s and exactly two 1s$\}$

  Even number of 0s $\rightarrow (1^*01^*01^*)^*$

  exactly two 1s $\rightarrow 0^*10^*10^*$

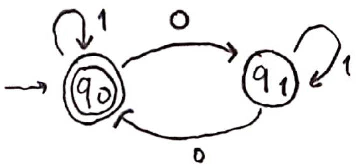
  $C \rightarrow (1^*01^*01^*)^* \cap (0^*10^*10^*)$

- d) $D = \{w \in \{0,1\}^* :$ w contains an even number of 0s and each 0 is followed by at least one$\}$

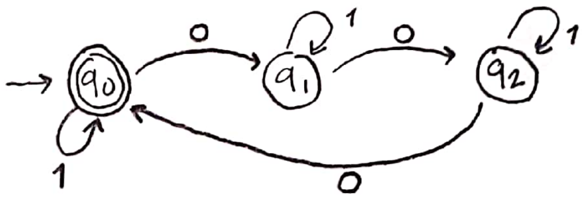
  $(1^*01^*01^*)^* \cap (1^*01^+1^*)^*$

(17) Design a DFA or NFA for the following languages. $n_0(w)$ denotes the number of zeros in the string w.

- a) $L_1 = \{w \in \{0,1\}^* : n_0(w) \% 2 = 0\}$



- b) $L_2 = \{w \in \{0,1\}^* : n_0(w) \% 3 = 0\}$



- c) $L_3 = \{w \in \{0,1\}^* : n_0(w) \% 6 = 0\}$