**1.** Give regular expressions generating the following languages:

```
a) {w: w contains exactly two 0s}

1*01*01*
```

**b)** {w : w contains at least two 0s}

```
\sum^* 0 \sum^* 0 \sum^*, here note that \sum =(0 \cup 1)
```

c)  $\{w \mid w \text{ begins with a 1 and ends with a 0}\}$ 

```
1\sum^*0
```

**d**) {w | w contains at least three 1s }

$$\sum^* 1 \sum^* 1 \sum^* 1 \sum^*$$

e) {w | the length of w is at most 5}

```
(\sum U \xi)(\sum U \xi)(\sum U \xi)(\sum U \xi)(\sum U \xi)
```

f) {w | w contains an even number of 0s, or contains exactly two 1s}

```
Even number of 0s: (1*01*01*)*
```

Contains Exactly two 1s: 0\*10\*10\*

Even number of 0s, or contains exactly two 1s: (1\*01\*01\*)\* U 0\*10\*10\*

2.

a) Convert the regular expression (((00)\*(11)) U 01)\* to an NFA.

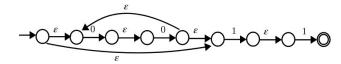
$$00 \longrightarrow 0 \longrightarrow 0 \longrightarrow 0$$

11 
$$\rightarrow \bigcirc^1 \bigcirc^{\epsilon} \bigcirc^1 \bigcirc^1$$

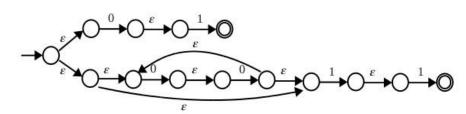
$$01$$
  $\longrightarrow \bigcirc \bigcirc \bigcirc \bigcirc \stackrel{\varepsilon}{\longrightarrow} \bigcirc \bigcirc \stackrel{1}{\longrightarrow} \bigcirc \bigcirc$ 

$$(00)^* \qquad \bullet \bigcirc \stackrel{\varepsilon}{\longrightarrow} \bigcirc \stackrel{\varepsilon}{\longrightarrow} \bigcirc 0$$

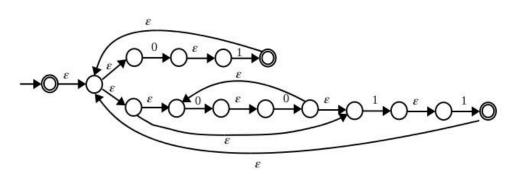
((00)\*(11))



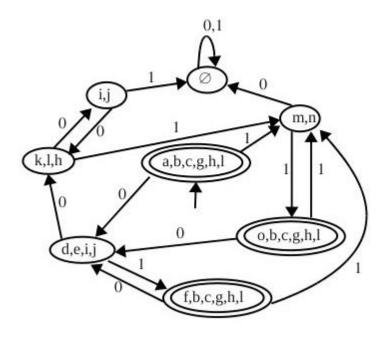
 $((00)*(11)) \cup 01$ 



 $(((00)*(11)) \cup 01)*$ 



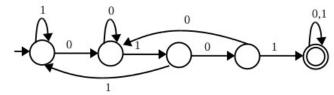
**b)** Convert the NFA (in part a)) to an equivalent DFA. Give only the portion of DFA that is reachable from the start state.



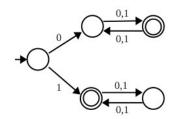
**3.** 

Give state diagrams of DFAs recognizing the following languages. In all parts, the alphabet is  $\{0,1\}$ .

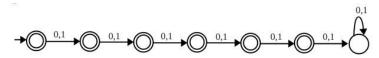
a)  $\{w \mid w \text{ contains the substring } 0101.$ 



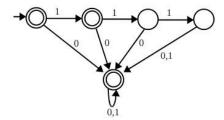
**b)**  $\{w \mid w \text{ starts with } 0 \text{ and has odd length, or starts with } 1 \text{ and has even length} \}$ .



c)  $\{w \mid \text{the length of } w \text{ is at most 5}\}.$ 

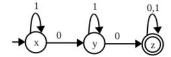


**d)**  $\{w \mid w \text{ is any string except } 11 \text{ and } 111\}.$ 

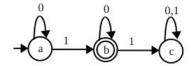


e)  $\{w \mid w \text{ contains at least two 0s and at most one 1}\}.$ 

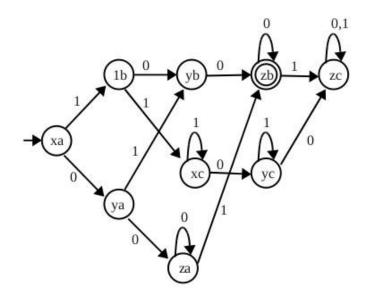
First draw the DFAs for strings consisting of at least two 0s.:



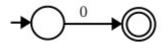
Strings consist of exactly one 1:



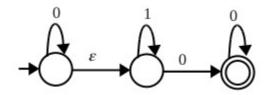
Now draw the final DFA which simulates these two DFAs and accepts only when both of these DFAs are in an accept state:



- **4.** Give state diagrams of NFAs with the specified number of states recognizing each of the following languages. In all parts, the alphabet is  $\{0,1\}$ .
- a) The language  $\{0\}$  with two states.



**b)** The language  $0*1*0^+$  with three states.



c) The language  $1*(001^+)*$  with three states

