Homework 1 (To be delivered during the 1st midterm exam)

BLM 2502: Theory of Computations — Spring 2020

Print family (or last) name: ——————————————————————————

Print given (or first) name: ——————————————————————————

Print given student number: ——————————————————————————

I see that this homework has 17 questions in total 16 pages.

I agree that I have to submit my homework solution before the deadline (i.e., the time of the 1st midterm exam) otherwise my homework solution will not be graded. I accept that ***I will add the signed version of this instruction page as a first page into my homework solution***; otherwise my homework solution will not be graded. I know that ***I have to give my solutions in the empty-white spaces just below the questions***; otherwise my homework solution will not be graded. I will take care of the readability of my solutions, from which I may lose 10 points. For any proofs, I am sure to provide a step-by-step argument, with justifications for every step. I understand that, during solving this homework, it is prohibited to exchange information about solutions with any other person in any way, including by talking or exchanging solutions / papers.

I know that the course book is "Introduction to the theory of computation, 2nd Ed., Massachusetts Institute of Technology, by Micheal Sipser."

I have read, understand and accept all of the instructions above. On my honor, I pledge that I have not violated the provisions of the Academic Integrity Code of Yıldız Technical University.

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Signature and Date

1	2	3	4	5	6	7	8	9	10
10 pts	**20 pts**	**15 pts**	**15 pts**	**15 pts**	**15 pts**	**10 pts**	**20 pts**	**20 pts**	**20 pts**

11	12	13	14	15	16	17
20 pts	**20 pts**	**20 pts**	**20 pts**	**20 pts**	**20 pts**	**20 pts**

Total
300 pts

1) **[10 Points]** Why do we need computation? Why do we need programming language for computation? Why do we need automats / machines that recognize/accept programming language?

2) **[20 Points]** For any $n \in \mathbb{N}$, prove that the following equality is valid.

$$1^6 + 2^6 + 3^6 + \cdots + n^6 = \frac{n}{42}(n+1)(2n+1)(3n^4 + 6n^3 - 3n + 1)$$

3) **[15 Points]** Let α and β be two positive integer numbers. If $\alpha^2 - \beta^2$ is not odd, then prove that $\alpha + \beta \geq 2$.

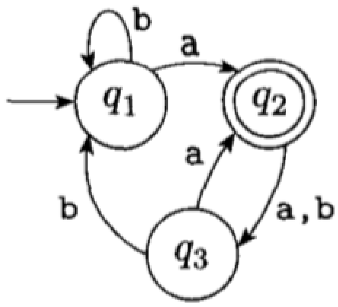
4) **[15 Points]** Let a, b, c, d be integers. If $a > c$ and $b > c$, then prove that $\max(a, b) - c$ is always positive.

5) **[15 points]** Given two sets X and Y. The Cartesian product of X and Y, written as $X \times Y$, is defined as the set of pairs (x, y) where $x \in X$ and $y \in Y$. Then, find a mathematical closed-form expression to write $|X \times Y|$ in terms of $|X|$ and $|Y|$.

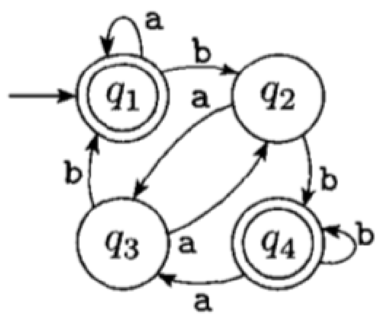
6) **[15 points]** Let us given two disjoint sets X and Y, and then their joint set S is $S = X \cup Y$. Sum of the elements in a set S is denoted by $\sum(S)$ while their product is by $\prod(S)$. Accordingly, what are $\sum(S)$ and $\prod(S)$ in terms of $\sum(X)$, $\sum(Y)$, $\prod(X)$ and $\prod(Y)$. Conclude from this what $\sum(\emptyset)$ and $\prod(\emptyset)$ should be ($\emptyset$ is the empty set).

7) **[10 points]** What is the relation between programming language and the power of a machine that recognizes / accepts that programming language? Give an example in your explanation.

8) **[20 Points]** The following are the state diagrams of two DFAs, M_1 and M_2 . Answer the follow- ing questions about each of these machines.



M_1



M_2

a) What is the start state?

b) What is the set of accept states?

c) What sequence of states does the machine go through on input $aabb$?

d) Does the machine accept the string $aabb$?

e) Does the machine accept the string ε?

9) **[20 Points]** The formal 5-tupple description of a DFA M is

$$(\{q_1, q_2, q_3, q_4, q_5\}, \{u, d\}, \delta, q_3, \{q_3\}),$$

where δ is given by the following table. Give the state diagram of this machine.

	u	d
q_1	q_1	q_2
q_2	q_1	q_3
q_3	q_2	q_4
q_4	q_3	q_5
q_5	q_4	q_5

10) **[20 Points]** Give state diagrams of DFAs recognizing the following languages. In all parts the alphabet is {0,1}.

a) {w | w begins with a 1 and ends with a 0}

b) {w | w contains at least three 1s}

c) {w | w contains the substring 0101, i.e., w = x0101y for some x and y}

d) {w |w has length at least 3 and its third symbol is a 0}

e) {w | w starts with 0 and has odd length, or starts with 1 and has even length}

f) {w | w doesn't contain the substring 1101}

g) {w | w the length of w is at most 5}

h) {w | w is any string except 11 and 1111}

i) {w | every odd position of w is a 1}

j) {w | w contains at least two 0s and at most one 1}

k) {ε, 0}

l) {w | w contains an even number of 0s, or contains exactly two ls}

m) The empty set

n) All strings except the empty string

11) **[20 Points]** Show by giving an example that if M is a DFA that recognizes language C, swapping the final and non-final states in M yields a new DFA that recognizes $\bar{C}$.

12) **[20 Points]** Design automata (DFA) to accept the following languages:

   a) A = {w ∈{0, 1}* : w has a 1 in the third position from the right}.

   b) B = {w ∈{0, 1}* : w contains at least two 0s}

c) C = {w $\in$ {0, 1}* : the length of w is divisible by three}

d) D = {w $\in$ {0, 1}* : w contains exactly two 0s and at least two 1s}.

13) **[20 Points]** Give state diagrams of NFAs with the specified number of states recognizing each of the following languages. In all parts the alphabet is {0,1}.

a) The language {w | w ends with 00} with three states

b) The language {w | w contains the substring 0101, i.e., w = x0101y for some x and y} with five states

c) The language {w | w contains an even number of 0s, or contains exactly two ls} with six states

d) The language {0} with two states

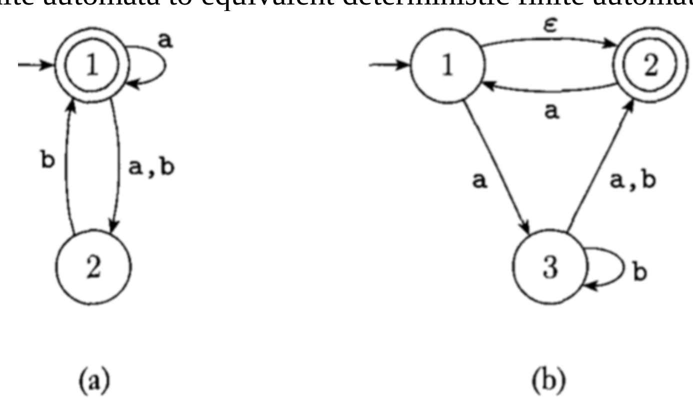
e) The language $0^*1^*0^+$ with three states

f) The language $1^*(001^+)^*$ with three states

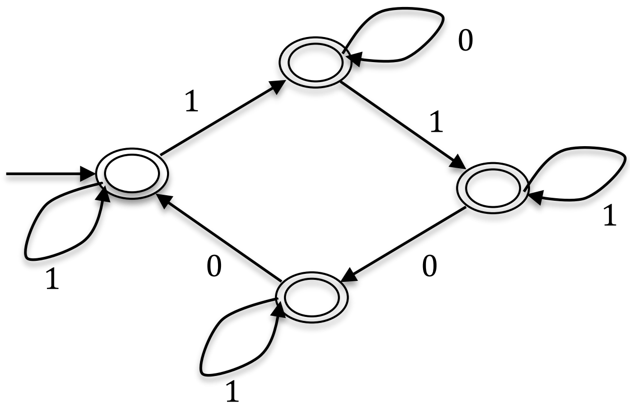
g) The language $\{\varepsilon\}$ with one state

h) The language 0^* with one state

14) **[20 Points]** Use the construction given in Theorem 1.39 in the book to convert the following two non-deterministic finite automata to equivalent deterministic finite automata.



15) **[20 points]** For the alphabet $\Sigma_1 = \{0,1\}$, answer the following questions for the automata machine shown below



a) Is the machine DFA or NFA? Why?

b) Give its regular expression.

c) Write the language which is a set consisting of strings that are recognized by this automaton.

16) **[20 Points]** Give regular expressions describing the following languages:

a) $A = \{w \in \{0,1\}^* : w \text{ contains at least three } 1s\}$.

b) $B = \{w \in \{0,1\}^* : w \text{ contains at least two } 1s \text{ and at most one } 0\}$,

c) $C = \{w \in \{0,1\}^* : w \text{ contains an even number of } 0s \text{ and exactly two } 1s\}$.

d) $D = \{w \in \{0, 1\}^* : w \text{ contains an even number of } 0s \text{ and each } 0 \text{ is followed by at least one}\}$

17) **[20 Points]** Design a DFA or NFA for the following languages. $n_0(w)$ denotes the number of zeros in the string w.

a) $L_1 = \{ w \in \{0, 1\}^* : n_0(w) \bmod 2 = 0 \}$,

b) $L_2 = \{ w \in \{0, 1\}^* : n_0(w) \bmod 3 = 0 \}$,

c) Based on using the NFA and DFA you designed in the options a and b, design an NFA that recognized the language $L_3 = \{ w \in \{0, 1\}^* : n_0(w) \bmod 6 = 0\}$.

Hint: De Morgan's Laws $L_1 \cap L_2 = \overline{(\overline{L_1} \cup \overline{L_2})}$ can be used for designing an NFA that recognizes the intersection of languages.