

# BLM2041 Signals and Systems

## Week 5

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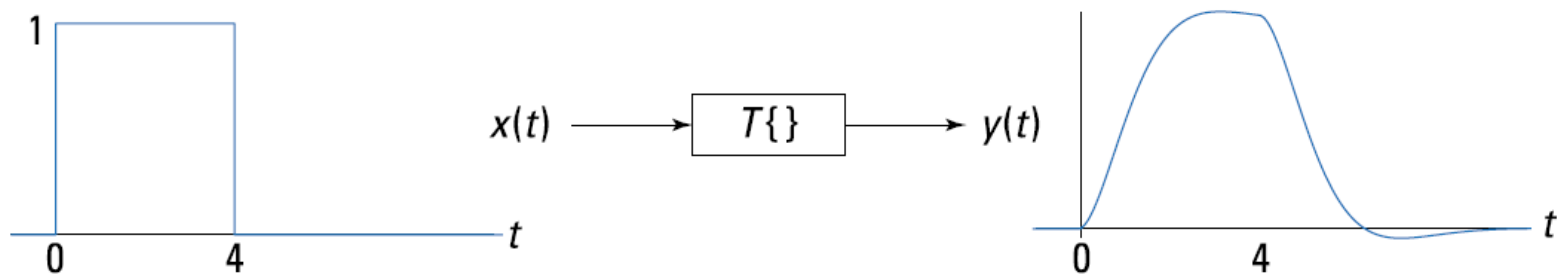
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# Responses to arbitrary signals

- Although we have focused on responses to simple signals ( $\delta[n], \delta(t)$ ) we are generally interested in responses to more complicated signals.
- How do we compute responses to a more complicated input signals?



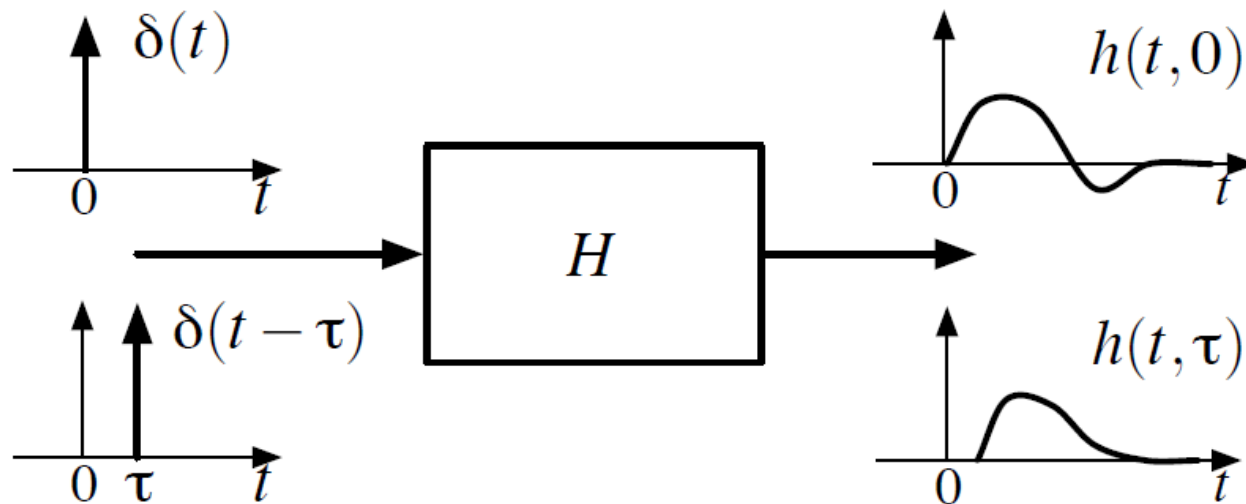
Block diagram depicting a general input/output relationship.

# Impulse Response

The *impulse response* of a linear system  $h_\tau(t)$  is the output of the system at time  $t$  to an impulse at time  $\tau$ . This can be written as

$$h_\tau = H(\delta_\tau)$$

Care is required in interpreting this expression!



**Note:** Be aware of potential confusion here:

When you write

$$h_{\tau}(t) = H(\delta_{\tau}(t))$$

the variable  $t$  serves different roles on each side of the equation.

- $t$  on the left is a specific value for time, the time at which the output is being sampled.
- $t$  on the right is varying over all real numbers, it is not the same  $t$  as on the left.
- The output at time specific time  $t$  on the left in general depends on the input at all times  $t$  on the right (the entire input waveform).

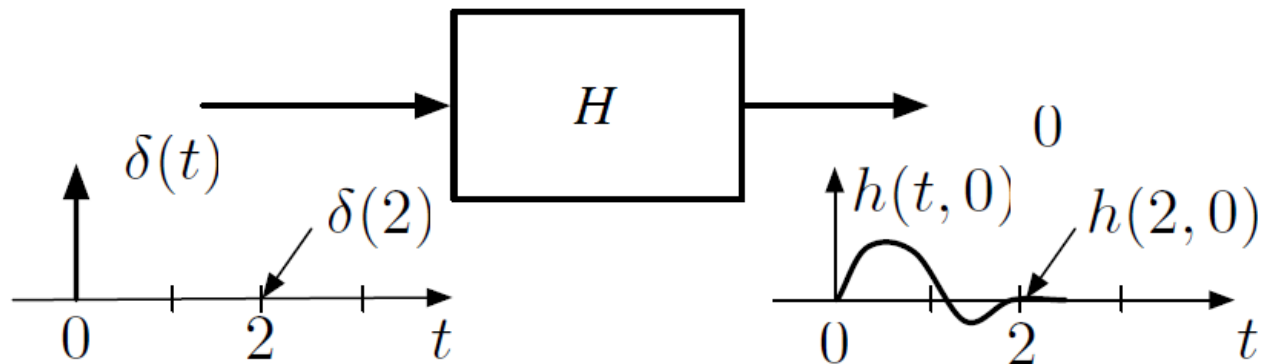
- Assume the input impulse is at  $\tau = 0$ ,

$$h = h_0 = H(\delta_0).$$

We want to know the impulse response at time  $t = 2$ . It doesn't make any sense to set  $t = 2$ , and write

$$h(2) = H(\delta(2)) \quad \Leftarrow \text{No!}$$

First,  $\delta(2)$  is something like zero, so  $H(0)$  would be zero. Second, the value of  $h(2)$  depends on the entire input waveform, not just the value at  $t = 2$ .

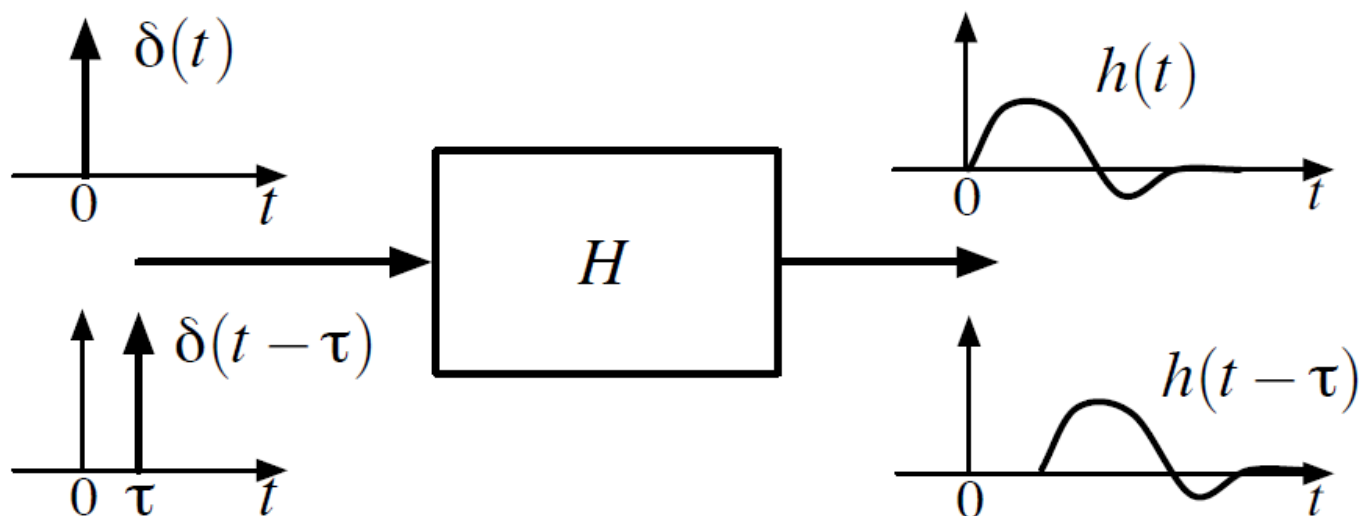


## Time-invariance

If  $H$  is time invariant, delaying the input and output both by a time  $\tau$  should produce the same response

$$h_{\tau}(t) = h(t - \tau).$$

In this case, we don't need to worry about  $h_{\tau}$  because it is just  $h$  shifted in time.



# Linearity and Extended Linearity

**Linearity:** A system  $S$  is linear if it satisfies both

- *Homogeneity:* If  $y = Sx$ , and  $a$  is a constant then

$$ay = S(ax).$$

- *Superposition:* If  $y_1 = Sx_1$  and  $y_2 = Sx_2$ , then

$$y_1 + y_2 = S(x_1 + x_2).$$

**Combined Homogeneity and Superposition:**

If  $y_1 = Sx_1$  and  $y_2 = Sx_2$ , and  $a$  and  $b$  are constants,

$$ay_1 + by_2 = S(ax_1 + bx_2)$$

## Extended Linearity

- *Summation*: If  $y_n = S(x_n)$  for all  $n$ , an integer from  $(-\infty < n < \infty)$ , and  $a_n$  are constants

$$\sum_n a_n y_n = S \left( \sum_n a_n x_n \right)$$

Summation and the system operator commute, and can be interchanged.

- *Integration (Simple Example)* : If  $y = S(x)$ ,

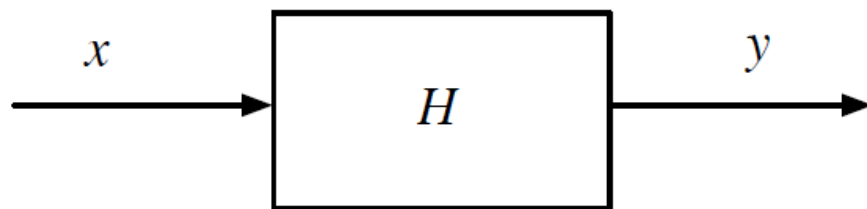
$$\int_{-\infty}^{\infty} a(\tau) y(t - \tau) d\tau = S \left( \int_{-\infty}^{\infty} a(\tau) x(t - \tau) d\tau \right)$$

Integration and the system operator commute, and can be interchanged.



# Output of an LTI System

We would like to determine an expression for the output  $y(t)$  of an linear time invariant system, given an input  $x(t)$



We can write a signal  $x(t)$  as a sample of itself

$$x(t) = \int_{-\infty}^{\infty} x(\tau) \delta_{\tau}(t) d\tau$$

This means that  $x(t)$  can be written as a weighted integral of  $\delta$  functions.

Applying the system  $H$  to the input  $x(t)$ ,

$$\begin{aligned} y(t) &= H(x(t)) \\ &= H\left(\int_{-\infty}^{\infty} x(\tau)\delta_{\tau}(t)d\tau\right) \end{aligned}$$

If the system obeys extended linearity we can interchange the order of the system operator and the integration

$$y(t) = \int_{-\infty}^{\infty} x(\tau)H(\delta_{\tau}(t))d\tau.$$

The impulse response is

$$h_{\tau}(t) = H(\delta_{\tau}(t)).$$

Substituting for the impulse response gives

$$y(t) = \int_{-\infty}^{\infty} x(\tau)h_{\tau}(t)d\tau.$$

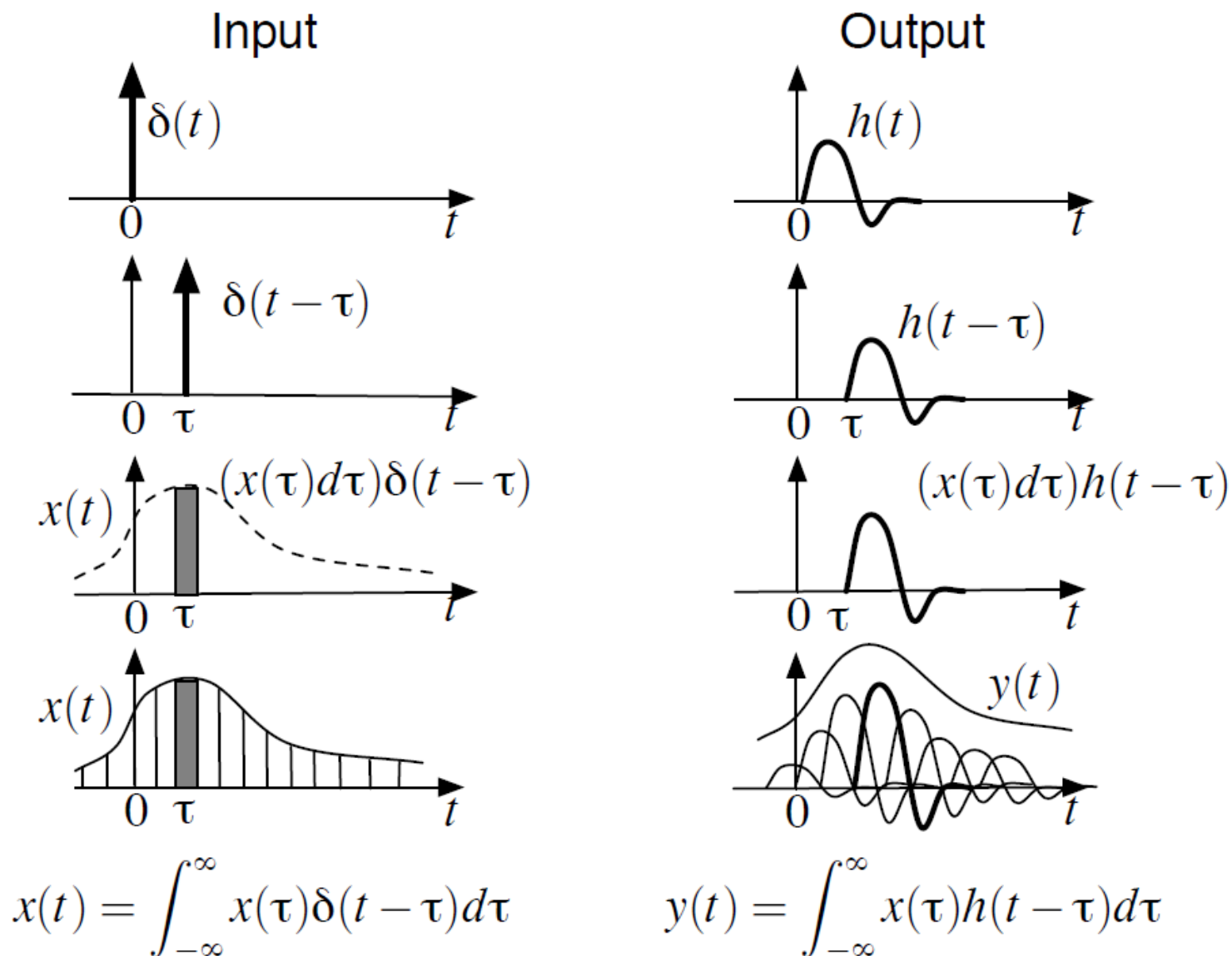
This is a *superposition integral*. The values of  $x(\tau)h(t, \tau)d\tau$  are superimposed (added up) for each input time  $\tau$ .

If  $H$  is time invariant, this written more simply as

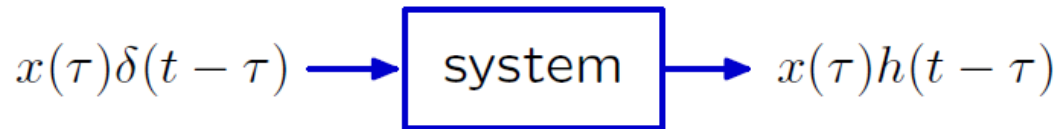
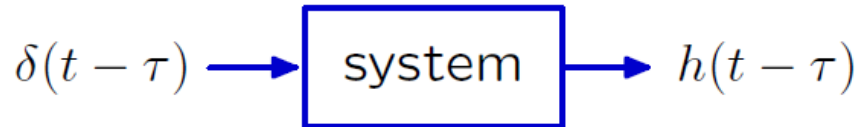
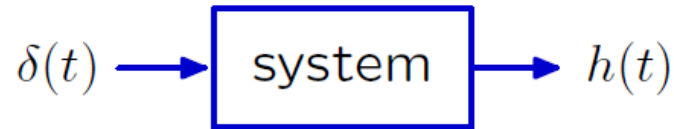
$$y(t) = \int_{-\infty}^{\infty} x(\tau)h_{\tau}(t)d\tau.$$

This is in the form of a *convolution integral*, which will be the subject of the next class.

Graphically, this can be represented as:



If a system is linear and time-invariant (LTI) then its output is the integral of weighted and shifted unit-impulse responses.

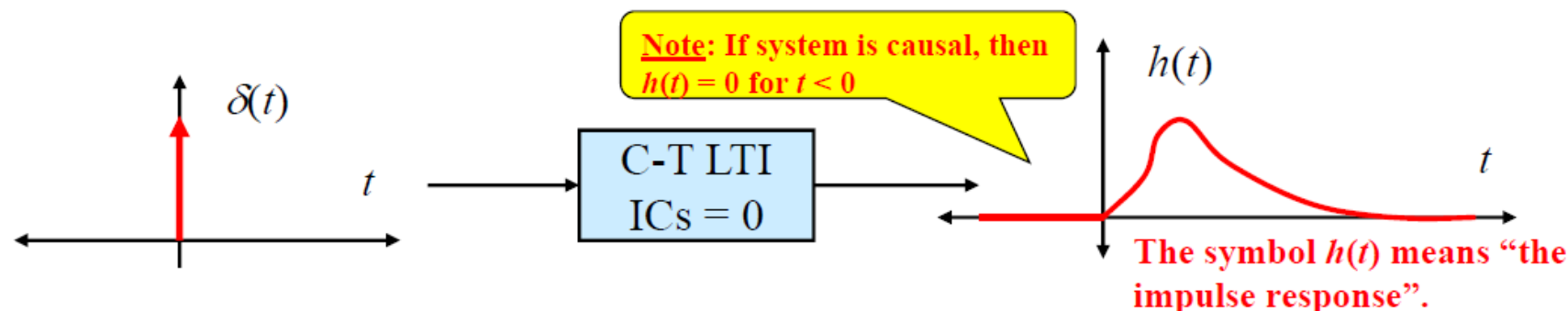


$$x(t) = \int_{-\infty}^{\infty} x(\tau)\delta(t - \tau)d\tau \rightarrow \text{system} \rightarrow y(t) = \int_{-\infty}^{\infty} x(\tau)h(t - \tau)d\tau$$

## Recall: Impulse Response

Earlier we introduced the concept of impulse response...

...what comes out of a system when the input is an impulse (delta function)



Noting that the LT of  $\delta(t) = 1$  and using the properties of the transfer function and the Z transform we said that

$$h(t) = \mathcal{L}^{-1} \{ H(s) \mathcal{L} \{ \delta(t) \} \}$$

$$h(t) = \mathcal{L}^{-1} \{ H(s) \}$$

$$h(t) = \mathcal{F}^{-1} \{ H(\omega) \}$$

So...once we have either  $H(s)$  or  $H(\omega)$  we can get the impulse response  $h(t)$

## Convolution Property and System Output

Let  $x(t)$  be a signal with CTFT  $X(\omega)$  and LT of  $X(s)$

$$x(t) \leftrightarrow X(\omega)$$

$$x(t) \leftrightarrow X(s)$$

Consider a system w/ freq resp  $H(\omega)$  & trans func  $H(s)$

$$h(t) \leftrightarrow H(\omega)$$

$$h(t) \leftrightarrow H(s)$$

We've spent much time using these tools to analyze system outputs this way:

$$Y(\omega) = H(\omega)X(\omega) \leftrightarrow y(t) = \mathcal{F}^{-1}\{H(\omega)X(\omega)\}$$

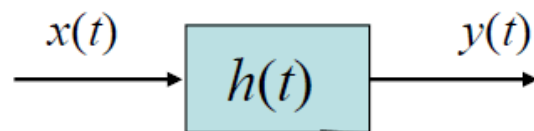
$$Y(s) = H(s)X(s) \leftrightarrow y[n] = \mathcal{L}^{-1}\{H(s)X(s)\}$$

**The convolution property of the CTFT and LT gives an alternate way to find  $y(t)$ :**

$$\mathcal{F}^{-1}\{X(\omega)H(\omega)\} = x(t) * h(t)$$

$$\mathcal{L}^{-1}\{X(s)H(s)\} = x(t) * h(t)$$

$$x(t) * h(t) = \int_{-\infty}^{\infty} x(\tau)h(t-\tau)d\tau$$



$$y(t) = \int_{-\infty}^{\infty} x(\tau)h(t-\tau)d\tau$$

**LTI System with impulse response  $h(t)$**

**“Convoluting”  
input  $x(t)$  with the  
impulse response  
 $h(t)$  gives the  
output  $y(t)$ !**

## Convolution for Causal System & with Causal Input

An arbitrary LTI system's output can be found using the general convolution form:

$$y(t) = \int_{-\infty}^{\infty} x(\tau)h(t-\tau)d\tau$$

**General LTI System**

If the system is causal then  $h(t) = 0$  for  $t < 0$ .... Thus  $h(t-\tau) = 0$  for  $t < \tau$  ... so:

$$y(t) = \int_{-\infty}^t x(\tau)h(t-\tau)d\tau$$

**Causal LTI System**

If the input is causal then  $x(t) = 0$  for  $t < 0$ .... so:

$$y(t) = \int_0^{\infty} x(\tau)h(t-\tau)d\tau$$

**Causal Input & General LTI System**

If the system & signal are both causal then

$$y(t) = \int_0^t x(\tau)h(t-\tau)d\tau$$

**Causal Input & Causal LTI System**



# Convolution Properties

## 1. Commutativity

$$x(t) * h(t) = h(t) * x(t)$$

## 2. Associativity

$$[x(t) * h_1(t)] * h_2(t) = x(t) * [h_1(t) * h_2(t)]$$

Associativity together with commutativity says we can interchange the order of two cascaded systems.

## 3. Distributivity

$$x(t) * [h_1(t) + h_2(t)] = x(t) * h_1(t) + x(t) * h_2(t)$$

## 4. Derivative Property:

$$\begin{aligned} \frac{d}{dt} [x(t) * v(t)] &= \dot{x}(t) * v(t) \\ &= x(t) * \dot{v}(t) \end{aligned}$$

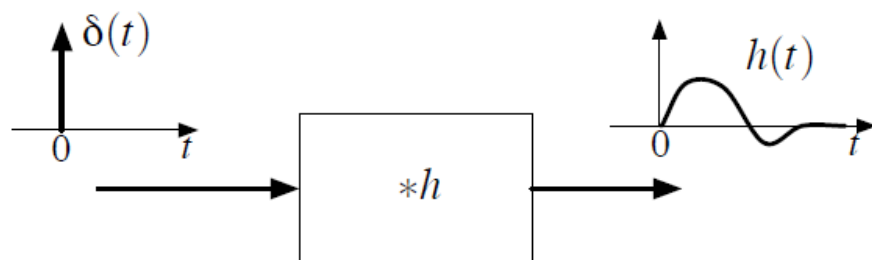
derivative

## 5. Integration Property Let $y(t) = x(t) * h(t)$ , then

$$\int_{-\infty}^t y(\lambda) d\lambda = \left[ \int_{-\infty}^t x(\lambda) d\lambda \right] * h(t) = x(t) * \left[ \int_{-\infty}^t h(\lambda) d\lambda \right]$$

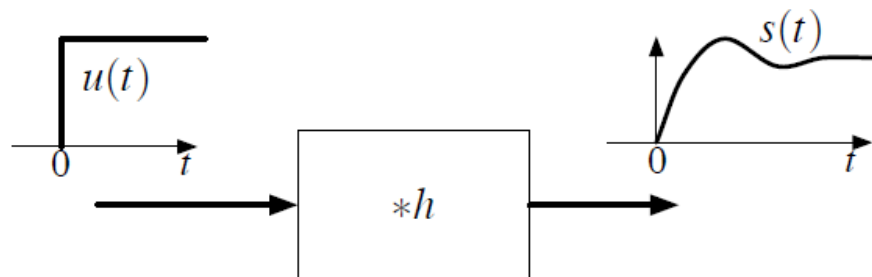
**Example:** Measuring the impulse response of an LTI system.

We would like to measure the impulse response of an LTI system, described by the impulse response  $h(t)$

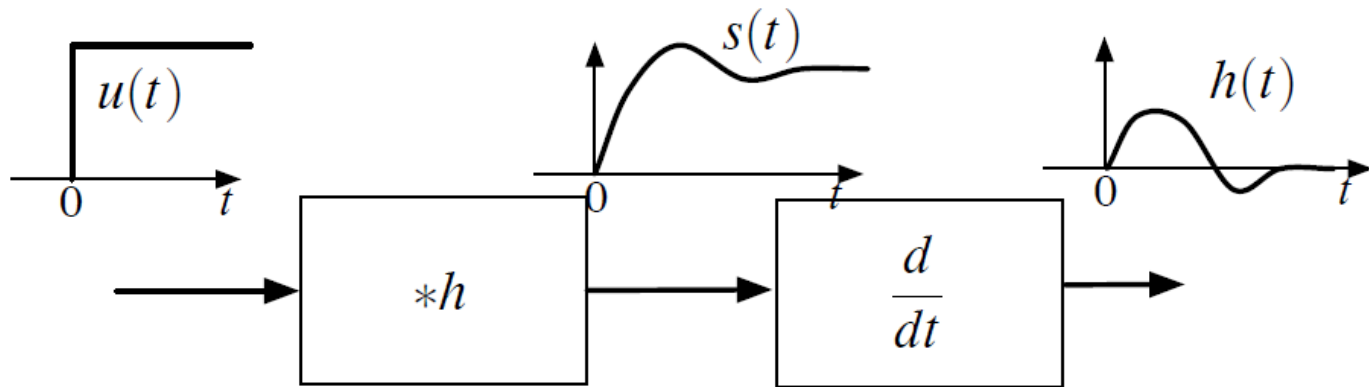


This can be practically difficult because input amplitude is often limited. A very short pulse then has very little energy.

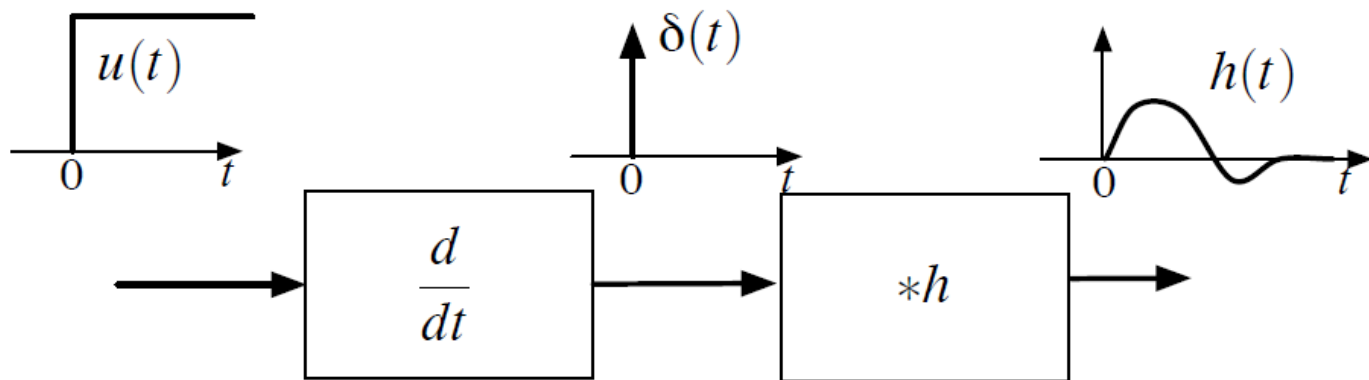
A common alternative is to measure the *step response*  $s(t)$ , the response to a unit step input  $u(t)$



The impulse response is determined by differentiating the step response,



To show this, commute the convolution system and the differentiator to produce a system with the same overall impulse response



## Steps for Graphical Convolution $x(t)*h(t)$

$$y(t) = \int_{-\infty}^{\infty} x(\tau)h(t - \tau)d\tau$$

1. **Re-Write the signals as functions of  $\tau$** :  $x(\tau)$  and  $h(\tau)$
2. **Flip** just one of the signals around  $t = 0$  to get either  $x(-\tau)$  or  $h(-\tau)$ 
  - a. It is usually best to flip the signal with shorter duration
  - b. For notational purposes here: we'll flip  $h(\tau)$  to get  $h(-\tau)$
3. **Find Edges** of the flipped signal
  - a. Find the left-hand-edge  $\tau$ -value of  $h(-\tau)$ : **call it  $\tau_{L,0}$**
  - b. Find the right-hand-edge  $\tau$ -value of  $h(-\tau)$ : **call it  $\tau_{R,0}$**
4. **Shift**  $h(-\tau)$  by an arbitrary value of  $t$  to get  $h(t - \tau)$  and **get its edges**
  - a. Find the left-hand-edge  $\tau$ -value of  $h(t - \tau)$  as a function of  $t$ : **call it  $\tau_{L,t}$** 
    - **Important**: It will always be...  **$\tau_{L,t} = t + \tau_{L,0}$**
  - b. Find the right-hand-edge  $\tau$ -value of  $h(t - \tau)$  as a function of  $t$ : **call it  $\tau_{R,t}$** 
    - **Important**: It will always be...  **$\tau_{R,t} = t + \tau_{R,0}$**

Note: If the signal you flipped is NOT finite duration,  
one or both of  $\tau_{L,t}$  and  $\tau_{R,t}$  will be infinite ( $\tau_{L,t} = -\infty$  and/or  $\tau_{R,t} = \infty$ )

## Steps Continued

### 5. Find Regions of $\tau$ -Overlap

- What you are trying to do here is find intervals of  $t$  over which the product  $x(\tau) h(t - \tau)$  has a single mathematical form in terms of  $\tau$
- In each region find: Interval of  $t$  that makes the identified overlap happen
- Working examples is the best way to learn how this is done

**Tips:** Regions should be contiguous with no gaps!!!  
Don't worry about  $<$  vs.  $\leq$  etc.

### 6. For Each Region: Form the Product $x(\tau) h(t - \tau)$ and Integrate

- Form product  $x(\tau) h(t - \tau)$
- Find the Limits of Integration by finding the interval of  $\tau$  over which the product is nonzero
  - Found by seeing where the edges of  $x(\tau)$  and  $h(t - \tau)$  lie
  - Recall that the edges of  $h(t - \tau)$  are  $\tau_{L,t}$  and  $\tau_{R,t}$ , which often depend on the value of  $t$ 
    - So... the limits of integration may depend on  $t$
- Integrate the product  $x(\tau) h(t - \tau)$  over the limits found in 6b
  - The result is generally a function of  $t$ , but is only valid for the interval of  $t$  found for the current region
  - Think of the result as a “time-section” of the output  $y(t)$

## Steps Continued

7. **“Assemble” the output** from the output time-sections for all the regions
  - a. Note: you do NOT add the sections together
  - b. You define the output “piecewise”
  - c. Finally, if possible, look for a way to write the output in a simpler form

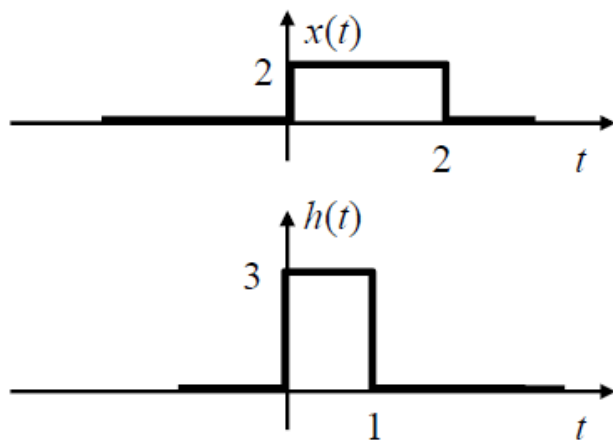
## Example: Graphically Convolve Two Signals

$$y(t) = \int_{-\infty}^{\infty} h(\tau)x(t - \tau)d\tau$$
$$= \int_{-\infty}^{\infty} x(\tau)h(t - \tau)d\tau$$

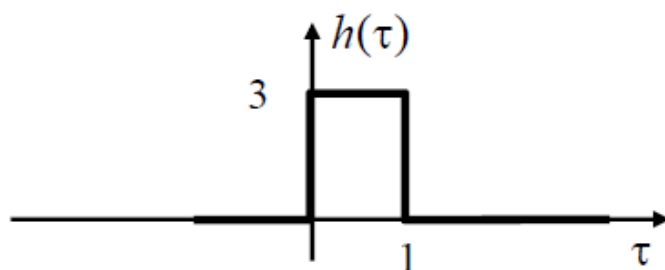
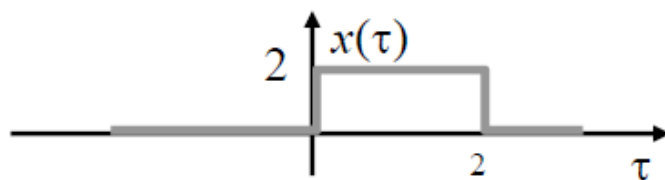
By “Properties of Convolution”...  
these two forms are  
Equal

This is why we can  
flip either signal

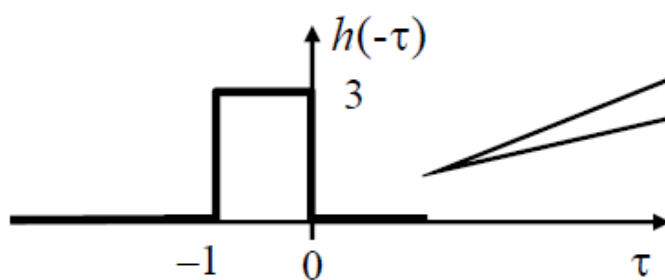
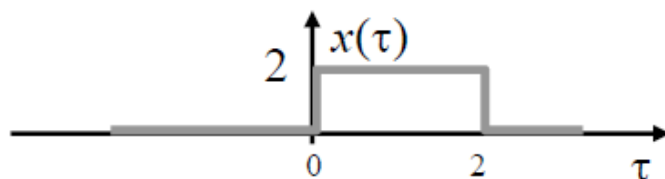
Convolve these two signals:



## Step #1: Write as Function of $\tau$



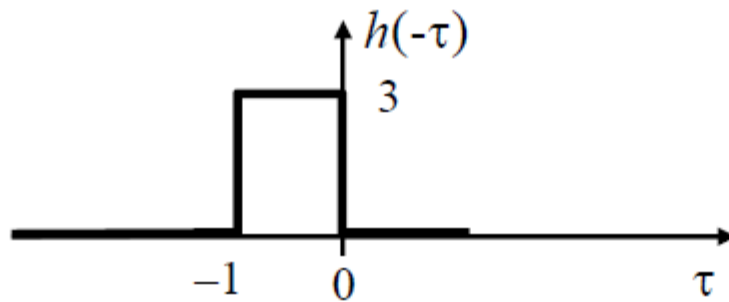
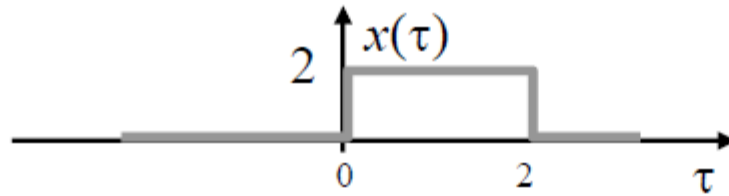
## Step #2: Flip $h(\tau)$ to get $h(-\tau)$



**Usually Easier  
to Flip the  
Shorter Signal**



### Step #3: Find Edges of Flipped Signal



$\tau_{L,0} = -1$

$\tau_{R,0} = 0$

## Motivating Step #4: Shift by $t$ to get $h(t-\tau)$ & Its Edges

Just looking at 2 “arbitrary”  $t$  values

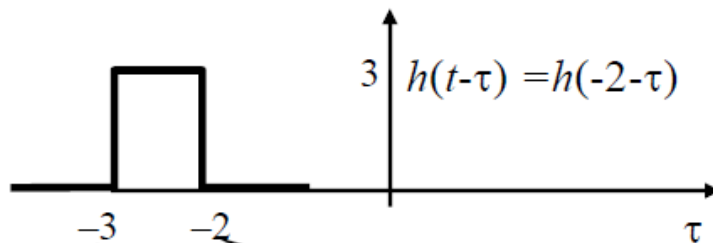
In Each Case We Get

$$\tau_{L,t} = t + \tau_{L,0}$$

$$\tau_{R,t} = t + \tau_{R,0}$$

For  $t = -2$

For  $t = 2$



$$\tau_{L,t} = t + \tau_{L,0}$$

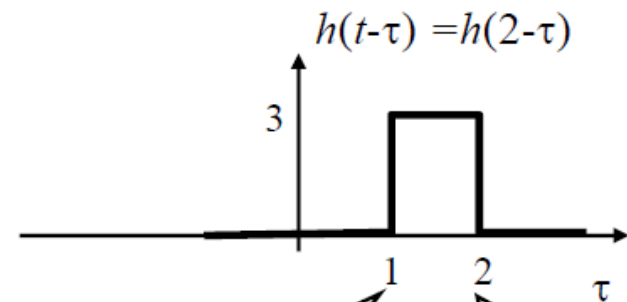
$$\tau_{L,t} = t - 1$$

$$\tau_{L,-2} = -2 - 1$$

$$\tau_{R,t} = t + \tau_{R,0}$$

$$\tau_{R,t} = t + 0$$

$$\tau_{R,-2} = -2 + 0$$



$$\tau_{L,t} = t + \tau_{L,0}$$

$$\tau_{L,t} = t - 1$$

$$\tau_{L,2} = 2 - 1$$

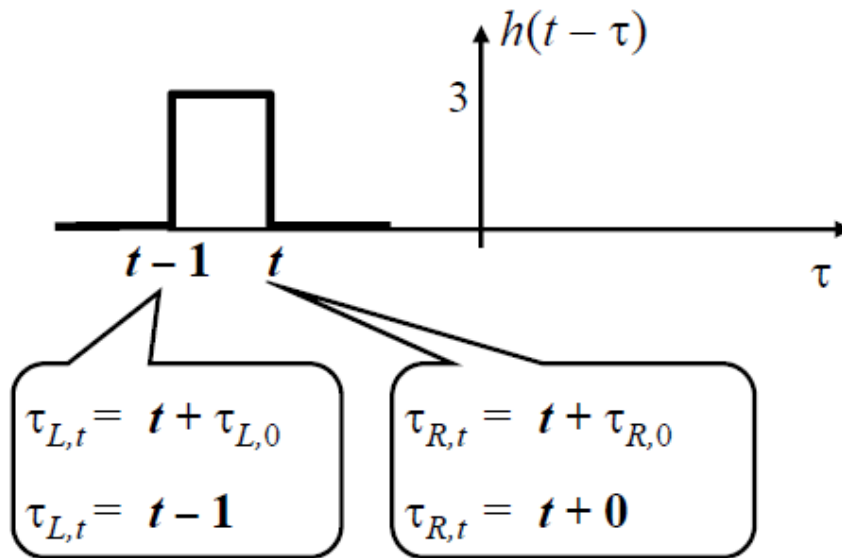
$$\tau_{R,t} = t + \tau_{R,0}$$

$$\tau_{R,t} = t + 0$$

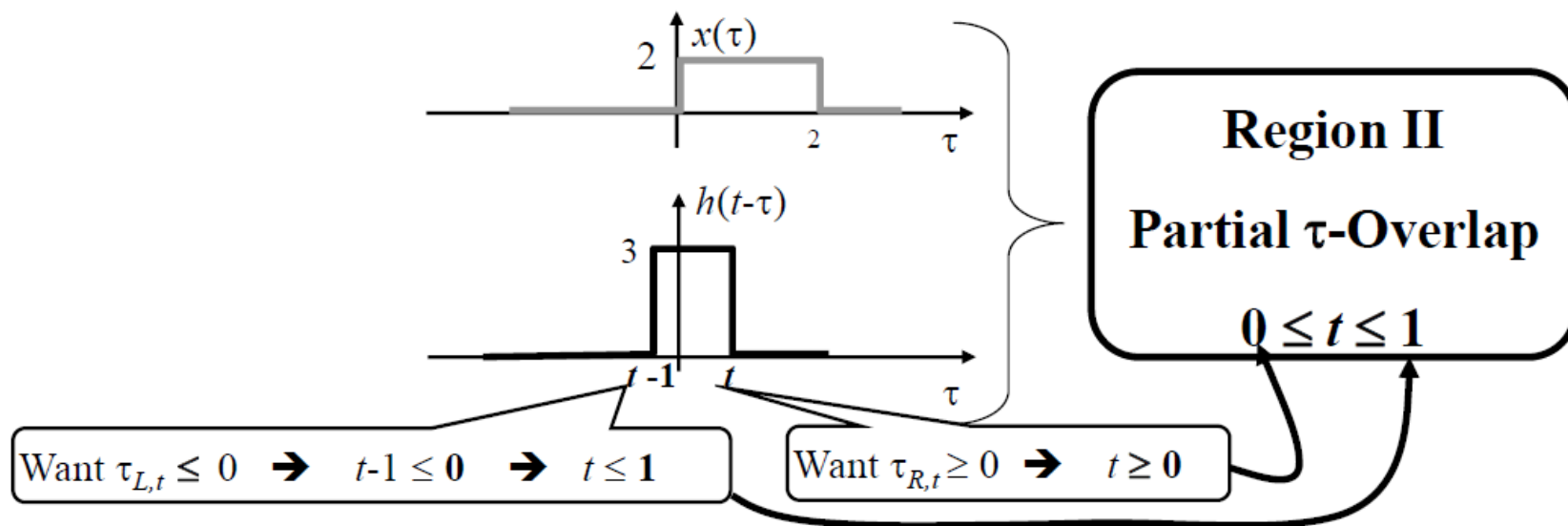
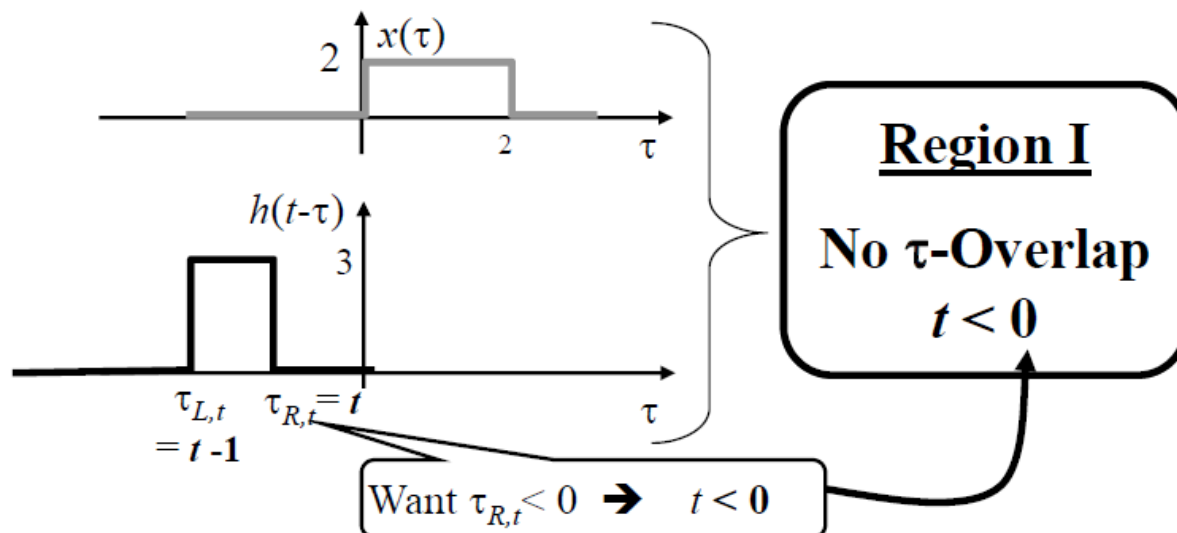
$$\tau_{R,2} = 2 + 0$$

**Doing Step #4: Shift by  $t$  to get  $h(t-\tau)$  & Its Edges**

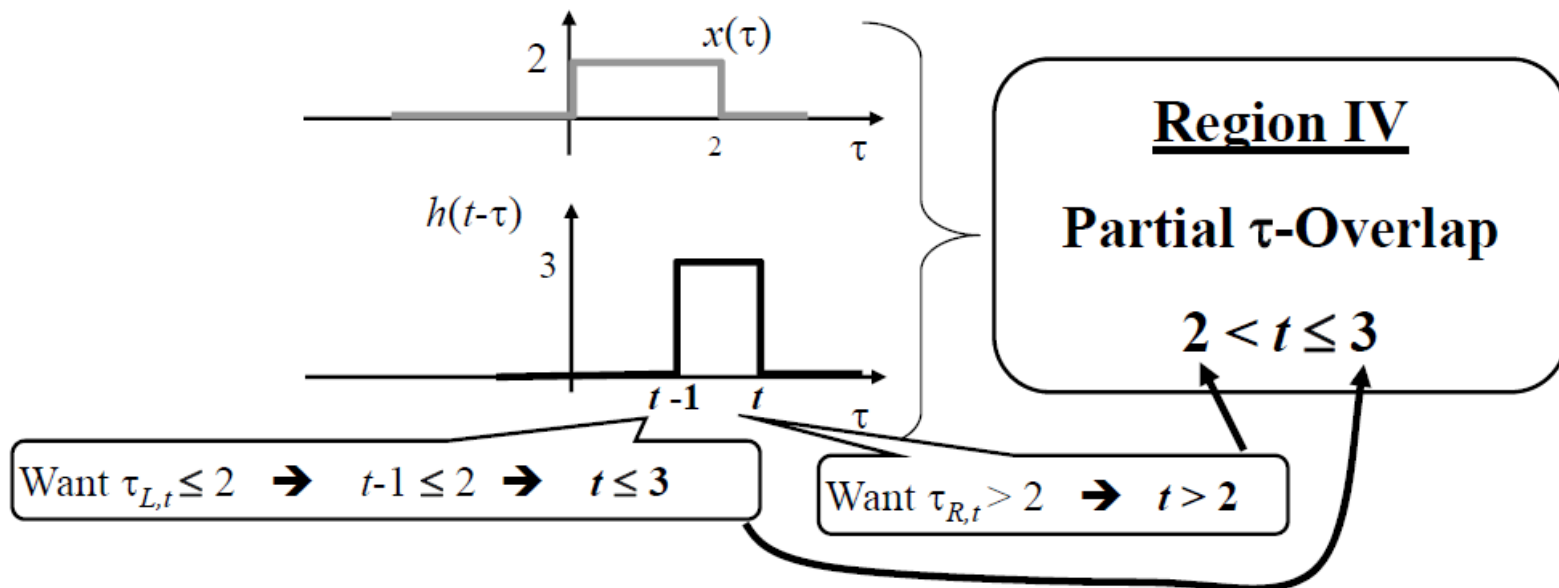
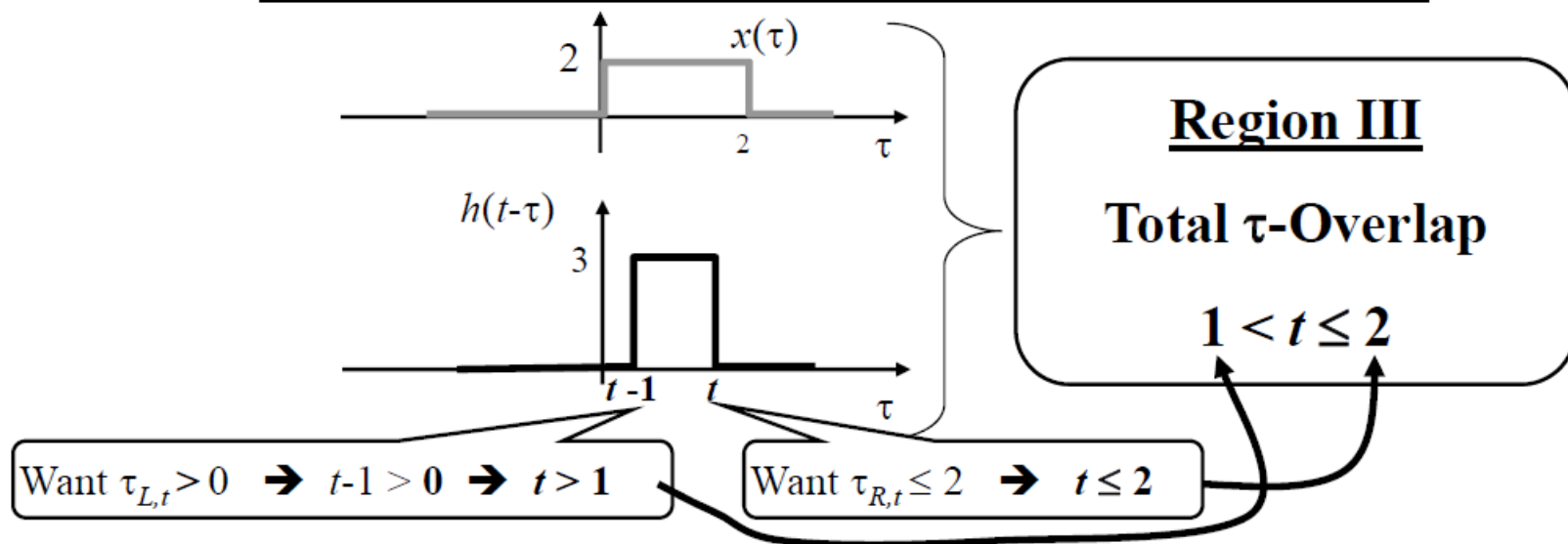
For Arbitrary Shift by  $t$



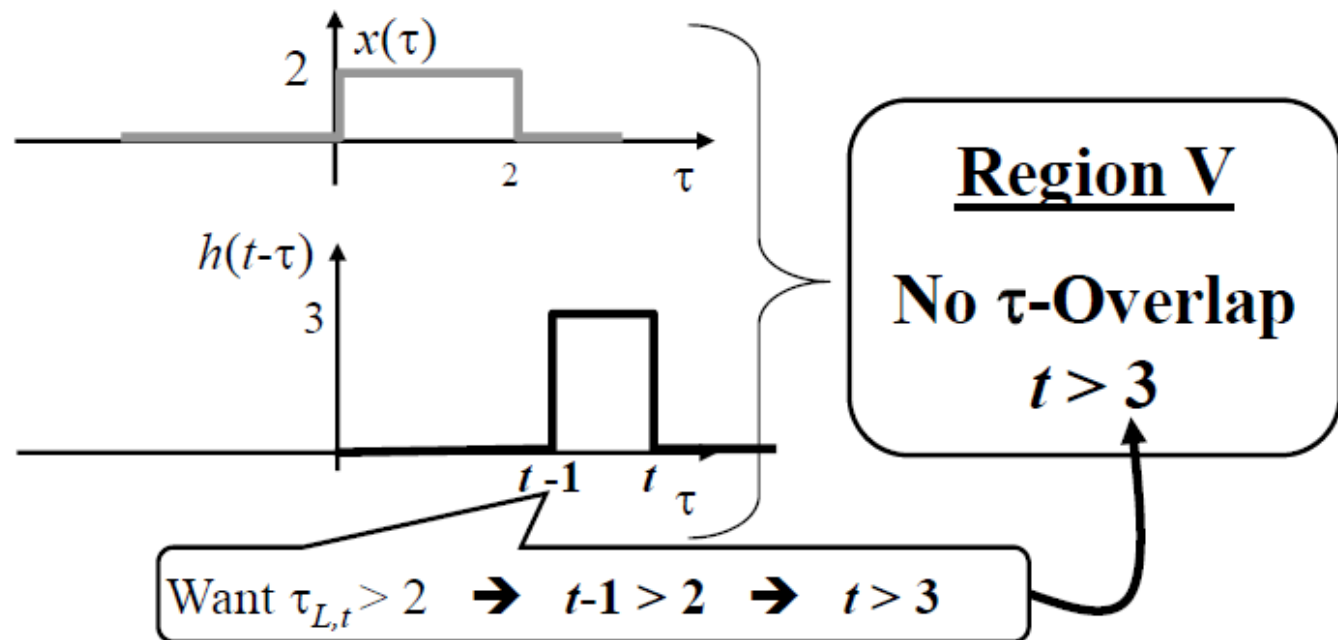
## Step #5: Find Regions of $\tau$ -Overlap



## Step #5 (Continued): Find Regions of $\tau$ -Overlap

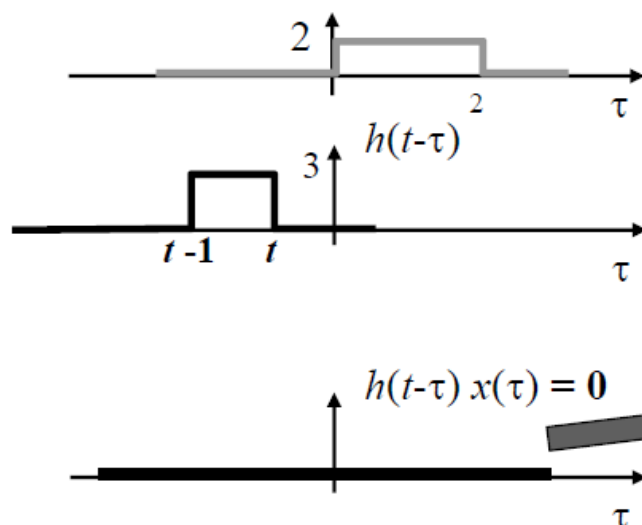


## Step #5 (Continued): Find Regions of $\tau$ -Overlap



## Step #6: Form Product & Integrate For Each Region

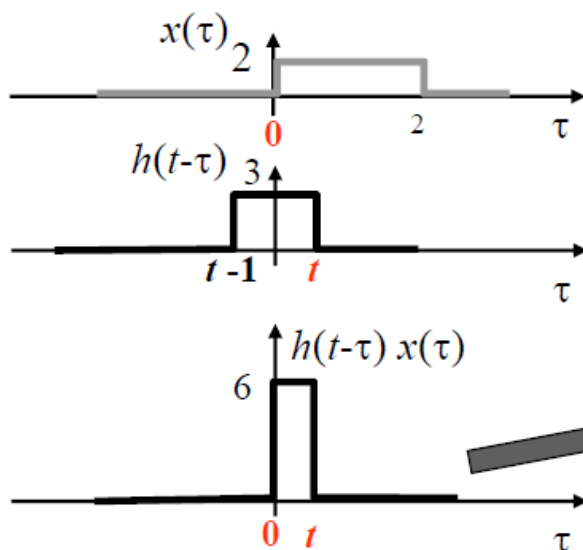
### Region I: $t < 0$



$$\begin{aligned}
 y(t) &= \int_{-\infty}^{\infty} x(\tau)h(t-\tau)d\tau \\
 &= \int_{-\infty}^{\infty} 0d\tau = 0 \\
 y(t) &= 0 \quad \text{for all } t < 0
 \end{aligned}$$

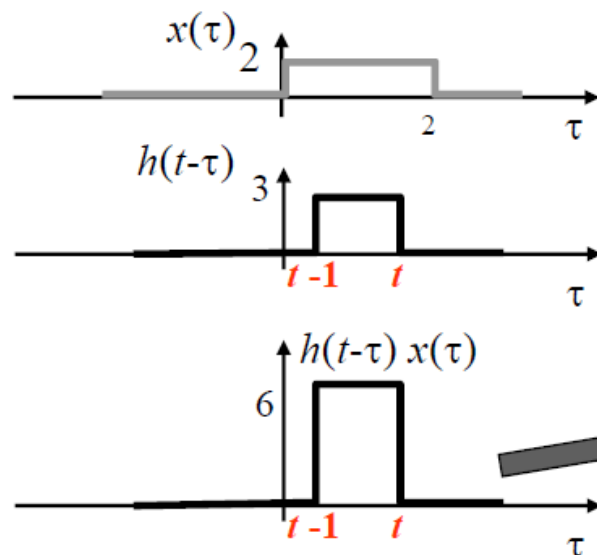
With 0 integrand  
the limits don't  
matter!!!

### Region II: $0 \leq t \leq 1$



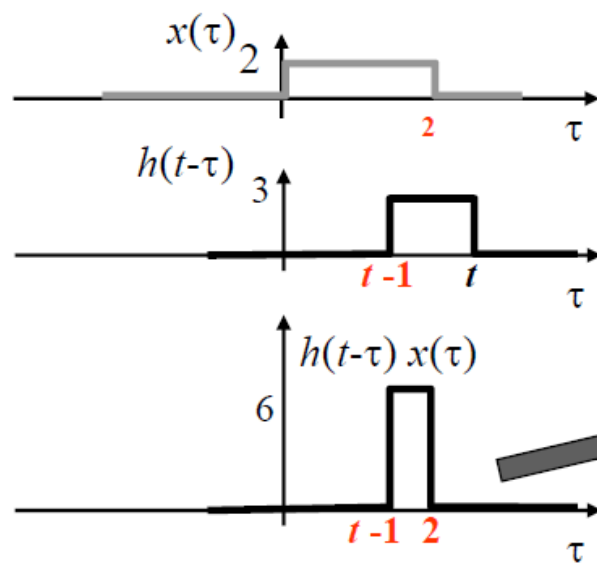
$$\begin{aligned}
 y(t) &= \int_{-\infty}^{\infty} x(\tau)h(t-\tau)d\tau \\
 &= \int_0^t 6d\tau = [6\tau]_0^t = 6t - 6 \times 0 = 6t \\
 y(t) &= 6t \quad \text{for } 0 \leq t \leq 1
 \end{aligned}$$

## Step #6 (Continued): Form Product & Integrate For Each Region



**Region III:  $1 < t \leq 2$**

$$\begin{aligned}
 y(t) &= \int_{-\infty}^{\infty} x(\tau)h(t-\tau)d\tau \\
 &= \int_{t-1}^t 6d\tau = [6\tau]_{t-1}^t = 6t - 6(t-1) = 6 \\
 y(t) &= 6 \quad \text{for all } t \text{ such that: } 1 < t \leq 2
 \end{aligned}$$



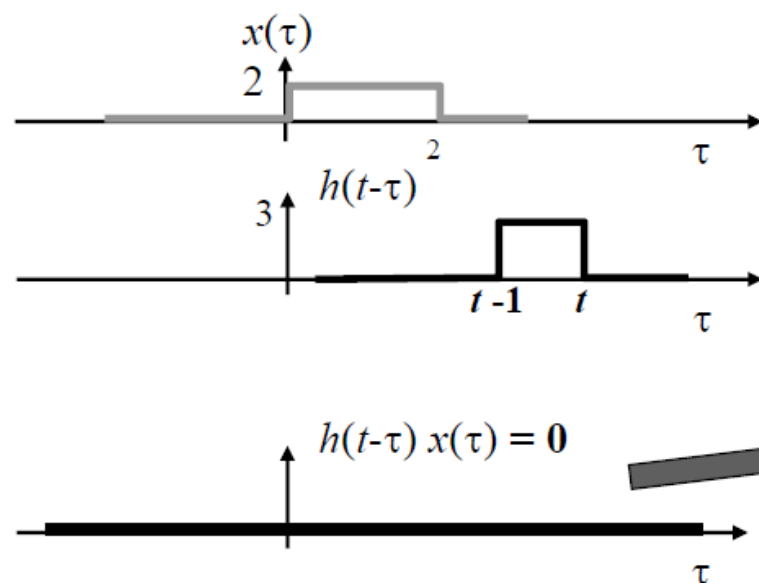
**Region IV:  $2 < t \leq 3$**

$$\begin{aligned}
 y(t) &= \int_{-\infty}^{\infty} x(\tau)h(t-\tau)d\tau \\
 &= \int_{t-1}^2 6d\tau = [6\tau]_{t-1}^2 = 6 \times 2 - 6(t-1) = -6t + 18 \\
 y(t) &= -6t + 18 \quad \text{for } 2 < t \leq 3
 \end{aligned}$$



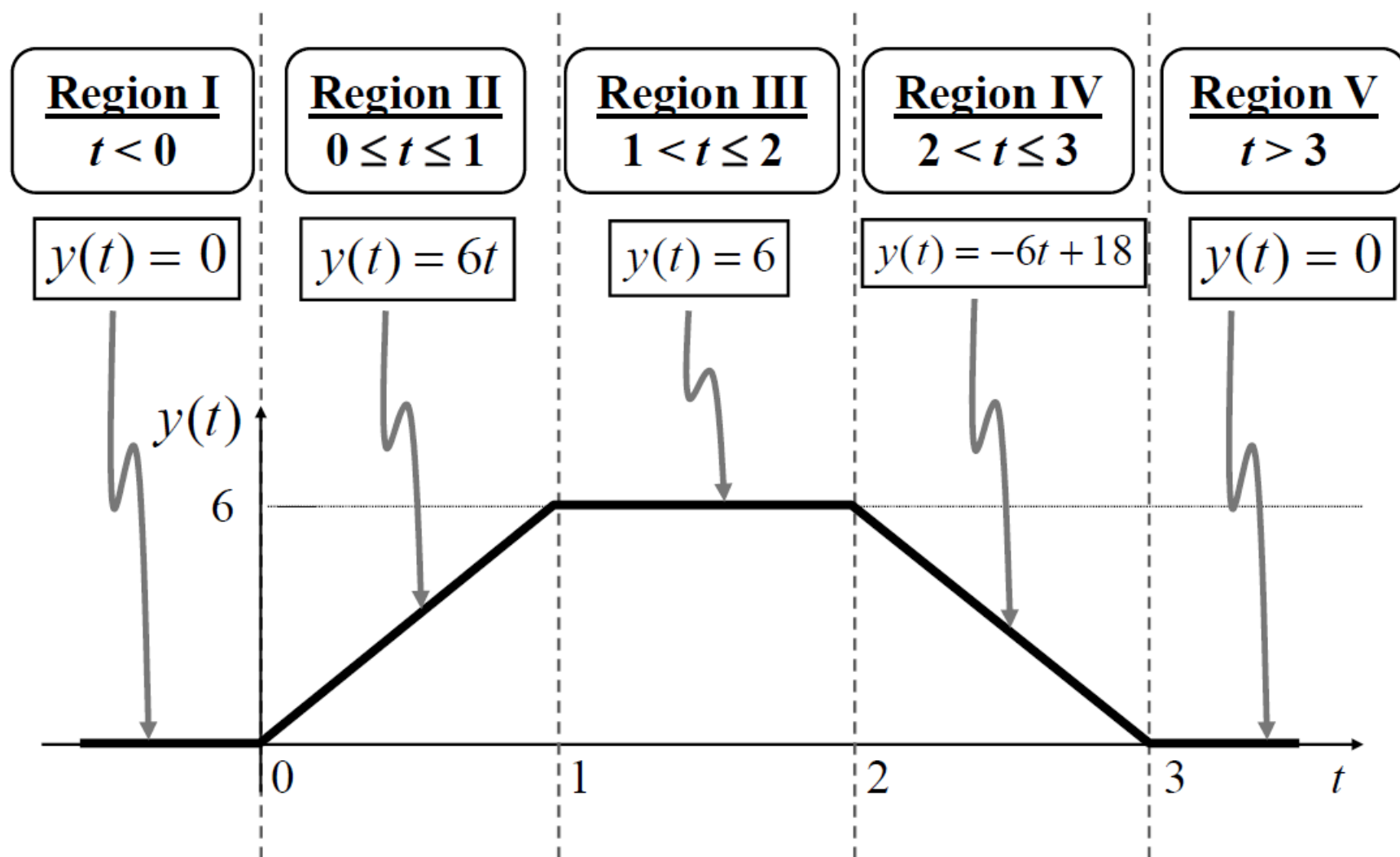
## Step #6 (Continued): Form Product & Integrate For Each Region

**Region V:  $t > 3$**



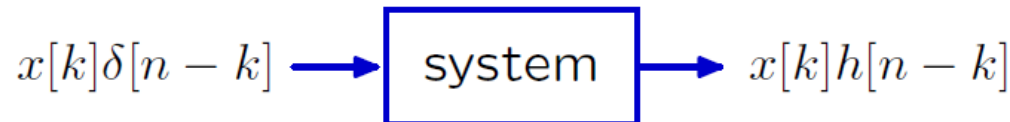
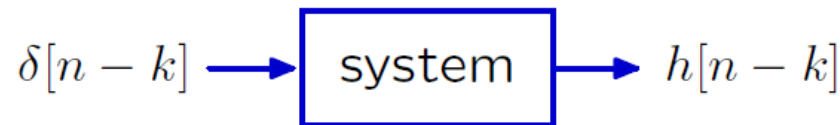
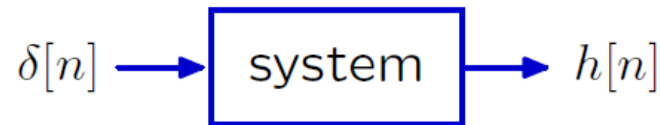
$$\begin{aligned} y(t) &= \int_{-\infty}^{\infty} x(\tau)h(t-\tau)d\tau \\ &= \int_{-\infty}^{\infty} 0d\tau = 0 \\ y(t) &= 0 \quad \text{for all } t > 3 \end{aligned}$$

### Step #7: “Assemble” Output Signal



# Discrete Convolution

If a system is linear and time-invariant (LTI) then its output is the sum of weighted and shifted unit-sample responses.



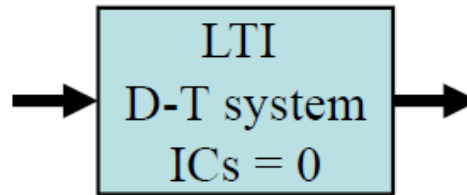
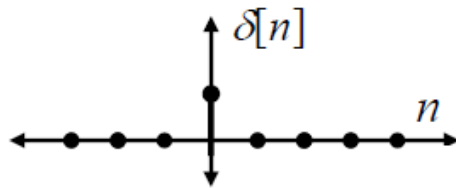
$$x[n] = \sum_{k=-\infty}^{\infty} x[k]\delta[n - k] \longrightarrow \text{system} \longrightarrow y[n] = \sum_{k=-\infty}^{\infty} x[k]h[n - k]$$

# Discrete Convolution

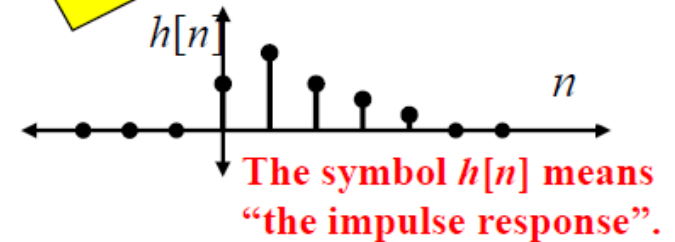
## Recall: Impulse Response

Earlier we introduced the concept of impulse response...

...what comes out of a system when the input is an impulse (delta sequence)



**Note:** If system is causal,  
then  $h[n] = 0$  for  $n < 0$



Noting that the ZT of  $\delta[n] = 1$  and using the properties of the transfer function and the Z transform we said that

$$h[n] = Z^{-1} \{ H(z) Z \{ \delta[n] \} \}$$

$$h[n] = Z^{-1} \{ H(z) \}$$

$$h[n] = IDTFT \{ H(\Omega) \}$$

So...once we have either  $H(z)$  or  $H(\Omega)$  we can get the impulse response  $h[n]$

## Convolution Property and System Output

Let  $x[n]$  be a signal with DTFT  $X(\Omega)$  and ZT of  $X(z)$

$$x[n] \leftrightarrow X(\Omega)$$

$$x[n] \leftrightarrow X(z)$$

Consider a system w/ freq resp  $H(\Omega)$  & trans func  $H(z)$

$$h[n] \leftrightarrow H(\Omega)$$

$$h[n] \leftrightarrow H(z)$$

We've spent much time using these tools to analyze system outputs this way:

$$Y(\Omega) = H(\Omega)X(\Omega) \leftrightarrow y[n] = DTFT^{-1}\{H(\Omega)X(\Omega)\}$$

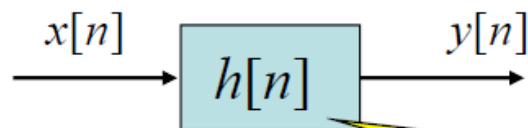
$$Y(z) = H(z)X(z) \leftrightarrow y[n] = Z^{-1}\{H(z)X(z)\}$$

**The convolution property of the DTFT and ZT gives an alternate way to find  $y[n]$ :**

$$DTFT^{-1}\{X(\Omega)H(\Omega)\} = x[n] * h[n]$$

$$Z^{-1}\{X(z)H(z)\} = x[n] * h[n]$$

$$x[n] * h[n] = \sum_{m=-\infty}^{\infty} x[m]h[n-m]$$



$$y[n] = \sum_{m=-\infty}^{\infty} x[m]h[n-m]$$

**LTI System with impulse response  $h[n]$**

**“Convoluting”  
input  $x[n]$  with the  
impulse response  
 $h[n]$  gives the  
output  $y[n]$ !**

## Convolution for Causal System & with Causal Input

An arbitrary LTI system's output can be found using the general convolution form:

$$y[n] = \sum_{m=-\infty}^{\infty} x[m]h[n-m]$$

**General LTI System**

If the system is causal then  $h[n] = 0$  for  $n < 0$ ... Thus  $h[n-m] = 0$  for  $m > n$  ... so:

$$y[n] = \sum_{m=-\infty}^n x[m]h[n-m]$$

**Causal LTI System**

If the input is causal then  $x[n] = 0$  for  $n < 0$ ... so:

$$y[n] = \sum_{m=0}^{\infty} x[m]h[n-m]$$

**Causal Input & General LTI System**

If the system & signal are both causal then

$$y[n] = \sum_{m=0}^n x[m]h[n-m]$$

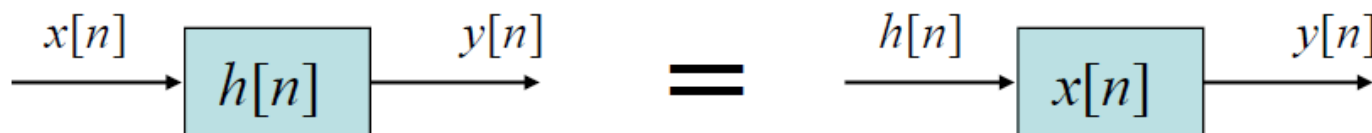
**Causal Input & Causal LTI System**

# Convolution Properties (can sometimes exploit to make things easier)

## 1. Commutativity

$$x[n] * h[n] = h[n] * x[n]$$

$$\sum_{m=-\infty}^{\infty} x[m]h[n-m] = \sum_{m=-\infty}^{\infty} h[m]x[n-m]$$



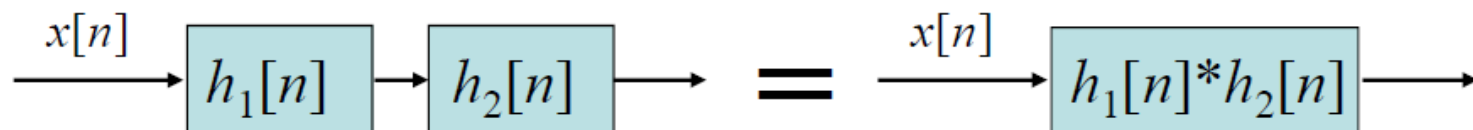
This is obvious from the frequency domain (or z domain) viewpoint:

$$x[n] * h[n] = h[n] * x[n] \quad \Rightarrow \quad X(\Omega)H(\Omega) = H(\Omega)X(\Omega)$$

## 2. Associativity

$$(x[n] * h_1[n]) * h_2[n] = x[n] * (h_1[n] * h_2[n])$$

$\Rightarrow$  Can combine cascade into single equivalent system

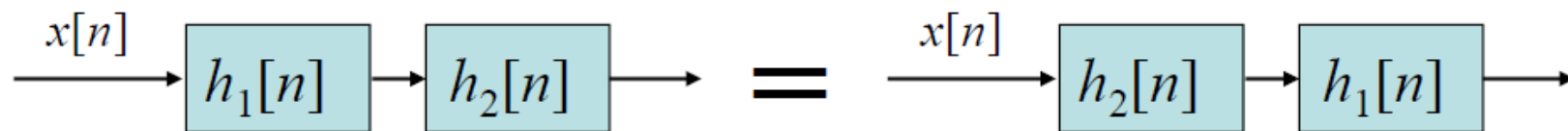


This is obvious from the frequency domain (or z domain) viewpoint:

$$[X(\Omega)H_1(\Omega)]H_2(\Omega) = X(\Omega)[H_1(\Omega)H_2(\Omega)]$$

Tells us what the Freq  
Resp is for a cascade

Associativity together with commutativity says we **can interchange the order of two cascaded systems**:

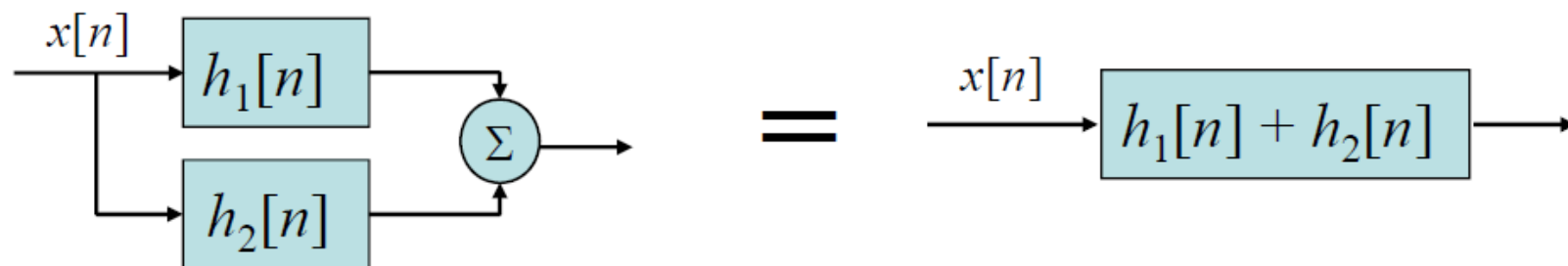


**Warning**: This holds in theory but in practice there may be physical issues that prevent this!!!

### 3. **Distributivity**

$$x[n] * (h_1[n] + h_2[n]) = x[n] * h_1[n] + x[n] * h_2[n]$$

$\Rightarrow$  can combine sum of two outputs into a single system (or vice versa)



With commutativity this says we can split a complicated input into sum of simple ones... which is nothing more than “linearity”!!



## Graphical Convolution – To Visualize & Test Real Systems

*Can do convolution this way when signals are known numerically or by equation*

- Convolution involves the sum of a product of two signals:  $x[i]h[n - i]$
- At each output index  $n$ , the product changes

“Commutativity” says we can flip either  $x[i]$  or  $h[i]$  and get the same answer

**Step 1:** Write both as functions of  $i$ :  $x[i]$  &  $h[i]$

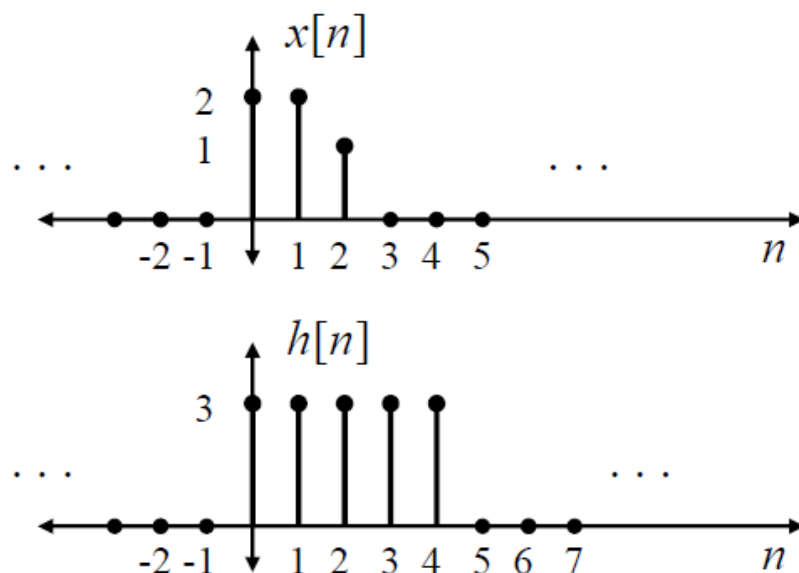
**Step 2:** Flip  $h[i]$  to get  $h[-i]$  (The book calls this “fold”)

**Repeat for each  $n$**  {

**Step 3:** For each output index  $n$  value of interest, shift by  $n$  to get  $h[n - i]$   
(Note: positive  $n$  gives right shift!!!!)

**Step 4:** Form product  $x[i]h[n - i]$  and sum its elements to get the number  $y[n]$

## Example of Graphical Convolution



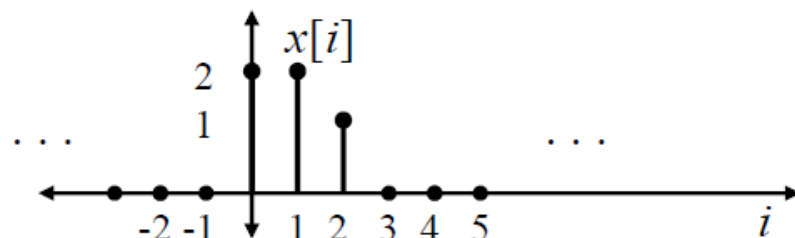
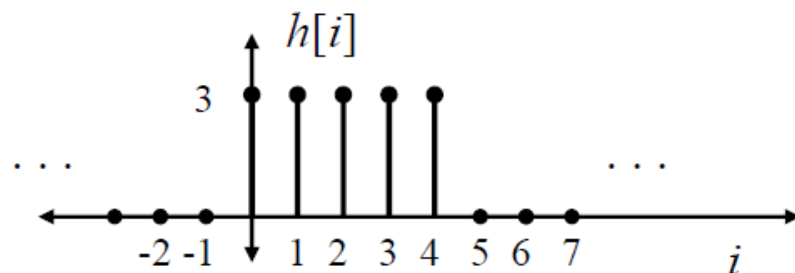
Find  $y[n]=x[n]*h[n]$   
for all  
integer values of  $n$

### Solution

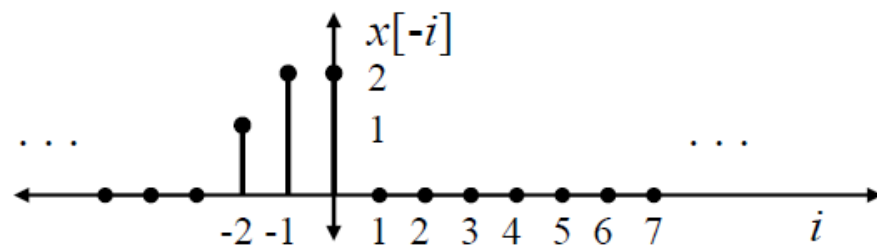
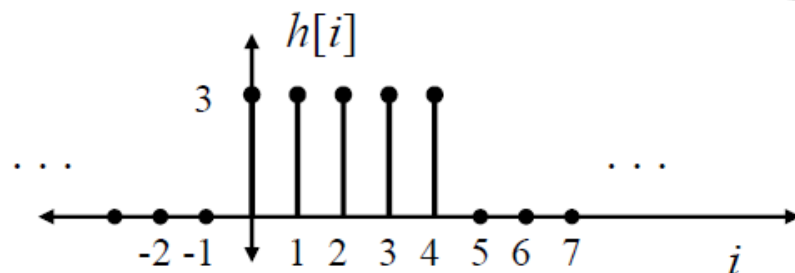
For this problem I choose to flip  $x[n]$

My personal preference is to flip the shorter signal although I sometimes don't follow that "rule"... only through lots of practice can you learn how to best choose which one to flip.

**Step 1:** Write both as functions of  $i$ :  $x[i]$  &  $h[i]$



**Step 2:** Flip  $x[i]$  to get  $x[-i]$



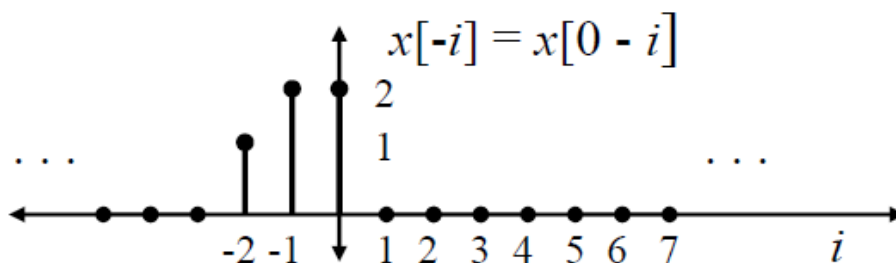
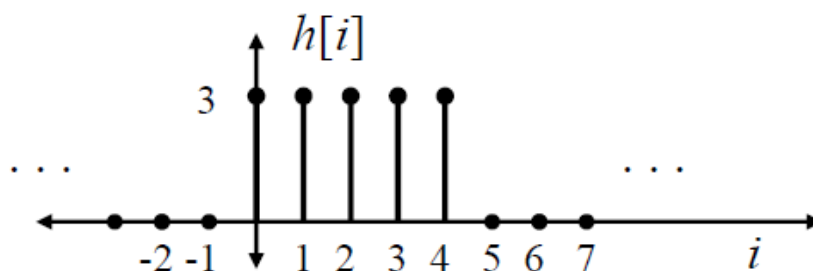
**“Commutativity”** says we can flip either  $x[i]$  or  $h[i]$  and get the same answer...  
**Here I flipped  $x[i]$**

We want a solution for  $n = \dots -2, -1, 0, 1, 2, \dots$  so must do Steps 3&4 for all  $n$ .

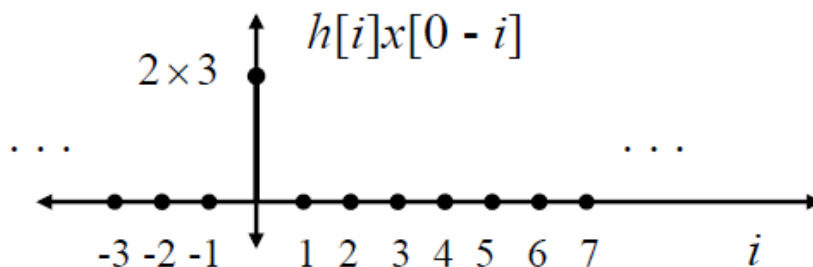
But... let's first do: **Steps 3&4 for  $n = 0$**  and then proceed from there.

**Step 3:** For  $n = 0$ , shift by  $n$  to get  $x[n - i]$

For  $n = 0$  case there is no shift!  
 $x[0 - i] = x[-i]$



**Step 4:** For  $n = 0$ , Form the product  $x[i]h[n - i]$  and sum its elements to give  $y[n]$



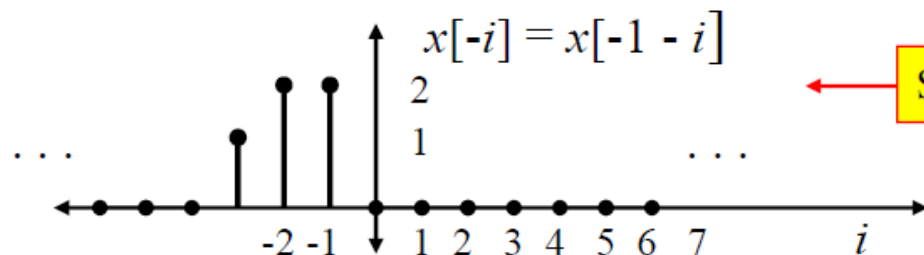
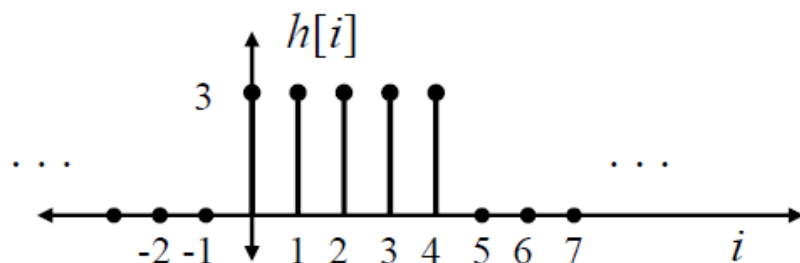
**Sum over  $i \Rightarrow$**

$$y[0] = 6$$

### Steps 3&4 for all $n < 0$

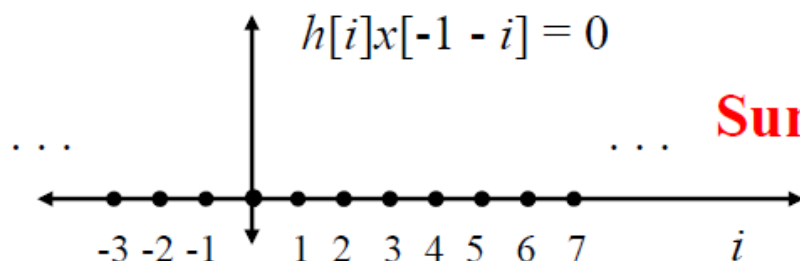
**Step 3:** For  $n < 0$ , shift by  $n$  to get  $x[n - i]$

Negative  $n$  gives a left-shift



Shown here for  $n = -1$

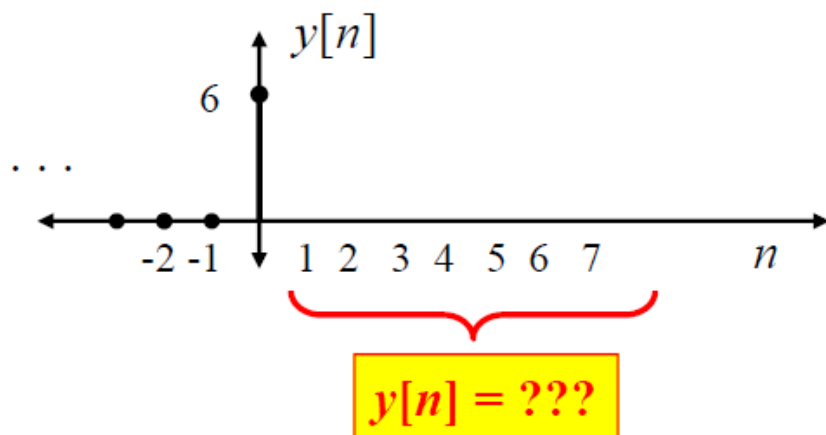
**Step 4:** For  $n < 0$ , Form the product  $x[i]h[n - i]$  and sum its elements to give  $y[n]$



Sum over  $i \Rightarrow y[n] = 0 \quad \forall n < 0$

So... what we know so far is that:

$$y[n] = \begin{cases} 0, & \forall n < 0 \\ 6, & n = 0 \end{cases}$$

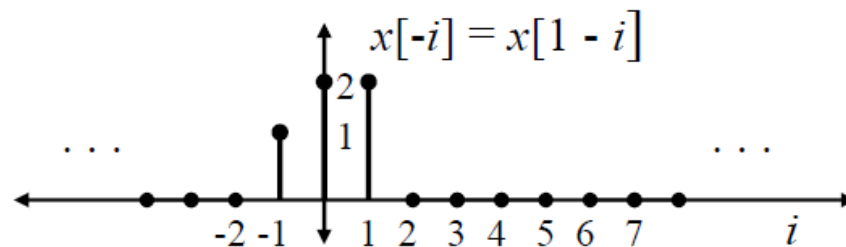
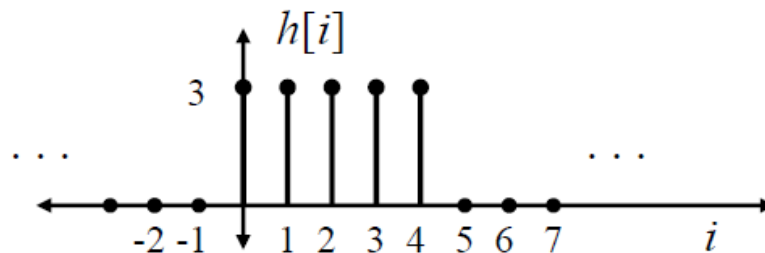


So now we have to do Steps 3&4 for  $n > 0$ ...

### Steps 3&4 for $n = 1$

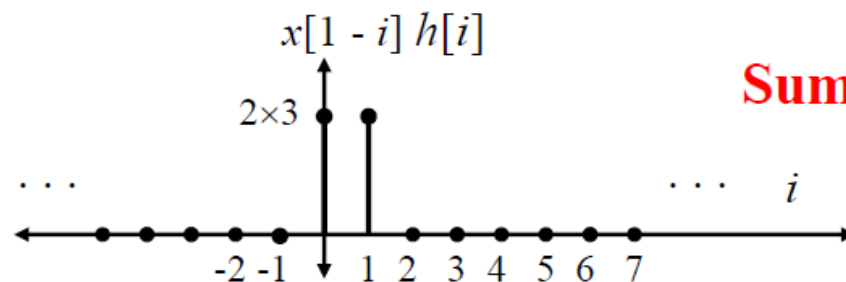
**Step 3:** For  $n = 1$ , shift by  $n$  to get  $x[n - i]$

Positive  $n$  gives a Right-shift



shifted to the right by one

**Step 4:** For  $n = 1$ , Form the product  $x[i]h[n - i]$  and sum its elements to give  $y[n]$

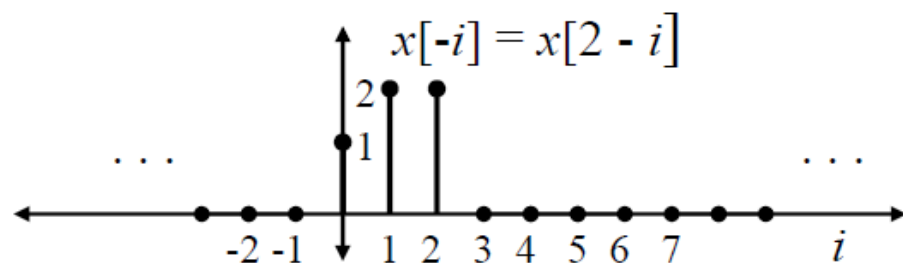
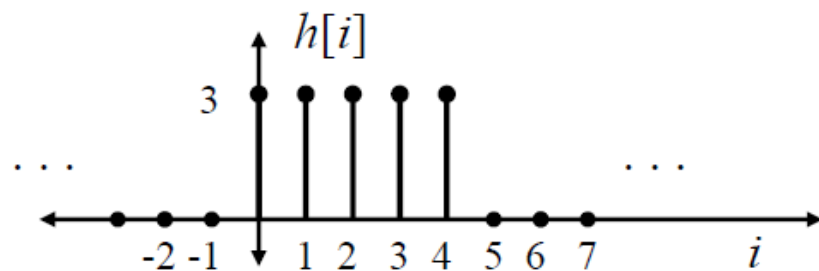


Sum over  $i \Rightarrow y[1] = 6 + 6 = 12$

## Steps 3&4 for $n = 2$

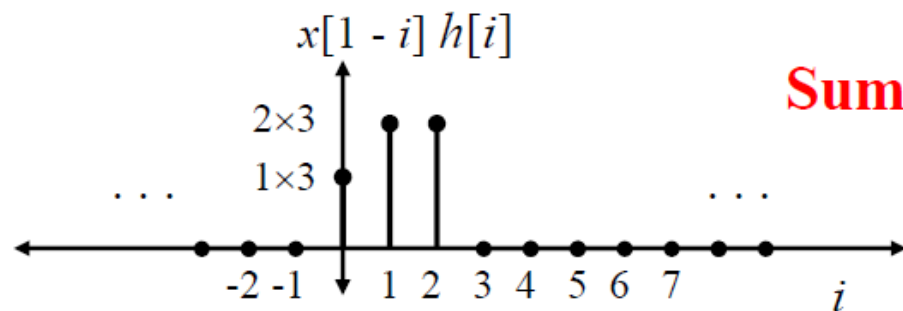
**Step 3:** For  $n = 2$ , shift by  $n$  to get  $x[n - i]$

Positive  $n$  gives a Right-shift



shifted to the right by two

**Step 4:** For  $n = 2$ , Form the product  $x[i]h[n - i]$  and sum its elements to give  $y[n]$



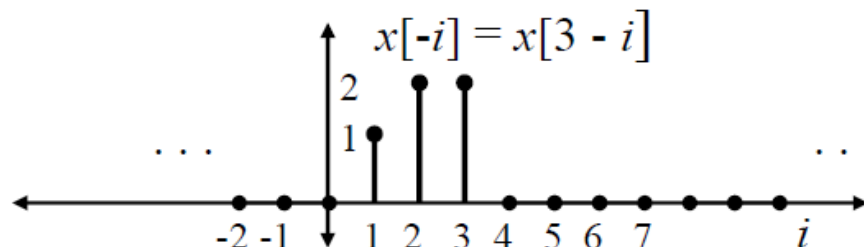
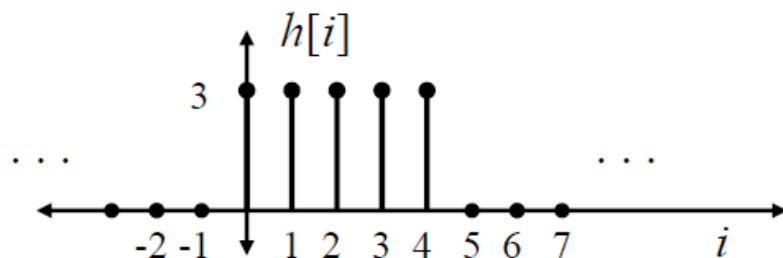
Sum over  $i \Rightarrow y[2] = 3 + 6 + 6 = 15$



### Steps 3&4 for $n = 3$

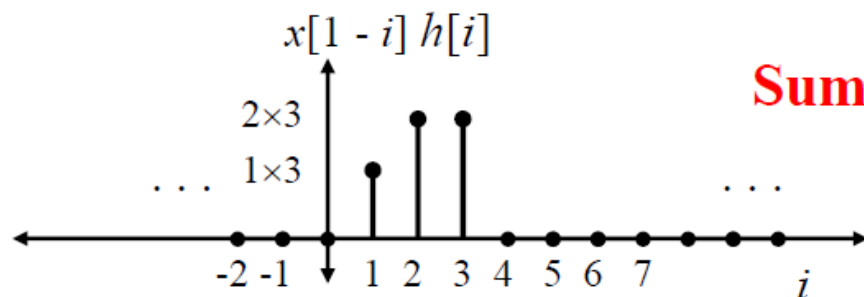
**Step 3:** For  $n = 3$ , shift by  $n$  to get  $x[n - i]$

Positive  $n$  gives a Right-shift



shifted to the  
right by three

**Step 4:** For  $n = 3$ , Form the product  $x[i]h[n - i]$  and sum its elements to give  $y[n]$

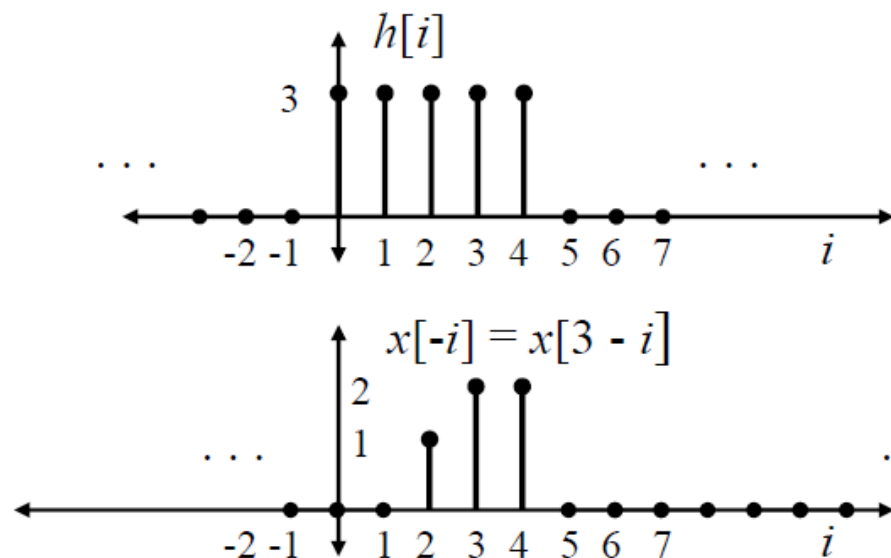


Sum over  $i \Rightarrow y[3] = 3 + 6 + 6 = 15$

## Steps 3&4 for $n = 4$

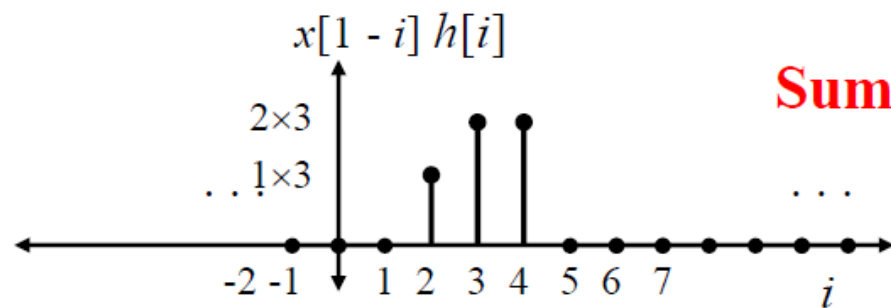
**Step 3:** For  $n = 4$ , shift by  $n$  to get  $x[n - i]$

Positive  $n$  gives a Right-shift



shifted to the right by four

**Step 4:** For  $n = 4$ , Form the product  $x[i]h[n - i]$  and sum its elements to give  $y[n]$

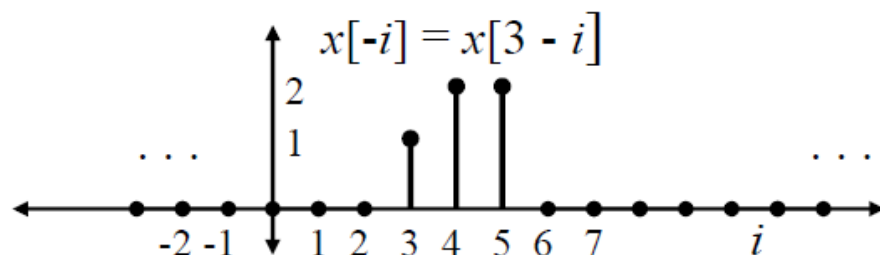
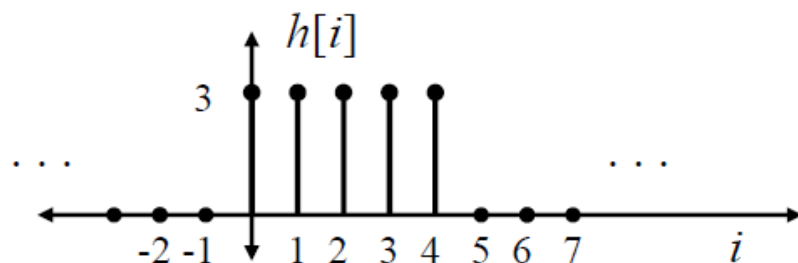


Sum over  $i \Rightarrow y[4] = 3 + 6 + 6 = 15$

### Steps 3&4 for $n = 5$

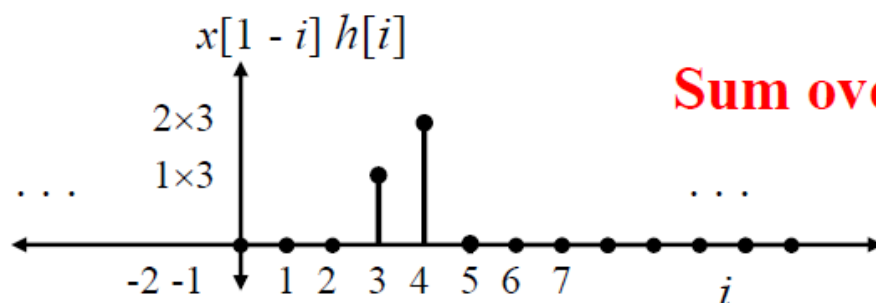
**Step 3:** For  $n = 5$ , shift by  $n$  to get  $x[n - i]$

Positive  $n$  gives a Right-shift



shifted to the  
right by five

**Step 4:** For  $n = 5$ , Form the product  $x[i]h[n - i]$  and sum its elements to give  $y[n]$

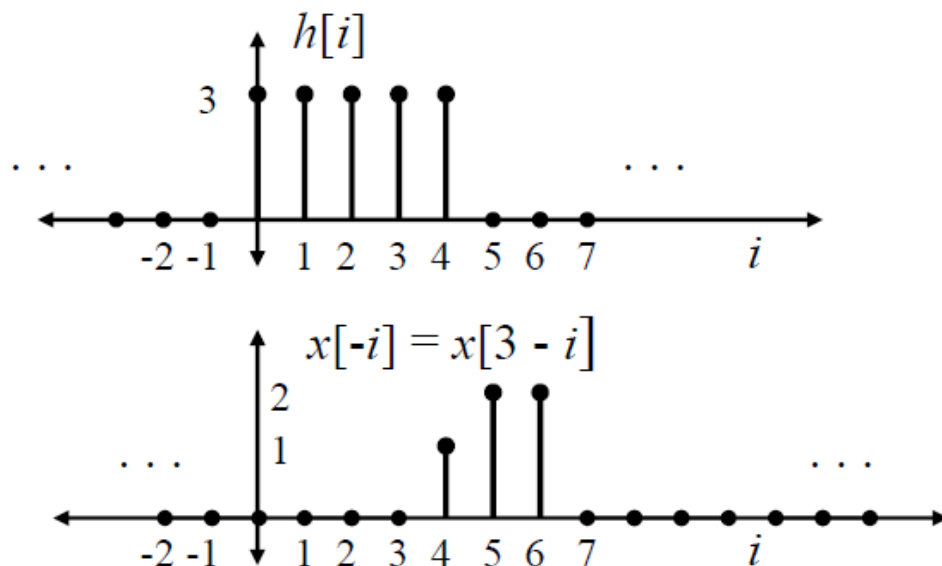


Sum over  $i \Rightarrow y[5] = 3 + 6 = 9$

### Steps 3&4 for $n = 6$

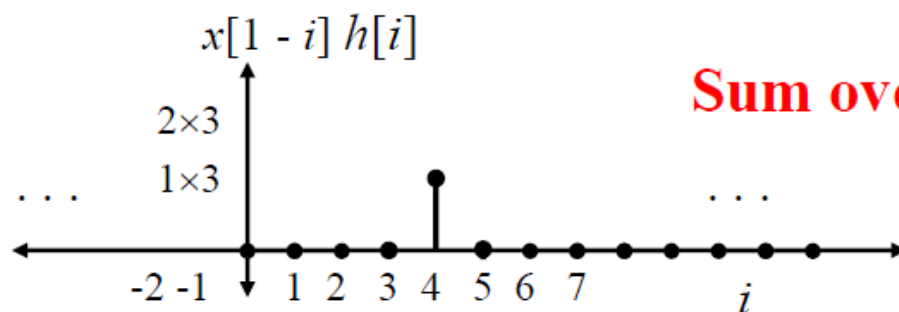
**Step 3:** For  $n = 6$ , shift by  $n$  to get  $x[n - i]$

Positive  $n$  gives a Right-shift



shifted to the  
right by six

**Step 4:** For  $n = 6$ , Form the product  $x[i]h[n - i]$  and sum its elements to give  $y[n]$

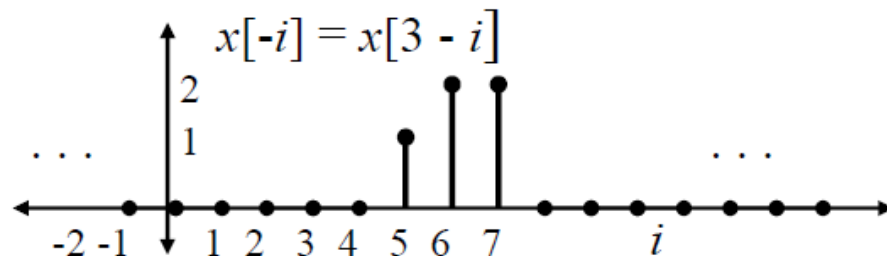
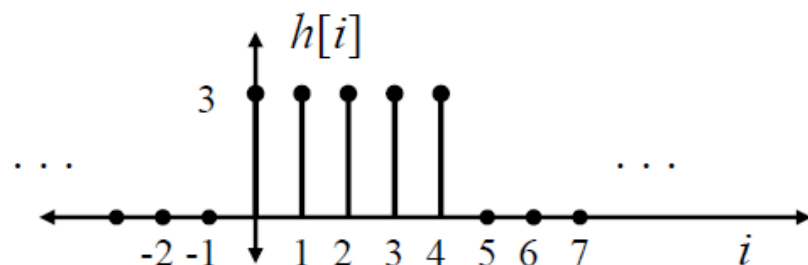


Sum over  $i \Rightarrow y[6] = 3$

## Steps 3&4 for all $n > 6$

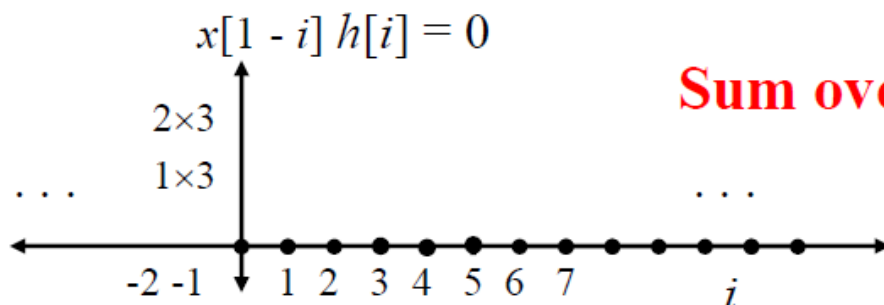
**Step 3:** For  $n > 6$ , shift by  $n$  to get  $x[n - i]$

Positive  $n$  gives a Right-shift



shifted to the  
right by seven

**Step 4:** For  $n > 6$ , Form the product  $x[i]h[n - i]$  and sum its elements to give  $y[n]$



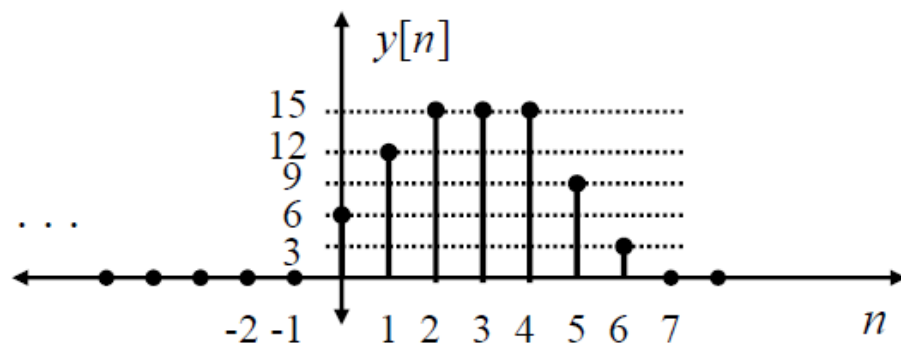
Sum over  $i \Rightarrow$

$$y[n] = 0 \quad \forall n > 6$$

So... now we know the values of  $y[n]$  for all values of  $n$

We just need to put it all together as a function...

Here it is easiest to just plot it... you could also list it as a table.



Note that convolving these kinds of signals gives a “ramp-up” at the beginning and a “ramp-down” at the end.

Various kinds of “transients” at the beginning and end of a convolution are common.