1) $P_0(-3,0,7)$'den geçen $\vec{n}=5\hat{i}+2\hat{j}-\hat{k}$'ya dik düzlem?

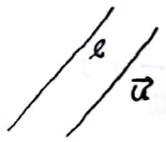
$$(x+3)\hat{i}+(y-0)\hat{j}+(z-7) \perp 5\hat{i}+2\hat{j}-\hat{k}$$

$$5(x+3)+2y+(7-z)=0$$

$$5x+2y-z+22=0$$

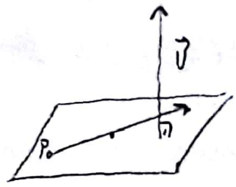
---

2) $P_0(2,1,4)$ noktasından geçen $l: \begin{cases} x=2+t \\ y=1+2t \\ z=3 \end{cases}$ doğrusuna dik düzlem?



$$\frac{x-2}{1}=\frac{y-1}{2}=\frac{z-3}{0}=t$$

$$\vec{u}=\hat{i}+2\hat{j}$$



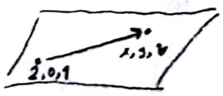
$$(x-2)\hat{i}+(y-1)\hat{j}+(z-4)\hat{k} \perp \hat{i}+2\hat{j}$$

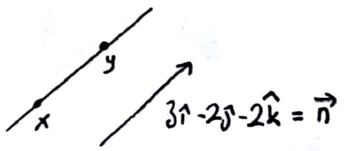
$$(x-2)+2(y-1)=0$$

$$x+2y-4=0$$

---

3) $(2,0,1)$'den geçen ve $x(1,1,0)$, $y(4,-1,-2)$ noktalarından geçen doğruya dik düzlem?





$$3(x-2)+(-2y)\hat{j}-2(z-1)\hat{k}$$

$$3y-2y-2z-4=0$$

4) $2x+3y+3z=6$ düzleminin normalini, düzlem üzerinde bir noktayı ve eksenlerin kesim noktası?

$\vec{n} = 2\hat{i} + 3\hat{j} + 3\hat{k}$

$x=y=1$, $z=1/3$

$x=0$ ...

---

5) $x-2y+5z-1=0$ düzlemi ile $\vec{r}(t) = \langle 2-t, 1+2t, t-1 \rangle$ doğrusu arasındaki ilişki?

$\frac{x-2}{-1} = \frac{y-1}{2} = \frac{z+1}{1} = t$ → doğrunun kanonik denklemi

$-\hat{i} + 2\hat{j} + \hat{k} = \vec{v}$ → doğruya paralel vektör.

$x-2y+5z-1=0$ düzleminin normali $\vec{n} = \hat{i} - 2\hat{j} + 5\hat{k}$

$\vec{n} \cdot \vec{v} = 0$, paraleller.

---

6) $3x-2y+z=2$ ve $x-y+3z=8$ düzlemlerinin arakesit doğrusunun parametrik denklemi!

$\vec{n_1} = 3\hat{i} - 2\hat{j} + \hat{k}$

$\vec{n_2} = \hat{i} - \hat{j} + 3\hat{k}$

$$\vec{n_3} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & -2 & 1 \\ 1 & -1 & 3 \end{vmatrix} = \hat{i}\begin{vmatrix} -2 & 1 \\ -1 & 3 \end{vmatrix} - \hat{j}\begin{vmatrix} 3 & 1 \\ 1 & 3 \end{vmatrix} + \hat{k}\begin{vmatrix} 3 & -2 \\ 1 & -1 \end{vmatrix}$$

$= -5\hat{i} - 8\hat{j} - \hat{k}$

arakesit üzerinde nokta → $x=1$, $y=2$, $z=3$

$\frac{x-1}{-5} = \frac{y-2}{-8} = \frac{z-3}{-1} = t$

$$l: \begin{cases} x = 1-5t \\ y = 2-8t \\ z = 3-t \end{cases}$$

$x = \frac{8}{3} + 2t$, $y = -2t$, $z = 1+t$ doğrusunun $3x+2y-6z-6=0$ düzlemi ile kesiştiği nokta?

$$3\left(\frac{8}{3}+2t\right) + 2.(-2t) + (1+t).-6 - 6 = 0$$

$t = -1$

$x = \frac{2}{3}$, $y = 2$, $z = 0$

---

8) $\vec{r}(t) = 3.\cos t\,\hat{i} + 3\sin t\,\hat{j} + t^2\hat{k}$ $\longrightarrow$ birim teğet vektörü?

$$\vec{t} = \frac{\frac{d\vec{r}}{dt}}{\left|\frac{d\vec{r}}{dt}\right|} = \frac{-3\sin t\,\hat{i} + 3\cos t\,\hat{j} + 2t\hat{k}}{\sqrt{9+4t^2}}$$

---

9) $x + \lambda y + 2z = 3$ ve $\lambda x + y - 2z = 1$ düzlemlerinin paralel olması için $\lambda = ?$

$\vec{n_1} = \hat{i} + \lambda\hat{j} + 2\hat{k}$

$\vec{n_2} = \lambda\hat{i} + \hat{j} - 2\hat{k}$

$\vec{n_1} \parallel \vec{n_2}$

$\hat{i} + \lambda\hat{j} + 2\hat{k} \parallel (\lambda\hat{i} + \hat{j} - 2\hat{k}).t$

$\frac{1}{\lambda} = \frac{\lambda}{1} = -1 = t$ ise $\lambda = -1$

---

10) $\lambda x - y + 3z = 1$ ve $2x + 3y - \lambda z = 4$ düzlemlerinin dik olması için $\lambda = ?$

$\vec{n_1} = \lambda\hat{i} - \hat{j} + 3\hat{k}$

$\vec{n_2} = 2\hat{i} + 3\hat{j} - \lambda\hat{k}$

, $\vec{n_1} \perp \vec{n_2} \Longrightarrow \vec{n_1}.\vec{n_2} = 0$

$2\lambda - 3 - 3\lambda = 0$

$-\lambda = -3$

11) $x-y+2z=3$ ve $x-y+3z=8$ ile verilen düzlemlerin paralel olmadığını göster ve arakesit doğrusunu parametrize et.

$\vec{n}_1 \parallel \vec{n}_2$?

$\vec{n}_1 = \hat{i}-\hat{j}+2\hat{k}$ $\longrightarrow$ $\vec{n}_1 \parallel \vec{n}_2$ için $\rightarrow (\hat{i}-\hat{j}+2\hat{k}) \parallel t.(\hat{i}-\hat{j}+3\hat{k})$

$\vec{n}_2 = \hat{i}-\hat{j}+3\hat{k}$

$1 = 1 = \frac{2}{3} = t \longrightarrow$ yanlış olduğundan paralel değildir.

$\vec{U} \perp (\vec{n}_1 \times \vec{n}_2)$

$$\vec{n}_1 \times \vec{n}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -1 & 2 \\ 1 & -1 & 3 \end{vmatrix} = \hat{i}\begin{vmatrix} -1 & 2 \\ -1 & 3 \end{vmatrix} - \hat{j}\begin{vmatrix} 1 & 2 \\ 1 & 3 \end{vmatrix} + \hat{k}\begin{vmatrix} 1 & -1 \\ 1 & -1 \end{vmatrix}$$

$$= -\hat{i}-\hat{j}$$

arakesit doğrusundan bir noktası

$x=0$

$-y+2z=3$
$-y+3z=8$

$z=5$
$y=7$
$x=0$

$\rightarrow \frac{x-0}{-1} = \frac{y-7}{-1} = \frac{z-5}{0} = t \rightarrow$ Kanonik denklem

$$l_8 \begin{cases} x = -t \\ y = 7-t \\ z = 5 \end{cases}$$

a) $L_1: r_1(t) = \langle t, 1+t, 0 \rangle$

$L_2: r_2(s) = \langle 1, s, 0 \rangle$

doğruların kesiştiği nokta ve aralarındaki açı?

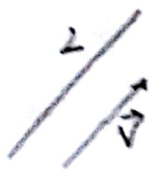
$t=1$
$s=2 \longrightarrow (1,2,0)$

$L_1: \left.\begin{array}{l} x=t \\ y=1+t \\ z=0 \end{array}\right\} \dfrac{x-0}{1} = \dfrac{y-1}{1} = \dfrac{z-0}{0} = t \longrightarrow \hat{i} + \hat{j}$

$L_2: \left.\begin{array}{l} x=1 \\ y=s \\ z=0 \end{array}\right\} \dfrac{x-1}{0} = \dfrac{y-0}{1} = \dfrac{z-0}{0} = s \longrightarrow \hat{j}$

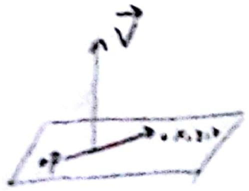
$$\cos\theta = \frac{\vec{a}\cdot\vec{b}}{|\vec{a}|\cdot|\vec{b}|} = \frac{1}{\sqrt{2}}, \quad \theta = \frac{\pi}{4}$$

B) $P(1,2,3)$ noktasından geçen ve $L: x = 1-t,\ y = 1+3t,\ z = 2t$ doğrusuna dik olan düzlemin denklemi



L'nin kanonik denklemi $\rightarrow \dfrac{x-1}{-1} = \dfrac{y-1}{3} = \dfrac{z-0}{2} = t$

paralel vektör $\rightarrow \vec{V} = -\hat{i} + 3\hat{j} + 2\hat{k}$

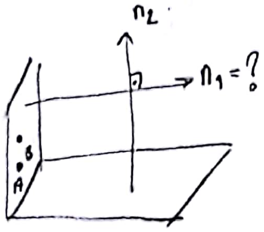


$(x-1)\hat{i} + (y-2)\hat{j} + (z-3)\hat{k} \perp \vec{V}$

$1 - x + 3y - 6 + 2z - 6 = 0$

$-x + 3y + 2z - 11 = 0$

14) $A(1,1,1)$ ve $B(2,0,3)$ noktalarından geçen ve $x+2y-3z=0$ düzlemine dik olan düzlemin denklemi =?



$$\vec{n_2} = \hat{i}+2\hat{j}-3\hat{k}$$

$$(2-1)\hat{i}+(0-1)\hat{j}+(3-1)\hat{k} \parallel t\left((x-1)\hat{i}+(y-1)\hat{j}+(z-1)\hat{k}\right)$$

$$\frac{x-1}{1} = \frac{y-1}{-1} = \frac{z-1}{2} = t$$

$$\vec{V} = \hat{i}-\hat{j}+2\hat{k}$$

$$(\vec{n_2} \times \vec{V}) = \vec{n_1}$$

$$n_2 \times \vec{V} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -1 & 2 \\ 1 & 2 & -3 \end{vmatrix} = -\hat{i}+5\hat{j}+3\hat{k} = \vec{n_1}$$

$$(x-1)\hat{i}+(y-1)\hat{j}+(z-1)k \perp \vec{n_1}$$

$$1-x+5y-5+3z-1=0$$

---

15) $x-z=1$, $y+2z=3$ düzlemlerinin arakesit doğrusunu içeren ve $x+y-2z=1$ düzlemine dik olan bir üçüncü düzlem denklemi?

$\vec{n_1} = \hat{i}-\hat{k}$

$\vec{n_2} = \hat{j}+2\hat{k}$

$$\rightarrow \vec{n_3} = \vec{n_1}\times\vec{n_2} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 0 & -1 \\ 0 & 1 & 2 \end{vmatrix} = \hat{i}\begin{vmatrix} 0 & -1 \\ 1 & 2 \end{vmatrix} - \hat{j}\begin{vmatrix} 1 & -1 \\ 0 & 2 \end{vmatrix} + \hat{k}\begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix}$$

$$= \boxed{\hat{i}-2\hat{j}+\hat{k}}$$

$x=1, x=2$

$y=3, y=1$

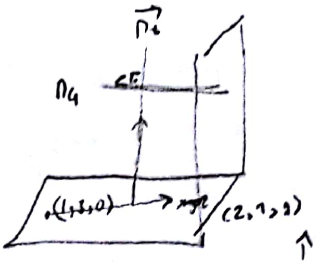
$z=0, z=1$

$$\frac{x-1}{1} = \frac{y-3}{-2} = \frac{z-0}{1} = t$$

$$l: \begin{cases} x = t+1 \\ y = 3-2t \\ z = t \end{cases}$$ arakesit doğrusu

$(1,3,0)$

$x+y-2z$ dik



$$\vec{n_4} = \hat{i}+\hat{j}-2\hat{k}$$

$$\hat{i}-2\hat{j}+\hat{k} = \vec{n_5}$$

$$\vec{n_6} = \vec{n_4}\times\vec{n_5} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & -2 \\ 1 & -2 & 1 \end{vmatrix} = \hat{i}\begin{vmatrix} 1 & -2 \\ -2 & 1 \end{vmatrix} - \hat{j}\begin{vmatrix} 1 & -2 \\ 1 & 1 \end{vmatrix} + \hat{k}\begin{vmatrix} 1 & 1 \\ 1 & -2 \end{vmatrix}$$

$$\boxed{x+y+z-4=0}$$

$$-3(x-1)+(y-3)-3+z-3$$

$$\boxed{-3x-3y-3z+12=0} = -3\hat{i}-3\hat{j}-3\hat{k}$$