### Solution: Removing leaves from BN20

- We have

$$
\begin{aligned}
p(z_{1:3}|x_1, x_2, x_4) &\propto \sum_{x_3} \sum_{x_5} p(z_{1:3}, x_{1:5}) && (1)\\
&= p(z_{1:3}) p(x_1|z_{1:3}) p(x_2|z_{1:3}) p(x_4|z_{1:3}) \left[\sum_{x_3} p(x_3|z_{1:3})\right] \left[\sum_{x_5} p(x_5|z_{1:3})\right] && (2)\\
&= p(z_{1:3}) p(x_1|z_{1:3}) p(x_2|z_{1:3}) p(x_4|z_{1:3}) && (3)
\end{aligned}
$$

  since $\sum_{x_3} p(x_3|z_{1:3}) = 1$. and $\sum_{x_5} p(x_3|z_{1:3}) = 1$. Note that we cannot remove hidden "leaves" from a UGM, since potentials do not necessarily sum to one locally.

- See (Jaakkola and Jordan, 1999) for the details.

## References


Jaakkola, T. and M. Jordan (1999). Variational probabilistic inference and the QMR-DT network. *J. of AI Research 10*, 291–322.