A New FastICA Algorithm for Blind Source Convolutive Mixtures

YunPeng Li

***Abstract*—In this paper,we introduce a modification of FastICA algorithm to extract source signals from the convolutive mixtures.The time domain algorithm employs a convolutive prewhitening stages as the replace of popular spatio-temporal prewhitening. The proposed contrast-function method utilizes the nonstationarity property of the source signals via second-order statistics and imposes the diagonalization constraints instead of the complicate para-unitary constraints to guarantee the uniqueness.The original source signals' contributions on each observation are acquired in a reconstruction procedure.Our algorithm enjoys the fast convergence,robustness as well as simplicity.Numerical simulations are implemented to verify the performance of the proposed method.**

***Index Terms*—FastICA,blind source separation(BSS), convolutive mixtures, decorrelation, singular value decomposition(SVD).**

I. Introduction

THE goal of blind source seperation is to recover m sources $\mathbf{s}(k) = (s_1(k), \cdots, s_m(k))^T$ from n observed mixtures $\mathbf{x}(k) = (x_1(k), \cdots, x_n(k))^T$ with N samples only,where the mixture parameters or the mixing characteristics are unknown.This problem can be distinguished by the mixing prcess $\mathbf{A}$ into two classes: instantaneous BSS and convolutive BSS.Instantaneous BSS or independent component analysis(ICA)[1] assumes the sources are mixed only in space,without any time delay or multipath reflection.FastICA[2] is one of the most efficient algorithms to the problem of ICA for its fast convergence and robust for outliers. The algorithm maximize entropy approximations of differential entropy as contrast functions along with a orthogonal constrains to guarantee uniqueness,after the prewhitening preprocessing.

Convolutive BSS arises in several signal processing problems,such as sonar array processing,seismic exploration and the "cocktail party problem". In convolutive BSS,the mixing procedure becomes more complex as concequence of time delays or multipath reflections.A multi-input/multi-output(MIMO) linear time invariant (LTI) system with impulse response $(\mathbf{A}(k)_{k\in Z})$ is used to describe the convolutive mixing process:

$$\mathbf{x}(k) = \mathbf{A}(k) * \mathbf{s}(k) = \sum_{l\in Z} \mathbf{A}(l)\mathbf{s}(k-l) \tag{1}$$

Recovering the estimated sources $\mathbf{y}(k) = (y_1(k), \cdots, y_m(k))^T$ from the convolutive mixtures $\mathbf{x}(k)$ is equivalent to finding a MIMO-LTI filter banks $(\mathbf{B}(k)_{k\in Z})$ to inverse the mixing system $(\mathbf{A}(k)_{k\in Z})$:

$$\mathbf{y}(k) = \mathbf{B}(k) * \mathbf{x}(k) = \sum_{l\in Z} \mathbf{B}(l)\mathbf{x}(k-l) \tag{2}$$

Each $y_i(k)$ is a scaling and filtering version of uniquely original source $s_{i'}(k)$ due to the ambiguities in ordering,scaling and filtering.The ordering and scaling ambiguity also exists in the instantaneous mixture,while the filtering ambiguity is the feature of the convolutive mixtures.These ambiguities lead to lots of local extremum points,making the convolutive BSS a difficult problem.

In this paper,we shall assume that:

1) H1 :The source signals $\mathbf{s}(k)$ are real-valued,zero-mean and are mutually statistically independent,at most one of the signals $s_i(k)$ is gaussian.
2) H2 :The filter banks $(\mathbf{A}(k)_{k\in Z})$,$(\mathbf{B}(k)_{k\in Z})$ are stable ,causal and finite impulse response(FIR).

Many Methods[3][4] have been proposed to solve the convolutive BSS problem.In frequency domain[5][6],convolutive BSS can be considered as instantaneous BSS for each frequency bin,each bin has own scaling and ordering indeterminacy as mentioned before.Complex value after discrete Fourier transform (DFT) and circularity problem happen in frequency domian.

Time domain algorithms can be cassified into density-matching approaches and contrast-function approaches.Density-matching methods apply the well-known InfoMax[7] approach to ICA into convolutive case[8]. The performance of the density-matching approaches depend on the prior knowledge on the selected pdf $p_s(s_i)$ of the under lying source $s_i(k)$.It's important to determine wether the source $s_i(k)$ has super-gaussian or sub-gaussian pdf.Density-approach approaches linearly transform the observed mixtures $\mathbf{x}(k)$ with the demixing system $(\mathbf{B}(k)_{k\in Z})$,making the $p_y(y_i)$ closely matches a selected model density $p_s(s_{i'})$.Most of those methods are based on the gradient optimization ,requiring appropriate learning rate and step direction choice.Natural gradient[9] and relative gradient[10] have been proposed to alleviate the drawbacks of stability and convergence above.Contrast-function approaches make the advantage of the statistical independence and the non-gaussianity of the source signals,whose maximization achieves the seperation.High order statistics(HOS)[11] of signal source $y_i(k)$ like cumulants, cross-cumulants,and cross-moments can be used as contrast-functions to optimize.Prewhitening stage and coefficents constraints are required to guarantee the

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uniqueness of the extracted components during the contrast-function approaches.For instantaneous case ,prewhitening stage is a spatial prewhitening process,which can be conducted by singular value decomposition. Numerical procedures like Gram–Schmidt orthogonalization or singular value decomposition can be used to preserve the orthogonal constraints for uniqueness.While in the convolutive mixtures,these two stages become more difficult tasks.

FastICA is a contrast-based algorithm to extract the sources from the mixtures via maximizing entropy approximations of differential entropy as contrast function.The method works well in instantaneous case for its fast convergence and robustness.It cannot directly be adapted to the convolutive BSS for the reason above.Several extensions have been proposed[12][13]. In [12],the algorithm conducts a convolutive prewhitening to the transformed original observation $\mathbf{x}(k)$ at first,then substracts the extracted estimated signals at previous steps from the mixtures in a deflation mode,leading to an accumulation of estimation errors which may become excessive after a certain number of source extractions.In [13],after a spatio-temporal prewhitening stage,the algorithm adapts a paraunitary constraints to ensure components' uniqueness ,the two stages above are both complicated and difficult.

In this paper we propose an extension of FastICA to convolutive BSS in convolutive mixtures,under the similar framework of original optimization problem in paper[2].The proposed time domain contrast-function approach combines a convolutive prewhitening stage for convolutive observations with fixed-point iterations under the diagonalization constraints in both deflation or symmetric mode. This method makes the advantgge of FastICA framework to guarantee the convegrence and robustness,without any particular step sizes to be selected. Unlike the frequency domain approaches ,our algorithm avoids the difficult permutation task.Compared with other contrast-function approaches in convolutive BSS[12][13],the unique feature of the proposed method is the straightforward and easy-implemented stages to guarantee the unqiueness of the extracted components.For the purpose of individually reconstructing the source signals' contributions in each observation,a regression routine is given.

The paper is organized as follows.In Section II,we state several assumptions for our method and some variables to rearrange the convolutive mixtures into instantaneous mixtures.An optimization problem is given based on the original FastICA in Section III. We describe the prewhitening strategy along with the diagonalization constraints,then we reconstruct the original source signals' contributions in Section IV.Experiments' results are presented to verify the new algorithm in Section V.We conclude our method in Section VI.

II. Problem statement

According to assumption H2,the mixing system can be described by the following FIR filter equation.

$$x_i(k) = \sum_{j=1}^{m}\sum_{l=0}^{P-1} a_{ij}(l)s_j(k-l) \qquad i=1,\cdots,n \tag{3}$$

where the $\mathbf{a}_{ij}$ are the mixing filters.Without loss of generality,we assume all the mixing filters have the same filter order P.

The outputs of the demixing $y_i(k)$ have the same form.

$$y_i(k) = \sum_{j=1}^{n}\sum_{l=0}^{Q-1} b_{ij}(l)x_j(k-l) \qquad i=1,\cdots,m \tag{4}$$

where the $\mathbf{b}_{ij}$ are the demixing filters,we assume all the demixing filters have the same filter order $Q.y_i(k)$ is a unique scaled,permuted,filtered vesion of$s_{i'}(k)$.

To reconstruct the contributions,another assumption is considered in our new algorithm:

1) H3 :Each signal source $s_i(k)$ is produced by an innovation process $u_i(k)$ via a stable FIR filters $F_i(k)$,where $u_i(k)$ is zero-mean,mutually independent and non-gaussian random process.

Under the above assumption,signal source $s_i(k)$ can be expressed in the following form.

$$s_i(k) = \sum_{l=-R+1}^{R-1} F_i(l)u_i(k-l) \qquad i=1,\cdots,m \tag{5}$$

The noncausal F_i FIR order is $2R$-1,all the filter banks $\mathbf{F}(k) = diag(F_1(k),\cdots,F_m(k))$ for innovation process $\mathbf{u}(k) = (u_1(k),\cdots,u_m(k))^T$ have the same order.In order to individually extract single source in $\mathbf{s}(k)$,we regard the $\mathbf{F}(k)$ as coloring filters for corresponding innovation process $\mathbf{u}(k)$.

In order to express the convolutive case more concise,we consider the column vector.

$$\underline{\mathbf{s}}(k) = \left(\mathbf{s}_1^T(k),\mathbf{s}_2^T(k),\cdots,\mathbf{s}_m^T(k)\right)^T \tag{6}$$

$$\mathbf{s}_i(k) = (s_i(k),s_i(k-1),\cdots,s_i(k-P+1))^T \tag{7}$$

where $\underline{\mathbf{s}}(k)$ is a $mP\times 1$ column vector,and $\mathbf{s}_i(k)$ is a $P\times 1$ column vector,the corresponding mixing system can be expressed in the matrix form.

$$\mathbf{A} = \begin{pmatrix} \mathbf{a}_{11}^T & \mathbf{a}_{12}^T & \cdots & \mathbf{a}_{1m}^T \\ \mathbf{a}_{21}^T & \mathbf{a}_{22}^T & \cdots & \mathbf{a}_{2m}^T \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{a}_{n1}^T & \mathbf{a}_{n2}^T & \ldots & \mathbf{a}_{nm}^T \end{pmatrix} \tag{8}$$

$$\mathbf{a}_{ij} = (a_{ij}(0),a_{ij}(1),\cdots,a_{ij}(P-1))^T \tag{9}$$

where $\mathbf{A}$ is a $n\times mP$ matrix,and $\mathbf{a}_{ij}$ is $P\times 1$ column vector,then the mixing system (3) can be described below

$$\mathbf{x}(k) = \mathbf{A}\underline{\mathbf{s}}(k) \tag{10}$$

Demixing procedure can be described in the same way.

$$\underline{\mathbf{x}}(k) = \left(\mathbf{x}_1^T(k),\mathbf{x}_2^T(k),\cdots,\mathbf{x}_n^T(k)\right)^T \tag{11}$$

$$\mathbf{x}_i(k) = (x_i(k),x_i(k-1),\cdots,x_i(k-Q+1))^T \tag{12}$$

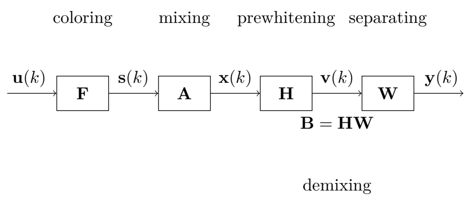


Fig. 1. Flow chart of the convolutive BSS.

where $\underline{\mathbf{x}}(k)$ is a $nQ \times 1$ column vector,and $\mathbf{x}_i(k)$ is a $Q \times 1$ column vector.

$$\mathbf{B} = \begin{pmatrix} \mathbf{b}_{11}^T & \mathbf{b}_{12}^T & \cdots & \mathbf{b}_{1n}^T \\ \mathbf{b}_{21}^T & \mathbf{b}_{22}^T & \cdots & \mathbf{b}_{2n}^T \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{b}_{m1}^T & \mathbf{b}_{m2}^T & \dots & \mathbf{b}_{mn}^T \end{pmatrix} \tag{13}$$

$$\mathbf{b}_{ij} = (b_{ij}(0), b_{ij}(1), \cdots, b_{ij}(Q-1))^T \tag{14}$$

where $\mathbf{B}$ is a $m \times nQ$ matrix,and $\mathbf{b}_{ij}$ is $Q \times 1$ column vector,the equation below describes the demixing procedure (4).

$$\mathbf{y}(k) = \mathbf{B}\underline{\mathbf{x}}(k) \tag{15}$$

Owing to the concise expressions of the mixing(10) and demixng(15) ,we have converted the m signals n observations covolutive BSS into a m signals nQ observations instantaneous BSS problem,for the sake of uniqueness in extraction,a prewhitening stage is required in contrast-function approaches.

$$\mathbf{v}(k) = \mathbf{H}(\mathbf{x}(k)) \tag{16}$$

we represent the prewhitening stage in function $\mathbf{H}()$,and the prewhitening ouput $\mathbf{v}(k)$ complies with some constraint.In instantaneous case,the particular step can be conducted via eigenvalue decomposition or PCA.

$$\mathbf{v}(k) = \mathbf{H}\mathbf{x}(k) \tag{17}$$

where $\mathbf{v}(k)$ conforms to below constraint.

$$\boldsymbol{E\{\mathbf{v}(k)\mathbf{v}^T(k)\} = \mathbf{I}_{n\times n}} \tag{18}$$

where $\mathbf{I}$ is the identity matrix.Unfortunately the above prewhitening method directly fails in the convolutive case in the sense of uniqueness,more details are discussed in Section IV.

$$\mathbf{y}(k) = \mathbf{B}\underline{\mathbf{x}}(k) = \mathbf{W}\mathbf{H}\underline{\mathbf{x}}(k) = \mathbf{W}\mathbf{v}(k) \tag{19}$$

After prewhitening stage, our method (19) adjusts coefficients in separation matrix $\mathbf{W}$ to recover the $y_i(k)$ as an estimation of a delayed scaled innovation process $u_{i'}(k-l)$.

Under several assumptions,the complete routine of the convolutive BSS problem can be concluded in Fig.1.The signal $\mathbf{s}(k)$ is produced by the innovation process $\mathbf{u}(k)$ through the coloring filters $\mathbf{F}$.Unknown mixing system $\mathbf{A}$ mixes the signal both in time and space.Given the observation $\mathbf{x}(k)$,the proposed algorithm conducts a demixing procedure $\mathbf{B}$ to ouput $\mathbf{u}(k)$'s scaled and filtered version $\mathbf{y}(k)$,which can be used to reconstruct the original contributions of $s_i(k)$ in each observation.The prewhitening $\mathbf{H}$ stages and seperating $\mathbf{W}$ stages are the keys in the demixing system,which avoid extracting same or equivalent solutions.

III. FastICA extension

In the instantaneous case ,FastICA applied to the observation looks for a sequence of orthogonal projections to maximize the negentropy $J(y_i)$ [14].With the prewhitened data,this amounts to seek components as independent as possible.

$$\begin{aligned} J(y_i) &= H(z_i) - H(y_i) \\ &\approx c\left[E\{G(y_i)\} - E\{G(z_i)\}\right]^2 \end{aligned} \tag{20}$$

where $J(y_i)$ is the negentropy,$H()$ is the random variable's entropy,c is an irrelevant constant,$G()$ is any non-quadratic function, z_i is a Gaussian random variable with the same variance as y_i. Mututal information $I(\mathbf{y})$ between the components of the random variable vector $\mathbf{y}(k)$ is a nature measure of dependence,since it is always non-negative and becomes zero only when the components are statistically independent.

$$\begin{aligned} I(\mathbf{y}) &= \sum_{i=1}^{m} H(y_i) - H(\mathbf{y}) \\ &= \sum_{i=1}^{m} H(y_i) - H(\mathbf{x}) - log\left|det(\mathbf{B})\right| \\ &= C - \sum_{i=1}^{m} J(y_i) \end{aligned} \tag{21}$$

where C is an irrelevant constant,the minimization of the mutual information $I(\mathbf{y})$ for independence (under the constraint of decorrelation) is equivalent to the maximization of the sum of the negentropies of the components $\sum_{i=1}^{m} J(y_i)$,and the original FastICA in instantaneous can be modeled as the optimization problem[2] below.

$$\begin{aligned} \max \quad & \sum_{i=1}^{m} J(y_i) \\ \text{s.t.} \quad & E\{y_i(k)y_j(k)\} = \delta_{ij}, \qquad i,j = 1,2,\ldots,m\,. \end{aligned} \tag{22}$$

where δ_{ij} is the item in identity matrix $\mathbf{I}_{m\times m}$. The objective function in (22) aims at the independence.In both the deflation and symmetric mode,the optimization problem in (22) is divided as single maximization of $J(y_i)$,and there are $2m$ extremums to this problem due to the ordering and scaling ambiguities in instantaneous case,so the constraint in (22) is designed to avoiding extracting the same solution for each extraction.

While in the convolutive case ,the filtering ambiguity introduces much more extremums as the increasing of the demixing filters $\mathbf{B}$'s filter order Q,resulting in the difficulty for avoiding extracting the same or equivalent solution.It's nature to adjust the constraint in (22) to derive the optimization problem in convolutive case.

$$\begin{aligned} \max \quad & \sum_{i=1}^{m} J(y_i) \\ \text{s.t.} \quad & E\{y_i(k)y_i(k)\} = 1, \qquad\qquad i = 1,2,\ldots,m\,, \\ & E\{y_i(k)y_j(k-l)\} = 0, \qquad i \neq j, -\infty < l < +\infty\,. \end{aligned} \tag{23}$$

The first constraint in (23) is for the scaling ambiguity,and the second constraint is designed to tack the ordering and filtering ambiguities.In order to simplify the second constraint,we choose $-L \le l \le L$,where L is a positive large enough integer.Prewhitening stage and diagonalization constraints in our proposed algorithm are conducted to satisfy these constraints above.

IV. Prewhitening and diagonalization constraints

Many contrast-function approaches for convolutive mixtures have a prewhiteing stage (16) in space and time as follow[3][13].

$$E\{\mathbf{v}(k)\mathbf{v}^T(k-l)\} = \delta_l \mathbf{I}_{n\times n} \quad \forall l \in Z \tag{24}$$

The whitening filter $\mathbf{H}$ is not unique and hard to determine,the contrast functions are required to be optimized under the constraint of complicate para-unitary filters $\mathbf{W}$,several methods have been proposed to solve such probelm[15][16].

We conduct a same prewhitening stage in instantaneous case[5] on $\underline{\mathbf{x}}(k)$ to produce the $nQ\times 1$ column vector $\mathbf{v}(k)$,and this procedure is regarded as a convolutive prewhitening.

$$E\{\mathbf{v}(k)\mathbf{v}^T(k)\} = \mathbf{I}_{nQ\times nQ} \tag{25}$$

After the above preprocessing stage,the main effort is to adjust the seperating filters $\mathbf{W}$ to solve the problem (23) in the form below.

$$\begin{aligned} \max \quad & \sum_{i=1}^{m} J(\mathbf{w}_i^T \mathbf{v}(k)) \\ \text{s.t.} \quad & \mathbf{w}_i^T\mathbf{w}_i = 1, && i = 1,2,\ldots,m, \\ & \mathbf{w}_i^T E\{\mathbf{v}^T(k)\mathbf{v}^T(k-l)\}\mathbf{w}_j = 0, && i \ne j, -L \le l \le L. \end{aligned} \tag{26}$$

where $\mathbf{w}_i^T$ is the ith row of $m\times nQ$ matrix $\mathbf{W}$.In pursuit of particular $\mathbf{w}_i$,the following iteration routine is carried out until convergence.

$$\begin{aligned} \mathbf{w}_i &= E\{\mathbf{v}(k)g(\mathbf{w}_i^T\mathbf{v}(k))\} - E\{g^{'}(\mathbf{w}_i^T\mathbf{v}(k))\}\mathbf{w}_i \\ \mathbf{w}_i &= \mathbf{w}_i/\|\mathbf{w}_i\|_2 \end{aligned} \tag{27}$$

where $g()$ is the derivative of particular non-quadratic function $G()$,coefficients constraints are required during the process of (27) in deflation and symmetric mode.

The validity of constraint in (23)(26) can also supported from [5][17] the fact:

If the source signals have unique temporal structures or nonstationary,simultaneous diagonalization of ouput correlation matrices over multiple time lags can seperate the independent sources from convolutive mixtures.

$$\begin{aligned} \mathbf{R}_y(\tau) &= E\{\mathbf{y}(k)\mathbf{y}^T(k-\tau)\} \\ &= \mathbf{W}E\{\mathbf{v}(k)\mathbf{v}^T(k-\tau)\}\mathbf{W}^T \\ &= \mathbf{W}\mathbf{R}_v(\tau)\mathbf{W}^T \end{aligned} \tag{28}$$

The output correlation matrix at time lag τ is represented as $\mathbf{R}_y(\tau)$,it is required to be a diagonal matrix,particularly,$\mathbf{R}_y(0)$ is the identity matrix $\mathbf{I}_{m\times m}$.For more intuitive explanation,we define several column vectors.

$$\underline{\mathbf{y}}(k) = \left(\mathbf{y}_1^T(k), \mathbf{y}_2^T(k), \cdots, \mathbf{y}_m^T(k)\right)^T \tag{29}$$

$$\mathbf{y}_i(k) = (y_i(k+L), y_i(k+L-1), \cdots, y_i(k-L))^T \tag{30}$$

where $\underline{\mathbf{y}}(k)$ is a $m(2L+1)\times 1$ column vector,and $\mathbf{y}_i(k)$ is a $(2L+1)\times 1$ column vector,the equivalent expression in (28) can be described below.

$$\begin{aligned} \mathbf{R}_{\underline{\mathbf{y}}} &= E\{\underline{\mathbf{y}}(k)\underline{\mathbf{y}}^T(k)\} \\ &= \begin{pmatrix} \mathbf{R}_{\mathbf{y}_1\mathbf{y}_1} & \mathbf{R}_{\mathbf{y}_1\mathbf{y}_2} & \cdots & \mathbf{R}_{\mathbf{y}_1\mathbf{y}_m} \\ \mathbf{R}_{\mathbf{y}_2\mathbf{y}_1} & \mathbf{R}_{\mathbf{y}_2\mathbf{y}_2} & \cdots & \mathbf{R}_{\mathbf{y}_2\mathbf{y}_m} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{R}_{\mathbf{y}_m\mathbf{y}_1} & \mathbf{R}_{\mathbf{y}_m\mathbf{y}_2} & \ldots & \mathbf{R}_{\mathbf{y}_m\mathbf{y}_m} \end{pmatrix} \end{aligned} \tag{31}$$

$$\mathbf{R}_{\mathbf{y}_i\mathbf{y}_j} = E\{\mathbf{y}_i(k)\mathbf{y}_j^T(k)\} \tag{32}$$

where $m(2L+1)\times m(2L+1)$ matrix $\mathbf{R}_{\underline{\mathbf{y}}}$ is the combination of correlation matrix concerning different time lags.According to the constraint in (23) and the nonstationarity property of source signals,the $(2L+1)\times(2L+1)$ matrix $\mathbf{R}_{\mathbf{y}_i\mathbf{y}_j}$ becomes nonzero only in the diagonal position of $\mathbf{R}_{\underline{\mathbf{y}}}$.We consider the coefficents constraints above as diagonalization constraints.

Compared with the existing para-unitary constraint,the diagonalization constraints are relaxed,it can be efficiently imposed via singular value decomposition in both deflation and symmetric mode.

A. deflation mode

When one of the sources' estimation $y_i(k)$ has been extracted,it's necessary to substract its contribution from the observations to obtain observations of $m-1$ sources,then we repeat this procedure to extract the remained sources one by one,until all the sources have been extracted.

Considering the constraints in (26),we construct the block matrix $\mathbf{O}$ during the extraction of the $y_i(k)$ to guarantee uniqueness.

$$\mathbf{O} = (\mathbf{R}_v(-L)\mathbf{w}_1, \mathbf{R}_v(-L+1)\mathbf{w}_1, \cdots, \mathbf{R}_v(L)\mathbf{w}_{i-1}) \tag{33}$$

$$\mathbf{w}_i^T\mathbf{O} = \mathbf{0} \tag{34}$$

The $\mathbf{w}_i$ in the ith extraction is required to be orthogonal to the column space of the block matrix $\mathbf{O}$,we represent the column space of $\mathbf{O}$ as $span\{\mathbf{O}\}$.

If the block matrix $\mathbf{O}$ is overdetermined,the column space $span\{\mathbf{O}\}$ is not full column rank,it's easy adjust the $\mathbf{w}_i$ via least square solution.

$$\mathbf{w}_i = \mathbf{w}_i - \left(\mathbf{O}^T\mathbf{O}\right)^{-1}\mathbf{O}^T\mathbf{w}_i \tag{35}$$

Unfortunately,$\mathbf{O}$ is often designed to be underdetermined due to choice of large L,so the strategy in (35) cannot work any longer.The proposed algorithm draws lessons from the principal component analysis based on singular value decomposition,only taking the directions with most variation in $\mathbf{O}$ into consideration.

$$\mathbf{O} = \mathbf{U}\boldsymbol{\Sigma}\mathbf{V}^T \tag{36}$$

Here $\mathbf{U}$ and $\mathbf{V}$ are orthogonal matrices,with the columns of $\mathbf{U}$ spanning the column space of $\mathbf{O}$,and the columns of $\mathbf{V}$ spanning the row space.$\boldsymbol{\Sigma}$ is a diagonal matrix,with diagonal entries in decreasing order.

$$\sigma_{11} \ge \sigma_{22} \ge \cdots \ge 0 \tag{37}$$

Algorithm 1 convolutive FastICA:deflation mode

$\mathbf{W} = \mathbf{0}_{m\times nQ}$
$i = 1$
repeat
  $\mathbf{w}_i = \mathbf{0}_{nQ\times 1}$
  $\mathbf{O} = (\mathbf{R}_v(-L)\mathbf{w}_1, \mathbf{R}_v(-L+1)\mathbf{w}_1, \cdots, \mathbf{R}_v(L)\mathbf{w}_{i-1})$
  calculate $\mathbf{U}_{(r)}$ from (36)(38) with threshold α
  repeat
    $\mathbf{w}_i^{'} = \mathbf{w}_i$
    $\mathbf{w}_i = E\{\mathbf{v}(k)g(\mathbf{w}_i^T\mathbf{v}(k))\} - E\{g^{'}(\mathbf{w}_i^T\mathbf{v}(k))\}\mathbf{w}_i$
    $\mathbf{w}_i = \mathbf{w}_i - \mathbf{U}_{(r)}\mathbf{U}_{(r)}^T\mathbf{w}_i$
    $\mathbf{w}_i = \mathbf{w}_i/\|\mathbf{w}_i\|_2$
  until $||\mathbf{w}_i^T\mathbf{w}_i^{'}| - 1.0| \leq tol$
  $\mathbf{W}[i,:] = \mathbf{w}_i$
  $i = i+1$
until $i > m$

Algorithm 2 convolutive FastICA:symmetric mode

$\mathbf{W} = \mathbf{0}_{m\times nQ}$
repeat
  $\mathbf{W}^{'} = \mathbf{W}$
  $i = 1$
  repeat
    $\mathbf{w}_i = \mathbf{W}[i,:]$
    $\mathbf{O} = (\mathbf{R}_v(-L)\mathbf{w}_1, \cdots, \mathbf{R}_v(L)\mathbf{w}_{i-1}, \mathbf{R}_v(-L)\mathbf{w}_{i+1}, \cdots)$
    calculate $\mathbf{U}_{(r)}$ from (36)(38) with threshold α
    $\mathbf{w}_i = E\{\mathbf{v}(k)g(\mathbf{w}_i^T\mathbf{v}(k))\} - E\{g^{'}(\mathbf{w}_i^T\mathbf{v}(k))\}\mathbf{w}_i$
    $\mathbf{w}_i = \mathbf{w}_i - \mathbf{U}_{(r)}\mathbf{U}_{(r)}^T\mathbf{w}_i$
    $\mathbf{w}_i = \mathbf{w}_i/\|\mathbf{w}_i\|_2$
    $\mathbf{W}[i,:] = \mathbf{w}_i$
    $i = i+1$
  until $i > m$
until $\||\mathbf{W}^{'}\mathbf{W}^T| - I_{m\times m}\|_2 \leq tol$

The algorithm picks the first r columns vectors in $\mathbf{U}$ to obtain the most variance in original $span\{\mathbf{O}\}$ based on the effective rank of $\mathbf{O}$, r is the minimum integer when $\mu(r)$ is greater than particular threshold α (such as 0.99995).

$$\mu(r) = \frac{\sqrt{\sigma_{11}^2 + \cdots + \sigma_{rr}^2}}{\|\Sigma\|_F} \tag{38}$$

The most variant r column vectors in $\mathbf{U}$ is represented as $\mathbf{U}_{(r)}$,and each column vector in $\mathbf{U}_{(r)}$ is orthogonal with each other, $\mathbf{U}_{(r)}$ is always high matrix,the (35) can be described in simple form.

$$\mathbf{w}_i = \mathbf{w}_i - \mathbf{U}_{(r)}\mathbf{U}_{(r)}^T\mathbf{w}_i \tag{39}$$

The FastICA convolutive algorithm in deflation mode is summarized in Alg.1.where tol is the threshold in iteration (such as 10^{-7}).

B. symmetric mode

Every extracted source has different priority in deflation mode,leading an accumulation of estimation errors.Symmetric mode is proposed to extract $\mathbf{w}_i$ equally.

Supposing only the $\mathbf{w}_i$ is in process,other extracted sources are fixed. It's nature to construct the block matrix from (26)(33).

$$\mathbf{O} = (\mathbf{R}_v(-L)\mathbf{w}_1, \cdots, \mathbf{R}_v(L)\mathbf{w}_{i-1}, \mathbf{R}_v(-L)\mathbf{w}_{i+1}, \cdots) \tag{40}$$

The FastICA convolutive algorithm in symmetric mode is similar to the deflation mode,and it can be summarized in Alg.2.

C. sources' reconstruction

After the extraction of $\mathbf{y}(k)$ in deflation or symmetric mode,we get the estimation of the ordered,scaled,and delayed version of innovation process $\mathbf{u}(k)$ (5),we then conduct a reconstruction process to recover the signals $\mathbf{s}(k)$'s contributions in the observation mixtures $\mathbf{x}(k)$.In this part,for the convenience of discussing,the ordering ambiguity is ignored.

For the purpose of achieve innovation $s_i(k)$'s contributions in observation $x_j(k)$,a $(2L+N)\times(2L+1)$ high matrix $\mathbf{T}$ is built to represent the column space via zero padding,including all the possible forward and backward time shift of y_i.

$$\mathbf{T} = \begin{pmatrix} y_i(0) & 0 & \cdots & 0 & 0 \\ y_i(1) & y_i(0) & \cdots & 0 & 0 \\ y_i(2) & y_i(1) & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & y_i(N-2) & y_i(N-3) \\ 0 & 0 & \cdots & y_i(N-1) & y_i(N-2) \\ 0 & 0 & \cdots & 0 & y_i(N-1) \end{pmatrix} \tag{41}$$

$N \times 1$ column vector x_j is padding with L zeros both forward and backward.

$$x_j^{(2L+N)} = (0, 0, \cdots, x_j(0), x_j(1), \cdots, 0, 0)^T \tag{42}$$

The reconstruction is based on the regression opinion:regress $x_j^{(2L+N)}$ on the column space of $\mathbf{T}$,finding the closest $\hat{s}_{ij}$ in the $span\{\mathbf{T}\}$ in least square sense.We describe the s_i contribution to observation x_j as $\hat{s}_{ij}$.

$$\hat{s}_{ij} = (\mathbf{T}^T\mathbf{T})^{-1}\mathbf{T}^T x_j^{(2L+N)} \tag{43}$$

V. EXPERIMENT RESULTS

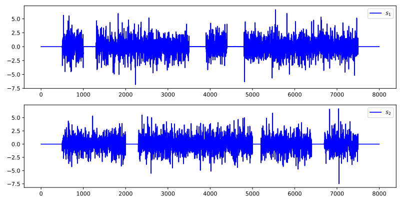
We conducted two experiments in this section to explore the performance of the proposed algorithm.

The first experiment mixed the given innovation process $u_1$andu_2 in Fig.2a,both satisfying the referred assumptions.After the convolutive prewhitening stage,the algorithm in symmetric mode recovered the y_1 and y_2,and calculated their contributions on each observation.

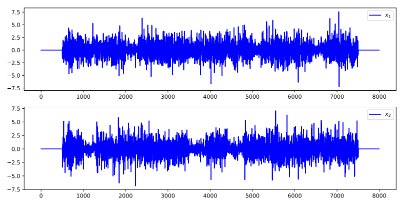
In Fig.3a the recovered estimation y_1 and y_2 are similar to the original innovation process,while there are little deviations in the orignal zero amplitude regions.The contributions in Fig.3b happened to be the same as innovation process.In this simulation.the algorithm accomplished the mission of convolutive BSS.

A more difficult task was considered in real recorded source signals from the public data of Salk Institute[18].

1) s_1 :speaker says the digits from one to ten in English.
2) s_2 :loud music in the background.

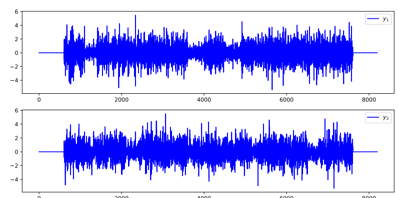


(a) Source signals

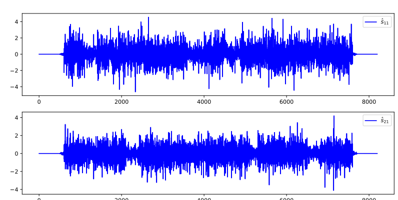


(b) Observations

Fig. 2. Source signals and observations in the 2×2 case(simulation).

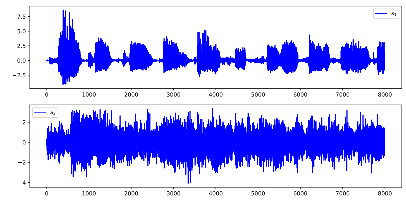


(a) Innovation process

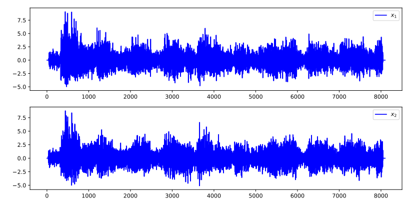


(b) Sources' contirbutions

Fig. 3. Innovation process and sources' contributions in the 2×2 case(symmetric mode).

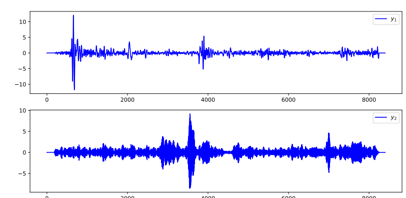


(a) Source signals

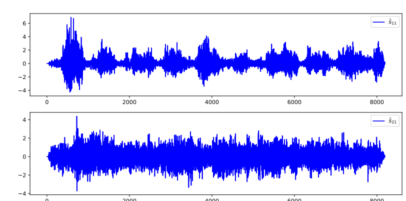


(b) Observations

Fig. 4. Source signals and observations in the 2×2 case(record).



(a) Innovation process



(b) Sources' contirbutions

Fig. 5. Innovation process and sources' contributions in the 2×2 case(deflation mode).

We conduct deflation mode in Alg.1 to produce the estimations and contributions.The results in Fig.5b showed the similarity between the calculated contributions and original source signals.

VI. Conclusion

In this paper,we have derived a novel extension of the FastICA for convolutive mixtures that enforces the diagnoalization constraints on the seperating system for uniqueness.Our algorithm has simple convolutive prewhitening stages and contributions' reconstruction procedure.Experiments are given to illustrate the performance of the proposed algorithm.

Our algorithm enjoys the robustness and fast convergence due to its fix-point iterations,no particular parameter tuning is required.Compared with spatio-temporal prewhitening stages and para-unitary filter constraints in other contrast-function approaches[3],the corresponding procedures in our algorithm are much more straightforward and simpler.

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