Charge symmetry breaking in hypernuclei within RMF model

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Abstract

We study the charge symmetry breaking (CSB) effect in the binding energy of mirror hypernuclei in the mass region $A = 7 \sim 48$ in relativistic mean field (RMF) models introducing NN and ΛN interactions. The phenomenological ΛN CSB interaction is introduced and the strength parameter is fitted to reproduce the experimental binding energy difference between the mirror hypernuclei $^{12}_{\Lambda}$B and $^{12}_{\Lambda}$C. This model is applied to calculate the CSB energy anomaly in mirror hypernuclei with the mass $A = 7 \sim 48$. The model is further applied to predict the binding energy difference of mirror hypernuclei of A=40 with the isospin $T = 1/2$, $3/2$ and $5/2$ nuclei together with various hyper Ca isotopes and their mirror hypernuclei. Finally the binding energy systematics of A =48 hypernuclei are predicted with/without the CSB effect by the PK1 and TM2 energy density functionals (EDFs).

Keywords: charge symmetry breaking, single-Λ hypernuclei, RMF model

1. Introduction

In hypernuclear physics, it is important to extract information on hyperon (Y)-nucleon(N) interaction. Historically, due to the difficulties of YN scattering experiments, we have been obtaining information on YN interaction by the studies of hypernuclear structures. For this purpose, in the case of the ΛN sector, high resolution γ-ray experiments have been performed in light Λ hypernuclei systematically [1].

Theoretically, several shell-model calculations for light Λ hypernuclei have been performed [2, 3]. Furthermore, microscopic calculations of three- and four-cluster system with sufficient numerical accuracy have been performed [4, 5, 6]. With these theoretical calculations and experimental data, we have obtained spin-dependent forces of ΛN interaction such as spin-spin term, spin-orbit term, and etc.

However, there is still an open important issue to be solved, i.e., charge symmetry breaking (CSB) component in the ΛN interaction. Historically, evidence for the CSB interaction has been observed in the single-Λ binding energies B_Λ of the $A = 4$ mirror hypernuclei $^4_\Lambda$H and $^4_\Lambda$He. This evidence is attributed to the energy difference $\Delta B_\Lambda \equiv B_\Lambda(^4_\Lambda\text{H}) - B_\Lambda(^4_\Lambda\text{He})$, which was measured to be -0.35 ± 0.06 MeV for the ground (0^+) state and -0.24 ± 0.06 MeV for the excited (1^+) state, respectively [7]. To reproduce the observed data, many theoretical efforts have been done. To understand the CSB effect, Dalitz and Von Hippel [8] pointed out that the Λ-Σ^0 mixing mechanism, which is related to ΛN-ΣN coupling, is important. Thereafter, several calculations taking account of ΛN-ΣN coupling have been performed for the $A = 4$ hypernuclei [9, 10, 11, 12]. However, it was difficult to reproduce the experimental data.

In 2015, the ground state of $^4_\Lambda$H has been observed with high accuracy at MAMI-C [13], and the obtained single-Λ separation energy $B_\Lambda = 2.21 \pm 0.01$ (stat) $\pm$ 0.09 (syst) MeV, which is consistent with the emulsion value [7]. Moreover, the γ-ray transition from the 1^+ excited state to the 0^+ ground state in $^4_\Lambda$He has been observed at J-PARC and the obtained excitation energy is 1.406 ± 0.002 (stat) $\pm$ 0.002 (syst) MeV [14], which was quite different with the old data. After these two observations, many works have discussed the CSB effect on the $A = 4$ light systems [15, 16, 17]. Still, it is difficult to reproduce the new data.

More information on CSB effect are required. For

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this purpose, one can also focus on the study of heavier Λ hypernuclei with mass number $A = 7, 10$ and larger. For the $A = 7$ single-Λ hypernuclei, the binding energies of ${}^{7}_{\Lambda}$He, ${}^{7}_{\Lambda}$Li, and ${}^{7}_{\Lambda}$Be with isospin $T = 1$, have been observed experimentally. Among these data, the binding energy B_Λ in ${}^{7}_{\Lambda}$He has been observed to be 5.68 ± 0.03 (stat) $\pm$ 0.25 (syst) MeV by the ${}^{7}\mathrm{Li}(e, e'K^+){}^{7}_{\Lambda}\mathrm{He}$ reaction at JLab [18]. Later, the observed binding energy of ${}^{7}_{\Lambda}$He has been updated to be 5.55 ± 0.10 (stat) $\pm$ 0.11 (syst) MeV with a better systematic error [19]. In the case of ${}^{7}_{\Lambda}$Be, there were old emulsion data giving $B_\Lambda = 5.16$ MeV [7]. As a result, the energy difference, $\Delta B_\Lambda \equiv B_\Lambda({}^{7}_{\Lambda}\mathrm{He}) - B_\Lambda({}^{7}_{\Lambda}\mathrm{Be})$, between ${}^{7}_{\Lambda}$He and ${}^{7}_{\Lambda}$Be is 0.39 MeV, which exhibits a larger energy difference compared to that of $A = 4$ hypernuclei.

Regarding the $A = 10$ mirror hypernuclei, there was one old emulsion data for ${}^{10}_{\Lambda}$Be which gives $B_\Lambda = 9.11 \pm 0.22$ MeV [7, 20]. In 2016, high-resolution experiment has been done at JLab using the $(e, e'K^+)$ reaction and reported the observed binding energy B_Λ of ${}^{10}_{\Lambda}$Be to be 8.60 ± 0.07 (stat)$\pm$0.11 (syst) MeV [21]. Meanwhile, the B_Λ for ${}^{10}_{\Lambda}$B is 8.89 ± 0.12 (stat) $\pm$ 0.04 (syst) MeV measured in emulsion [22] while 8.1 ± 0.1 MeV measured by the (π^+, K^+) reaction at KEK [23], which was corrected to be $B_\Lambda = 8.64 \pm 0.1$ MeV in Ref. [21]. Thus, based on those experimental data, there are two suggested possible binding energy differences $\Delta B_\Lambda \equiv B_\Lambda({}^{10}_{\Lambda}\mathrm{Be}) - B_\Lambda({}^{10}_{\Lambda}\mathrm{B})$ between the $A = 10$ mirror hypernuclei. One value is $\Delta B_\Lambda = 9.11 - 8.89 = 0.22$ MeV, while the other is $8.60 - 8.64 = -0.04$ MeV. For more heavier Λ hypernuclei, we have data for the $A = 12$ mirror hypernuclei ${}^{12}_{\Lambda}$B and ${}^{12}_{\Lambda}$C, and the observed B_Λs are 11.529 ± 0.025 MeV [24] and 11.30 ± 0.19 MeV [21, 22, 25], respectively[1]. Therefore, the experimental value of $\Delta B_\Lambda \equiv B_\Lambda({}^{12}_{\Lambda}\mathrm{B}) - B_\Lambda({}^{12}_{\Lambda}\mathrm{C})$ is 0.229 MeV, although the error bar in ${}^{12}_{\Lambda}$C is still large. A comprehensive summary of B_Λ values for Λ-hypernuclei with $A \leq 16$ has been provided in Ref. [27].

It is noted that in mirror hypernuclei, the binding energies B_Λs on the neutron-rich side are larger compared to those on the proton-rich side for p-shell Λ hypernuclei. In contrast, the behavior of s-shell Λ hypernuclei is opposite to that of p-shell Λ hypernuclei. In order to reproduce both the s-shell and p-shell hypernuclear data, in Ref. [6], one of the present authors, E.H., introduced a phenomenological odd-state CSB interaction which has an opposite sign to the even-state CSB interaction to reproduce the observed data of ${}^{4}_{\Lambda}$H and ${}^{4}_{\Lambda}$He. The odd-state CSB interaction was adjusted so as to reproduce the observed B_Λs of $T = 1$ isotriplet hypernuclei ${}^{7}_{\Lambda}$He, ${}^{7}_{\Lambda}$Li, and ${}^{7}_{\Lambda}$Be. The CSB interaction was also applied to calculate the binding energies of the $A = 10$ mirror hypernuclei, ${}^{10}_{\Lambda}$B and ${}^{10}_{\Lambda}$Be, and the obtained ΔB_Λ was consistent with the data, provided that the odd-state CSB interaction has an opposite sign to that of the even-state CSB interaction.

To further study CSB effect, we need information on heavier Λ hypernuclei. For this purpose, it is planned at JLab to produce ${}^{40}_{\Lambda}$K and ${}^{48}_{\Lambda}$K via $(e, e'K^+)$ reactions using ^{40}Ca and ^{48}Ca targets. In this paper, based on the relativistic mean field (RMF) model, we discuss the CSB effect on the single-Λ hypernuclei with mass numbers ranging from $A = 7$ to 48 and also predict the possibility to observe the CSB effect at JLab and J-PARC. In Section 2, the theoretical framework is given. The calculated results and discussions are presented in Section 3 and the summary is drawn in Section 4.

2. Theoretical framework

2.1. RMF model for Λ hypernuclei

RMF models have achieved great successes in the descriptions of ordinary nuclei [28], hypernuclei [29, 30, 31, 32, 33] as well as baryon matter [34, 35]. The starting point of the meson-exchange RMF model for the Λ hypernuclei is the following covariant Lagrangian density,

$$\mathcal{L} = \mathcal{L}_N + \mathcal{L}_\Lambda. \tag{1}$$

Here $\mathcal{L}_N$ is the standard RMF Lagrangian density for nucleons [28], in which the couplings with the scalar-isoscalar σ, vector-isoscalar ω_μ, and vector-isovector $\vec{\rho}_\mu$ mesons, and the photon A_μ are included, i.e.,

$$\begin{aligned}
\mathcal{L}_N = & \sum_{i=n,p} \bar{\psi}_i \Big[i\gamma^\mu \partial_\mu - M_i - g_{\sigma i}\sigma - g_{\omega i}\gamma^\mu \omega_\mu \\
& - g_{\rho i}\gamma^\mu \vec{\tau}_i \cdot \vec{\rho}_\mu - e\gamma^\mu A_\mu \frac{1-\tau_{i,3}}{2} \Big] \psi_i \\
& + \frac{1}{2}\partial_\mu \sigma \partial^\mu \sigma - \frac{1}{2} m_\sigma^2 \sigma^2 - \frac{1}{3} g_2 \sigma^3 - \frac{1}{4} g_3 \sigma^4 \\
& - \frac{1}{4}\Omega_{\mu\nu}\Omega^{\mu\nu} + \frac{1}{2} m_\omega^2 \omega_\mu \omega^\mu + \frac{1}{4} c_3 (\omega_\mu \omega^\mu)^2 \\
& - \frac{1}{4}\vec{R}_{\mu\nu} \cdot \vec{R}^{\mu\nu} + \frac{1}{2} m_\rho^2 \vec{\rho}_\mu \cdot \vec{\rho}^\mu - \frac{1}{4} F_{\mu\nu} F^{\mu\nu},
\end{aligned} \tag{2}$$

where M_i $(i = n, p)$ denotes the nucleon mass, $\vec{\tau}_i$ is the isospin with the 3rd component $\tau_{i,3}$ (+1 for neutrons and -1 for protons), and m_ϕ $(\phi = \sigma, \omega, \rho)$ and $g_{\phi i}$ are the masses and coupling constants for mesons, respectively.

[1] According to Mainz compilation [26], the data is 11.335 ± 0.126 MeV, which is similar with Ref. [21]. It should be noted that Ref. [24] provides high statics. Thus, we cite Ref. [21].

$\Omega_{\mu\nu}$, $\vec{R}_{\mu\nu}$, and $F_{\mu\nu}$ are the field tensors for the ω and $\vec{\rho}$ mesons and photons. g_2, g_3, and c_3 are the parameters introduced in the nonlinear self-coupling terms.

The Lagrangian density $\mathcal{L}_\Lambda$ represents the contributions from Λ hyperons, in which only the couplings with the σ and ω_μ mesons are included because of Λ hyperons being charge neutral and zero isospin, i.e.,

$$\begin{aligned}\mathcal{L}_\Lambda =&\bar{\psi}_\Lambda \Big[i\gamma^\mu\partial_\mu - M_\Lambda - g_{\sigma\Lambda}\sigma - g_{\omega\Lambda}\gamma^\mu\omega_\mu \\ &-\frac{f_{\omega\Lambda\Lambda}}{2m_\Lambda}\sigma^{\mu\nu}\partial_\nu\omega_\mu\Big]\psi_\Lambda,\end{aligned} \tag{3}$$

where M_Λ is the mass of the Λ hyperon, $g_{\sigma\Lambda}$ and $g_{\omega\Lambda}$ are the coupling constants with the σ and ω meson, respectively. To reproduce the small Λ spin-orbit splitting, a term of $\omega\Lambda\Lambda$ tensor coupling is introduced with $f_{\omega\Lambda\Lambda}$ being the coupling constant.

For a system with time-reversal symmetry, the space-like components of the vector fields ω_μ and $\vec{\rho}_\mu$ vanish, leaving only the time components ω_0 and $\vec{\rho}_0$. Meanwhile, charge conservation guarantees that only the third component $\rho_{0,3}$ in the isospin space of $\vec{\rho}_0$ exists. With the mean-field and no-sea approximations, the single-particle Dirac equations for nucleons and hyperons and the Klein-Gordon equations for mesons and photons can be obtained by the variational procedure.

With spherical symmetry, the Dirac spinor for nucleons and hyperons can be expanded as

$$\psi_{n\kappa m}(\boldsymbol{r}) = \frac{1}{r}\begin{pmatrix} iG_{n\kappa}(r) \\ -F_{n\kappa}(r)\boldsymbol{\sigma}\cdot\hat{\boldsymbol{r}} \end{pmatrix} Y^l_{jm}(\theta,\phi), \tag{4}$$

where $G_{n\kappa}(r)/r$ and $F_{n\kappa}(r)/r$ are the radial wave functions for the upper and lower components, $Y^l_{jm}(\theta,\phi)$ is the spinor spherical harmonic. The quantum number κ is defined as $\kappa = (-1)^{j+l+1/2}(j+1/2)$. With the radial wave functions, the radial Dirac equations for baryons ($i = n, p, \Lambda$) can be obtained as,

$$\begin{pmatrix} V_i + S_i & -\dfrac{d}{dr} + \dfrac{\kappa}{r} + T_i \\ \dfrac{d}{dr} + \dfrac{\kappa}{r} + T_i & V_i - S_i - 2M_i \end{pmatrix}\begin{pmatrix} G_{n\kappa} \\ F_{n\kappa}\end{pmatrix} = \varepsilon_{n\kappa}\begin{pmatrix} G_{n\kappa} \\ F_{n\kappa}\end{pmatrix}, \tag{5}$$

where S_i, V_i, and T_i are respectively the mean-field scalar potential, vector potential, and the $\omega\Lambda\Lambda$ tensor potential,

$$S_i = g_{\sigma i}\sigma, \tag{6a}$$

$$V_i = g_{\omega i}\omega_0 + g_{\rho i}\tau_{i,3}\rho_{0,3} + \frac{1}{2}e(1-\tau_{i,3})A_0, \tag{6b}$$

$$T_i = -\frac{f_{\omega\Lambda\Lambda}}{2M_i}\partial_r\omega_0. \tag{6c}$$

Note that the terms related to $\rho_{0,3}$ and A_0 in Eq. (6b) are zero for Λ hyperons, while the tensor potential in Eq. (6c) is zero for nucleons. The $\omega\Lambda\Lambda$ tensor interaction was introduced to reproduce the experimentally observed small spin-orbit splitting for Λ hyperon [36, 37].

The Klein-Gordon equations for mesons and photons are

$$(\partial^\mu\partial_\mu + m_\phi^2)\phi = S_\phi, \tag{7}$$

with the source terms

$$S_\phi = \begin{cases} \sum\limits_{i=n,p,\Lambda} -g_{\sigma i}\rho_{si} - g_2\sigma^2 - g_3\sigma^3, & \phi = \sigma; \\ \sum\limits_{i=n,p,\Lambda} g_{\omega i}\rho_{vi} + \frac{f_{\omega\Lambda\Lambda}}{2M_\Lambda}\partial_k j^{0k}_{T\Lambda} - c_3\omega_0^3, & \phi = \omega; \\ \sum\limits_{i=n,p} g_{\rho i}\tau_{i,3}\rho_{vi}, & \phi = \rho; \\ e\rho_c, & \phi = A; \end{cases} \tag{8}$$

where ρ_{si} and ρ_{vi} are the scalar and vector densities for nucleons and hyperons, $j^{0k}_{T\Lambda}$ is the tensor density for Λ hyperons, and ρ_c is the charge density for protons.

With the radial wave functions, those densities in Eq. (8) can be obtained as

$$\rho_{si}(r) = \frac{1}{4\pi r^2}\sum_{k=1}^{A_i}\left[G^2_{ki}(r) - F^2_{ki}(r)\right], \tag{9a}$$

$$\rho_{vi}(r) = \frac{1}{4\pi r^2}\sum_{k=1}^{A_i}\left[G^2_{ki}(r) + F^2_{ki}(r)\right], \tag{9b}$$

$$\rho_c(r) = \frac{1}{4\pi r^2}\sum_{k=1}^{A_p}\left[G^2_{kp}(r) + F^2_{kp}(r)\right], \tag{9c}$$

$$\boldsymbol{j}^0_{T\Lambda}(r) = \frac{1}{4\pi r^2}\sum_{k=1}^{A_\Lambda}\left[2G_{k\Lambda}(r)F_{k\Lambda}(r)\right]\boldsymbol{n}, \tag{9d}$$

where $\boldsymbol{n}$ is the angular unit vector. The baryon number A_i ($i = n, p, \Lambda$) can be calculated by integrating the baryon density $\rho_{vi}(r)$ in coordinate space as

$$A_i = \int 4\pi r^2 dr\rho_{vi}(r). \tag{10}$$

2.2. ΛN CSB interaction

In analogy of non-relativistic CSB interaction [43], we introduce a simple ΛN charge symmetry breaking (CSB) interaction as

$$V^{\Lambda N}_{CSB} = \frac{1}{2}V^{\Lambda N}_0\sum_{k=1}^{A}\bar{\psi}_\Lambda\gamma_\mu\psi_\Lambda\bar{\psi}_k\gamma^\mu\tau_3\psi_k, \tag{11}$$

where $V^{\Lambda N}_0$ is the strength of CSB interaction, τ_3 is the 3rd component of isospin for neutrons or protons. The energy density functional is obtained for the interaction (11) as,

$$\varepsilon_{\Lambda N} = \frac{1}{2} V_0^{\Lambda N} \rho_\Lambda (\rho_n - \rho_p), \qquad (12)$$

where $\rho_n(r)$ and $\rho_p(r)$ are the baryon densities of neutron and proton, respectively. With this CSB interaction, the attractive Λn interaction is strengthen while Λp interaction is weaken for a negative $V_0^{\Lambda N}$ value.

With or without the ΛN CSB interaction, the Dirac equations for baryons (n, p, Λ), the Klein-Gordon equations for mesons and photon, the mean-field potentials, and densities in the RMF model are solved by iteration procedure in the coordinate space. The single-Λ binding energy B_Λ is calculated by using a formula,

$$B_\Lambda(Z, N, 1) = B({}^{A}_{\Lambda}Z) - B({}^{A-1}Z), \qquad (13)$$

where $B({}^{A}_{\Lambda}Z)$ and $B({}^{A-1}Z)$ are the binding energies for the single-Λ hypernuclei and the corresponding core nuclei, respectively. The difference in single-Λ binding energies between mirror hypernuclei is then defined for $N \geq Z$ as

$$\Delta B_\Lambda(A = N + Z) = B_\Lambda(Z, N, 1) - B_\Lambda(N, Z, 1). \qquad (14)$$

Equations in the RMF model are solved in the coordinate space with a size of $R = 20$ fm and a step size of $\Delta r = 0.05$ fm. For the NN interaction, the PK1 [38] and TM2 [39] parameter sets are adopted. For the ΛN interactions, the parameters are listed in Table 1. With those NN and ΛN interactions, the single-Λ binding energy in ${}^{12}_{\Lambda}$C [40] and the Λ $1p$ spin-orbit splitting in ${}^{13}_{\Lambda}$C [41] are well reproduced as B_Λ=11.3 MeV and $E(1/2^-) - E(3/2^-)$=0.152 MeV, respectively. The single-Λ binding energy in the heavier hypernucleus ${}^{40}_{\Lambda}$Ca can also be well given, and it is predicted to be 19.24 and 18.45 MeV with PK1 and TM2 EDFs, respectively, while the experimental value is reported as 18.7 ± 1.1 MeV [42]. The parameter $V_0^{\Lambda N}$ in the CSB interaction is determined to reproduce the Λ binding energy difference of the $A = 12$ mirror hypernuclei ${}^{12}_{\Lambda}$B and ${}^{12}_{\Lambda}$C, i.e., $\Delta B_\Lambda = 0.229$ MeV given by the experimental values of $B_\Lambda = 11.529 \pm 0.025$ MeV for ${}^{12}_{\Lambda}$B [24] and 11.30 ± 0.19 MeV for ${}^{12}_{\Lambda}$C [22], both in their ground states. It is noted that the error bar of B_Λ for ${}^{12}_{\Lambda}$C is much larger than that for ${}^{12}_{\Lambda}$B. Here we focus on the centroid values of B_Λs in these two hypernuclei with CSB effect. To reproduce this value, $V_0^{\Lambda N}$ is determined as given in Table 1. The upper and lower limits of the ΛN CSB strength are also evaluated by considering the experimental uncertainties of binding energies.

Table 1: Parameters of the ΛN interactions in the RMF model.

	PK1	TM2
$g_{\sigma\Lambda}/g_{\sigma N}$	0.6206	0.6233
$g_{\omega\Lambda}/g_{\omega N}$	0.6666	0.6666
$f_{\omega\Lambda\Lambda}/f_{\omega N}$	-1.1174	-1.1210
$V_0^{\Lambda N}$ (MeV fm^3)	-33.05	-38.00
upper limit	-63.15	-72.00
lower limit	-2.85	-3.70

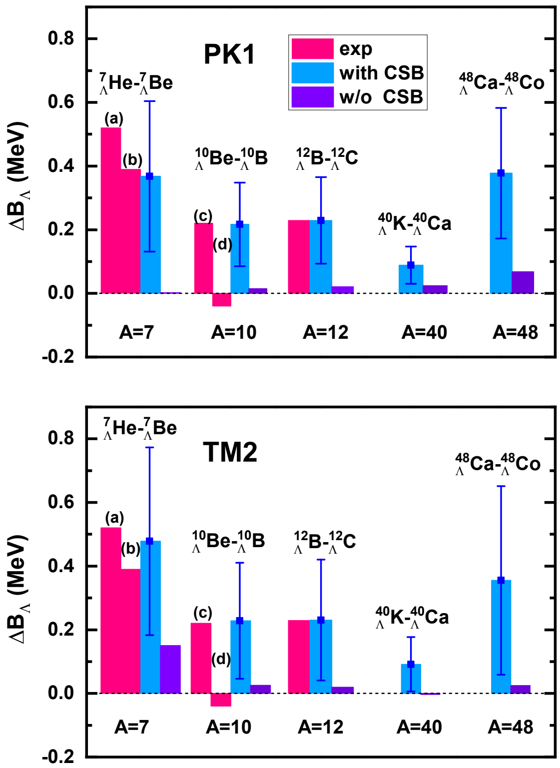

Figure 1: The differences ΔB_Λ of the single-Λ binding energy between the mirror hypernuclei (${}^{7}_{\Lambda}$He, ${}^{7}_{\Lambda}$Be), (${}^{10}_{\Lambda}$Be, ${}^{10}_{\Lambda}$B), (${}^{12}_{\Lambda}$B, ${}^{12}_{\Lambda}$C), (${}^{40}_{\Lambda}$K, ${}^{40}_{\Lambda}$Ca), and (${}^{48}_{\Lambda}$Ca, ${}^{48}_{\Lambda}$Co) obtained by the RMF models with and without the ΛN CSB interaction, in comparison with the experimental data. The upper (lower) panel shows results with PK1(TM2) EDF. In ${}^{10}_{\Lambda}$Be-${}^{10}_{\Lambda}$B, we cite two experimental data specified in (a) and (b).

3. Results and discussions

Using CSB ΛN interaction, we first examine the lighter mirror Λ hypernuclei with $A = 7$ and $A = 10$, such as (${}^{7}_{\Lambda}$He, ${}^{7}_{\Lambda}$Be) and (${}^{10}_{\Lambda}$Be, ${}^{10}_{\Lambda}$B), as shown in Fig. 1. Among these, ${}^{7}_{\Lambda}$He and ${}^{7}_{\Lambda}$Be are the lightest p-shell Λ hypernuclei for the discussions on CSB effect. The ${}^{7}_{\Lambda}$Be was observed by emulsion and the binding energy in the ground state is 5.16 ± 0.08 MeV [7]. The binding energy B_Λ of ${}^{7}_{\Lambda}$He in the ground state has been measured to be 5.55 ± 0.10 (stat) ± 0.11 (syst) MeV [19] at JLab. Thus, $\Delta B_\Lambda = B_\Lambda({}^{7}_{\Lambda}\text{He}) - B_\Lambda({}^{7}_{\Lambda}\text{Be}) = 0.39$ MeV. As shown in Fig. 1, our calculated ΔB_Λ is about 0.37 MeV by PK1 EDF, which is close to the data of 0.39 MeV when the ΛN interaction is fitted to reproduce the centroid data of the ΔB_Λ for $A = 12$ mirror Λ hypernuclei. We also fit the ΛN interaction strengths to reproduce the binding energy differences across the range of observed values. These results are shown by the blue-colored bars. Our results for $A = 7$ hypernuclei are consistent with the data within one σ deviation. Especially, it is in good agreement with the data if we use PK1 EDF. For $A = 10$ mirror hypernuclei, our calculated ΔB_Λ is about 0.2 MeV by both PK1 and TM2 EDFs. As previously mentioned, there are two interpretations of the experimental data for $A = 10$ Λ hypernuclei. One interpretation, based on Refs. [21, 22, 23], yields a value of 0.22 MeV, which is consistent with our results (see Fig. 1(a)). The other interpretation, corresponding to Fig. 1(b), gives a value of -0.04 MeV, which is inconsistent with our findings. Due to these conflicting interpretations, it is challenging to give definitive conclusions on the CSB effect in $A = 10$ Λ hypernuclei. Therefore, further high-resolution experimental data are needed. In fact, it is already planned to measure the binding energy of ${}^{10}_{\Lambda}$B with improved accuracy in the J-PARC E94 experiment [44].

Next, based on these results, we discuss the ΔB_Λ values for mirror Λ hypernuclei with $A = 40$ and 48. For the combination of ${}^{40}_{\Lambda}$K and ${}^{40}_{\Lambda}$Ca, Fig. 1 shows that $\Delta B_\Lambda \sim 0$ without CSB. When CSB interaction is included, the calculated $\Delta B_\Lambda = 0.1$ MeV. For ${}^{40}_{\Lambda}$K, at JLab, it is planned to produce this hypernucleus by the $(e, e'K^+)$ reaction using a ^{40}Ca target and measure the binding energy of the ground state with a resolution of 100 keV. Meanwhile, it is possible to produce ${}^{40}_{\Lambda}$Ca by the (π^+, K^+) reaction at J-PARC using a ^{40}Ca target. Thus, it would be possible to see the CSB effect by these experiments.

Moreover, at JLab, they plan to use a ^{48}Ca target to produce ${}^{48}_{\Lambda}$K by $(e, e'K^+)$ reaction. The calculated binding energy of ${}^{48}_{\Lambda}$K is 21.08 (20.68) MeV for PK1 EDF with (without) ΛN CSB interaction, from which we can also see the effect of CSB. We also calculate the binding energy difference between $A = 48$ mirror hypernuclei ${}^{48}_{\Lambda}$Ca and ${}^{48}_{\Lambda}$Co to be 0.378 (0.068) MeV with (without) CSB. We see a more pronounced CSB effect compared to the case of $A = 40$ hypernuclei. At J-PARC, by (π^+, K^+) reaction with a ^{48}Ca target, ${}^{48}_{\Lambda}$Ca can be produced and the large CSB effect of 0.378 MeV shown in Fig. 1 might be proved in comparison with the energy of ${}^{48}_{\Lambda}$Co. However, it would be difficult to produce ${}^{48}_{\Lambda}$Co

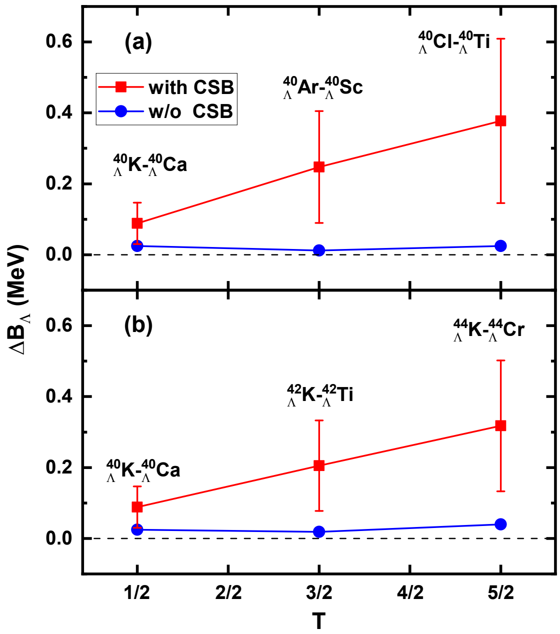

Figure 2: The single-Λ binding energy differences ΔB_Λ between the mirror hypernuclei with $A = 40$ and those with different isospins, i.e., $T = 1/2$ multiplet (${}^{40}_{\Lambda}$K, ${}^{40}_{\Lambda}$Ca), $T = 3/2$ multiplets (${}^{40}_{\Lambda}$Ar, ${}^{40}_{\Lambda}$Sc) and (${}^{42}_{\Lambda}$K, ${}^{42}_{\Lambda}$Ti), and $T = 5/2$ multiplets (${}^{40}_{\Lambda}$Cl, ${}^{40}_{\Lambda}$Ti) and (${}^{44}_{\Lambda}$K, ${}^{44}_{\Lambda}$Cr), obtained by the PK1 EDF with and without ΛN CSB interaction.

due to the lack of an appropriate target. From this fact, it would be better to see ΔB_Λ in the mirror hypernuclei ${}^{40}_{\Lambda}$K and ${}^{40}_{\Lambda}$Ca, using the ^{40}Ca target by $(e, e'K^+)$ reaction at JLab and by (π^+, K^+) reaction at J-PARC.

Recently, the importance of the spin dependent hyperon-nucleon CSB interaction was suggested in light hypernuclear systems in Refs. [17, 45, 46]. For instances, Ref. [46] demonstrated that the effects of this spin dependence can vary significantly, even in sign, for $A = 7$ and $A = 8$ hypernuclei. In this work, we also examine the impact of spin-dependent CSB by introducing the following spin-spin interaction and spin-spin CSB interaction,

$$V^{N\Lambda}_{\sigma\sigma} = g_{N\Lambda}\vec{\sigma}_N \cdot \vec{\sigma}_\Lambda, \tag{15}$$

$$V^{\text{CSB}}_{N\Lambda} = g^{\text{CSB}}_{N\Lambda}\vec{\sigma}_N \cdot \vec{\sigma}_\Lambda \frac{1}{2}(\tau^N_z + \tau^\Lambda_z), \tag{16}$$

where the interaction strength parameters $g_{N\Lambda}$ and $g^{\text{CSB}}_{N\Lambda}$ are fixed to reproduce the spin-doublet states in $A = 12$ mirror Λ hypernuclei [24]. The observed energy splitting between the 1^- and 2^- spin-doublet states is 0.162 MeV in ${}^{12}_{\Lambda}$C and 0.179 MeV in ${}^{12}_{\Lambda}$B, resulting in a small contribution from the spin-spin CSB interaction of only 0.017 MeV. Using the spin-spin and spin-spin CSB interactions, we further predict the energy splittings for

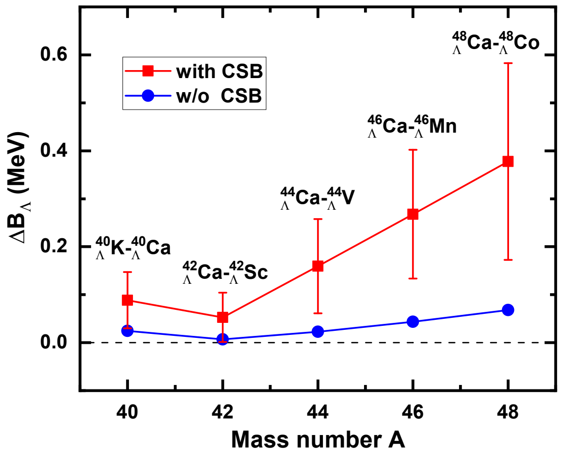

Figure 3: The single-Λ binding energy differences ΔB_Λ between the mirror hypernuclei with different mass number, i.e., (${}^{40}_{\Lambda}$K, ${}^{40}_{\Lambda}$Ca), (${}^{42}_{\Lambda}$Ca, ${}^{42}_{\Lambda}$Sc), (${}^{44}_{\Lambda}$Ca, ${}^{44}_{\Lambda}$V), (${}^{46}_{\Lambda}$Ca, ${}^{46}_{\Lambda}$Mn), (${}^{48}_{\Lambda}$Ca, ${}^{48}_{\Lambda}$Co), obtained with the PK1 EDF with and without ΛN CSB interaction.

the ground state doublets in ${}^{40}_{\Lambda}$K-${}^{40}_{\Lambda}$Ca, and ${}^{48}_{\Lambda}$Ca-${}^{48}_{\Lambda}$Co, respectively. The calculated energy splitting for ${}^{40}_{\Lambda}$K and ${}^{40}_{\Lambda}$Ca are 0.1074MeV and 0.0972MeV, respectively, while those for ${}^{48}_{\Lambda}$Ca and ${}^{48}_{\Lambda}$Co are 0.1389 MeV and 0.1534 MeV, respectively. Then the contributions from the spin-spin CSB term are finally obtained to be 0.0102 for $A = 40$ and -0.0145 MeV for $A = 48$, which are negligibly small compared to the obtained ΔB_Λ values of 0.089 ± 0.059 MeV and 0.378 ± 0.205 MeV, respectively.

Second, let us explain the reason why CSB effect of (${}^{40}_{\Lambda}$Ca, ${}^{40}_{\Lambda}$K) is rather small, i.e., ΔB_Λ=0.09 MeV. It is noted that the isospin of (${}^{40}_{\Lambda}$Ca, ${}^{40}_{\Lambda}$K) is $T = 1/2$. Generally speaking, it is better to see ΔB_Λ for study of CSB in systems with larger total isospins. In Fig. 2, we show the calculated ΔB_Λ for (${}^{40}_{\Lambda}$Ar, ${}^{40}_{\Lambda}$Sc) with $T = 3/2$ and for (${}^{40}_{\Lambda}$Cl, ${}^{40}_{\Lambda}$Ti) with $T = 5/2$. We find that the calculated ΔB_Λ becomes larger with increasing T, that is, $\Delta B_\Lambda \sim 0.25$ MeV for (${}^{40}_{\Lambda}$Ar, ${}^{40}_{\Lambda}$Sc) and $\Delta B_\Lambda \sim 0.4$ MeV for (${}^{40}_{\Lambda}$Cl, ${}^{40}_{\Lambda}$Ti) with $T = 5/2$, as increasing total isospin T. To produce the mirror hypernuclei (${}^{40}_{\Lambda}$Ar, ${}^{40}_{\Lambda}$Sc) and (${}^{40}_{\Lambda}$Cl and ${}^{40}_{\Lambda}$Ti), ^{40}Ar, ^{40}Sc, ^{40}Cl, and ^{40}Ti target can be used by (π^+, K^+) reaction and for the production of ${}^{40}_{\Lambda}$Cl and ${}^{40}_{\Lambda}$Sc, we also can use ^{40}Ar and ^{40}Ti targets by $(e, e'K^+)$ reaction at JLab.

As another way to see CSB effect, we propose to use Ca target. Because, several Ca targets such as ^{40}Ca, ^{42}Ca, ^{44}Ca, ^{46}Ca, and ^{48}Ca are available for experiments to produce hypernuclei. By (π^+, K^+) reaction, ${}^{40}_{\Lambda}$Ca, ${}^{42}_{\Lambda}$Ca, ${}^{44}_{\Lambda}$Ca, ${}^{46}_{\Lambda}$Ca, and ${}^{48}_{\Lambda}$Ca can be produced.

In Fig. 3, we show the calculated ΔB_Λ between hyper Ca isotopes and the corresponding mirror hypernuclei.

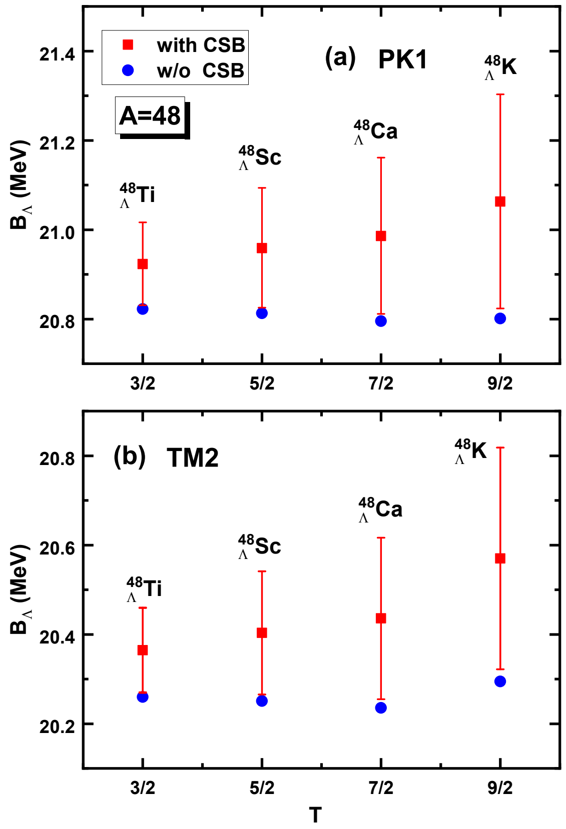

Figure 4: The single-Λ binding energies B_Λ for the $A = 48$ hypernuclei, i.e., ${}^{48}_{\Lambda}$Ti, ${}^{48}_{\Lambda}$Sc, ${}^{48}_{\Lambda}$Ca, ${}^{48}_{\Lambda}$K with isospin $T = 3/2, 5/2, 7/2, 9/2$, obtained by the RMF model with and without ΛN CSB interaction. The upper (lower) panel shows results with PK1(TM2) EDF.

The calculated ΔB_Λ becomes larger with larger neutron number. For $A = 48$, the calculated ΔB_Λ is about 0.4 MeV with CSB effect, which is significantly large. If we could obtain the observed binding energies of ${}^{48}_{\Lambda}$Ca and ${}^{48}_{\Lambda}$Co, it is useful for the study of CSB effect in ΛN interaction. However, it is difficult to produce ${}^{48}_{\Lambda}$Co due to the lack of an appropriate target.

In this way, we understand that the larger CSB effect is associated with nuclei with larger isospins, that is, it is better to see the binding energies of $A = 48$ hypernuclei if there are some target. Along this line, let us examine nuclei with $A = 48$ isobars. Using ^{48}Ti, ^{48}Sc, ^{48}Ca, and ^{48}K as targets, we could produce ${}^{48}_{\Lambda}$Ti, ${}^{48}_{\Lambda}$Sc, ${}^{48}_{\Lambda}$Ca, and ${}^{48}_{\Lambda}$K by (π^+, K^+) reaction. Figure 4 shows the binding energies of the ground states of these Λ hypernuclei with/without CSB ΛN interaction. We see that calculated B_Λ without CSB interaction for these hypernuclei are about 20.8 MeV with PK1 EDF in Fig. 4(a), and the isospin dependence is small. This is also the case in the TM2 EDF shown in Fig. 4(b), having the largest CSB effect in ${}^{48}_{\Lambda}$K. When we include the CSB interaction, it is seen that the calculated B_Λs increase with larger number of total isospin T. It is interesting to see that calculated B_Λ of ${}^{48}_{\Lambda}$K is the largest among $A = 48$ Λ hypernuclei. Using ^{48}Ca target, it is possible to produce ${}^{48}_{\Lambda}$K by $(e, e'K^+)$ reaction at JLab.

Table 2: The single-Λ binding energies B_Λ as well as the difference ΔB_Λ in mirror hypernuclei with the same total isospin T and opposite the third components T_z calculated by PK1 EDF with or without $N\Lambda$ CSB interaction. The contributions from the Coulomb interaction for B_Λ and ΔB_Λ are also listed. For comparison, available experimental data are listed. All energies are in MeV.
* The B_Λ value, 5.064 ± 0.332 MeV, for ${}^{7}_{\Lambda}$He [26] is taken by Refs. [19, 47, 48]. It should be noted that the statics of data in Refs. [47, 48] are poor. Thus, we do not mention the data in the present paper.

				B_Λ					ΔB_Λ				
		T	T_z	Expt.	with CSB		w/o CSB		Expt.	with CSB		w/o CSB	
					total	coul.	total	coul.		total	coul.	total	coul.
$A=7$	${}^{7}_{\Lambda}$He	1	1	$5.55 \pm 0.10 \pm 0.11$ [19] $5.064 \pm 0.332^*$ [26]	$5.260^{+0.179}_{-0.176}$	0.007	5.068	0.013	0.39	0.368±0.236	−0.229	0.003	−0.264
	${}^{7}_{\Lambda}$Be	1	−1	5.16 ± 0.08 [7]	$4.893^{+0.158}_{-0.155}$	0.236	5.065	0.277					
$A=10$	${}^{10}_{\Lambda}$Be	1/2	1/2	9.11±0.22 [7, 20] 8.60±0.07 ± 0.16 [21]	$9.019^{+0.105}_{-0.102}$	0.032	8.908	0.055	0.22/-0.04	0.217 ± 0.132	-0.029	0.015	-0.044
	${}^{10}_{\Lambda}$B	1/2	−1/2	8.89±0.12 [22] 8.64±0.1 [23, 21]	$8.803^{+0.082}_{-0.080}$	0.061	8.893	0.099					
$A=12$	${}^{12}_{\Lambda}$B	1/2	1/2	11.529±0.025 [24]	$11.439^{+0.111}_{-0.108}$	0.043	11.321	0.079	0.23±0.19	0.229 ± 0.136	-0.027	0.021	-0.045
	${}^{12}_{\Lambda}$C	1/2	−1/2	11.30±0.19 [21, 22] 11.335 ± 0.126 [26]	$11.209^{+0.082}_{-0.079}$	0.070	11.300	0.124					
$A=40$	${}^{40}_{\Lambda}$K	1/2	1/2	-	$19.328^{+0.059}_{-0.058}$	0.012	19.265	0.074	-	0.089 ± 0.059	0.037	0.024	0.030
	${}^{40}_{\Lambda}$Ca	1/2	−1/2	18.7 ± 1.1 [42]	$19.239^{+0.001}_{-0.001}$	-0.025	19.240	0.044					
	${}^{40}_{\Lambda}$Ar	3/2	3/2	-	$19.232^{+0.138}_{-0.135}$	-0.008	19.084	0.045	-	0.248 ± 0.158	-0.015	0.012	-0.036
	${}^{40}_{\Lambda}$Sc	3/2	−3/2	-	$18.984^{+0.080}_{-0.078}$	0.007	19.072	0.081					
	${}^{40}_{\Lambda}$Cl	5/2	5/2	-	$19.544^{+0.192}_{-0.190}$	0.009	19.336	0.058	-	0.378 ± 0.232	-0.031	0.025	-0.068
	${}^{40}_{\Lambda}$Ti	5/2	−5/2	-	$19.166^{+0.132}_{-0.130}$	0.040	19.311	0.126					
$A=42$	${}^{42}_{\Lambda}$Ca	1/2	1/2	-	$19.519^{+0.051}_{-0.050}$	-0.003	19.464	0.060	-	0.053 ± 0.051	0.014	0.007	-0.009
	${}^{42}_{\Lambda}$Sc	1/2	−1/2	-	$19.466^{+0.009}_{-0.008}$	-0.017	19.458	0.051					
	${}^{42}_{\Lambda}$K	3/2	3/2	-	$19.613^{+0.115}_{-0.114}$	-0.014	19.489	0.041	-	0.205 ± 0.127	-0.022	0.019	-0.043
	${}^{42}_{\Lambda}$Ti	3/2	−3/2	-	$19.408^{+0.057}_{-0.055}$	0.008	19.470	0.084					
$A=44$	${}^{44}_{\Lambda}$Ca	3/2	3/2	-	$20.025^{+0.094}_{-0.092}$	-0.017	19.924	0.042	-	0.159 ± 0.098	-0.027	0.023	-0.046
	${}^{44}_{\Lambda}$V	3/2	−3/2	-	$19.865^{+0.033}_{-0.031}$	0.010	19.901	0.088					
	${}^{44}_{\Lambda}$K	5/2	5/2	-	$20.114^{+0.158}_{-0.157}$	-0.004	19.943	0.048	-	0.318 ± 0.184	-0.047	0.040	-0.081
	${}^{44}_{\Lambda}$Cr	5/2	−5/2	-	$19.797^{+0.097}_{-0.095}$	0.043	19.903	0.129					
$A=46$	${}^{46}_{\Lambda}$Ca	5/2	5/2	-	$20.513^{+0.135}_{-0.134}$	-0.008	20.367	0.047	-	0.268 ± 0.134	-0.051	0.043	-0.083
	${}^{46}_{\Lambda}$Mn	5/2	−5/2	-	$20.246^{+0.071}_{-0.069}$	0.043	20.324	0.130					
$A=48$	${}^{48}_{\Lambda}$Ca	7/2	7/2	-	$20.986^{+0.175}_{-0.174}$	0.001	20.795	0.053	-	0.378 ± 0.205	-0.079	0.068	-0.124
	${}^{48}_{\Lambda}$Co	7/2	−7/2	-	$20.608^{+0.109}_{-0.106}$	0.080	20.727	0.177					

4. Summary

We studied the CSB effect in the binding energy of hyperon in the mass region of $A = 7 \sim 48$ in the RMF models introducing NN, and ΛN interactions. The phenomenological ΛN CSB interaction is introduced and the strength parameter is fitted to reproduce the experimental binding energy difference between the mirror hypernuclei $^{12}_{\Lambda}$B and $^{12}_{\Lambda}$C. This model is applied to calculate the CSB energy anomaly in A=7 and 10 mirror hypernuclei. We found that our result for $A = 7$ mirror hypernuclei is consistent with the data. On the other hand, in $A = 10$ hypernuclei, due to uncertainties of the experimental data is large, it is difficult to conclude whether our results are consistent or not with experimental data. In the future at J-PARC, it is planned to measure the binding energy of $^{10}_{\Lambda}$Be with better accuracy [44]. It might be possible to discuss on CSB effect of $A = 10$ Λ hypernuclei with future experimental data. The model is further applied to predict the binding energy difference of A=40 mirror hypernuclei with the isospin $T = 1/2$, $3/2$, and $5/2$ nuclei together with various hyper Ca isotopes and the corresponding mirror hypernuclei. Finally the binding energy systematics of A =48 isobars of hypernuclei are predicted with/without the CSB EDF. The future experimental observations in JLab and J-PARC are desperately wanted to confirm the CSB effect in hypernuclei especially in the medium-heavy hypernuclei, $A = 40 \sim 48$.

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