

# model.rmd

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```
library(mice)
```

```
##
```

```
## Attaching package: 'mice'
```

```
## The following object is masked from 'package:stats':
```

```
##
```

```
## filter
```

```
## The following objects are masked from 'package:base':
```

```
##
```

```
## cbind, rbind
```

```
library(tidyr)
```

```
library(dplyr)
```

```
##
```

```
## Attaching package: 'dplyr'
```

```
## The following objects are masked from 'package:stats':
```

```
##
```

```
## filter, lag
```

```
## The following objects are masked from 'package:base':
```

```
##
```

```
## intersect, setdiff, setequal, union
```

```
library(visdat)
```

```
library(naniar)
```

```
library(ggplot2)
```

```
library(lme4)
```

```
## Loading required package: Matrix
```

```
## Warning: package 'Matrix' was built under R version 4.2.3
```

```
##
```

```
## Attaching package: 'Matrix'
```

```
## The following objects are masked from 'package:tidyr':  
##  
##   expand, pack, unpack
```

```
library(broom.mixed)  
library(kableExtra)
```

```
## Warning: package 'kableExtra' was built under R version 4.2.3
```

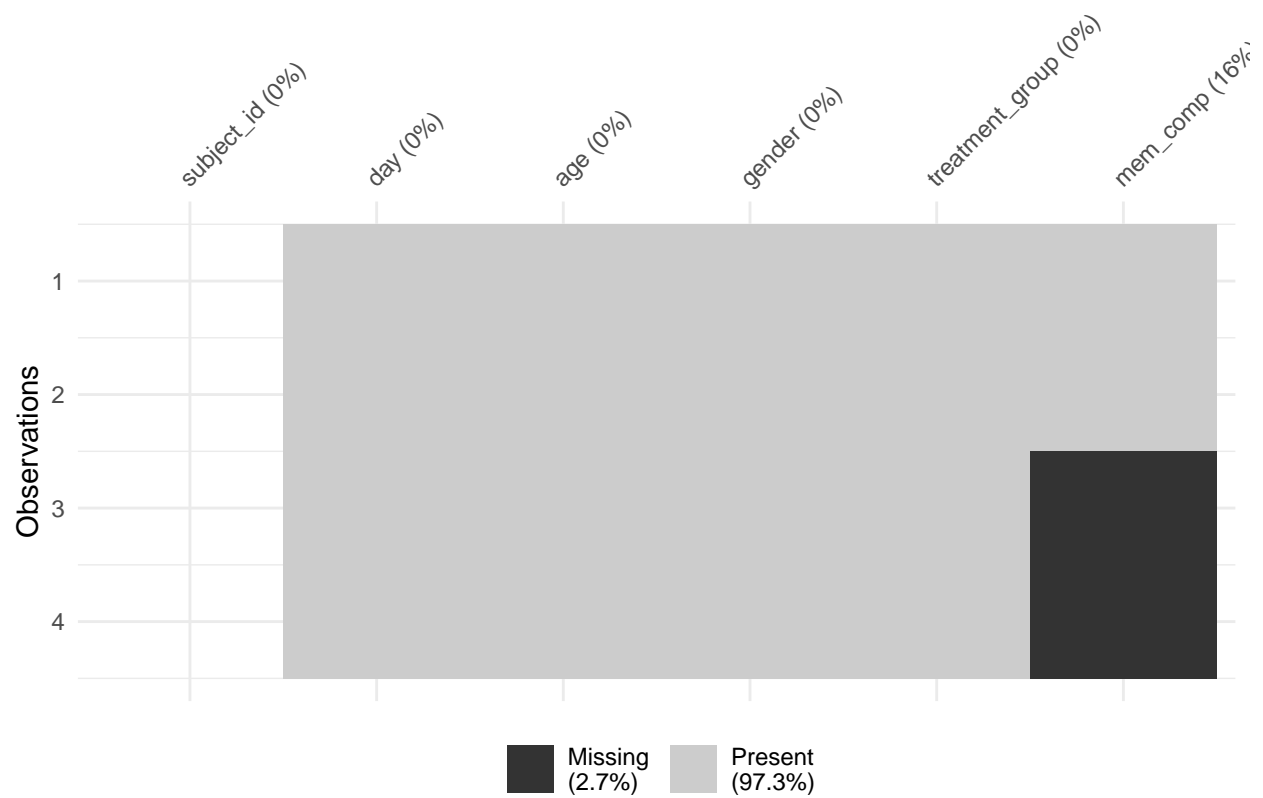
```
##  
## Attaching package: 'kableExtra'
```

```
## The following object is masked from 'package:dplyr':  
##  
##   group_rows
```

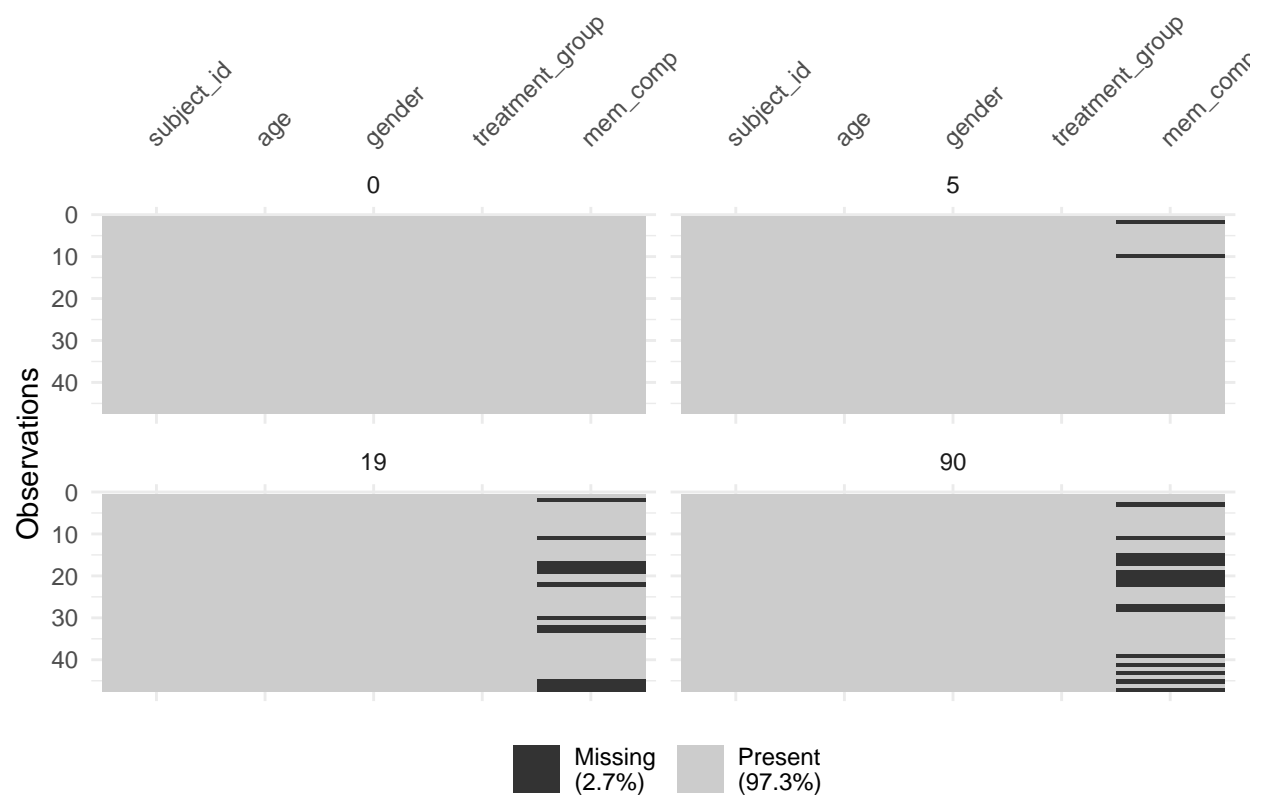
```
data <- read.csv("data.csv") %>%  
  janitor::clean_names() %>%  
  mutate(day = as.factor(day))
```

```
# pre-processing: expand design matrix and fill in NAs  
full_data <- data %>%  
  complete(day,  
            nesting(subject_id, treatment_group)) %>%  
  relocate(day, .after = subject_id) %>%  
  relocate(treatment_group, .after = gender) %>%  
  group_by(subject_id) %>%  
  fill(age, gender, .direction = "downup") %>%  
  arrange(subject_id, day)
```

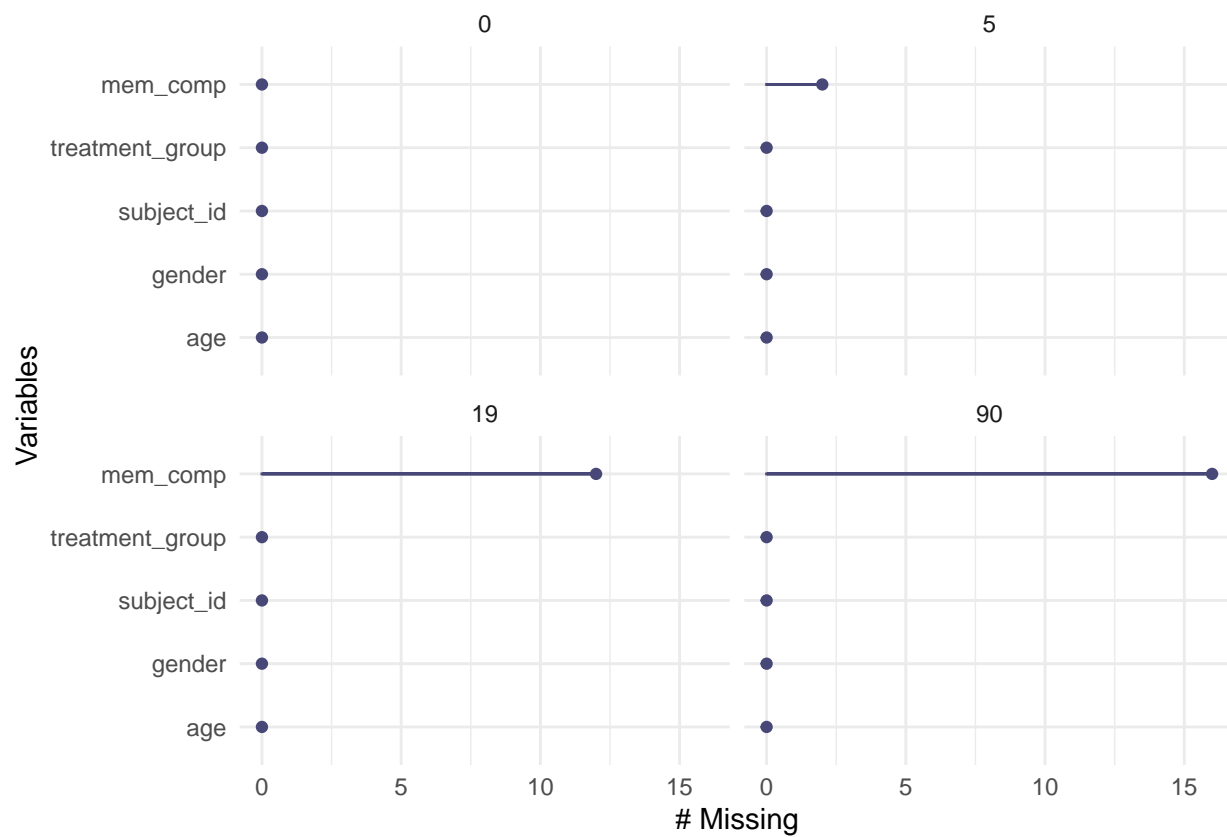
```
vis_miss(full_data)
```



```
vis_miss(full_data, facet = day)
```

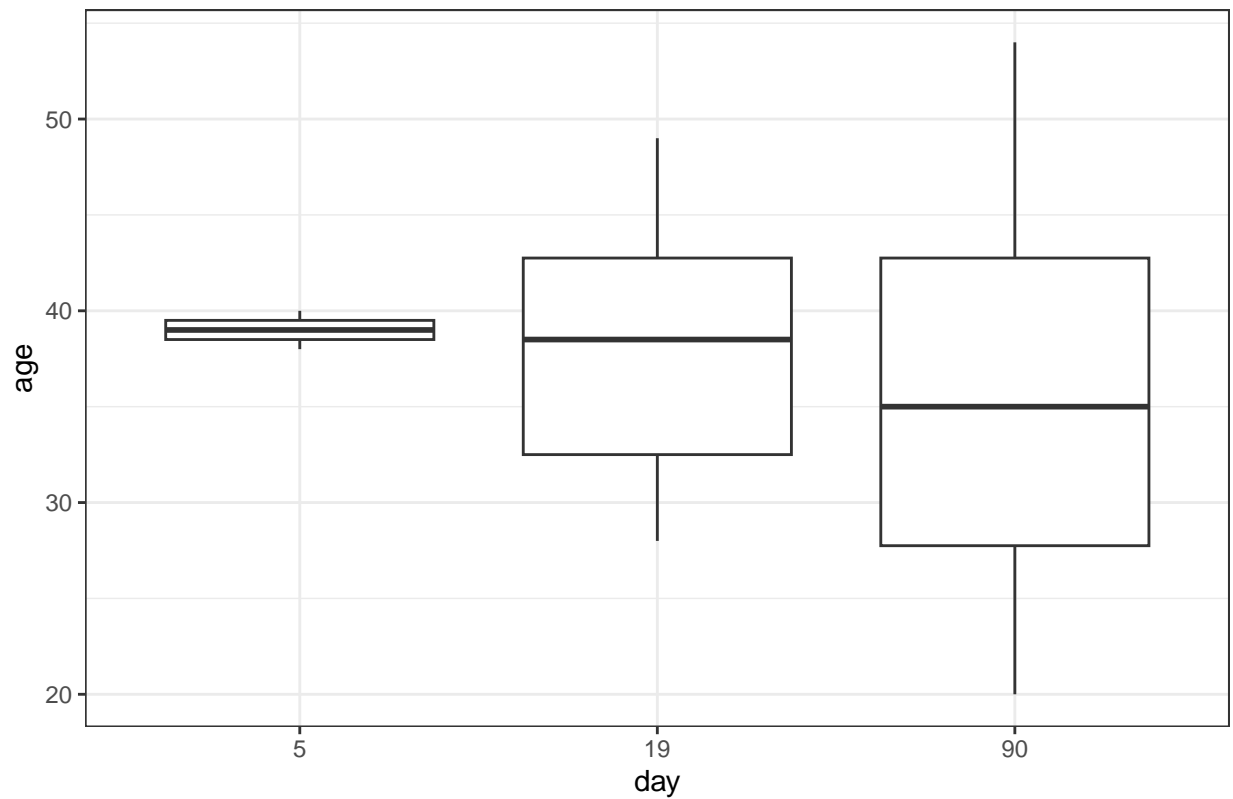


```
gg_miss_var(full_data,
  facet = day)
```



```
full_data %>% filter(is.na(mem_comp)) %>%
  ggplot(aes(x=as.factor(day)))+
  geom_boxplot(aes(y=age))+
  theme_bw()+
  xlab("day")+
  ggtitle("Relationship of age and missing values")
```

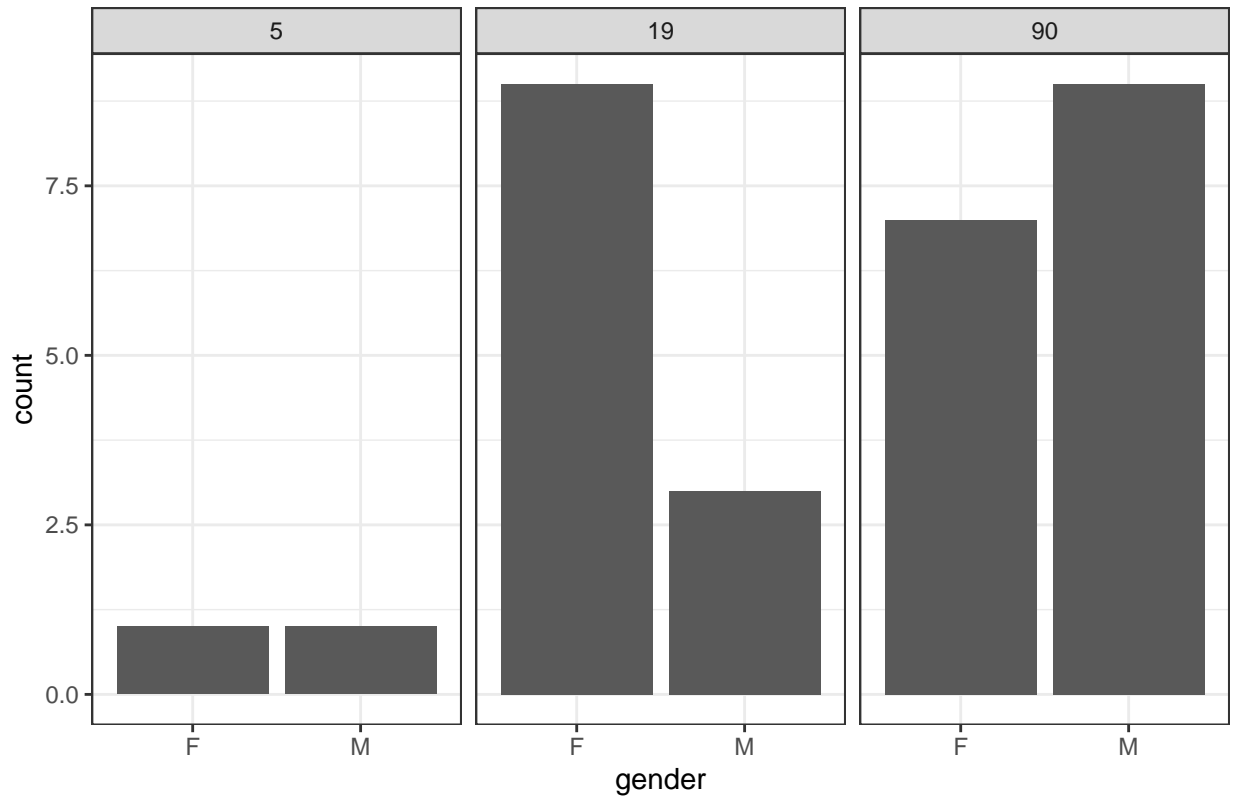
Relationship of age and missing values



```
full_data %>% filter(is.na(mem_comp)) %>%  
  ggplot()+  
  geom_bar(aes(x=gender, stat = "count"))+  
  facet_wrap(day~.)+  
  theme_bw()+  
  xlab("gender")+  
  ggtitle("Relationship of gender and missing values")
```

```
## Warning in geom_bar(aes(x = gender, stat = "count")): Ignoring unknown  
## aesthetics: stat
```

## Relationship of gender and missing values



```
# assume MAR
# multiple imputations
imputed_data <- mice::mice(data = full_data, m = 6, seed = 2024, print = FALSE)
```

```
## Warning: Number of logged events: 2
```

```
imputed_data
```

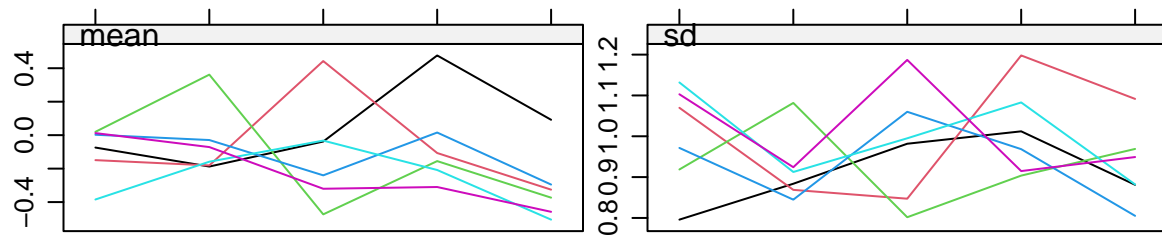
```
## Class: mids
## Number of multiple imputations: 6
## Imputation methods:
##   subject_id      day      age      gender treatment_group
##           ""           ""           ""           ""           ""
##   mem_comp
##   "pmm"
## PredictorMatrix:
##           subject_id day age gender treatment_group mem_comp
## subject_id      0  1  1  0      0      1
## day             1  0  1  0      0      1
## age             1  1  0  0      0      1
## gender          1  1  1  0      0      1
## treatment_group 1  1  1  0      0      1
## mem_comp        1  1  1  0      0      0
## Number of logged events: 2
##   it im dep      meth      out
```

```
## 1 0 0    constant    gender
## 2 0 0    constant treatment_group
```

```
# used pmm method
```

```
# check convergence
```

```
plot(imputed_data)
```



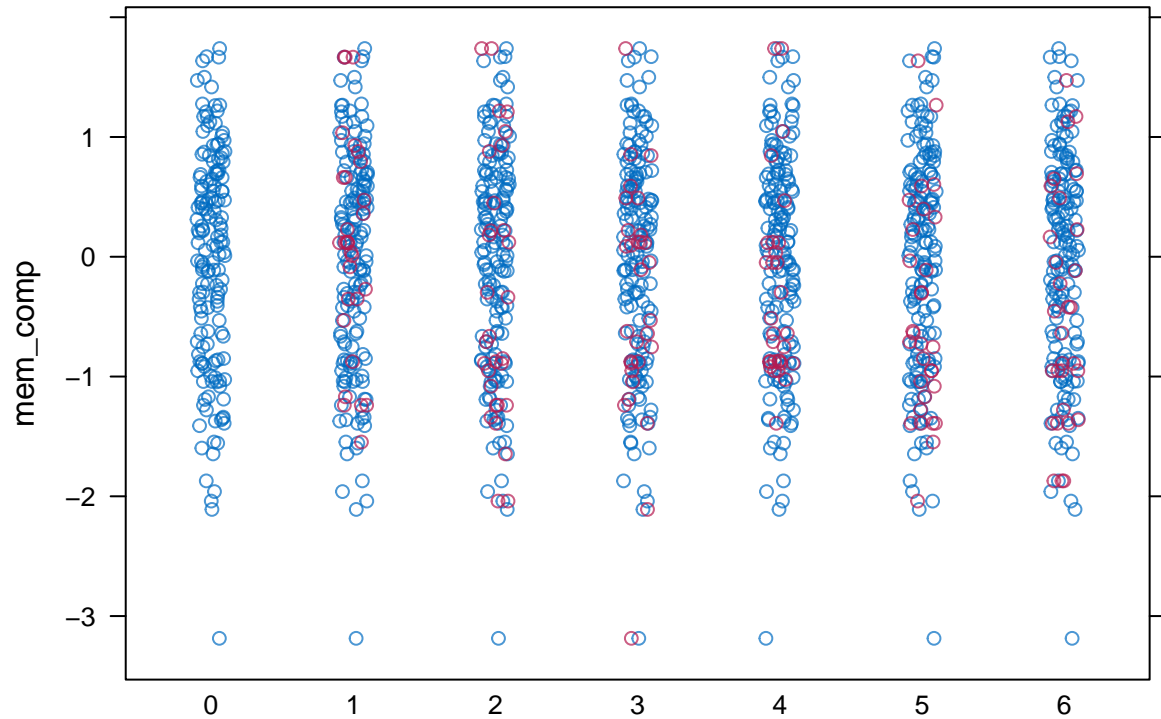
Iteration

```
# check if imputed data makes sense (ie. in a reasonable bound)
```

```
# red is imputed, blue is observed
```

```
mice::stripplot(imputed_data, mem_comp)
```





```
# fit model to each imputed datasets
```

```
model <- with(imputed_data, lmer(mem_comp ~ age + gender + treatment_group+day+treatment_group:day + (1
```

```
pool.fit <- pool(model)
```

```
summary(pool.fit)
```

```
##           term      estimate std.error statistic      df
## 1      (Intercept)  0.61259148 0.58498731  1.0471876 142.40918
## 2             age -0.01435861 0.01378527 -1.0415912 125.55072
## 3          genderM  0.07118804 0.25591736  0.2781681  88.04616
## 4  treatment_groupB -0.07762312 0.34501125 -0.2249872 169.48896
## 5  treatment_groupC -0.18796858 0.33052182 -0.5687025 169.82702
## 6           day5 -0.09577277 0.20913497 -0.4579472 170.01722
## 7           day19 -0.29378401 0.21785002 -1.3485609 131.38638
## 8           day90 -0.32331261 0.32115763 -1.0067100  12.46684
## 9  treatment_groupB:day5 -0.06276973 0.32782242 -0.1914748 117.93381
## 10 treatment_groupC:day5  0.34998520 0.30725190  1.1390823 152.67981
## 11 treatment_groupB:day19 0.09954652 0.36771610  0.2707157  41.12777
## 12 treatment_groupC:day19 0.57107906 0.31251365  1.8273732 132.74876
## 13 treatment_groupB:day90 0.12771299 0.40146403  0.3181181  24.01220
## 14 treatment_groupC:day90 0.31860668 0.34760696  0.9165716  48.25563
##      p.value
## 1 0.29678712
```

```
## 2 0.29960283
## 3 0.78153553
## 4 0.82226025
## 5 0.57030935
## 6 0.64757513
## 7 0.17979804
## 8 0.33321463
## 9 0.84848291
## 10 0.25645285
## 11 0.78796372
## 12 0.06988959
## 13 0.75314553
## 14 0.36392529
```

```
# check the pooled model covariance
# get the full matrix ubar (instead of only the diagonal)
m <- pool.fit$m
ubar <- Reduce("+", lapply(model$analyses, vcov)) / (m)
b <- pool.fit$pooled$b # this one is still provided by mice

# or by hand as well
# qbar <- getqbar(fil.pooled) # pooled estimates
# b <- 1 / (m-1) * rowSums((sapply(fit.mi$analyses, coef) - qbar)^2)

t <- ubar + (1 + 1 / (m)) * b # this is t as it used to be

# check versus the diagonal of t that is still provided
all.equal(as.numeric(diag(t)), pool.fit$pooled$t) # check
```

```
## [1] TRUE
```

```
calc_pooled_covar<- function(pool.fit,model){
  # check the pooled model covariance
  # get the full matrix ubar (instead of only the diagonal)
  m <- pool.fit$m
  ubar <- Reduce("+", lapply(model$analyses, vcov)) / (m)
  b <- pool.fit$pooled$b # this one is still provided by mice

  # or by hand as well
  # qbar <- getqbar(fil.pooled) # pooled estimates
  # b <- 1 / (m-1) * rowSums((sapply(fit.mi$analyses, coef) - qbar)^2)

  t <- ubar + (1 + 1 / (m)) * b # this is t as it used to be

  # check versus the diagonal of t that is still provided
  check = all.equal(as.numeric(diag(t)), pool.fit$pooled$t) # check
  if (check) {
    return(t)
  } else{
    stop("error!!!!!!")
  }
}
```

```

smy<-summary(pool.fit)
# Hypothesis test to see whether treatment effects at day 5, 19, and 90 are significant

output <- data.frame(
  Estimate = round(c(smy$estimate[c(6:8, 9, 11, 13, 10, 12, 14)]), smy$estimate[6]+smy$estimate[9], smy$
  `Standard error` = round(c(smy$std.error[c(6:8, 9, 11, 13, 10, 12, 14)]), sqrt((t[6,6]) + (t[9,9]) + 2
  `95% CI` = rep(NA,15)
)
colnames(output)[c(2,3)] <- c("Standard error", "95% CI")
output$`95% CI` <- paste0( "(", as.character(round(output$Estimate - 1.96*output$`Standard error`, 3)),
output$Significance <- ifelse((output$Estimate - 1.96*output$`Standard error`) * (output$Estimate + 1.9
rownames(output) <- c("$\\beta_5$", "$\\beta_6$", "$\\beta_7$", "$\\beta_8$", "$\\beta_9$", "$\\beta_{10}$", "$\\beta_{11}$", "$\\beta_{12}$", "$\\beta_{13}$", "$\\beta_5 + \\beta_8$", "$\\beta_6 + \\beta_9$", "$\\beta_7 + \\beta_{10}$", "$\\beta_5 + \\beta_{11}$", "$\\beta_6 + \\beta_{12}$", "$\\beta_7 + \\beta_{13}$")

# Print the table using kable
knitr::kable(output,caption = "Summary table")

```

Table 1: Summary table

|                        | Estimate | Standard error | 95% CI         | Significance |
|------------------------|----------|----------------|----------------|--------------|
| $\beta_5$              | -0.096   | 0.209          | (-0.506,0.314) | FALSE        |
| $\beta_6$              | -0.294   | 0.218          | (-0.721,0.133) | FALSE        |
| $\beta_7$              | -0.323   | 0.321          | (-0.952,0.306) | FALSE        |
| $\beta_8$              | -0.063   | 0.328          | (-0.706,0.58)  | FALSE        |
| $\beta_9$              | 0.100    | 0.368          | (-0.621,0.821) | FALSE        |
| $\beta_{10}$           | 0.128    | 0.401          | (-0.658,0.914) | FALSE        |
| $\beta_{11}$           | 0.350    | 0.307          | (-0.252,0.952) | FALSE        |
| $\beta_{12}$           | 0.571    | 0.313          | (-0.042,1.184) | FALSE        |
| $\beta_{13}$           | 0.319    | 0.348          | (-0.363,1.001) | FALSE        |
| $\beta_5 + \beta_8$    | -0.159   | 0.252          | (-0.653,0.335) | FALSE        |
| $\beta_6 + \beta_9$    | -0.194   | 0.320          | (-0.821,0.433) | FALSE        |
| $\beta_7 + \beta_{10}$ | -0.196   | 0.544          | (-1.262,0.87)  | FALSE        |
| $\beta_5 + \beta_{11}$ | 0.254    | 0.225          | (-0.187,0.695) | FALSE        |
| $\beta_6 + \beta_{12}$ | 0.277    | 0.255          | (-0.223,0.777) | FALSE        |
| $\beta_7 + \beta_{13}$ | -0.005   | 0.505          | (-0.995,0.985) | FALSE        |

```

# pool coef estimates and standard errors to create 95% CIs
pool_coef_ci_table <- function(pool.fit, t, caption){
  smy<-summary(pool.fit)
  # Hypothesis test to see whether treatment effects at day 5, 19, and 90 are significant

output <- data.frame(
  Estimate = round(c(smy$estimate[c(6:8, 9, 11, 13, 10, 12, 14)]), smy$estimate[6]+smy$estimate[9], smy$
  `Standard error` = round(c(smy$std.error[c(6:8, 9, 11, 13, 10, 12, 14)]), sqrt((t[6,6]) + (t[9,9]) + 2
  `95% CI` = rep(NA,15)
)
colnames(output)[c(2,3)] <- c("Standard error", "95% CI")
output$`95% CI` <- paste0( "(", as.character(round(output$Estimate - 1.96*output$`Standard error`, 3)),
output$Significance <- ifelse((output$Estimate - 1.96*output$`Standard error`) * (output$Estimate + 1.9
rownames(output) <- c("$\\beta_5$", "$\\beta_6$", "$\\beta_7$", "$\\beta_8$", "$\\beta_9$", "$\\beta_{10}$", "$\\beta_{11}$", "$\\beta_{12}$", "$\\beta_{13}$", "$\\beta_5 + \\beta_8$", "$\\beta_6 + \\beta_9$", "$\\beta_7 + \\beta_{10}$", "$\\beta_5 + \\beta_{11}$", "$\\beta_6 + \\beta_{12}$", "$\\beta_7 + \\beta_{13}$")

# Print the table using kable

```

```

return (knitr::kable(output, caption = caption, escape = TRUE))
}

```

```

# sensitivity analysis for each data missing assumptions

```

```

# mcar - fit data with completers data

```

```

completers_data <- full_data %>%

```

```

  group_by(subject_id) %>%

```

```

  mutate(missing = any(is.na(mem_comp))) %>%

```

```

  ungroup()

```

```

completers_fit <- lmer(mem_comp ~ age + gender + treatment_group+day +treatment_group:day + (1|subject_id), data=completers_data)
summary(completers_fit)

```

```

## Linear mixed model fit by REML ['lmerMod']

```

```

## Formula:

```

```

## mem_comp ~ age + gender + treatment_group + day + treatment_group:day +

```

```

## (1 | subject_id)

```

```

## Data: completers_data %>% filter(missing == FALSE)

```

```

##

```

```

## REML criterion at convergence: 173.8

```

```

##

```

```

## Scaled residuals:

```

```

##      Min       1Q   Median       3Q      Max

```

```

## -1.7688 -0.5756  0.1020  0.6102  1.7294

```

```

##

```

```

## Random effects:

```

```

## Groups      Name      Variance Std.Dev.

```

```

## subject_id (Intercept) 0.6790  0.8240

```

```

## Residual              0.1597  0.3997

```

```

## Number of obs: 96, groups: subject_id, 24

```

```

##

```

```

## Fixed effects:

```

```

##              Estimate Std. Error t value

```

```

## (Intercept)    -0.2777878  0.9983716  -0.278

```

```

## age              0.0004539  0.0227681   0.020

```

```

## genderM          0.2159847  0.3696430   0.584

```

```

## treatment_groupB  0.0579773  0.4876764   0.119

```

```

## treatment_groupC -0.0553875  0.4461694  -0.124

```

```

## day5             -0.2121626  0.1998282  -1.062

```

```

## day19            -0.3476050  0.1998282  -1.740

```

```

## day90            -0.2289847  0.1998282  -1.146

```

```

## treatment_groupB:day5  0.0978170  0.2925186   0.334

```

```

## treatment_groupC:day5  0.3899459  0.2746377   1.420

```

```

## treatment_groupB:day19 0.3457121  0.2925186   1.182

```

```

## treatment_groupC:day19 0.6461374  0.2746377   2.353

```

```

## treatment_groupB:day90 0.3018569  0.2925186   1.032

```

```

## treatment_groupC:day90 0.2610949  0.2746377   0.951

```

```

##

```

```

## Correlation matrix not shown by default, as p = 14 > 12.

```

```

## Use print(x, correlation=TRUE) or

```

```

##      vcov(x)          if you need it

```

```
t <-vcov(completers_fit)
```

```
completers_fit_se <- summary(completers_fit)[["coefficients"]][,2]
# calculate CIs
output <- data.frame(
  Estimate = round(c(completers_fit@beta[c(6:8, 9, 11, 13, 10, 12, 14)], completers_fit@beta[6]+completers_fit@beta[7]), 3),
  `Standard error` = round(c(completers_fit_se[c(6:8, 9, 11, 13, 10, 12, 14)], sqrt((t[6,6]) + (t[9,9]))), 3),
  `95% CI` = rep(NA,15)
)
colnames(output)[c(2,3)] <- c("Standard error", "95% CI")
output$`95% CI` <- paste0( "(", as.character(round(output$Estimate - 1.96*output$`Standard error`, 3)),
output$Significance <- ifelse((output$Estimate - 1.96*output$`Standard error`) * (output$Estimate + 1.96*output$`Standard error`) < 0, "TRUE", "FALSE")
rownames(output) <- c("$\\beta_5$", "$\\beta_6$", "$\\beta_7$", "$\\beta_8$", "$\\beta_9$", "$\\beta_{10}$", "$\\beta_{11}$", "$\\beta_{12}$", "$\\beta_{13}$",
"$\\beta_5 + \\beta_8$", "$\\beta_6 + \\beta_9$", "$\\beta_7 + \\beta_{10}$", "$\\beta_5 + \\beta_{11}$", "$\\beta_6 + \\beta_{12}$", "$\\beta_7 + \\beta_{13}$")

# Print the table using kable
knitr::kable(output,caption = "Completers' Model Summary table")
```

Table 2: Completers' Model Summary table

|                        | Estimate | Standard error | 95% CI         | Significance |
|------------------------|----------|----------------|----------------|--------------|
| $\beta_5$              | -0.212   | 0.200          | (-0.604,0.18)  | FALSE        |
| $\beta_6$              | -0.348   | 0.200          | (-0.74,0.044)  | FALSE        |
| $\beta_7$              | -0.229   | 0.200          | (-0.621,0.163) | FALSE        |
| $\beta_8$              | 0.098    | 0.293          | (-0.476,0.672) | FALSE        |
| $\beta_9$              | 0.346    | 0.293          | (-0.228,0.92)  | FALSE        |
| $\beta_{10}$           | 0.302    | 0.293          | (-0.272,0.876) | FALSE        |
| $\beta_{11}$           | 0.390    | 0.275          | (-0.149,0.929) | FALSE        |
| $\beta_{12}$           | 0.646    | 0.275          | (0.107,1.185)  | TRUE         |
| $\beta_{13}$           | 0.261    | 0.275          | (-0.278,0.8)   | FALSE        |
| $\beta_5 + \beta_8$    | -0.114   | 0.214          | (-0.533,0.305) | FALSE        |
| $\beta_6 + \beta_9$    | -0.002   | 0.214          | (-0.421,0.417) | FALSE        |
| $\beta_7 + \beta_{10}$ | 0.073    | 0.214          | (-0.346,0.492) | FALSE        |
| $\beta_5 + \beta_{11}$ | 0.178    | 0.188          | (-0.19,0.546)  | FALSE        |
| $\beta_6 + \beta_{12}$ | 0.299    | 0.188          | (-0.069,0.667) | FALSE        |
| $\beta_7 + \beta_{13}$ | 0.032    | 0.188          | (-0.336,0.4)   | FALSE        |

```
# mmar - fill na's with different delta --> fit model --> CI's

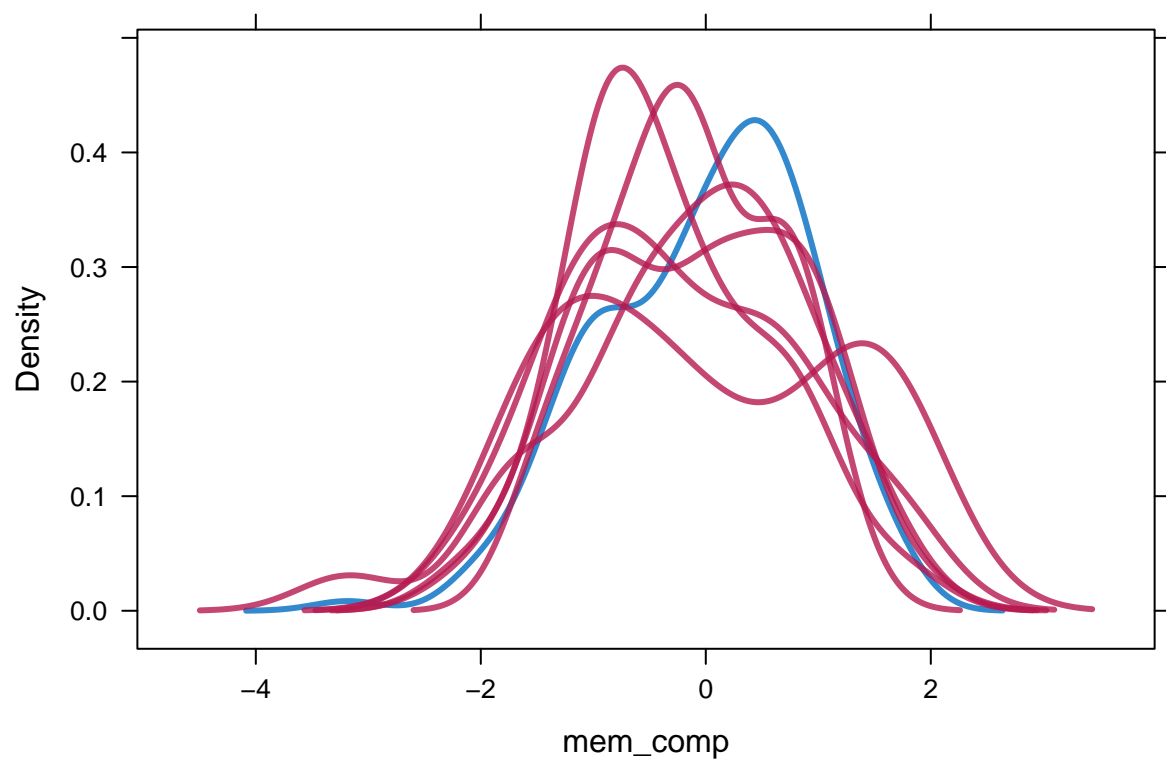
# Create a delta vector that represent the following adjustment values for mmHg: 0 for MAR, and -0.7, -0.2, 0.2, 0.7
delta <- c(0, -0.7, -0.2, 0.2, 0.7 )

imp.all <- vector("list", length(delta))
post <- imputed_data$post
for (i in 1:length(delta)){
  d <- delta[i]
  cmd <- paste("imp[[j]][,i] <- imp[[j]][,i] +", d)
  post["mem_comp"] <- cmd
  imp <- mice::mice(full_data, post = post,m = 6,seed = i, print = FALSE)
  imp.all[[i]] <- imp
}
```

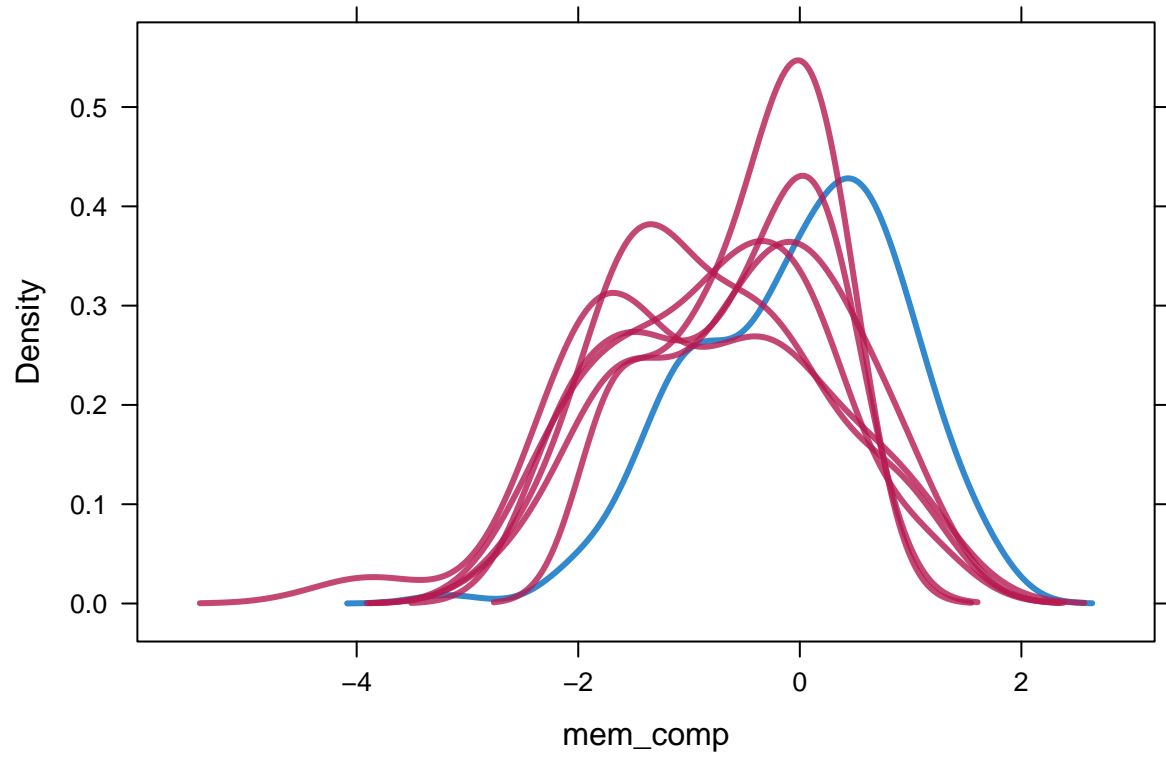
```
## Warning: Number of logged events: 2
```

```
## Warning: Number of logged events: 2
## Warning: Number of logged events: 2
## Warning: Number of logged events: 2
## Warning: Number of logged events: 2
```

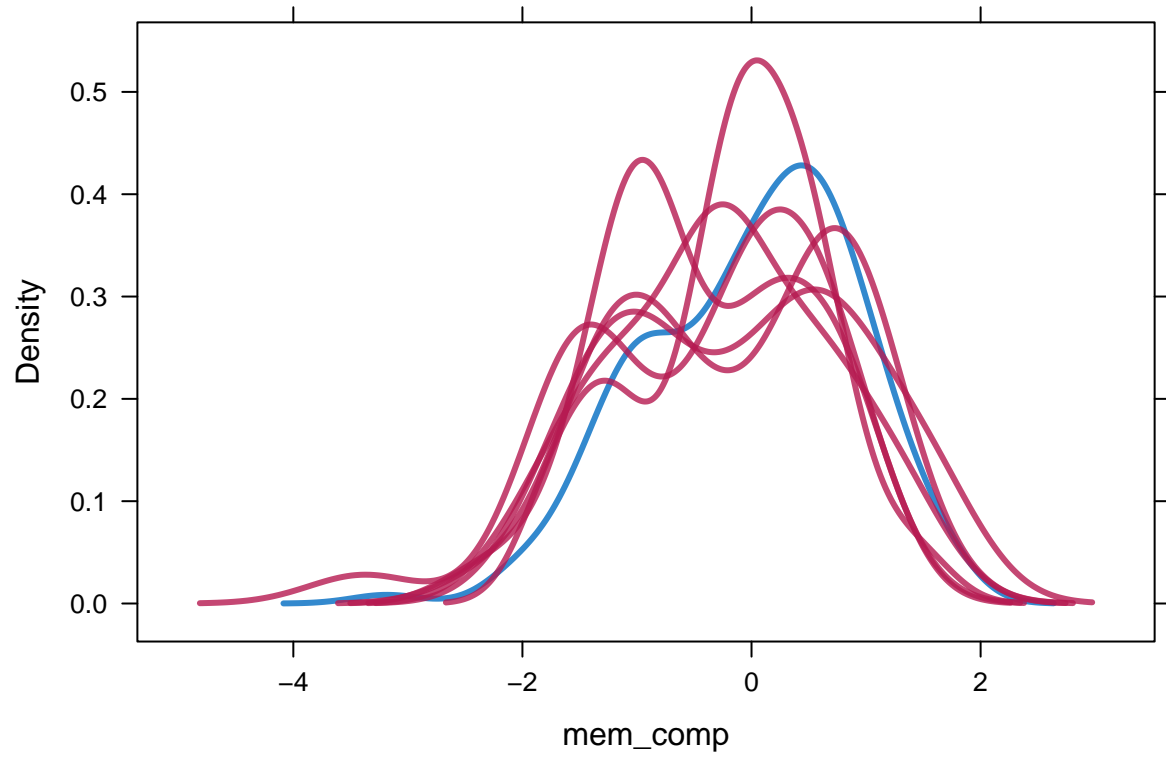
```
# imputation with no adjustment (delta = 0)
densityplot(imp.all[[1]], lwd = 3)
```



```
# imputation with adjustment (delta = -0.7)
densityplot(imp.all[[2]], lwd = 3)
```

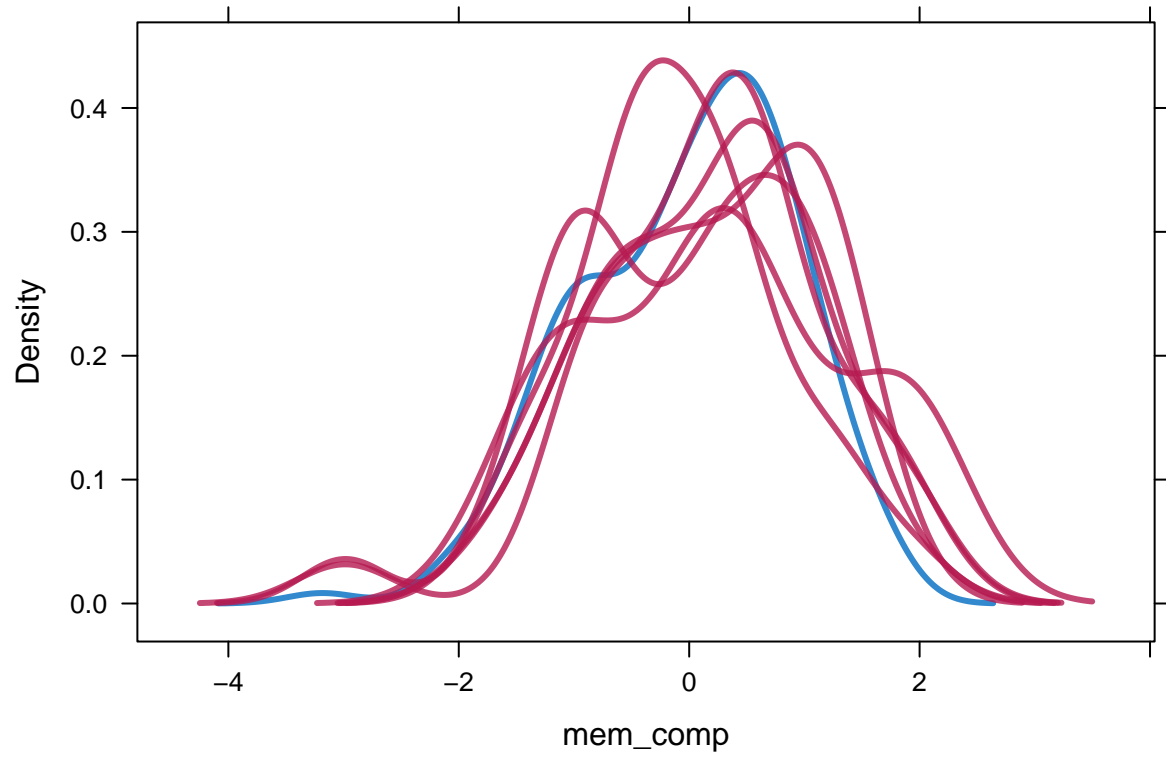


```
# imputation with adjustment (delta = -0.2)
densityplot(imp.all[[3]], lwd = 3)
```

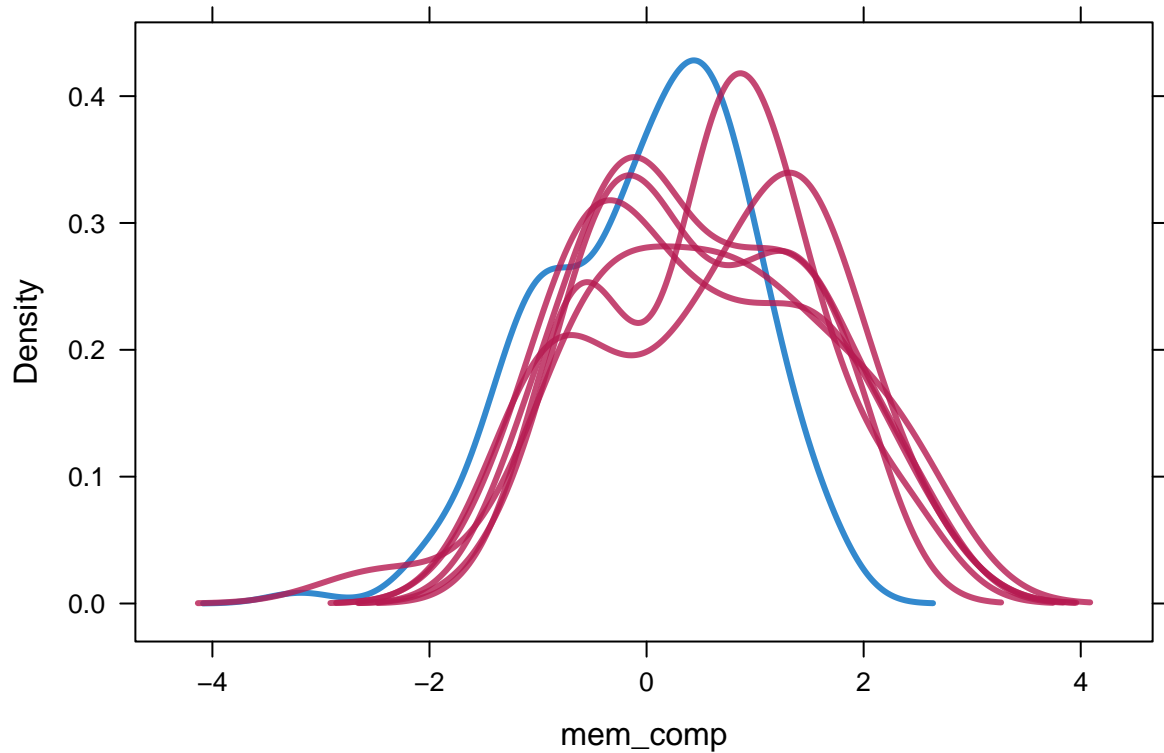


```
# imputation with adjustment (delta = 0.2)  
densityplot(imp.all[[4]], lwd = 3)
```





```
# imputation with adjustment (delta = 0.7)
densityplot(imp.all[[5]], lwd = 3)
```



```
# calculate results for imputation+adjustment datasets
for (i in 2:length(delta)){
  curr_data <- imp.all[[i]]

  # fit model
  curr_model <- with(curr_data, lmer(mem_comp ~ age + gender + treatment_group+day+treatment_group:day +
  (1|treatment_group)))

  curr.pool.fit <- pool(curr_model)

  # pool covar matrix
  curr_t<-calc_pooled_covar(curr.pool.fit,curr_model)

  # generate ci table
  curr_table <- pool_coef_ci_table(curr.pool.fit, curr_t, caption = paste0("delta=", delta[i], " Summary
  Statistics"))

  print(curr_table)
}
```

Table 3: delta=-0.7 Summary Table

|                        | Estimate | Standard error | 95% CI         | Significance |
|------------------------|----------|----------------|----------------|--------------|
| $\beta_5$              | -0.096   | 0.231          | (-0.549,0.357) | FALSE        |
| $\beta_6$              | -0.361   | 0.259          | (-0.869,0.147) | FALSE        |
| $\beta_7$              | -0.446   | 0.254          | (-0.944,0.052) | FALSE        |
| $\beta_8$              | -0.040   | 0.352          | (-0.73,0.65)   | FALSE        |
| $\beta_9$              | 0.039    | 0.409          | (-0.763,0.841) | FALSE        |
| $\beta_{10}$           | 0.107    | 0.365          | (-0.608,0.822) | FALSE        |
| $\beta_{11}$           | 0.303    | 0.336          | (-0.356,0.962) | FALSE        |
| $\beta_{12}$           | 0.622    | 0.379          | (-0.121,1.365) | FALSE        |
| $\beta_{13}$           | 0.200    | 0.377          | (-0.539,0.939) | FALSE        |
| $\beta_5 + \beta_8$    | -0.135   | 0.265          | (-0.654,0.384) | FALSE        |
| $\beta_6 + \beta_9$    | -0.322   | 0.394          | (-1.094,0.45)  | FALSE        |
| $\beta_7 + \beta_{10}$ | -0.339   | 0.337          | (-1,0.322)     | FALSE        |
| $\beta_5 + \beta_{11}$ | 0.207    | 0.245          | (-0.273,0.687) | FALSE        |
| $\beta_6 + \beta_{12}$ | 0.261    | 0.364          | (-0.452,0.974) | FALSE        |
| $\beta_7 + \beta_{13}$ | -0.245   | 0.350          | (-0.931,0.441) | FALSE        |

Table 4: delta=-0.2 Summary Table

|                        | Estimate | Standard error | 95% CI         | Significance |
|------------------------|----------|----------------|----------------|--------------|
| $\beta_5$              | -0.096   | 0.214          | (-0.515,0.323) | FALSE        |
| $\beta_6$              | -0.275   | 0.236          | (-0.738,0.188) | FALSE        |
| $\beta_7$              | -0.326   | 0.255          | (-0.826,0.174) | FALSE        |
| $\beta_8$              | -0.022   | 0.326          | (-0.661,0.617) | FALSE        |
| $\beta_9$              | 0.206    | 0.355          | (-0.49,0.902)  | FALSE        |
| $\beta_{10}$           | 0.143    | 0.374          | (-0.59,0.876)  | FALSE        |
| $\beta_{11}$           | 0.295    | 0.313          | (-0.318,0.908) | FALSE        |
| $\beta_{12}$           | 0.584    | 0.332          | (-0.067,1.235) | FALSE        |
| $\beta_{13}$           | 0.289    | 0.388          | (-0.471,1.049) | FALSE        |
| $\beta_5 + \beta_8$    | -0.118   | 0.246          | (-0.6,0.364)   | FALSE        |
| $\beta_6 + \beta_9$    | -0.069   | 0.333          | (-0.722,0.584) | FALSE        |
| $\beta_7 + \beta_{10}$ | -0.183   | 0.390          | (-0.947,0.581) | FALSE        |
| $\beta_5 + \beta_{11}$ | 0.199    | 0.228          | (-0.248,0.646) | FALSE        |
| $\beta_6 + \beta_{12}$ | 0.309    | 0.307          | (-0.293,0.911) | FALSE        |
| $\beta_7 + \beta_{13}$ | -0.037   | 0.404          | (-0.829,0.755) | FALSE        |

Table 5: delta=0.2 Summary Table

|              | Estimate | Standard error | 95% CI         | Significance |
|--------------|----------|----------------|----------------|--------------|
| $\beta_5$    | -0.096   | 0.201          | (-0.49,0.298)  | FALSE        |
| $\beta_6$    | -0.260   | 0.231          | (-0.713,0.193) | FALSE        |
| $\beta_7$    | -0.116   | 0.234          | (-0.575,0.343) | FALSE        |
| $\beta_8$    | -0.018   | 0.326          | (-0.657,0.621) | FALSE        |
| $\beta_9$    | 0.199    | 0.366          | (-0.518,0.916) | FALSE        |
| $\beta_{10}$ | 0.080    | 0.352          | (-0.61,0.77)   | FALSE        |
| $\beta_{11}$ | 0.384    | 0.293          | (-0.19,0.958)  | FALSE        |
| $\beta_{12}$ | 0.598    | 0.349          | (-0.086,1.282) | FALSE        |
| $\beta_{13}$ | 0.271    | 0.330          | (-0.376,0.918) | FALSE        |

|                        | Estimate | Standard error | 95% CI         | Significance |
|------------------------|----------|----------------|----------------|--------------|
| $\beta_5 + \beta_8$    | -0.114   | 0.256          | (-0.616,0.388) | FALSE        |
| $\beta_6 + \beta_9$    | -0.061   | 0.363          | (-0.772,0.65)  | FALSE        |
| $\beta_7 + \beta_{10}$ | -0.036   | 0.354          | (-0.73,0.658)  | FALSE        |
| $\beta_5 + \beta_{11}$ | 0.288    | 0.213          | (-0.129,0.705) | FALSE        |
| $\beta_6 + \beta_{12}$ | 0.339    | 0.346          | (-0.339,1.017) | FALSE        |
| $\beta_7 + \beta_{13}$ | 0.156    | 0.332          | (-0.495,0.807) | FALSE        |

Table 6: delta=0.7 Summary Table

|                        | Estimate | Standard error | 95% CI         | Significance |
|------------------------|----------|----------------|----------------|--------------|
| $\beta_5$              | -0.096   | 0.220          | (-0.527,0.335) | FALSE        |
| $\beta_6$              | -0.072   | 0.226          | (-0.515,0.371) | FALSE        |
| $\beta_7$              | -0.079   | 0.263          | (-0.594,0.436) | FALSE        |
| $\beta_8$              | 0.051    | 0.336          | (-0.608,0.71)  | FALSE        |
| $\beta_9$              | 0.216    | 0.405          | (-0.578,1.01)  | FALSE        |
| $\beta_{10}$           | 0.205    | 0.419          | (-0.616,1.026) | FALSE        |
| $\beta_{11}$           | 0.365    | 0.325          | (-0.272,1.002) | FALSE        |
| $\beta_{12}$           | 0.630    | 0.362          | (-0.08,1.34)   | FALSE        |
| $\beta_{13}$           | 0.178    | 0.424          | (-0.653,1.009) | FALSE        |
| $\beta_5 + \beta_8$    | -0.045   | 0.254          | (-0.543,0.453) | FALSE        |
| $\beta_6 + \beta_9$    | 0.144    | 0.354          | (-0.55,0.838)  | FALSE        |
| $\beta_7 + \beta_{10}$ | 0.126    | 0.437          | (-0.731,0.983) | FALSE        |
| $\beta_5 + \beta_{11}$ | 0.269    | 0.239          | (-0.199,0.737) | FALSE        |
| $\beta_6 + \beta_{12}$ | 0.558    | 0.303          | (-0.036,1.152) | FALSE        |
| $\beta_7 + \beta_{13}$ | 0.099    | 0.441          | (-0.765,0.963) | FALSE        |