Modeling of carbon cycle

```
from scipy.integrate import odeint
import numpy as np
import pandas as pd
import matplotlib.pyplot as plt
import scipy.optimize as optimize
from matplotlib.ticker import MaxNLocator
import math
from scipy.integrate import odeint
from math import e
```

1.1Following equation 1-2 (without the buffer effect), build a two-box model to compute the atmospheric CO2 level in ppm (parts per million) from 1987 to 2004.

```
df=pd.read_csv('global.1751_2008.ems.txt',sep='\s+',skiprows=27,     #Skip 27 lines
               names=['year','total','gas','liquids','cement solids','gas production','per flaring','capita'])
df.head(5)
```

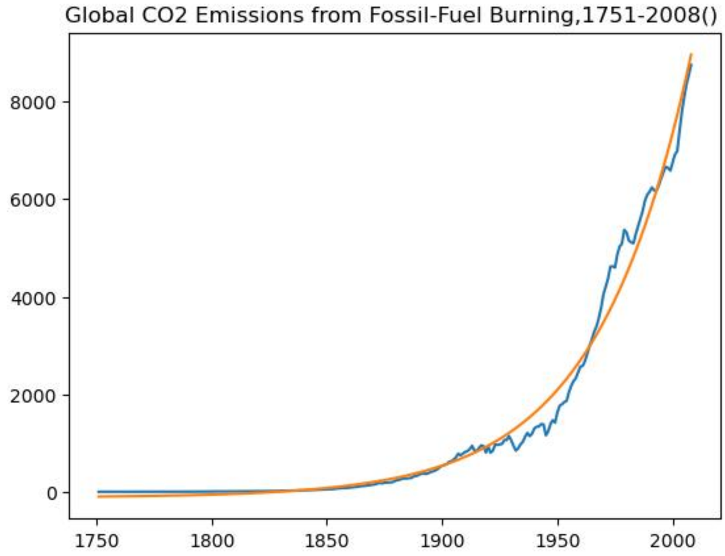
	year	total	gas	liquids	cement solids	gas production	per flaring	capita
0	1751	3	0	0	3	0	0	NaN
1	1752	3	0	0	3	0	0	NaN
2	1753	3	0	0	3	0	0	NaN
3	1754	3	0	0	3	0	0	NaN
4	1755	3	0	0	3	0	0	NaN

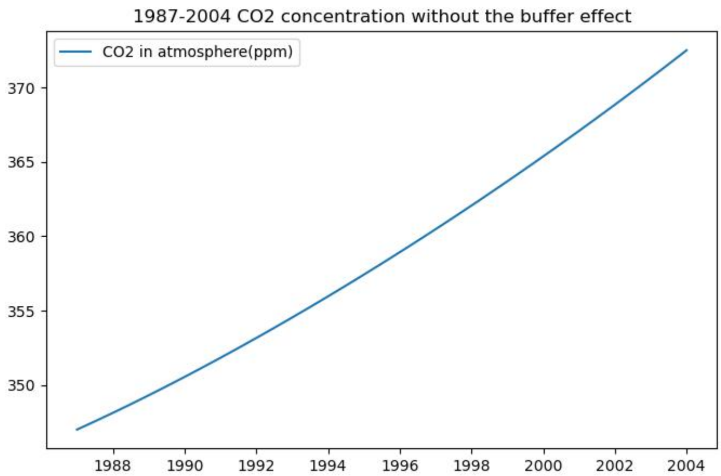
```
# The emission was observed to change rapidly over time and was simulated exponentially
def CO2_emis(t,r1,r2,r3): # Define the function that fits
    return np.exp(r1*t+r2)+r3
r1=0.1
r2=0.1
r3=0
p0=[r1,r2,r3] #Set the initial value of the fit
para,cov=optimize.curve_fit(CO2_emis,df['year'],df['total'],p0=p0)#Call the fitting function
emis_fit=[CO2_emis(a,*para) for a in df['year']] #Calculate the result after fitting
print(para)
plt.plot(df['year'],df['total'])
plt.plot(df['year'],emis_fit)
plt.title('Global CO2 Emissions from Fossil-Fuel Burning,1751-2008() ')
plt.show()                          # Comparison graph between true value and fitting
def model1(y,t,k12,k21,para):     #Define a function without the buffer effect
    N1,N2,r = y
    dydt=[-k12*N1+k21*N2+r,k12*N1-k21*N2,para[0]*(r-para[2])]
    return dydt

t=np.linspace(1987, 2004, 100)#independent variable
r=CO2_emis(t,*para)
k12=105/740
k21=102/900
N1=740*1000
N2=900*1000
y0=[N1,N2,r[0]] #initial value
sol = odeint(model1, y0, t, args=(k12,k21,para))/1000/740*347 #Call the carbon models function, the result is translated into ppm units
plt.figure(figsize=(8,5))
plt.plot(t, sol[:, 0], label='CO2 in atmosphere(ppm)')
plt.gca().xaxis.set_major_locator(MaxNLocator(integer=True)) # Set the scale of the horizontal axis to an integer
plt.legend(loc='best')
plt.title('1987-2004 CO2 concentration without the buffer effect')
plt.show()
```

The curve_fit function is used to fit the exponential growth model of CO2 emissions from the given data. The second is to simulate the first order linear differential equation using odeint function to predict the change of CO2 concentration with time in the absence of buffering effect. It is used to fit and predict changes in CO2 emissions and concentrations over time.

Result:





#1.2 [20 points] Following equation 3-4 (with the buffer effect), build a two-box model to compute the atmospheric CO2 level in ppm from 1987 to 2004.

```
def model2(y,t,k12,k21,N0,para):     #Define a function with the buffer effect
    N1,N2,r2 = y
    bf=3.69+1.86e-2*(N1/740/1000*347)-1.8e-6*((N1/740/1000*347)**2) #buffer factor
    dydt=[-k12*N1+k21*(N0+bf*(N2-N0))+r2,     #  Due N0 is the equilibrium value of carbon in the surface ocean In the preindustrial era,
          k12*N1-k21*(N0+bf*(N2-N0)),         # so the independent variables(t) should start in the preindustrial era
          para[0]*(r2-para[2])]
    return dydt
t2=np.linspace(1751, 2004, 253) #independent variable(start in the preindustrial era)
r2=CO2_emis(t2,*para)
k12=105/740
k21=102/900
N0=821*1000
N1=618*1000
N2=821*1000
y0=[N1,N2,r2[0]] #initial value
sol2 = odeint(model2, y0, t2, args=(k12,k21,N0,para))/740/1000*347 #Call the carbon models function, the result is translated into ppm units
plt.figure(figsize=(8,5))
plt.plot(t2[235:253],sol2[235:253,0], label='CO2 in atmosphere(ppm)')
plt.gca().xaxis.set_major_locator(MaxNLocator(integer=True)) # Set the scale of the horizontal axis to an integer
plt.legend(loc='best')
plt.title('1987-2004 CO2 concentration with the buffer effect')
plt.show()
```

This code is used to simulate the changes of CO2 concentration over time while considering the buffer effect using a first-order linear differential equation.

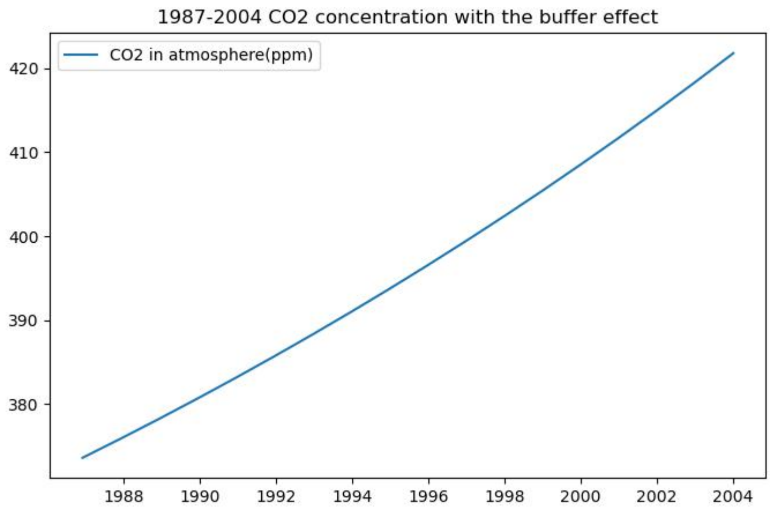
First, a function named `model2` is defined, which is used to calculate the changes of CO2 concentration over time with the buffer effect. The buffer effect is reflected by the `bf`, which is a function related to the CO2 concentration and decreases with the increase of CO2 concentration. `N0` is the equilibrium value of carbon in the surface ocean, which is a constant before the Industrial Revolution.

Then, the `odeint` function is used to solve this differential equation. This function takes the initial value `y0` and time period `t2` as input and returns the CO2 concentration at each time point.

Finally, the simulated CO2 concentration is plotted to show the changes of CO2 concentration from 1987 to 2004.

In summary, this code is mainly used for simulating the changes of CO2 concentration over time while considering the buffer effect.

Result:



#1.3 [5 points] Based on your results from 1.1 and 1.2, reproduce Figure 2 in Tomizuka (2009) as much as you can.

```
df2=pd.read_csv('co2_annmean_mlo.csv',sep=',',skiprows=57,usecols=[0,1], # obversed data
                names=['year','CO2'])
plt.figure(figsize=(8,5))
plt.plot(t, sol[:, 0]) # without buffer effect
plt.plot(t2[235:253], sol2[235:253,0], 'r') # with buffer effect
plt.scatter(df2['year'][28:45],df2['CO2'][28:45]) # observed data
plt.legend(['without buffer effect','with buffer effect','observation'],loc='best')
plt.xlabel('year')
plt.ylabel('CO2(ppm)')
plt.gca().xaxis.set_major_locator(MaxNLocator(integer=True))
plt.title('CO2 concentration(ppm)')
plt.show()
```

The main purpose of this code is to plot the changes of CO2 concentration over time, including the cases with and without considering the buffer effect, as well as the actual observed data.

1. First, the `pd.read_csv` function is used to read a CSV file named 'co2_annmean_mlo.csv', which should contain the observed CO2 concentration data. These data are stored in the `df2` variable.

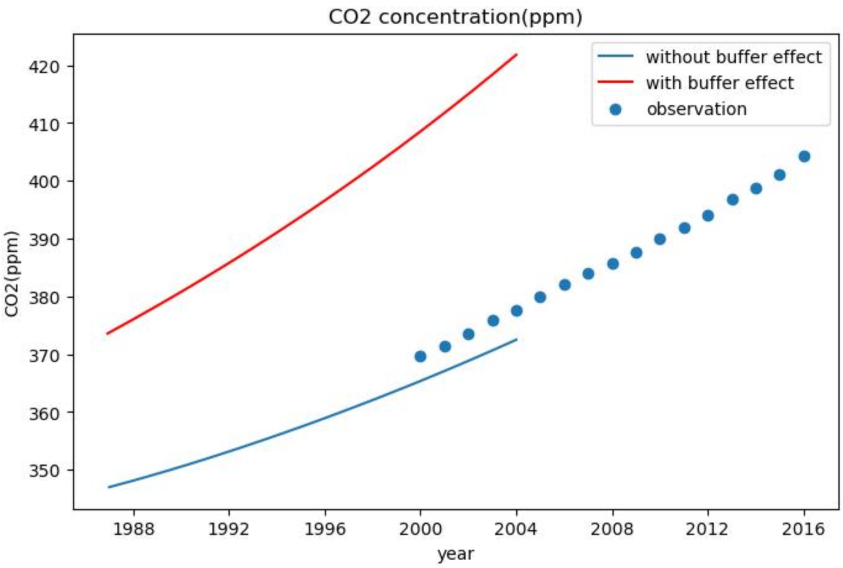
2. Then, the `plt.plot` function is used to draw two curves: one is the curve of CO2 concentration over time considering the buffer effect (red), and the other is the curve of CO2 concentration over time without considering the buffer effect. These data are provided by the

`sol2` and `sol` variables, respectively.

3. Next, the `plt.scatter` function is used to mark the actual observed data points on the curves. These data points are extracted from the `df2` variable.

4. Finally, the `plt.legend` function is used to add a legend to the graph, explaining the meaning of the three curves. The labels for the x-axis and y-axis are set, as well as the title of the graph.

Result:



#[Bonus] [15 points] Following equation 5-13, compute the atmospheric CO2 level in ppm and reproduce Figure 4 in Tomizuka (2009).

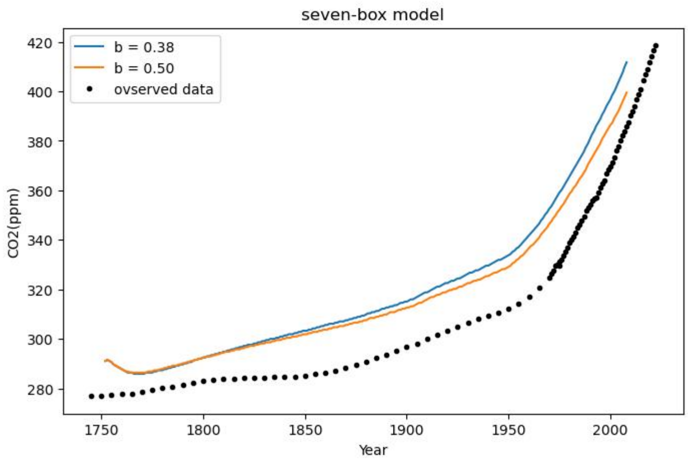
```
def model3(N,t,rr,bf,f,da):   #defintion seveb-box model
    N02 = 821
    k12 = 60/615
    k21 = 60/842
    k23 = 9/842
    k24 = 43/842
    k32 = 52/9744
    k34 = 162/9744
    k43 = 205/26280
    k45 = 0.2/26280
    k51 = 0.2/90000000
    k67 = 62/731
    k71 = 62/1328
    N1,N2,N3,N4,N5,N6,N7 = N
    dN1dt = [-k12 * N1 + k21 * (N02 + bf*(N2 - N02)) + rr - f + da + k51 * N5 + k71 * N7,
            k12 * N1 - k21 * (N02 + bf*(N2 - N02)) - k23 * N2 + k32 * N3 - k24 * N2,
            k23*N2 - k32*N3 - k34*N3 +k43*N4,
            k34*N3 - k43*N4 + k24*N2 -k45*N4,
            k45*N4 - k51*N5,
            f - k67*N6 - 2*da,
            k67*N6 - k71*N7 + da]
    return dN1dt
def buffer(CO2):   #defintion buffer
    bf = 3.69 + 1.86 * 10**-2 * CO2 - 1.80 * 10**-6 * CO2**2
    return bf
def ff(P,beta):
    f0 = 62
    P0=290.21
    f = f0 * (1 + beta * math.log(P/P0))
    return f
```

```
# read the data
rr = np.loadtxt("global.1751_2008.csv", delimiter=",", skiprows = 2, usecols = 1)/10**3
da = np.empty_like(rr)
da[0:100] = np.linspace(0.2,0.5,100)
da[99:-3]= np.loadtxt("Global_land-use_flux-1850_2005.csv", delimiter=",", skiprows = 1, usecols = 1)/10**3
da[-3:] = da[-4]
annualco2 = np.loadtxt("co2_annmean_mlo.csv", delimiter=",", skiprows = 56 )[:,1]
T = np.loadtxt("co2_annmean_mlo.csv", delimiter=",", skiprows = 56 )[:,0]
ynum = 2008 - 1751 + 1

t = np.linspace(1751,1751+ynum-1,ynum,dtype='int')
co2_obs = np.loadtxt("co2.csv", delimiter=",", skiprows = 149)
model3_b1 = np.empty_like(t)
model3_b2 = np.empty_like(t)
# situation of Bita=0.38    #reference shenshao
bb = 0.38
NO = [615, 842, 9744, 26280, 90000000, 731, 1238]
model3_b1[0] = NO[0]
co2p = model3_b1[0]/2.13
bf = buffer(co2p)
f = ff(co2p,bb)
for i in range(1,ynum):# slove the ODE
    dt = [0,1]
    N = odeint(model3,NO,dt,args=(rr[i-1],bf,f,da[i-1])) # solve the equal
    model3_b1[i] = N[1][0]     # print the result
    co2p = model3_b1[i]/2.13 # initiation
    NO = N[1]
    bf = buffer(co2p)
    f = ff(co2p,bb)
# situation of Bita=0.5
bb = 0.5
NO = [615, 842, 9744, 26280, 90000000, 731, 1238]
model3_b2[0] = NO[0]
co2p = model3_b2[0]/2.13
bf = buffer(co2p)
f = ff(co2p,bb)
for i in range(1,ynum):
    dt = [0,1]
    N = odeint(model3,NO,dt,args=(rr[i-1],bf,f,da[i-1])) # solve the equal
    model3_b2[i] = N[1][0]         # print the result
    co2p = model3_b2[i]/2.13   # initiation
    NO = N[1]

    bf = buffer(co2p)
    f = ff(co2p,bb)
```

```
plt.figure(figsize=(8,5))
plt.plot(t[1:],model3_b1[1:]/2.13,label='b = 0.38') # plot Beta=0.38
plt.plot(t[1:],model3_b2[1:]/2.13,label='b = 0.50')  #plot Beta=0.5
plt.plot(co2_obs[:,0], co2_obs[:,1],'k.')
plt.plot(T,annualco2,'k.',label='ovserved data')    # ovserved data
plt.ylabel('CO2(ppm)')
plt.xlabel('Year')
plt.title('seven-box model')
plt.legend(loc='best')
plt.show()
```



The main purpose of this code is to define a seven-box model to simulate the changes of CO2 concentration over time, considering the buffer effect and the observed data of CO2.

1. The `model3` function defines the seven-box model, which includes the CO2 concentrations (N1, N2, N3, N4, N5, N6, N7) in seven different states, as well as the rates of change of these states over time. This model considers the conversion of CO2 in different environments, such as the atmosphere, ocean surface, deep sea, etc.

2. The `buffer` function defines the buffer effect, which is a function related to the CO2 concentration and decreases with the increase of CO2 concentration.

3. The `ff` function defines the source-sink term of CO_2, which is related to atmospheric pressure and temperature.
4. Next, the code reads the observed data of CO_2, including CO_2 concentration, atmospheric pressure, temperature, etc.
5. Then, the code solves the changes of CO_2 concentration over time using the `odeint` function, based on the seven-box model and the buffer effect. This process considers the conversion of CO_2 in different environments and the buffer effect.
6. Finally, the code plots the curve of simulated CO_2 concentration over time, as well as the actual observed data points. These curve charts show the changes of CO_2 concentration under different buffer effects (b = 0.38 and b = 0.5).