Modeling of carbon cycle

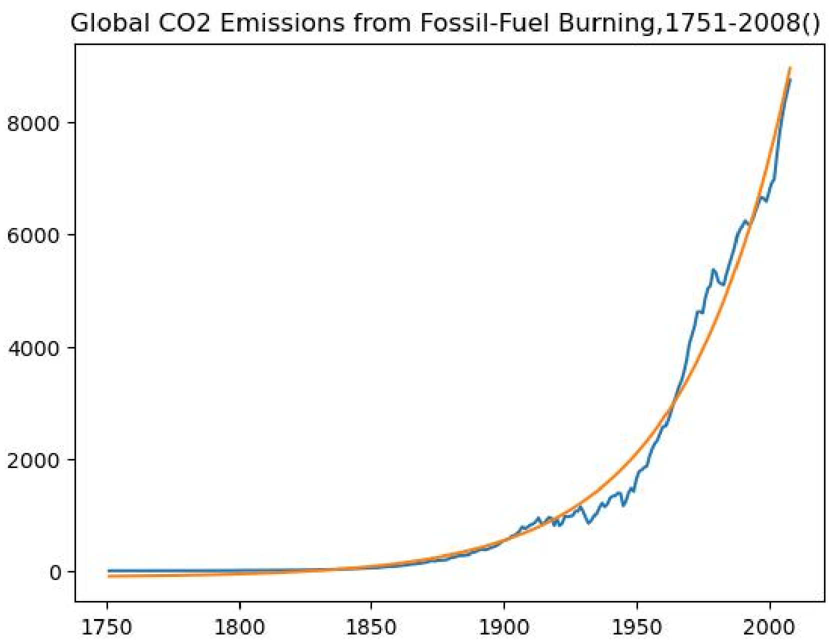
```
from scipy.integrate import odeint
import numpy as np
import pandas as pd
import matplotlib.pyplot as plt
import scipy.optimize as optimize
from matplotlib.ticker import MaxNLocator
import math
from scipy.integrate import odeint
from math import e
```

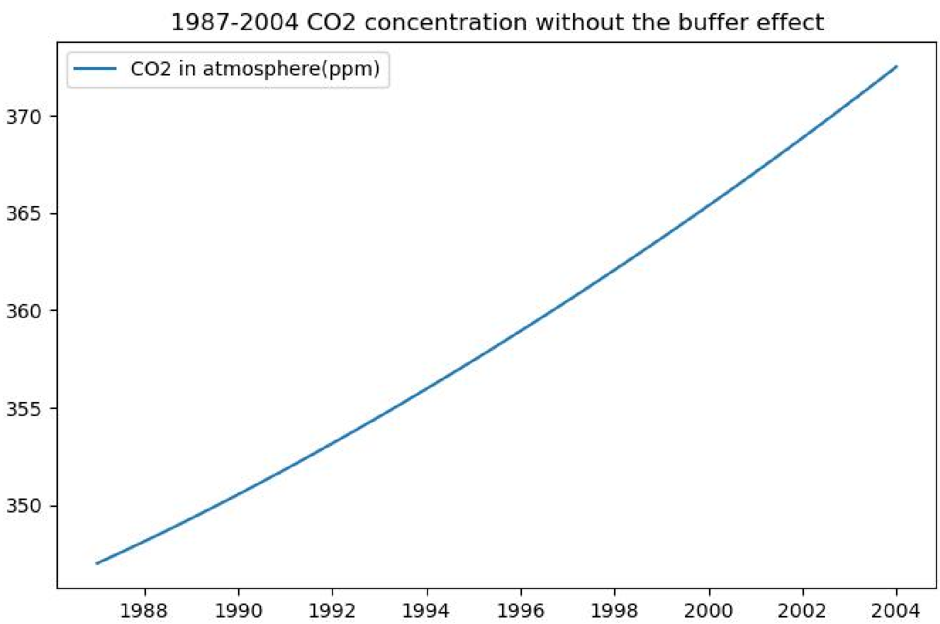
1.1Following equation 1-2 (without the buffer effect), build a two-box model to compute the atmospheric CO2 level in ppm (parts per million) from 1987 to 2004.

```
df=pd.read_csv('global.1751_2008.ems.txt',sep='\s+',skiprows=27,     #Skip 27 lines
                 names=['year','total','gas','liquids','cement solids','gas production','per flaring','capita'])
df.head(5)
```

	year	total	gas	liquids	cement solids	gas production	per flaring	capita
0	1751	3	0	0	3	0	0	NaN
1	1752	3	0	0	3	0	0	NaN
2	1753	3	0	0	3	0	0	NaN
3	1754	3	0	0	3	0	0	NaN
4	1755	3	0	0	3	0	0	NaN

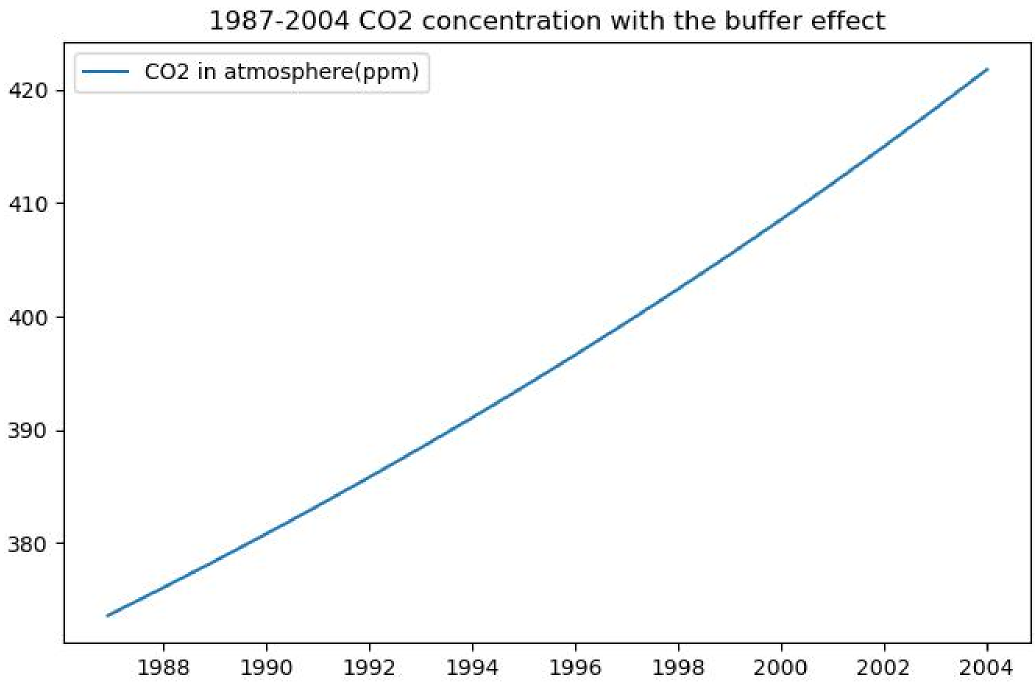
```
# The emission was observed to change rapidly over time and was simulated exponentially
def CO2_emis(t,r1,r2,r3): # Define the function that fits
    return np.exp(r1*t+r2)+r3
r1=0.1
r2=0.1
r3=0
p0=[r1,r2,r3] #Set the initial value of the fit
para,cov=optimize.curve_fit(CO2_emis,df['year'],df['total'],p0=p0)#Call the fitting function
emis_fit=[CO2_emis(a,*para) for a in df['year']] #Calculate the result after fitting
print(para)
plt.plot(df['year'],df['total'])
plt.plot(df['year'],emis_fit)
plt.title('Global CO2 Emissions from Fossil-Fuel Burning,1751-2008() ')
plt.show()                              # Comparison graph between true value and fitting
def model1(y,t,k12,k21,para):     #Define a function without the buffer effect
    N1,N2,r = y
    dydt=[-k12*N1+k21*N2+r,k12*N1-k21*N2,para[0]*(r-para[2])]
    return dydt

t=np.linspace(1987, 2004, 100)#independent variable
r=CO2_emis(t,*para)
k12=105/740
k21=102/900
N1=740*1000
N2=900*1000
y0=[N1,N2,r[0]] #initial value
sol = odeint(model1, y0, t, args=(k12,k21,para))/1000/740*347 #Call the carbon models function, the result is translated into ppm units
plt.figure(figsize=(8,5))
plt.plot(t, sol[:, 0], label='CO2 in atmosphere(ppm)')
plt.gca().xaxis.set_major_locator(MaxNLocator(integer=True)) # Set the scale of the horizontal axis to an integer
plt.legend(loc='best')
plt.title('1987-2004 CO2 concentration without the buffer effect')
plt.show()
```





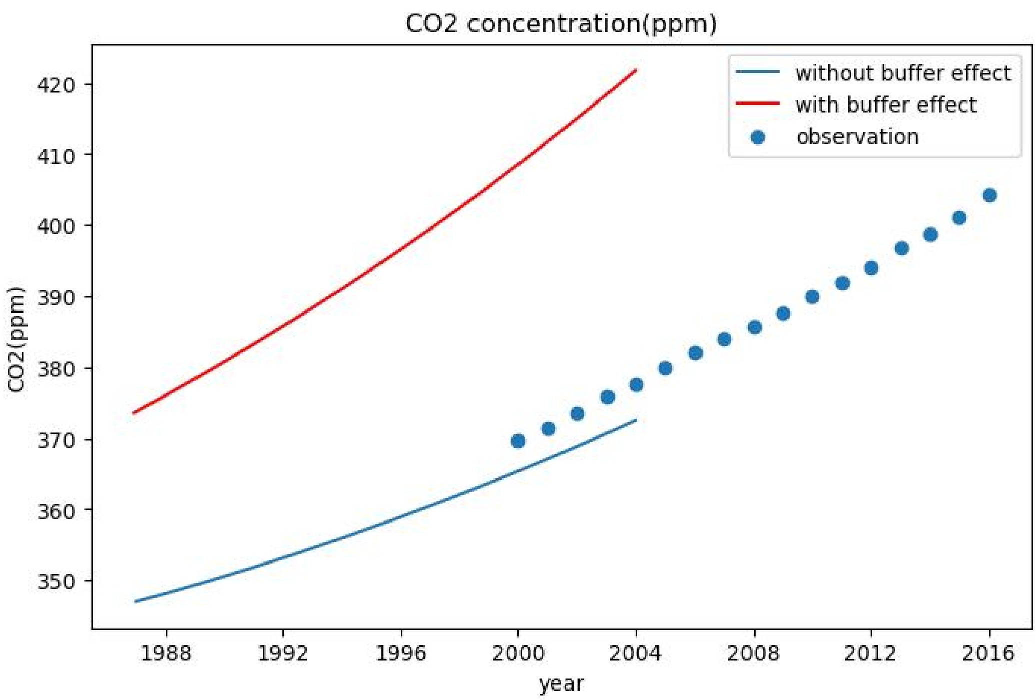
#1.2 [20 points] Following equation 3-4 (with the buffer effect), build a two-box model to compute the atmospheric CO2 level in ppm from 1987 to 2004.

```
def model2(y,t,k12,k21,N0,para):     #Define a function with the buffer effect
    N1,N2,r2 = y
    bf=3.69+1.86e-2*(N1/740/1000*347)-1.8e-6*((N1/740/1000*347)**2) #buffer factor
    dydt=[-k12*N1+k21*(N0+bf*(N2-N0))+r2,     #  Due N0 is the equilibrium value of carbon in the surface ocean In the preindustrial era,
          k12*N1-k21*(N0+bf*(N2-N0)),         # so the independent variables(t) should start in the preindustrial era
          para[0]*(r2-para[2])]
    return dydt
t2=np.linspace(1751, 2004, 253) #independent variable(start in the preindustrial era)
r2=CO2_emis(t2,*para)
k12=105/740
k21=102/900
N0=821*1000
N1=618*1000
N2=821*1000
y0=[N1,N2,r2[0]] #initial value
sol2 = odeint(model2, y0, t2, args=(k12,k21,N0,para))/740/1000*347 #Call the carbon models function, the result is translated into ppm units
plt.figure(figsize=(8,5))
plt.plot(t2[235:253],sol2[235:253,0], label='CO2 in atmosphere(ppm)')
plt.gca().xaxis.set_major_locator(MaxNLocator(integer=True)) # Set the scale of the horizontal axis to an integer
plt.legend(loc='best')
plt.title('1987-2004 CO2 concentration with the buffer effect')
plt.show()
```



#1.3 [5 points] Based on your results from 1.1 and 1.2, reproduce Figure 2 in Tomizuka (2009) as much as you can.

```
df2=pd.read_csv('co2_annmean_mlo.csv',sep=',',skiprows=57,usecols=[0,1],  # obversed data
                names=['year','CO2'])
plt.figure(figsize=(8,5))
plt.plot(t, sol[:, 0]) # without buffer effect
plt.plot(t2[235:253], sol2[235:253,0], 'r') # with buffer effect
plt.scatter(df2['year'][28:45],df2['CO2'][28:45]) # observed data
plt.legend(['without buffer effect','with buffer effect','observation'],loc='best')
plt.xlabel('year')
plt.ylabel('CO2(ppm)')
plt.gca().xaxis.set_major_locator(MaxNLocator(integer=True))
plt.title('CO2 concentration(ppm)')
plt.show()
```



#[Bonus] [15 points] Following equation 5-13, compute the atmospheric CO2 level in ppm and reproduce Figure 4 in Tomizuka (2009).

```
5]: def model3(N,t,rr,bf,f,da):   #defintion seveb-box model
        N02 = 821
        k12 = 60/615
        k21 = 60/842
        k23 = 9/842
        k24 = 43/842
        k32 = 52/9744
        k34 = 162/9744
        k43 = 205/26280
        k45 = 0.2/26280
        k51 = 0.2/90000000
        k67 = 62/731
        k71 = 62/1328
        N1,N2,N3,N4,N5,N6,N7 = N
        dN1dt = [-k12 * N1 + k21 * (N02 + bf*(N2 - N02)) + rr - f + da + k51 * N5 + k71 * N7,
                k12 * N1 - k21 * (N02 + bf*(N2 - N02)) - k23 * N2 + k32 * N3 - k24 * N2,
                k23*N2 - k32*N3 - k34*N3 +k43*N4,
                k34*N3 - k43*N4 + k24*N2 -k45*N4,
                k45*N4 - k51*N5,
                f - k67*N6 - 2*da,
                k67*N6 - k71*N7 + da]
        return dN1dt
    def buffer(CO2):    #defintion buffer
        bf = 3.69 + 1.86 * 10**-2 * CO2 - 1.80 * 10**-6 * CO2**2
        return bf
    def ff(P,beta):
        f0 = 62
        P0=290.21
        f = f0 * (1 + beta * math.log(P/P0))
        return f
```

```
: # read the data
rr = np.loadtxt("global.1751_2008.csv", delimiter=",", skiprows = 2, usecols = 1)/10**3
da = np.empty_like(rr)
da[0:100] = np.linspace(0.2,0.5,100)
da[99:-3]= np.loadtxt("Global_land-use_flux-1850_2005.csv", delimiter=",", skiprows = 1, usecols = 1)/10**3
da[-3:] = da[-4]
annualco2 = np.loadtxt("co2_annmean_mlo.csv", delimiter=",", skiprows = 56 )[:,1]
T = np.loadtxt("co2_annmean_mlo.csv", delimiter=",", skiprows = 56 )[:,0]
ynum = 2008 - 1751 + 1

t = np.linspace(1751,1751+ynum-1,ynum,dtype='int')
co2_obs = np.loadtxt("co2.csv", delimiter=",", skiprows = 149)
model3_b1 = np.empty_like(t)
model3_b2 = np.empty_like(t)
# situation of Bita=0.38    #reference shenshao
bb = 0.38
N0 = [615, 842, 9744, 26280, 90000000, 731, 1238]
model3_b1[0] = N0[0]
co2p = model3_b1[0]/2.13
bf = buffer(co2p)
f = ff(co2p,bb)
for i in range(1,ynum):# slove the ODE
    dt = [0,1]
    N = odeint(model3,N0,dt,args=(rr[i-1],bf,f,da[i-1])) # solve the equal
    model3_b1[i] = N[1][0]     # print the result
    co2p = model3_b1[i]/2.13 # initiation
    N0 = N[1]
    bf = buffer(co2p)
    f = ff(co2p,bb)
# situation of Bita=0.5
bb = 0.5
N0 = [615, 842, 9744, 26280, 90000000, 731, 1238]
model3_b2[0] = N0[0]
co2p = model3_b2[0]/2.13
bf = buffer(co2p)
f = ff(co2p,bb)
for i in range(1,ynum):
    dt = [0,1]
    N = odeint(model3,N0,dt,args=(rr[i-1],bf,f,da[i-1])) # solve the equal
    model3_b2[i] = N[1][0]         # print the result
    co2p = model3_b2[i]/2.13   # initiation
    N0 = N[1]

    bf = buffer(co2p)
    f = ff(co2p,bb)
```

```
: plt.figure(figsize=(8,5))
  plt.plot(t[1:],model3_b1[1:]/2.13,label='b = 0.38') # plot Beta=0.38
  plt.plot(t[1:],model3_b2[1:]/2.13,label='b = 0.50')  #plot Beta=0.5
  plt.plot(co2_obs[:,0], co2_obs[:,1],'k.')
  plt.plot(T,annualco2,'k.',label='ovserved data')   # ovserved data
  plt.ylabel('CO2(ppm)')
  plt.xlabel('Year')
  plt.title('seven-box model')
  plt.legend(loc='best')
  plt.show()
```