1. Random variables y_t are independent and normally distributed $\mathcal{N}(0;1)$. We define random variables $z_t = y_t \cdot y_{t-1}$.
   (a) Are z_t independent?
   (b) Find $\gamma_0 = \mathrm{Var}(z_t)$, $\gamma_1 = \mathrm{Cov}(z_t, z_{t-1})$, $\gamma_2 = \mathrm{Cov}(z_t, z_{t-3})$.
   (c) Is z_t a white noise process?

2. Consider stationary process $y_t = 3 + 0.5y_{t-1} + u_t + u_{t-1}$, where u_t is a white noise with $\mathrm{Var}(u_t) = \sigma_u^2$. Find the following:
   (a) $\mathrm{E}(y_t)$, $\gamma_0 = \mathrm{Var}(y_t)$, $\gamma_1 = \mathrm{Cov}(y_t, y_{t-1})$, $\gamma_2 = \mathrm{Cov}(y_t, y_{t-2})$;
   (b) Find the first two values of autocorrelation function, ρ_1, ρ_2;
   (c) Find the first two values of partial autocorrelation function, ϕ_{11}, ϕ_{22};

3. Consider the equation $y_t = 3 + 0.5y_{t-1} + u_t + u_{t-1}$.
   (a) If it is possible to express y_t in terms of past u_{t-i} then calculate coefficients before u_{t-1}, u_{t-2} and u_{t-3}.
   (b) If it is possible to express u_t in terms of past y_{t-i} then calculate coefficients before y_{t-1}, y_{t-2} and y_{t-3}.

4. This is the output of seasonal ARIMA model estimation in R for Russian population income. Quarterly data from 1992Q4 to 2015Q4 are used.

```
Series: y
ARIMA(0,1,1)(0,1,0)[4]

Coefficients:
          ma1
      -0.6611
s.e.   0.0811
```

   (a) Write down the estimated equation.
   (b) Is the series of population income stationary?

5. It is known that $u_{100} = 0.5$, $y_{100} = 4.5$, $\mathrm{Var}(u_t) = 9$ and y_t is defined by equation $y_t = 3 + 0.5y_{t-1} + u_t + u_{t-1}$, where u_t is a white noise.
   (a) Make one-step and two-steps point forecasts: find $\mathrm{E}(y_{101}|\mathcal{F}_{100})$ and $\mathrm{E}(y_{102}|\mathcal{F}_{100})$.
   (b) Assuming normal distribution of u_t construct 95% prediction intervals for y_{101} and y_{102}.

6. Processes u_t and v_t are independent white noises. The process y_t is defined by equation

$$\begin{cases} y_0 = 0; \\ y_t = y_{t-1} + u_t. \end{cases}$$

   The process z_t is defined by equation $z_t = 7 + 0.5z_{t-1} + 2y_{t-1} - 4y_t + v_t$;
   (a) Find the order of integration of y_t and z_t;
   (b) Are y_t and z_t cointegrated?