1. Consider stationary process $y_t = 2 + 0.3y_{t-1} + u_t - u_{t-1}$, where u_t is a white noise with $\mathrm{Var}(u_t) = \sigma_u^2$.

   (a) Find $\mathrm{E}(y_t)$, $\mathrm{Var}(y_t)$ and first values of autocovariance function γ_1, γ_2;

   (b) Find the first values of autocorrelation function, ρ_1, ρ_2;

   (c) Find the first values of partial autocorrelation function, ϕ_{11}, ϕ_{22};

2. It is known that $u_{100} = 0.5$, $y_{100} = 4.5$, $\mathrm{Var}(u_t) = 9$ and y_t is defined by equation $y_t = 2+0.3y_{t-1}+u_t-u_{t-1}$, where u_t is a white noise.

   (a) Make one-step and two-steps point forecasts: find $\mathrm{E}(y_{101}|\mathcal{F}_{100})$ and $\mathrm{E}(y_{102}|\mathcal{F}_{100})$.

   (b) Assuming normal distribution of u_t construct 95% prediction intervals for y_{101} and y_{102}.

   (c) What is higher $\mathbb{P}(y_{101} > 3|\mathcal{F}_{100})$ or $\mathbb{P}(y_{102} > 3|\mathcal{F}_{100})$? Briefly argue why.

3. Find the equation of the process with partial autocorrelation function given by $\phi_{11} = 0.9$, $\phi_{22} = -0.9$, $\phi_{kk} = 0$ for $k \geq 3$.

4. This is the output of seasonal ARIMA model estimation in R for the number of marriages in Russia. Data from 2006 to 2018 are used.

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ARIMA(0,1,1)(1,1,2)[12]

Coefficients:
          ma1     sar1     sma1     sma2
      -0.8544  -0.2677  -0.5076  -0.3289
s.e.   0.0540   0.5048   0.5103   0.4107

sigma^2 estimated as 150821423:  log likelihood=-1545.24
AIC=3100.49   AICc=3100.93   BIC=3115.26
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   (a) Is the number of marriages stationary? Which transformation will make it stationary?

   (b) Write down the estimated equation.

5. Consider Holt's linear model with additive errors, $\varepsilon_t \sim \mathcal{N}(0; \sigma^2)$ are independent.

$$\begin{cases} y_t = \ell_{t-1} + b_{t-1} + \varepsilon_t \\ \ell_t = \ell_{t-1} + b_{t-1} + \alpha\varepsilon_t \\ b_t = b_{t-1} + \beta\varepsilon_t \end{cases}$$

Here y_t — is the observed value of the variable of interest, ℓ_t — unobserved ideal level of the variable of interest, b_t — unobserved current growth rate of the variable.

Represent this model as $ARIMA(0, 2, 2)$-model for y_t. Find all the coefficients of this $ARIMA$ representation.

6. Using 200 observations James Bond has estimated the regression

$$\Delta\hat{y}_t = \underset{(0.2)}{4} - \underset{(0.3)}{0.8} y_{t-1} + \underset{(0.2)}{0.9} \Delta y_{t-1}$$

Standard errors are in brackets. Check whether the process y_t is stationary.

Critical values for Dickey-Fuller statistic for 5% critical level are provided below:

	τ_0	τ_c	τ_t
$n = 100$	-1.95	-2.89	-3.45
$n = 200$	-1.95	-2.88	-3.43