Promo-code activation :)

You have two options!

You can try to solve all the proposed problems and submit your solutions as a pdf file. In this case I will honestly grade them.

You can write a promo-code #arma_da. In this case I will ignore your solutions and grade your exam as 4/10.

Problems

1. Consider a stationary solution of the equation $y_t = 3 + 0.4y_{t-1} + u_t - 0.5u_{t-1}$, where u_t is a white noise process.

   (a) Calculate $\mathrm{E}(y_t)$, first two values of autocorrelation and partial autocorrelation functions.

   (b) Assuming normality and independence of u_t, $u_t \sim \mathcal{N}(0; 9)$, $u_{100} = -1$, $y_{100} = 5$, calculate short-term 95% predictive interval for y_{101} and long-term 95% predictive interval for y_{100+h} where $h \to \infty$.

2. Consider a stationary solution of the equation $y_t = 3 + 0.7y_{t-1} - 0.12y_{t-2} + u_t$, where u_t is a white noise process.

   James Bond assumes the wrong model $y_t = \beta_1 + \beta_2 y_{t-1} + u_t$ and estimate the regression $\hat{y}_t = \hat{\beta}_1 + \hat{\beta}_2 y_{t-1}$ by OLS.

   (a) Find the probability limit of $\hat{\beta}_1$ and $\hat{\beta}_2$.

   (b) Using $\hat{\beta}_1$, $\hat{\beta}_2$ and his wrong assumption James Bond tries to estimate $\mu = \mathrm{E}(y_t)$. Find the probability limit of this estimator $\hat{\mu}$. Will it be consistent?

3. Consider a $ETS(AAA)$ model for monthly data described by the system

$$
\begin{cases}
y_t = \ell_{t-1} + b_{t-1} + s_{t-12} + \varepsilon_t \\
\ell_t = \ell_{t-1} + b_{t-1} + \alpha\varepsilon_t \\
b_t = b_{t-1} + \beta\varepsilon_t \\
s_t = s_{t-12} + \gamma\varepsilon_t
\end{cases}
$$

   Represent it as ARIMA model. Find all the coefficients of this $ARIMA$ representation.

4. Consider two independent $MA(1)$ process with zero expected value and unit variance, A_t and B_t. The first values of autocorrelation function are $\rho_1^A = 0.3$ and $\rho_1^B = 0.3$.

   The process S_t is just the sum of these two, $S_t = A_t + B_t$.

   (a) Classify S_t as $ARIMA(p, d, q)$ process. Find p, d and q.

   (b) Calculate the autocorrelation function for S_t.

5. Consider the process $y_t = u_1 \sin 2t + u_2 \cos 2t$, where u_t — is a white noise.

   (a) Is y_t stationary?

   (b) Can y_t be represented as $ARIMA(p, d, q)$ process? Find p, d, q if possible.

   Hint: $\cos(a + b) = \cos a \cos b - \sin a \sin b$, $\sin(a + b) = \sin a \cos b + \sin b \cos a$.

6. Variables (x_t) are independent and are equal to 0 or 1 with equal probability, u_t are independent $\mathcal{N}(0; 1)$. Consider the process $z_t = x_t^2(1 - x_{t-1})u_t$.

   (a) Is z_t a stationary process? Find its autocorrelation function if it is stationary.

   (b) You know that $z_{100} = 2.3$. Find one and two step ahead point and interval forecasts. What is special about interval forecasts in this case?