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Derivations for problem 1

Derivation of the functional:

$$\min_d \int_\Omega \|\Delta d\|^2 \, d\Omega$$

using Gateaux derivative

$$\frac{d}{dh} \int_\Omega \|\Delta(d + hv)\|^2 \, d\Omega \Big|_{h=0} = 0$$

$$\frac{d}{dh} \int_\Omega \|\Delta d + h\Delta v\|^2 \, d\Omega$$

$$\frac{d}{dh} \int_\Omega \langle \Delta d + h\Delta v, \Delta d + h\Delta v \rangle \, d\Omega$$

$$\frac{d}{dh} \int_\Omega \cancel{\langle \Delta d, \Delta d \rangle} + 2h \langle \Delta d \, \Delta v \rangle + \cancel{\langle \Delta v, \Delta v \rangle} \, d\Omega$$

$$2\int_\Omega \langle \Delta d \Delta v, \Delta d \Delta v \rangle \, d\Omega = 0$$

$$\int_\Omega \Delta^2 d \, \Delta^2 v \, d\Omega = 0$$

$u' = \Delta^2 v \quad v = \nabla(\Delta v)$ by integration

$u = \Delta^2 d \quad u' = \nabla(\Delta^2 d)$ by parts

$$\cancel{\nabla(\Delta v)}\, \Delta^2 d \Big|_{\partial\Omega} - \int_\Omega \nabla(\Delta v)\, \nabla(\Delta^2 d)\, d\Omega = 0$$

$$\int_\Omega \nabla(\Delta v\, \Delta^2 d)\, d\Omega = 0$$

$$\cancel{\Delta v}\ \Delta^2 d = 0$$

$$\boxed{\Delta^2 d = 0}$$

Derivation of the matrix form:

$$\min_d \ d^T K d$$

$$\frac{d}{dh} (d + hv)^T K (d + hv) \Big|_{h=0} = 0$$

$$\frac{d}{dh} (d^T + hv^T)(Kd + hKv) \Big|_{h=0} = 0$$

$$\frac{d}{dh}\ \cancel{d^T K d} + h d^T K v + h v^T K d + h^2 v^T K v \Big|_{h=0}$$

$$d^T K v + v^T K d + \cancel{h v^T K v} \Big|_{h=0}$$

$$\cancel{2v^T} K d = 0$$

$$\boxed{Kd = 0}$$