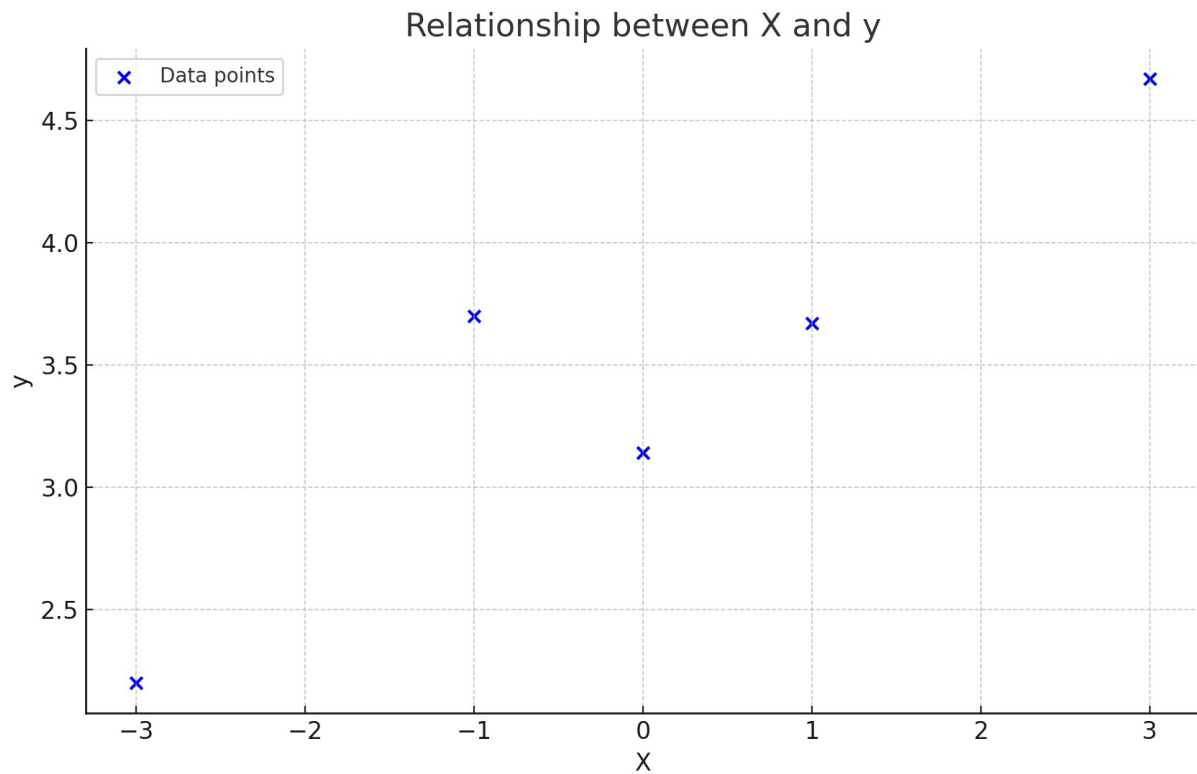


### Assignment 3: Maximum Likelihood Estimation

#### Step 1: Graph - Relationship between X and y



```
# Step 1: Plotting relationship
plt.scatter(X, y, color='blue', label='Data points')
plt.xlabel('X')
plt.ylabel('y')
plt.title('Relationship between X and y')
plt.legend()
plt.grid(True)
plt.show()
```

## Assignment 3: Maximum Likelihood Estimation

### Step 2: Applying the Derived Equation

The calculated Theta (slope) is approximately 0.369.

```
# Step 2: Computing Theta without bias  
Theta = np.linalg.inv(X.T @ X) @ X.T @ y  
print(Theta)
```

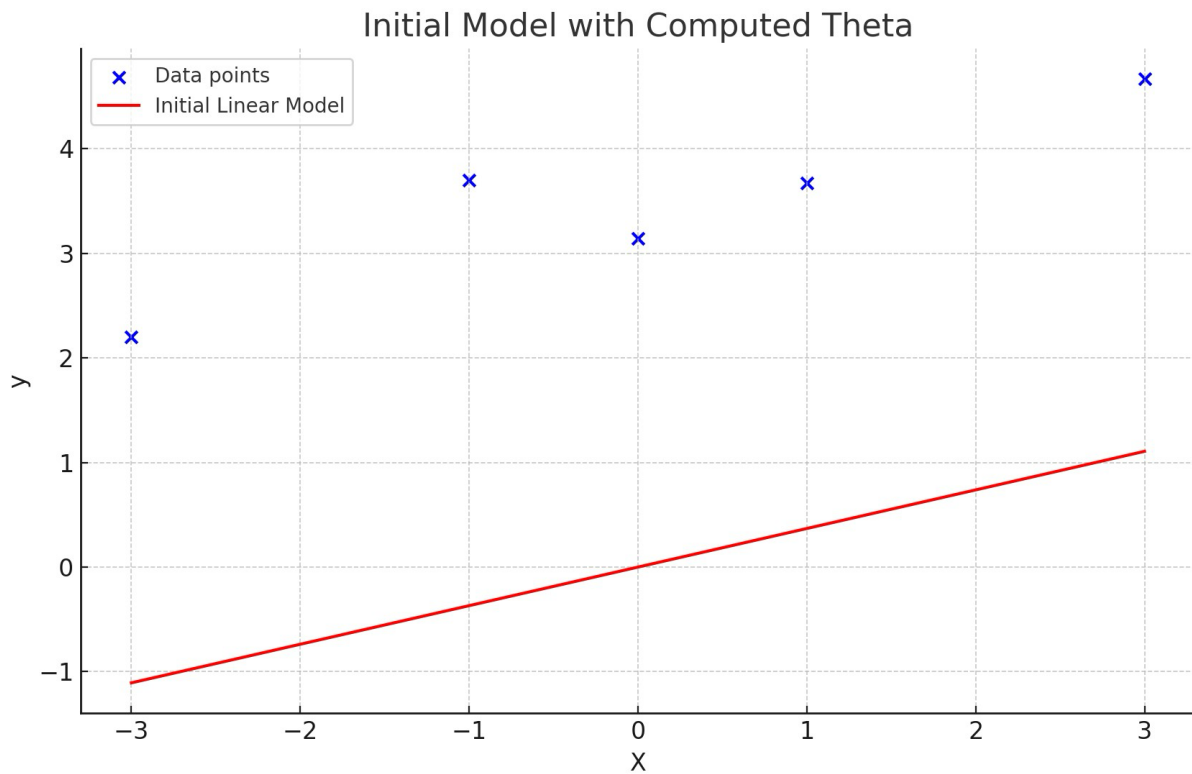
### Step 3: Plotting the Initial Model

The initial linear model (red line) plotted with the original data points (blue dots) is shown below.

```
# Step 3: Plotting initial model  
y_pred_initial = X @ Theta  
plt.scatter(X, y, color='blue', label='Data points')  
plt.plot(X, y_pred_initial, color='red', label='Initial Linear Model')  
plt.xlabel('X')  
plt.ylabel('y')  
plt.title('Initial Model with Computed Theta')  
plt.legend()  
plt.grid(True)  
plt.show()
```

## Assignment 3: Maximum Likelihood Estimation

### Step 3: Initial Model Plot



### Step 4: Improving the Model

The updated Theta values after including the bias term are:

Theta\_0 (bias): 3.476

Theta\_1 (slope): 0.369

```
# Step 4: Including bias term
X_with_bias = np.hstack((np.ones((X.shape[0], 1)), X))
Theta_with_bias = np.linalg.inv(X_with_bias.T @ X_with_bias) @ X_with_bias.T @ y
```

## Assignment 3: Maximum Likelihood Estimation

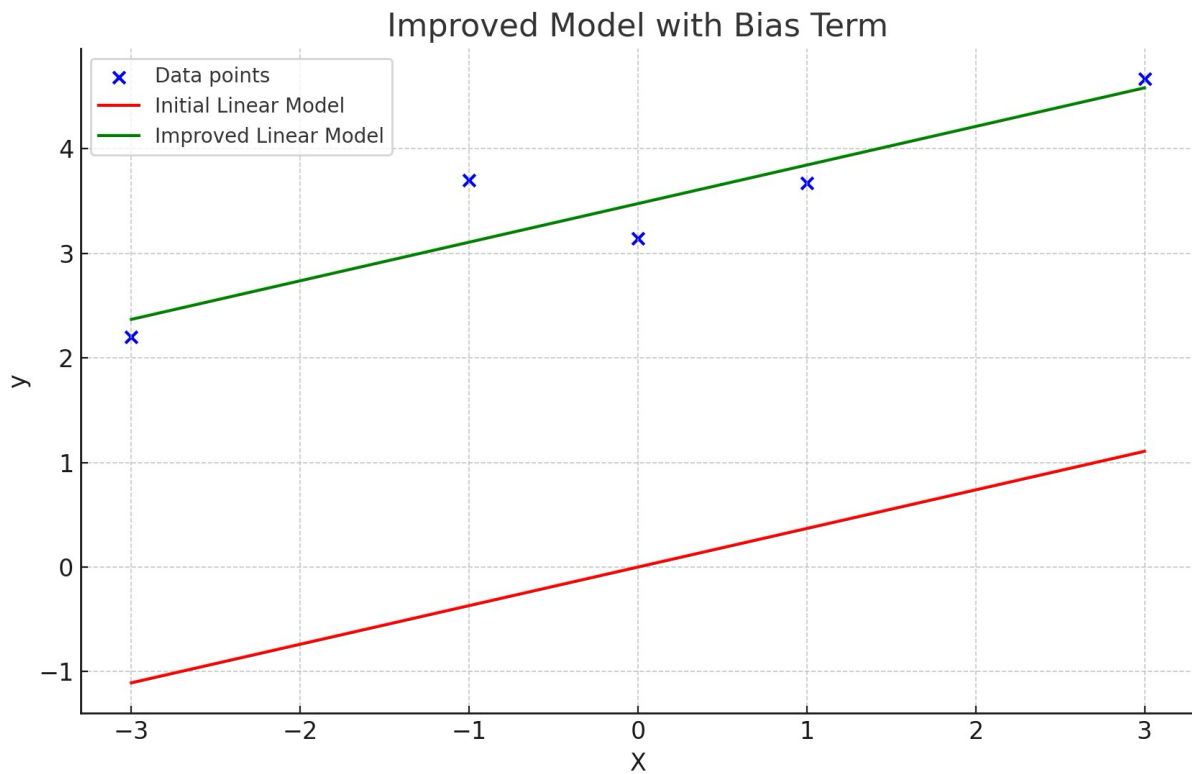
### Step 5: Plotting the Improved Model

The improved linear model (green line) fits the data better, as shown in the plot below.

```
# Step 5: Plotting improved model
y_pred_improved = X_with_bias @ Theta_with_bias
plt.scatter(X, y, color='blue', label='Data points')
plt.plot(X, y_pred_initial, color='red', label='Initial Linear Model')
plt.plot(X, y_pred_improved, color='green', label='Improved Linear Model')
plt.xlabel('X')
plt.ylabel('y')
plt.title('Improved Model with Bias Term')
plt.legend()
plt.grid(True)
plt.show()
```

## Assignment 3: Maximum Likelihood Estimation

### Step 5: Improved Model Plot



### Step 6: Analytical Explanation

Including a bias term improves the model by allowing it to better align with the data points. Without a bias, the regression line is constrained to pass through the origin, which is often not ideal for real-world data. Adding a bias term enables the line to adjust vertically, leading to a better fit, as demonstrated in the improved model plot.