Exercise 02 - Functions & Relations

Create a text file named '`exercises02.txt`' that contains the solutions to each problem below.

1. Given $\mathbf{A} = \{1, 2, 3, 4, 5\}$, $\mathbf{B} = \{6, 7, 8, 9, 10\}$, $\mathbf{C} = \{11, 12, 13, 14, 15, 16\}$, and $\mathbf{D} = \{17, 18, 19\}$, construct the following functions as a list with the specified constraint if possible; otherwise, state why the construction is impossible.

    a) $f : \mathbf{A} \longrightarrow \mathbf{D}$ that is a surjection.
    b) $g : \mathbf{B} \longrightarrow \mathbf{C}$ that is an injection.
    c) $h : \mathbf{C} \longrightarrow \mathbf{D}$ that is not a surjection.
    d) $i : \mathbf{A} \longrightarrow \mathbf{B}$ that is a bijection.
    e) $j : \mathbf{C} \longrightarrow \mathbf{B}$ that is a bijection.

2. Given the sets $\mathbf{A} = \{2, 3, 5, 7\}$ and $\mathbf{B} = \{0, 1, 2, 3\}$, and the functions $f : \mathbf{A} \longrightarrow \mathbf{B}$ and $g : \mathbf{A} \times \mathbf{B} \longrightarrow \mathbf{B}$ defined as

$$f(x) = (3x + 1) \setminus 4$$
$$g(x, y) = (x^2 + y^2 - xy) \setminus 4$$

    where $\setminus$ outputs the integer remainder of division, find

    a) the domain of **F** and **G** (write as a list).
    b) the codomain of **F** and **G** (write as a list).
    c) the range of **F** and **G** (write as a list).
    d) $f(x)$ for $x \in \mathbf{A}$.
    e) $g(x, y)$ for $x \in \{5, 7\}$ and $y \in \{2, 3\}$

3. Given $\mathbf{A} = \{1, 2, 3, 4\}$, write the partition of the set of permutations on **A** into the sets

$$\{f : f = f^{-1}\} \text{ (permutation f is its own inverse)}$$
$$\{f : f \neq f^{-1}\} \text{ (permutation f is not its own inverse)}$$

4. Given $\mathbf{A} = \{x : x \in \mathbb{N} \wedge 1 \leq x \leq 10\}$, give a relation with at least three elements that satisfies the condition.

    a) reflexive and symmetric but not transitive
    b) reflexive and transitive but not symmetric
    c) symmetric and transitive but not reflexive
    d) an equivalence relation
    e) a partial ordering

5. Given the relation $R = \{(a, a), (a, b), (b, d), (d, c)\}$ on the set $\mathbf{A} = \{a, b, c, d\}$, modify R so that it satisfies the condition.

    a) reflexive
    b) symmetric
    c) transitive