2)

1)

$$A = \begin{bmatrix} 1 & -2 \\ -1 & 1 \\ 2 & -1 \\ 3 & 1 \end{bmatrix}$$

$$A^T = \begin{bmatrix} 1 & -1 & 2 & 3 \\ -2 & 1 & -1 & 1 \end{bmatrix}$$

$$\underset{2\times 4}{A^T} \cdot \underset{4\times 2}{A} = \begin{bmatrix} 1\cdot 1 + (-1)\cdot(-1) + 2\cdot 2 + 3\cdot 3 & 1\cdot(-2) + (-1)\cdot 1 + 2\cdot(-1) + 3\cdot 1 \\ -2\cdot 1 + 1\cdot(-1) + (-1)\cdot 2 + 1\cdot 3 & -2\cdot(-2) + 1\cdot 1 + (-1)\cdot(-1) + 1\cdot 1 \end{bmatrix}$$

$$= \begin{bmatrix} 15 & 2 \\ -2 & 7 \end{bmatrix}$$

$\text{Det } A^T A \neq 0$

$\text{Det } A^T A = 15\cdot 7 - (2\cdot(-2)) = \underline{107} \neq 0$

$\underline{\underline{\text{Det } A^T A \neq 0 \Rightarrow A^T A \text{ er inverterbar}}}$

ii) $B = \begin{bmatrix} 1 & 2 \\ 3 & -1 \end{bmatrix}$

$C_{11} = -1 \qquad C_{12} = 3$

$C_{21} = 2 \qquad C_{22} = 1$

$C = \begin{bmatrix} -1 & 3 \\ 2 & 1 \end{bmatrix}$

Bruker Gauss-Jordan på $[C \mid I]$ matrisen.

$C^{-1} = [C \mid I]$

$= \left[\begin{array}{cc|cc} -1 & 3 & 1 & 0 \\ 2 & 1 & 0 & 1 \end{array}\right] \cdot 2 \quad = \left[\begin{array}{cc|cc} -1 & 3 & 1 & 0 \\ 0 & 7 & 2 & 1 \end{array}\right] \cdot \frac{1}{7}$

$= \left[\begin{array}{cc|cc} -1 & 3 & 1 & 0 \\ 0 & 1 & \frac{2}{7} & \frac{1}{7} \end{array}\right] \cdot 3 \quad = \left[\begin{array}{cc|cc} -1 & 0 & \frac{1}{7} & -\frac{3}{7} \\ 0 & 1 & \frac{2}{7} & \frac{1}{7} \end{array}\right] \cdot -1$

$= \left[\begin{array}{cc|cc} 1 & 0 & -\frac{1}{7} & \frac{3}{7} \\ 0 & 1 & \frac{2}{7} & \frac{1}{7} \end{array}\right]$

$$C^{-1} = \begin{bmatrix} -\frac{1}{7} & \frac{3}{7} \\ \frac{2}{7} & \frac{1}{7} \end{bmatrix}$$