ISLR - 4.7.3 . Ans.

Given that for each $k \in \{1, \dots, K\}$

$$f_k(x) = \frac{1}{\sqrt{2\pi}\sigma_k} \exp\left(\frac{-1}{2\sigma_k^2}(x-\mu_k)^2\right)$$

then the posterior probability distribution is:

① $$P_k(x) = \frac{\pi_k \frac{1}{\sqrt{2\pi}\sigma_k} \exp\left(-\frac{1}{2\sigma_k^2}(x-\mu_k)^2\right)}{\sum_{l=1}^{K} \pi_l \frac{1}{\sqrt{2\pi}\sigma_l} \exp\left(\frac{-1}{2\sigma_l^2}(x-\mu_l)^2\right)}$$

② $$\delta_k = x \cdot \frac{\mu_k}{\sigma^2} - \frac{\mu_k^2}{2\sigma^2} + \log(\pi_k)$$

③ $$\log(p_k(x)) = \log\left(\frac{\pi_k \frac{1}{\sqrt{2\pi}\sigma} \exp\left(-\frac{1}{2\sigma_k^2}(x-\mu_k)^2\right)}{\sum_{l=1}^{K} \pi_l \frac{1}{\sqrt{2\pi}\sigma} \exp\left(\frac{-1}{2\sigma_k^2}(x-\mu_l)^2\right)}\right)$$

$$= \log\left(\pi_k \frac{1}{\sqrt{2\pi}\sigma} \exp\left(-\frac{1}{2\sigma_k^2}(x-\mu_k)^2\right)\right)$$
$$- \log\left(\sum_{l=1}^{K} \pi_l \frac{1}{\sqrt{2\pi}\sigma} \exp\left(-\frac{1}{2\sigma_k^2}(x-\mu_l)^2\right)\right)$$

④ $$K = \log\left(\sum_{l=1}^{K} \pi_l \frac{1}{\sqrt{2\pi}\sigma} \exp\left(-\frac{1}{2\sigma_k^2}(x-\mu_l)^2\right)\right)$$

from ③.④

$$\Rightarrow \log(p_k(x)) = \log\left(\pi_k \frac{1}{\sqrt{2\pi}\sigma} \exp\left(-\frac{1}{2\sigma_k^2}(x-\mu_k)^2\right)\right) - K$$

$$= \log \pi_k + \log\left(\frac{1}{\sqrt{2\pi}\sigma}\right) + \log\left(\exp\left(-\frac{1}{2\sigma_k^2}(x-\mu_k)^2\right)\right) - K$$

$$= -\frac{1}{2\sigma^2}(x-\mu_k)^2 + \log(\pi_k) + \log\left(\frac{1}{\sqrt{2\pi}\sigma}\right) - K$$

$$= -\frac{1}{2\sigma^2}(x^2 - 2x\mu_k + \mu_k^2) + \log(\pi_k) + \log\left(\frac{1}{\sqrt{2\pi}\sigma}\right) - k$$

⑤ $$= x \cdot \frac{\mu_k}{\sigma^2} + \frac{\mu_k^2}{2\sigma^2} + \log(\pi_k) - \frac{x^2}{2\sigma^2} + \log\left(\frac{1}{\sqrt{2\pi}\sigma}\right) - k$$

Assume that: the Bayes classifer assigns an observation to whichever k in $p_k(x)$ is MAX:

if $p_j(x) > p_i(x)$ for all $i \neq j$

from ⑤ $\Rightarrow \log(p_j(x)) > \log(p_i(x))$ for all $i \neq j$

$$\Rightarrow x \cdot \frac{\mu_j}{\sigma^2} + \frac{\mu_j^2}{2\sigma^2} + \log(\pi_j) - \frac{x^2}{2\sigma^2} + \log\left(\frac{1}{\sqrt{2\pi}\sigma}\right) - k$$

$$> x \cdot \frac{\mu_i}{\sigma^2} + \frac{\mu_i^2}{2\sigma^2} + \log(\pi_i) - \frac{x^2}{2\sigma^2} + \log\left(\frac{1}{\sqrt{2\pi}\sigma} - k\right)$$

⑥ $$\Rightarrow x \cdot \frac{\mu_j}{\sigma^2} + \frac{\mu_j^2}{2\sigma^2} + \log(\pi_j) > x \cdot \frac{\mu_i}{\sigma^2} + \frac{\mu_i^2}{2\sigma^2} + \log(\pi_i)$$

for all $i \neq j$ Since $\sigma_1 = \sigma_2 = \sigma_3 = \dots = \sigma_n$

the Bayes classifer assigns an observation to the discriminant function which is MAX.

⑦ $$\delta_k = x \cdot \frac{\mu_k}{\sigma^2} - \frac{\mu_k^2}{2\sigma^2} + \log(\pi_k)$$

from ⑦

$$k = \log(\pi_k) - \frac{(x^2)}{2\sigma^2} + \frac{2x\mu_k}{2\sigma^2} - \frac{\mu_k^2}{2\sigma^2}$$

$\therefore$ obviously, $\delta_k(x)$ is quadratic in x.

ISLR 4.7.7 Ans:

Known:

$\because$ ① Mean value of X, which issued a dividend $= \bar{X} = 10$

② Mean value of X, which didn't issue a dividend $- \bar{X} = 0$

③ variance of X for 2 sets of companies was $\hat{\sigma}^2 = 36$

④ X follows normal distribution.

⑤ Percentage profit was $X = 4$ in last year

⑥ $f(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-(x-\mu)^2/2\hat{\sigma}^2}$

$$\therefore P_k(x) = \frac{\pi_k \frac{1}{\sqrt{2\pi\sigma^2}} e^{\left(-\frac{(x-\mu)^2}{2\hat{\sigma}^2}\right)}}{\sum_{l=1}^{k} \frac{1}{\sqrt{2\pi}\,\sigma} e^{\left(-\frac{(x-\mu_k)^2}{2\hat{\sigma}^2}\right)} + \pi_l \frac{1}{\sqrt{2\pi}\,\sigma} e^{\left(\frac{-(x-\mu_l)^2}{2\hat{\sigma}^2}\right)}}$$

$\because X = 4, \mu_k = 10, \mu_l = 0, \hat{\sigma}^2 = 36, \pi_k = 0.8, \pi_l = 0.2$

$$\Rightarrow \frac{0.8 \times \frac{1}{\sqrt{2\times3.14\times6}} \times e^{\left(\frac{-(4-10)^2}{2\times36}\right)}}{0.8 \times \frac{1}{\sqrt{2\times3.14\times6}} \times e^{\left(\frac{-(4-10)^2}{2\times36}\right)} + 0.2 \times \frac{1}{\sqrt{2\times3.14\times6}} \times e^{\frac{-(4-0)}{2\times36}}}$$

$$= \frac{0.8 \times e^{\left(\frac{-36}{36\times2}\right)}}{0.8 \times e^{\frac{-36}{36\times2}} + 0.2 \times e^{\frac{-16}{36\times2}}}$$

$$= \frac{0.8 \times e^{-0.5}}{0.8 \times e^{-0.5} + 0.2 \times e^{-0.22}}$$

$$= \frac{0.485}{0.485 + 0.1675}$$

$$= \frac{0.485}{0.645503} \approx \underline{0.751}$$

$\therefore$ Probality that X should issue a dividend is $\underline{\underline{0.751}}$