

10. Интегриращо и идеално диференциращо звено

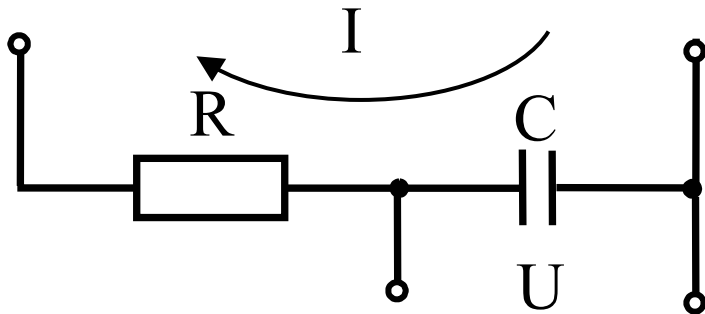
1. Интегриращо звено

ДУ: $\frac{dy(t)}{dt} = ku(t)$

ПФ: $pY(p) = kU(p), \quad W(p) = \frac{Y(p)}{U(p)} = \frac{k}{p}$

(Други записи: $W(p) = \frac{k}{Tp}, \quad W(p) = \frac{1}{Tp}$)

Пример:



$$W(p) = \frac{U(p)}{I(p)} = \frac{\frac{1}{pC} I(p)}{I(p)} = \frac{1}{pC} = \frac{k}{p}$$

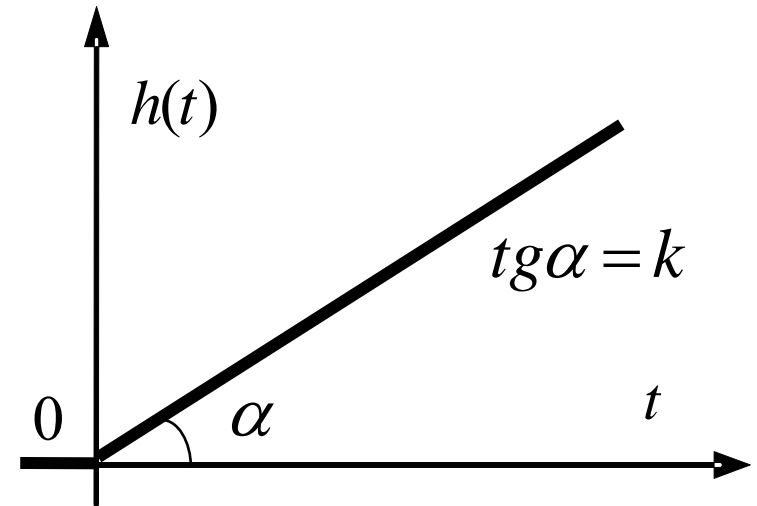
$$k = \frac{1}{C}$$

10. Интегриращо и идеално диференциращо звено

ПХ: $u(t) = 1(t), \quad \frac{dy(t)}{dt} = k \cdot 1(t)$

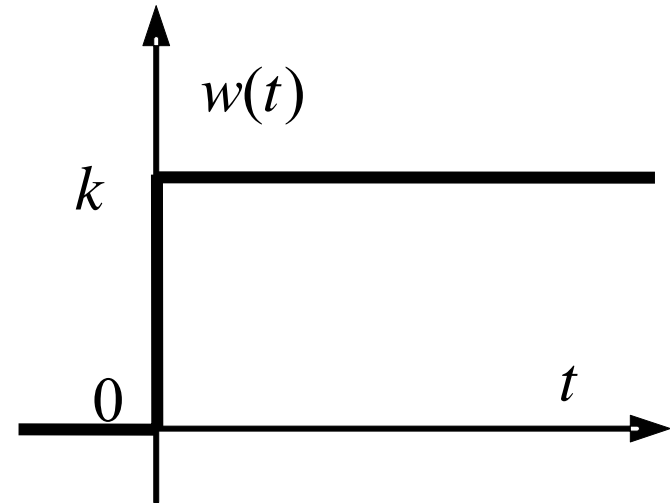
$$dy = k dt, \quad \Rightarrow \quad y(t) = k t$$

(при нулеви начални условия)



ТХ: $u(t) = \delta(t),$

$$w(t) = \frac{dh(t)}{dt} = \frac{d}{dt}(k t) = k$$



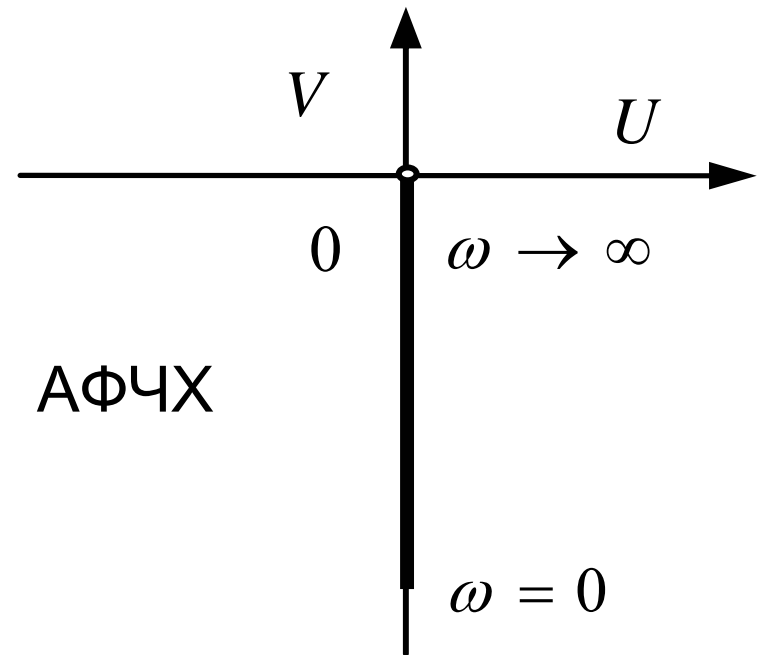
10. Интегриращо и идеално диференциращо звено

ЧПФ:
$$W(j\omega) = \frac{k}{j\omega} \cdot \frac{j}{j} = -j \frac{k}{\omega}$$

РЧФ:
$$U(\omega) = \operatorname{Re} W(j\omega) = 0$$

ИЧФ:
$$V(\omega) = \operatorname{Im} W(j\omega) = -\frac{k}{\omega}$$

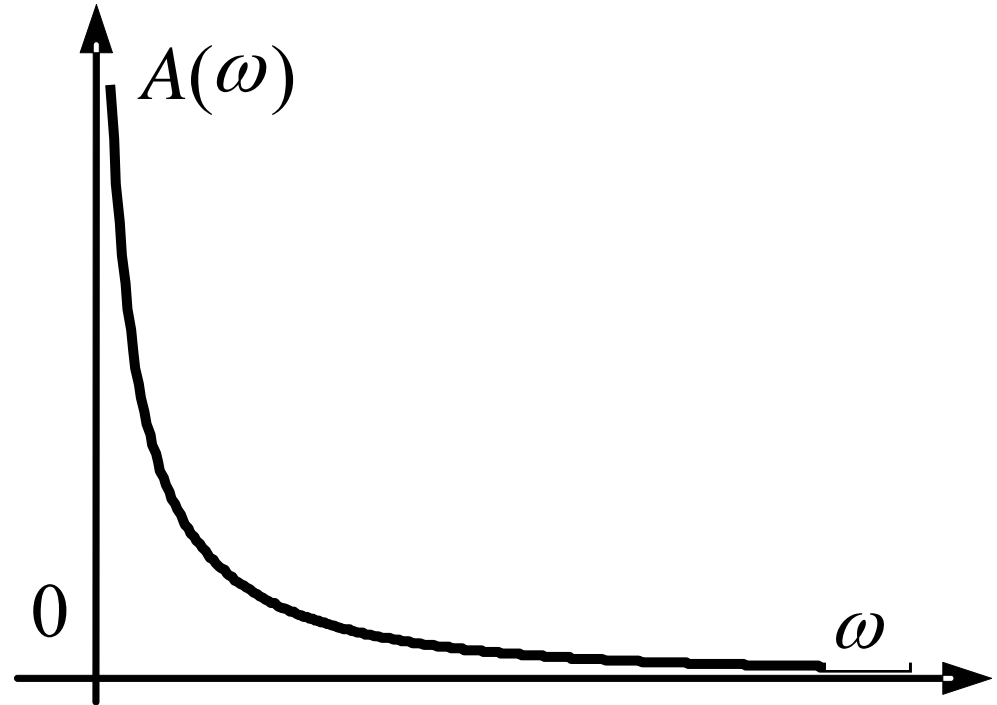
ω	0	∞
U	0	0
V	$-\infty$	0



10. Интегриращо и идеално диференциращо звено

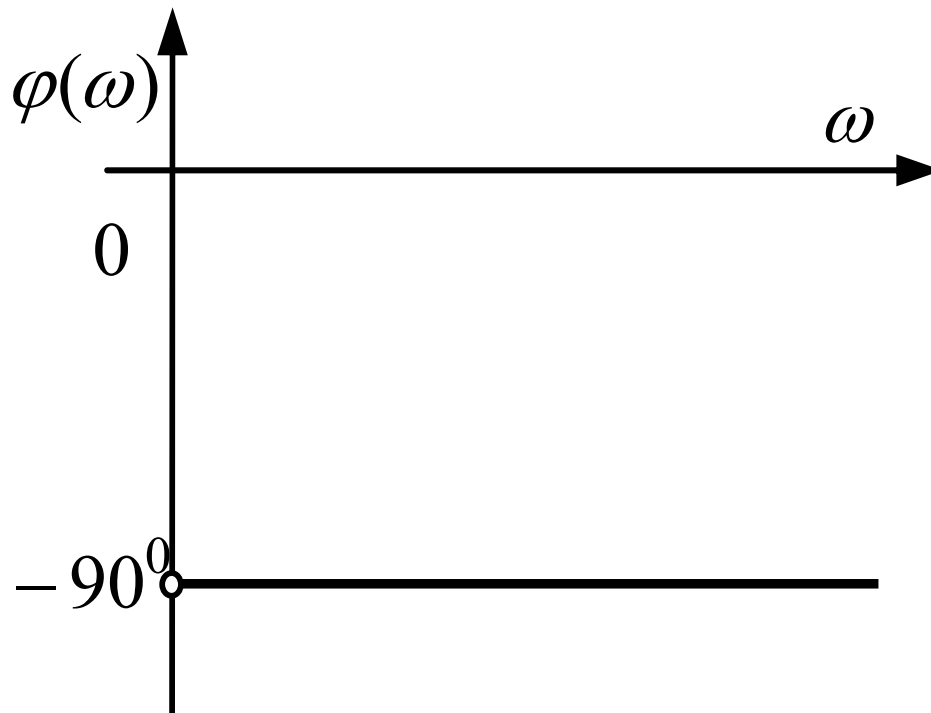
АЧХ:
$$A(\omega) = \sqrt{U^2(\omega) + V^2(\omega)} = \sqrt{0^2 + \left(\frac{-k}{\omega}\right)^2} = \frac{k}{\omega}$$

ω	0	∞
A	∞	0



10. Интегриращо и идеално диференциращо звено

ФЧХ:
$$\varphi(\omega) = \operatorname{arctg} \frac{V(\omega)}{U(\omega)} = \operatorname{arctg} \frac{-k}{\omega} = -\frac{\pi}{2}$$

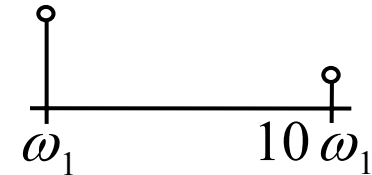


10. Интегриращо и идеално диференциращо звено

ЛАЧХ: $L(\omega) = 20 \lg A(\omega) = 20 \lg \frac{k}{\omega} = 20 \lg k - 20 \lg \omega$

т($\omega = 1; L = 20 \lg k$), $\frac{\Delta L}{\Delta \omega} = ?$

Нека $\Delta \omega = 1 \text{ dec}$



$$L(\omega_1) = 20 \lg k - 20 \lg \omega_1$$

$$L(10\omega_1) = 20 \lg k - 20 \lg 10\omega_1$$

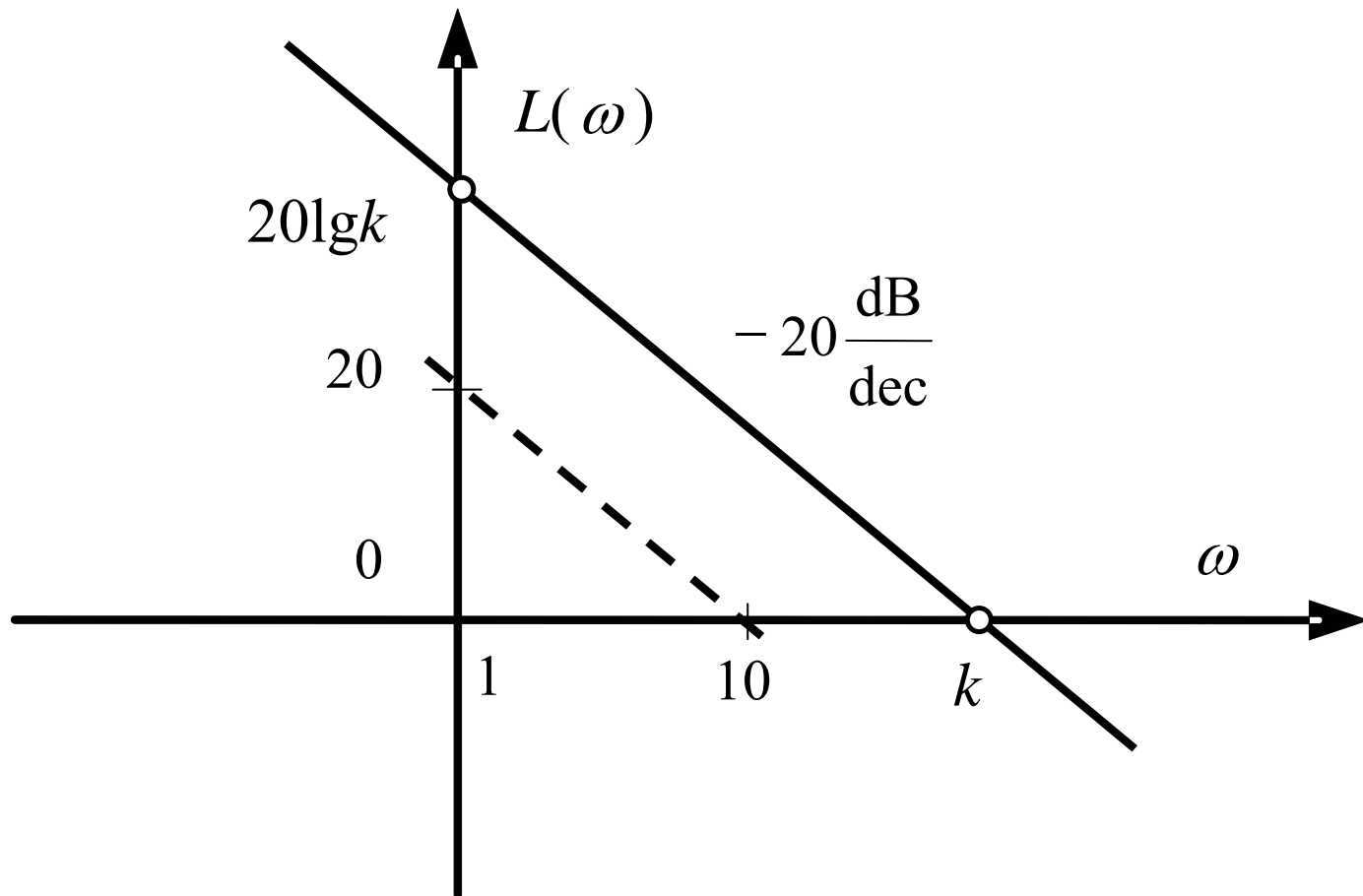
$$\Delta L = L(10\omega_1) - L(\omega_1) =$$

$$= 20 \lg k - 20 \lg 10\omega_1 - 20 \lg k + 20 \lg \omega_1 =$$

$$= -20 \lg 10 = -20 \text{ dB}$$

10. Интегриращо и идеално диференциращо звено

ЛАЧХ: $\frac{\Delta L}{\Delta \omega} = -20 \frac{\text{dB}}{\text{dec}}; \quad \omega = 1, \quad L(1) = 20 \lg k$



2. Идеално диференциращо звено

ДУ: $y(t) = k \frac{du(t)}{dt}$

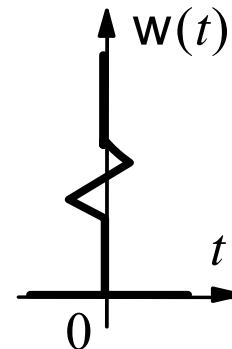
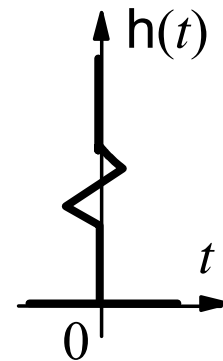
ПФ: $Y(p) = kpU(p), \quad W(p) = \frac{Y(p)}{U(p)} = kp$

ПХ: $u(t) = 1(t)$

$$h(t) = k \frac{d1(t)}{dt} = k\delta(t)$$

ТХ: $u(t) = \delta(t)$

$$w(t) = k \frac{d\delta(t)}{dt} = k\dot{\delta}(t)$$



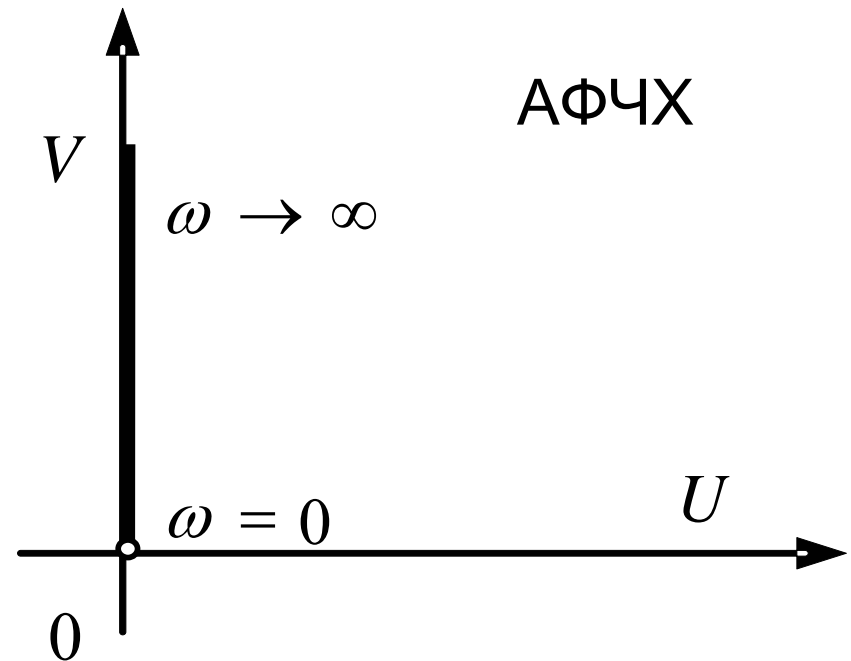
10. Интегриращо и идеално диференциращо звено

ЧПФ: $W(j\omega) = jk\omega = U(\omega) + jV(\omega)$

РЧФ: $U(\omega) = 0$

ИЧФ: $V(\omega) = k\omega$

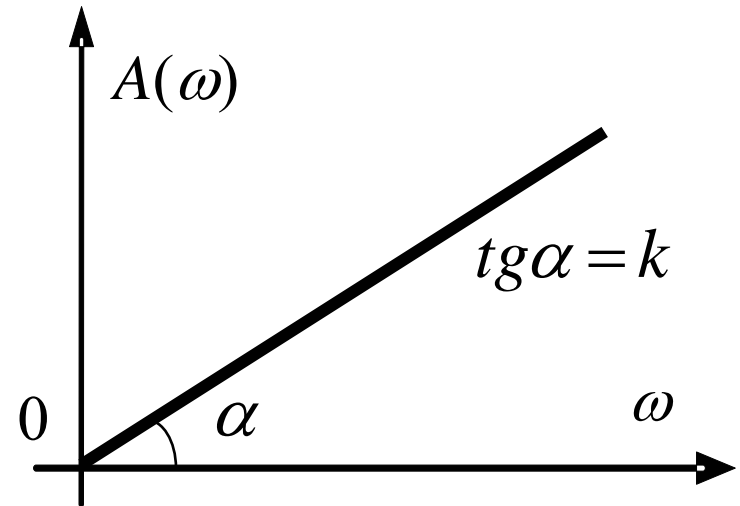
ω	0	∞
U	0	0
V	0	∞



10. Интегриращо и идеално диференциращо звено

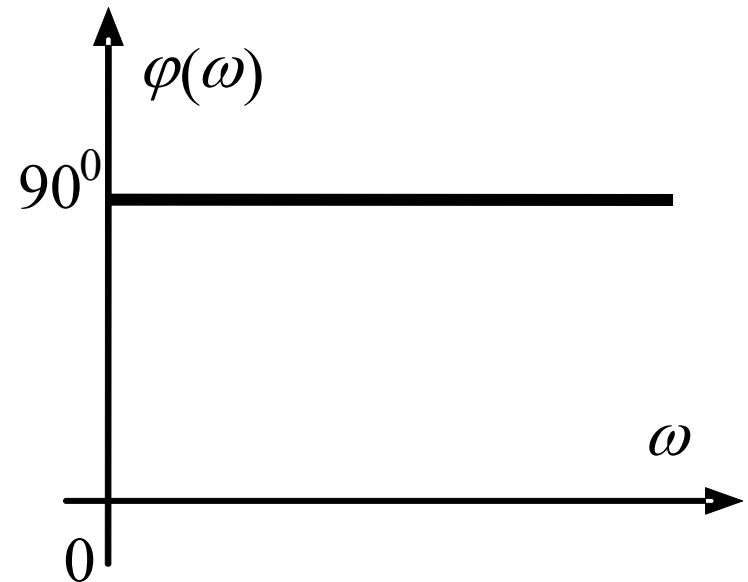
АЧХ:

$$\begin{aligned} A(\omega) &= \sqrt{U^2(\omega) + V^2(\omega)} = \\ &= \sqrt{0^2 + (k\omega)^2} = k\omega \end{aligned}$$



ФЧХ:

$$\begin{aligned} \varphi(\omega) &= \text{arctg} \frac{V(\omega)}{U(\omega)} = \\ &= \text{arctg} \frac{k\omega}{0} = \frac{\pi}{2} \end{aligned}$$

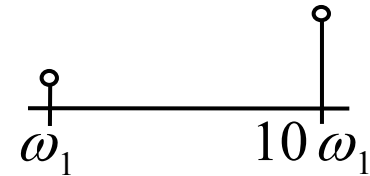


10. Интегриращо и идеално диференциращо звено

ЛАЧХ: $L(\omega) = 20 \lg A(\omega) = 20 \lg k\omega = 20 \lg k + 20 \lg \omega$

т($\omega = 1; L = 20 \lg k$), $\frac{\Delta L}{\Delta \omega} = ?$

Нека $\Delta \omega = 1 \text{ dec}$



$$L(\omega_1) = 20 \lg k + 20 \lg \omega_1$$

$$L(10\omega_1) = 20 \lg k + 20 \lg 10\omega_1$$

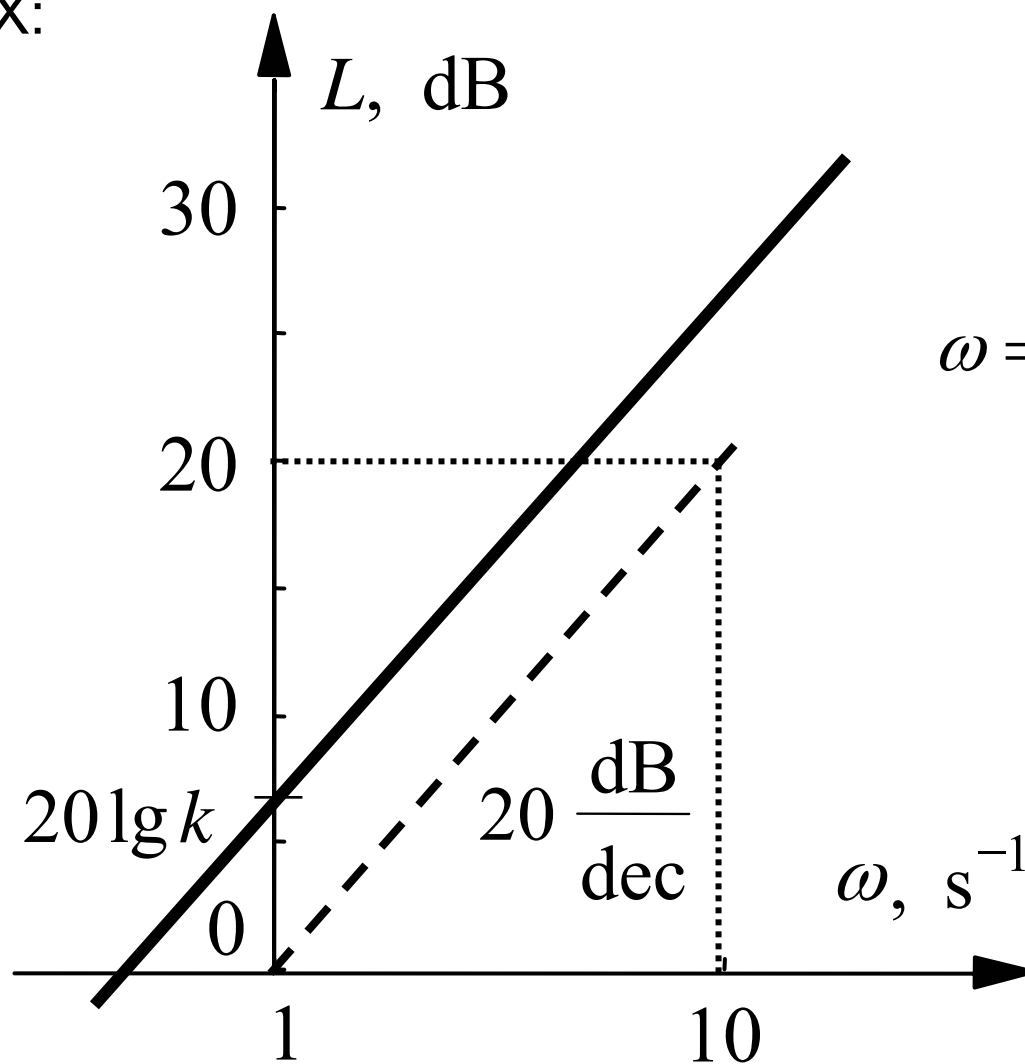
$$\Delta L = L(10\omega_1) - L(\omega_1) =$$

$$= 20 \lg k + 20 \lg 10\omega_1 - 20 \lg k - 20 \lg \omega_1 =$$

$$= 20 \lg 10 = 20 \text{ dB}$$

10. Интегриращо и идеално диференциращо звено

ЛАЧХ:



$$\frac{\Delta L}{\Delta \omega} = 20 \frac{\text{dB}}{\text{dec}};$$

$$\omega = 1, \quad L(1) = 20 \lg k$$