# 9. Колебателно звено

ДУ: $$T^2 \frac{d^2 y(t)}{dt^2} + 2\xi T \frac{dy(t)}{dt} + y(t) = ku(t), \qquad 0 < \xi < 1,$$

$k$ - предавателен коефициент,
$\xi$ - коефициент на затихване,
$T\,[s]$ - времеконстанта.

ПФ: $$T^2 p^2 Y(p) + 2\xi T p Y(p) + Y(p) = kU(p)$$

$$(T^2 p^2 + 2\xi T p + 1) Y(p) = kU(p)$$

$$W(p) = \frac{Y(p)}{U(p)} = \frac{k}{T^2 p^2 + 2\xi T p + 1}$$

## *9. Колебателно звено*

Времеви характеристики: $$T^2\lambda^2 + 2\xi T\lambda + 1 = 0$$

**(1)** $0 < \xi < 1$ - типично колебателно звено (затихващи колебания)

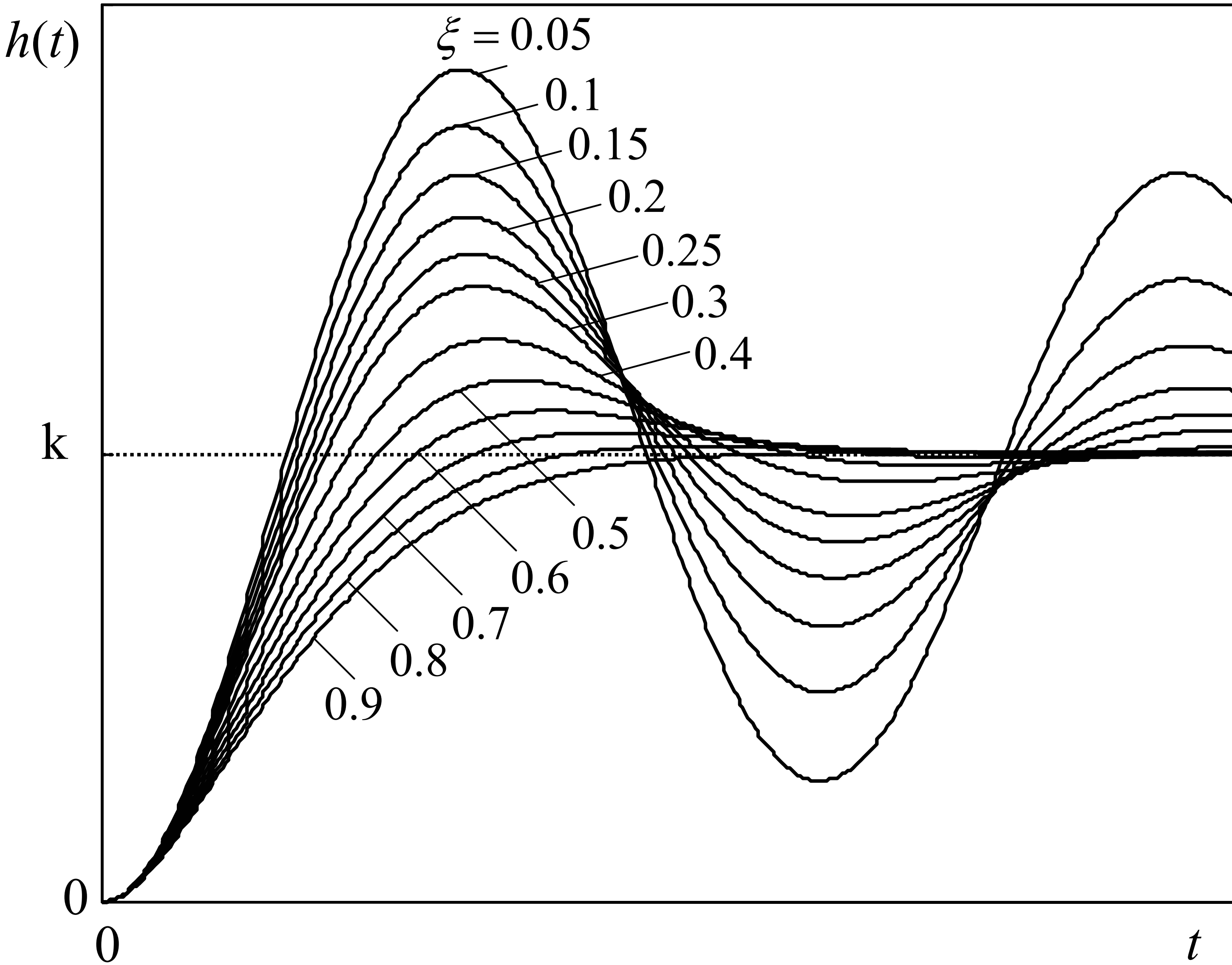
$$\lambda_{1,2} = -\frac{\xi}{T} \pm \frac{\sqrt{\xi^2 - 1}}{T} = \alpha \pm j\beta,$$

ПХ: $$h(t) = k\left[1 - \frac{1}{\sqrt{1-\xi^2}} e^{\alpha t} \sin\left(\beta t + \theta\right)\right],$$

$$\alpha = -\frac{\xi}{T}, \qquad \beta = \frac{\sqrt{1-\xi^2}}{T}, \qquad \theta = \operatorname{arctg}\frac{\sqrt{1-\xi^2}}{\xi}.$$

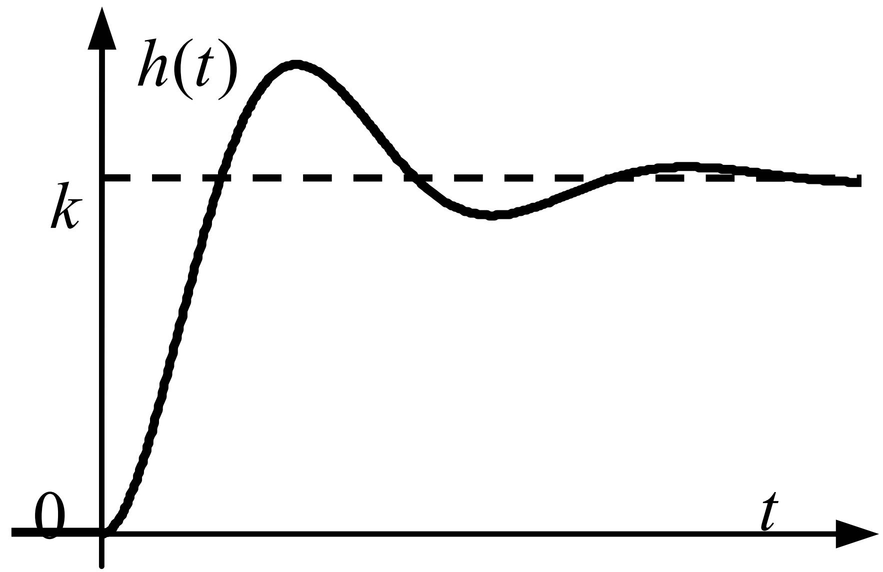
ТХ: $$w(t) = \frac{dh(t)}{dt} = \frac{k}{T\sqrt{1-\xi^2}} e^{\frac{-\xi}{T}\cdot t} \sin\frac{\sqrt{1-\xi^2}}{T} t\ .$$

# ПХ при различни стойности на $\xi$:

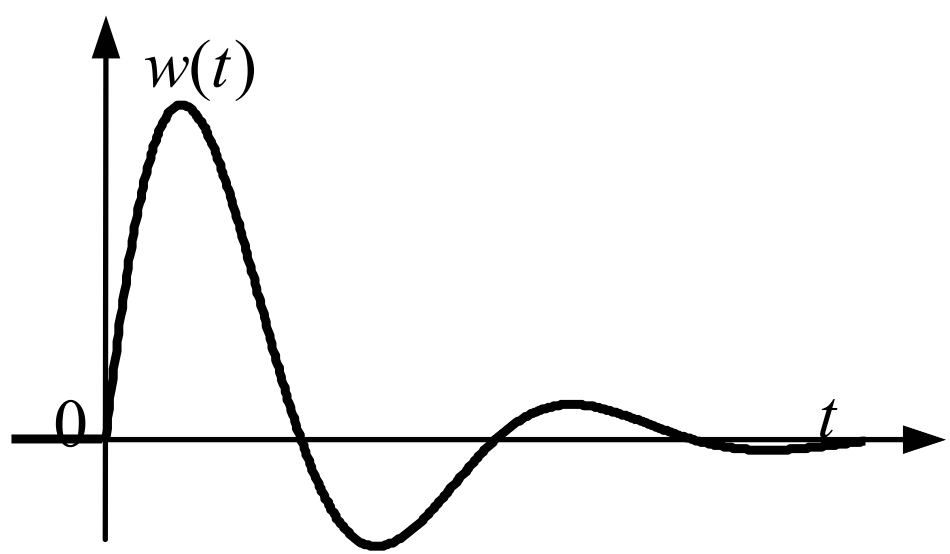


# 9. Колебателно звено

ПХ:




ТХ:

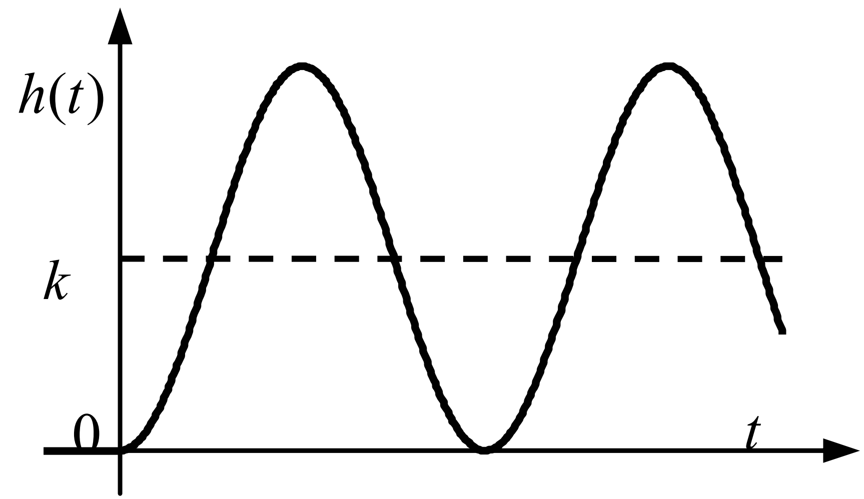


# 9. Колебателно звено

**(2)** $\xi = 0$ - консервативно звено (незатихващи колебания)

$$W(p) = \frac{k}{T^2 p^2 + 1}$$

$$h(t) = k(1 - \cos\frac{1}{T}t)$$

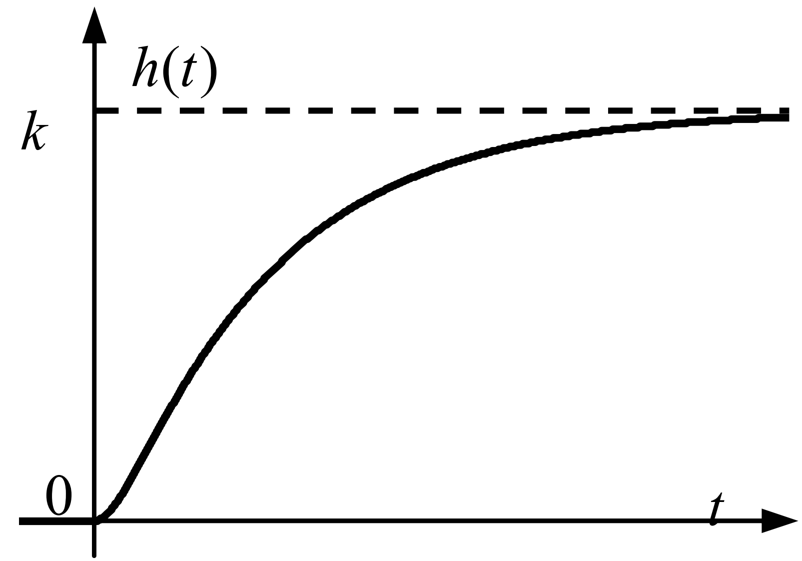



**(3)** $\xi = 1$ - критично-апериодичен преходен процес

$$W(p) = \frac{k}{(Tp+1)^2}$$

**(4)** $\xi > 1$ - апериодично звено от II ред (реални различни корени на характеристичното уравнение)

$$W(p) = \frac{k}{(T_1 p + 1)(T_2 p + 1)}$$

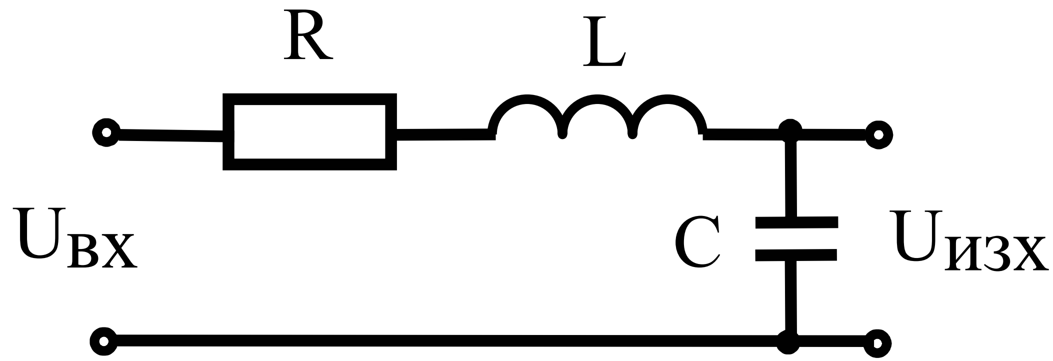



**(5)** $\xi >> 1, \quad T_2 << T_1, \quad W(p) = \dfrac{k}{(T_1 p + 1)}$

**(6)** $-1 < \xi < 0$ - неустойчиво колебателно звено (незатихващи колебания с нарастваща амплитуда)

## 9. Колебателно звено

Пример:




$$W(p) = \frac{U_{\text{изх}}(p)}{U_{\text{вх}}(p)} = \frac{\dfrac{1}{pC}}{R + pL + \dfrac{1}{pC}} = \frac{1}{LCp^2 + RCp + 1} =$$

$$= \frac{1}{T^2 p^2 + 2\xi T p + 1}$$

$$T = \sqrt{LC}\,; \quad 2\xi T = RC \quad \Rightarrow \quad \xi = \frac{RC}{2T} = \frac{RC}{2\sqrt{LC}} = \frac{R}{2}\sqrt{\frac{C}{L}} < 1.$$

## 9. Колебателно звено

ЧПФ:

$$W(j\omega) = \frac{k}{1-\omega^2 T^2 + j2\xi\omega T} \cdot \frac{1-\omega^2 T^2 - j2\xi\omega T}{1-\omega^2 T^2 - j2\xi\omega T} =$$

$$= \frac{k(1-\omega^2 T^2)}{(1-\omega^2 T^2)^2 + 4\xi^2\omega^2 T^2} - j\frac{2k\xi\omega T}{(1-\omega^2 T^2)^2 + 4\xi^2\omega^2 T^2} =$$

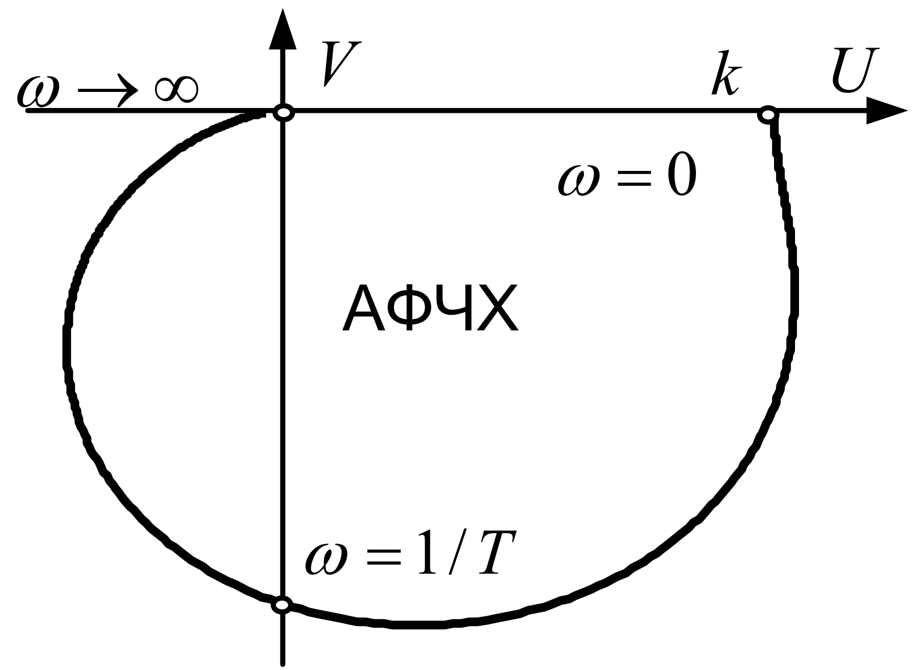
$$= U(\omega) + jV(\omega)$$

# 9. Колебателно звено

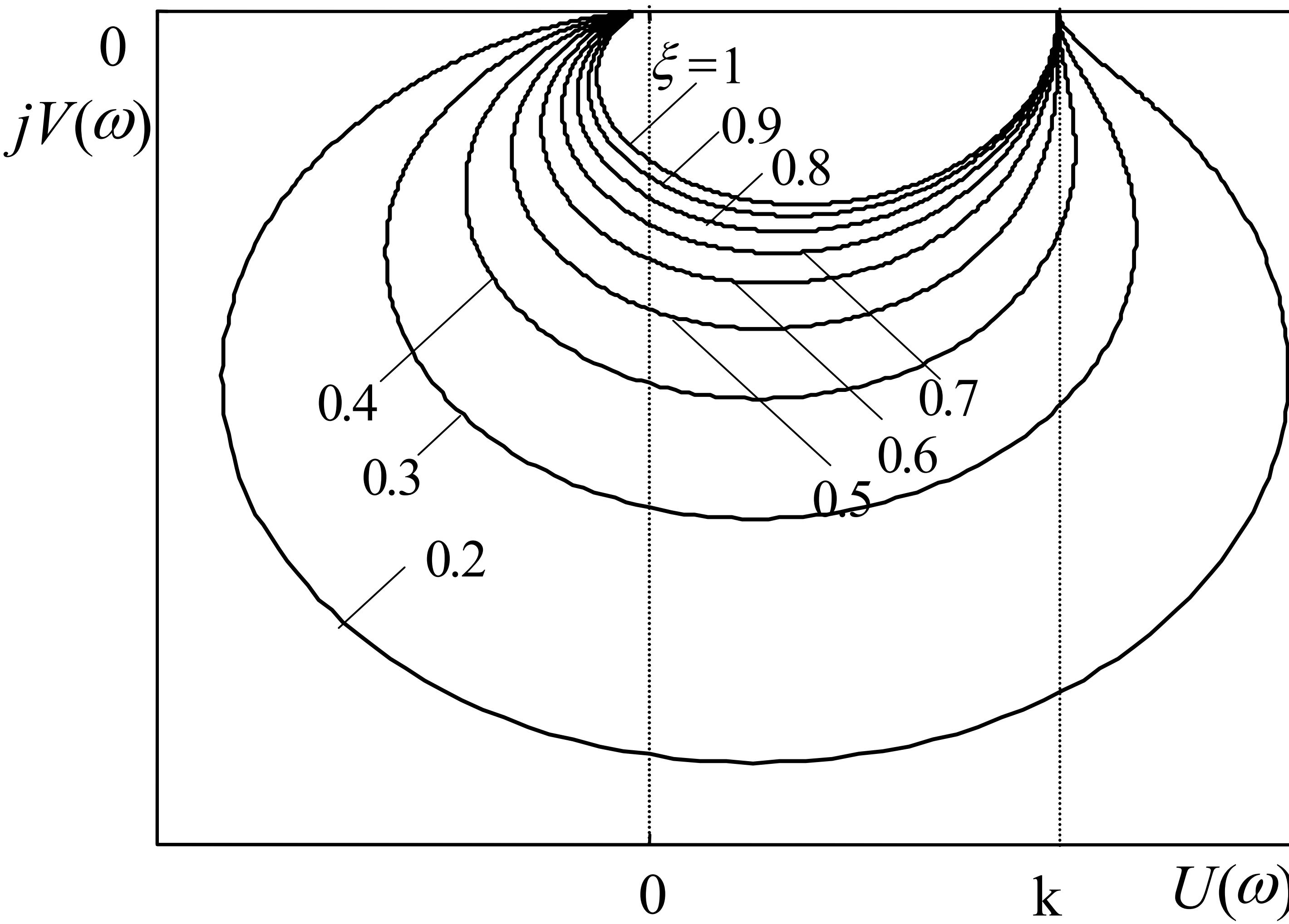
РЧФ: $$U(\omega) = \frac{k(1-\omega^2 T^2)}{(1-\omega^2 T^2)^2 + 4\xi^2 \omega^2 T^2}$$

ИЧФ: $$V(\omega) = \frac{-2k\xi\omega T}{(1-\omega^2 T^2)^2 + 4\xi^2 \omega^2 T^2}$$

| ω | 0 | 1/T | ∞ |
|---|---|---|---|
| U | k | 0 | 0 |
| V | 0 | -k/(2ξ) | 0 |



# Влияние на $\xi$ върху АФЧХ:



# *9. Колебателно звено*

АЧХ:

$$A(\omega)=\sqrt{U^2(\omega)+V^2(\omega)}=$$

$$=\sqrt{\left(\frac{k(1-\omega^2T^2)}{(1-\omega^2T^2)^2+4\xi^2\omega^2T^2}\right)^2+\left(\frac{-2k\xi\omega T}{(1-\omega^2T^2)^2+4\xi^2\omega^2T^2}\right)^2}=$$

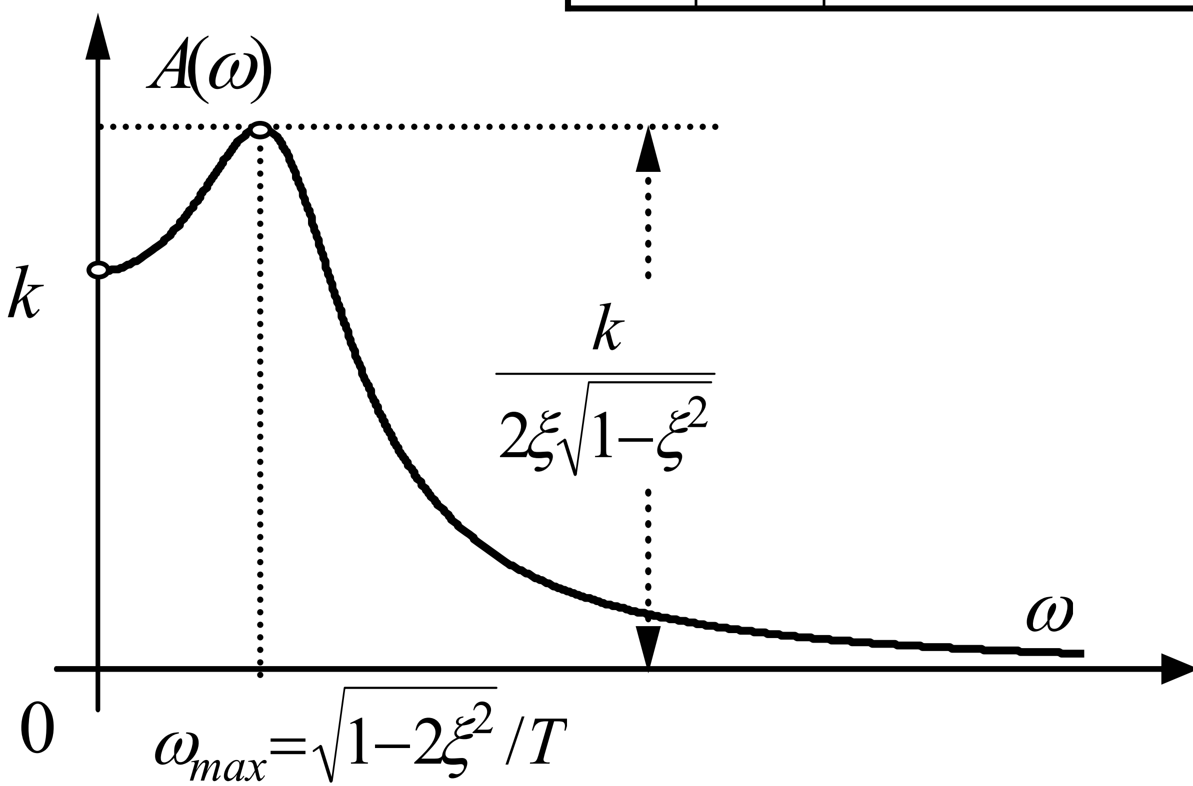
$$=\frac{k}{(1-\omega^2T^2)^2+4\xi^2\omega^2T^2}\sqrt{(1-\omega^2T^2)^2+4\xi^2\omega^2T^2}=$$

$$=\frac{k}{\sqrt{(1-\omega^2T^2)^2+4\xi^2\omega^2T^2}}$$

# 9. Колебателно звено

АЧХ: $$A(\omega)=\frac{k}{\sqrt{(1-\omega^2T^2)^2+4\xi^2\omega^2T^2}}$$

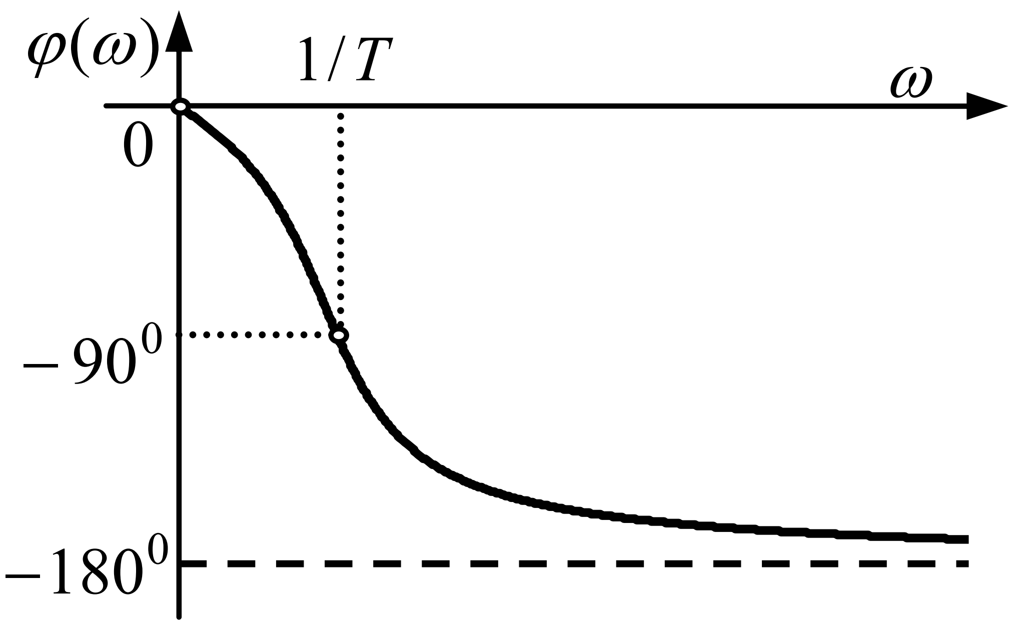
| | | | | |
|---|---|---|---|---|
| $\omega$ | $0$ | $\omega_{\max}=\sqrt{1-2\xi^2}/T$ | $1/T$ | $\infty$ |
| $A$ | $k$ | $A_{\max}=k/(2\xi\sqrt{1-\xi^2})$ | $k/2\xi$ | $0$ |



ФЧХ:

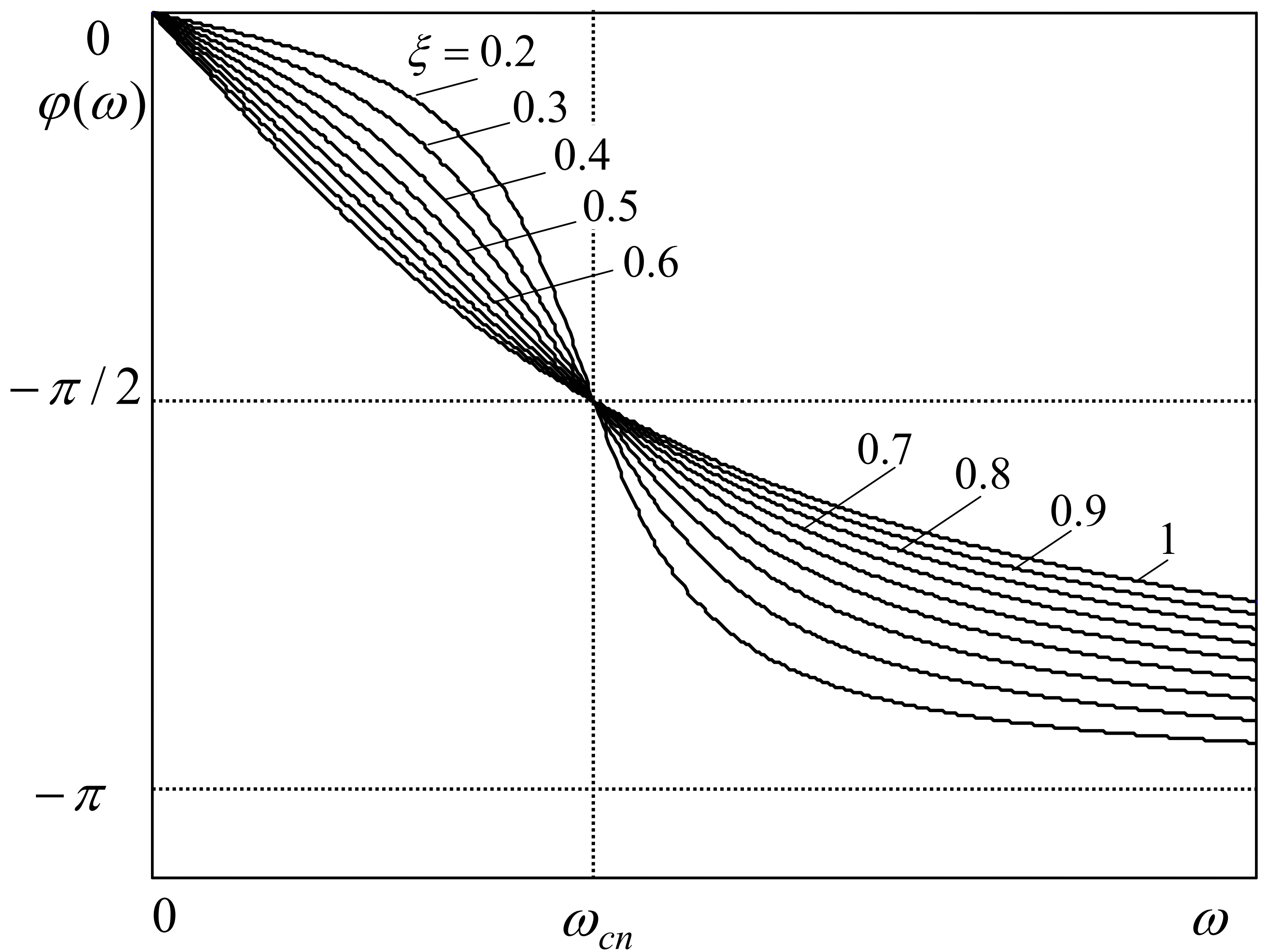
$$\varphi(\omega) = \operatorname{arctg}\frac{V(\omega)}{U(\omega)} = \operatorname{arctg}\frac{-2\xi\omega T}{1-\omega^2 T^2},$$

$$\varphi(\omega) = \begin{cases} -\operatorname{arctg}\dfrac{2\xi\omega T}{1-\omega^2 T^2}, & \text{при} \quad \omega \le \dfrac{1}{T}; \\ -\pi - \operatorname{arctg}\dfrac{2\xi\omega T}{1-\omega^2 T^2}, & \text{при} \quad \omega > \dfrac{1}{T}. \end{cases}$$




| ω | 0 | 1/T | ∞ |
|---|---|---|---|
| φ | 0 | $-\pi/2$ | $-\pi$ |

# Влияние на $\xi$ върху ФЧХ:



# 9. Колебателно звено

ЛАЧХ: $L(\omega) = 20\lg A(\omega) = 20\lg \dfrac{k}{\sqrt{(1-\omega^2 T^2)^2 + 4\xi^2\omega^2 T^2}} =$

$$= 20\lg k - 20\lg \sqrt{(1-\omega^2 T^2)^2 + 4\xi^2\omega^2 T^2}$$

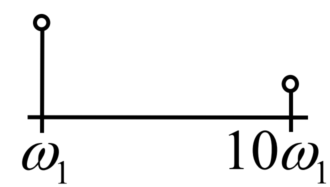
*НЧ:* $\omega << \dfrac{1}{T};\ \omega T << 1,$ пренебрегва се $\omega^2 T^2$ :

$$L_{НЧ}(\omega) = 20\lg k$$

*ВЧ:* $\omega >> \dfrac{1}{T};\ \omega T >> 1,$ пренебрегва се 1 и $4\xi^2\omega^2 T^2$ :

$$L_{ВЧ}(\omega) = 20\lg k - 20\lg \omega^2 T^2 = 20\lg k - 40\lg \omega T$$

$\dfrac{\Delta L_{ВЧ}}{\Delta\omega} = ?$ Нека $\Delta\omega = 1\,\mathrm{dec}$




$$L_{ВЧ}(\omega_1) = 20\lg k - 40\lg \omega_1 T$$

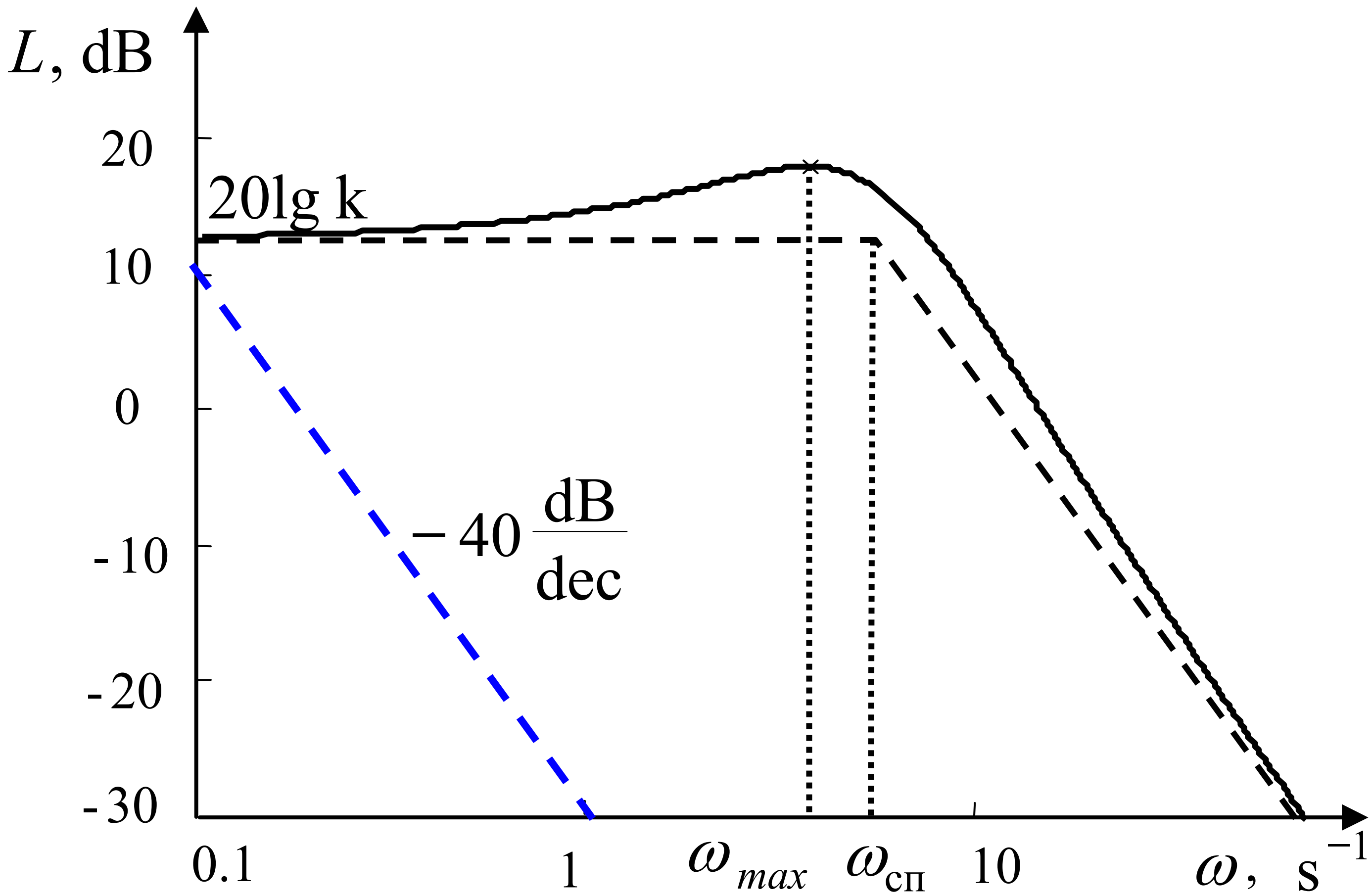
$$L_{ВЧ}(10\omega_1) = 20\lg k - 40\lg 10\omega_1 T$$

$$\Delta L_{ВЧ} = L_{ВЧ}(10\omega_1) - L_{ВЧ}(\omega_1) =$$

$$= 20\lg k - 40\lg 10\omega_1 T - 20\lg k + 40\lg \omega_1 T = -40\lg 10 = -40\,\mathrm{dB}$$

## 9. Колебателно звено

ЛАЧХ: $\dfrac{\Delta L_{ВЧ}}{\Delta\omega} = -40\,\dfrac{\text{dB}}{\text{dec}};\qquad \omega_{\text{сп}} = 1/T,\quad L_{ВЧ}(1/T) = 20\lg k.$




За $0.4 < \xi < 0.7 \qquad \Delta L_{\max} \le 3$ dB

# Влияние на $\xi$ върху ЛАЧХ: