

$$\begin{aligned} \text{min} \quad & -10x_1 - 12x_2 - 12x_3 \\ & x_1 + 2x_2 + 2x_3 \leq y_1 \\ & 2x_1 + x_2 + 2x_3 \leq y_2 \\ & 2x_1 + 2x_2 + x_3 \leq y_3 \\ & x_i, y_i \geq 0 \end{aligned}$$

$$\begin{aligned} & -10(2) - 12(0) - 12(0) - (-20) \\ & 20 - 20 = 0 \checkmark \\ & 2 \leq 2 \\ & 4 \leq 5 \\ & 4 \leq 4 \end{aligned}$$

$$\begin{aligned} \text{min} \quad & -10x_1 - 12x_2 - 12x_3 - z = 0 \\ & x_1 + 2x_2 + 2x_3 + s_1 = 2 \\ & 2x_1 + x_2 + 2x_3 + s_2 = 5 \\ & 2x_1 + x_2 + 2x_3 + s_3 = 4 \\ & x_i, s_i \geq 0 \end{aligned}$$

$$\begin{aligned} y_1 &= z ; y_1 = \frac{z}{2} + 1 = 2 \\ y_2 &= \left[ \frac{z}{2} \right] + 2 ; y_2 = 5 \\ 3s &= 3 + 2 \end{aligned}$$

I TABLEAU

$$y_3 = 1 + 3 = 4 = y_3$$

$$\begin{array}{ccccccc|c} x_1 & x_2 & x_3 & s_1 & s_2 & s_3 & z & b \\ -10 & -12 & -12 & 0 & 0 & 0 & -1 & 0 \end{array}$$

$$s_1 \circled{1} \quad 2 \quad 2 \quad 1 \quad 0 \quad 0 \quad 0 \quad 2$$

$$s_2 \quad 2 \quad 1 \quad 2 \quad 0 \quad 1 \quad 0 \quad 0 \quad 5$$

$$s_3 \quad 2 \quad 1 \quad 2 \quad 0 \quad 0 \quad 1 \quad 0 \quad 4$$

$$R_0' : R_0 + 10R_1 = 0 \quad 8 \quad 8 \quad 10 \quad 0 \quad 0 \quad -1 \quad 20$$

$$R_2' : R_2 - 2R_1 = 0 \quad -3 \quad -2 \quad -2 \quad 1 \quad 0 \quad 0 \quad 1$$

$$R_3' : R_3 - 2R_1 = 0 \quad -3 \quad -2 \quad -2 \quad 0 \quad 1 \quad 0 \quad 0$$

II TABLEAU

$$\begin{array}{ccccccc|c} x_1 & x_2 & x_3 & s_1 & s_2 & s_3 & z & b \\ 0 & 8 & 8 & 10 & 0 & 0 & -1 & 20 \end{array}$$

$$x_1 \quad 1 \quad 2 \quad 2 \quad 1 \quad 0 \quad 0 \quad 0 \quad 2$$

$$s_2 \quad 0 \quad -3 \quad -2 \quad -2 \quad 1 \quad 0 \quad 0 \quad 1$$

$$s_3 \quad 0 \quad -3 \quad -2 \quad -2 \quad 0 \quad 1 \quad 0 \quad 0$$

$$x = (2, 0, 0, 1, 0) \quad z_{\min} = -20$$

Soluzione verificata  
e corretta

22/01

## FORMULAZIONE DEL DUALE

$$\begin{array}{l} \text{Max } 5\mu_1 + 2\mu_2 - \mu_3 \\ -3\mu_1 - 2\mu_2 \leq 2 \\ -3\mu_1 + 2\mu_2 + 2\mu_3 \geq -1 \\ \mu_1 - \mu_3 = 1 \\ \mu_1 - 3\mu_2 \leq 0 \\ \mu_1 \geq 0 \\ \mu_2 \leq 0 \\ \mu_3 \text{ libera} \end{array}$$

] per minimi primale  
 ] segno opposto di segno delle variabili  
nel primale  
 ] il segno viene sottratto  
 ] dei segni dei vincoli nel primale

$$\begin{array}{l} \text{Max } 5x_1 + 4x_2 + 4x_3 \\ -4x_1 + 5x_2 - x_3 \leq y_1 \\ 2x_1 + 2x_2 - x_3 \leq y_2 \\ 3x_2 + 4x_3 \leq y_3 \\ x_i \geq 0 \end{array}$$

$$y_1 = \left(\frac{2}{5}\right) + 1 = 1$$

$$y_2 = \left(\frac{7}{5}\right) + 1 = 2$$

$$y_3 = \frac{3}{4} = 1$$

$$\begin{array}{l} \min -5x_1 - 4x_2 - 4x_3 - z = 0 \checkmark \\ -4x_1 + 5x_2 - x_3 + s_1 = 1 \checkmark \\ 2x_1 + 2x_2 - x_3 + s_2 = 2 \checkmark \\ 3x_2 + 4x_3 + s_3 = 1 \checkmark \\ x_i, s_i \geq 0 \end{array}$$

$$x_1 = 9/8$$

$$x_3 = 1/4$$

$$s_1 = 23/4$$

## TABEAU

$x_1$	$x_2$	$x_3$	$s_1$	$s_2$	$s_3$	$\geq$	b
-5	-4	-4	0	0	0	-1	0
$s_1$	-4	5	-1	1	0	0	1
$x_1$	(2)	2	-1	0	1	0	2
$s_3$	0	3	4	0	0	1	0

$$R_0^1 : -5 + 2k = 0 ; k = \frac{5}{2} ; R_0^1 = R_0 + \frac{5}{2} R_2 = 0 \quad 1 \quad -\frac{13}{2} \quad 0 \quad \frac{5}{2} \quad 0 \quad -1 \quad 5$$

$$R_1^1 : -4 + 2k = 0 ; k = 2 ; R_1^1 = R_1 + 2R_2 = 0 \quad 9 \quad -3 \quad 1 \quad 2 \quad 0 \quad 0 \quad 5$$

$$R_2^1 : \frac{1}{2} R_2 = 1 \quad 1 \quad -1/2 \quad 0 \quad 1/2 \quad 0 \quad 0 \quad 1$$

II TABLEAU

	$x_1$	$x_2$	$x_3$	$s_1$	$s_2$	$s_3$	$z$	b
	0	1	-1/2	0	5/2	0	-1	5
$s_1$	0	9	-3	1	2	0	0	5
$x_1$	1	1	-1/2	0	1/2	0	0	1
$x_3$	0	3	( $k$ )	0	0	1	0	1

$$R_0' = -\frac{13}{2} + 4k = 0 ; k = \frac{13}{2} \cdot \frac{1}{4} ; k = \frac{13}{8}$$

$$R_0' = R_0 + \frac{13}{8} R_3 = 0 \quad \frac{39}{8} \quad 0 \quad 0 \quad \frac{5}{2} \quad \frac{13}{8} \quad -1 \quad \frac{53}{8}$$

$$R_1' = -3 + hk = 0 ; k = \frac{3}{h} ; R_1' = R_1 + \frac{3}{h} R_3 = 0 \quad \frac{45}{4} \quad 0 \quad 1 \quad 2 \quad \frac{3}{4} \quad 0 \quad \frac{23}{4}$$

$$R_2' = -\frac{1}{2} + hk = 0 ; k = \frac{1}{8} ; R_2' = R_2 + \frac{1}{8} R_3 = 1 \quad \frac{11}{8} \quad 0 \quad 0 \quad 1/2 \quad 1/8 \quad 0 \quad 9/8$$

$$R_3' = \frac{1}{h} R_3 = 0 \quad \frac{3}{4} \quad 1 \quad 0 \quad 0 \quad 1/4 \quad 0 \quad 1/4$$

III TABLEAU

	$x_1$	$x_2$	$x_3$	$s_1$	$s_2$	$s_3$	$z$	b
	0	39/8	0	0	5/2	13/8	-1	53/8
$s_1$	0	45/4	0	1	2	3/4	0	25/4
$x_1$	1	11/8	0	0	1/2	1/8	0	9/8
$x_3$	0	3/4	1	0	0	1/4	0	1/4

$$X = (9/8, 0, 1/4, 25/4, 0, 0) \quad z_{\text{min}} = -\frac{53}{8}$$

Quesito 4 16/02/2022

$$\begin{aligned} \text{Max } & 100x_1 + 50x_2 \\ & 6x_1 + 4x_2 \leq 160 \\ & 3x_1 + 2x_2 \leq 200 \\ & x_1, x_2 \geq 0 \end{aligned}$$

$$y_1 = M_1/3 + h = 6$$

$$y_2 = M_2/3 + b = 3$$

$$y_3 = M_3/3 + z = h$$

$$y_4 = M_4/3 + 1 = 2$$

Quesito 4 4/04/21

$$\begin{aligned} \text{Max } & 5x_1 + 10x_2 \\ & 4x_1 + 2x_2 \leq 10 \\ & x_1 \leq 3 \\ & x_2 \leq 2 \\ & x_1, x_2 \geq 0 \end{aligned}$$

19/02/2021

$$\begin{aligned} \text{Max } & 3x_1 + 2x_2 - 5x_3 \\ & 4x_1 - 2x_2 + 2x_3 \leq 4 \quad \Rightarrow \quad 4 \cdot \frac{1}{2} - 2 \leq 4 \checkmark \quad \text{la soluzione} \\ & -2x_1 + x_2 - x_3 \geq -1 \quad \checkmark \quad \text{proposta e'} \\ & x_i \geq 0 \quad \text{ammisibili} \end{aligned}$$

Verificare che la soluzione  $[x_1, x_2, x_3] = [1/2, 0, 0]$  sia ottima

$$\begin{aligned} \text{Min } & 4M_1 - M_2 \\ & 4M_1 - 2M_2 \geq 3 \\ & -2M_1 + M_2 \geq 2 \quad \Rightarrow \quad 4 \cdot 0 - 2 \left( -\frac{3}{2} \right) \geq 3 ; \checkmark \\ & 2M_1 - M_2 \geq -5 \quad -2 \cdot 0 + \left( -\frac{3}{2} \right) \geq 2 \quad X \\ & M_1 \geq 0 \\ & M_2 \leq 0 \end{aligned}$$

Condizione di ortogonalità:

$$u^T (Ax - b) = 0$$

$$x^T (c^T - u^T A) = 0$$

$$M_1 \underbrace{(Ax - b)}_{\substack{\text{vincolo primario}}} = (4x_1 - 2x_2 + 2x_3 - 4) \quad \begin{aligned} & \text{Sostituisco la soluzione proposta} \\ & 2 \cdot \frac{1}{2} - 2(0) + 2(0) - 4 = -2 \end{aligned}$$

$$M_2 (Ax - b) = (-2x_1 + x_2 - x_3 + 1)$$

$$-2 \cdot \frac{1}{2} + 0 - 0 + 1 \quad M_2 \text{ e' ok per i valori di } x$$

$$x_1 \underbrace{(c^T - u^T A)}_{\substack{\text{vincolo doppio}}} = x_1 (4M_1 - 2M_2 - 3)$$

$$x_2 (c^T - u^T A) = x_2 (-2M_1 + M_2 - 2) = \text{e' ok per } x_2 = 0$$

$$x_3(C^T - \mu^T A) = x_3(2\mu_1 - \mu_2 + 5) = \text{e' on per } x_3 = 0$$

$$\begin{cases} \mu_1 = 0 \\ 4\mu_1 - 2\mu_2 = 3 \end{cases} ; \quad \begin{cases} \mu_1 = 0 \\ -2\mu_2 = 3 \end{cases} ; \quad \begin{cases} \mu_1 = 0 \\ \mu_2 = -\frac{3}{2} \end{cases}$$

Verifichiamo che la soluzione  $\mu_1, \mu_2$  sia ammissibile per il duale

Non e' ammissibile, quindi la soluzione  $[x_1, x_2, x_3] = [1/2, 0, 0]$   
non e' ottima

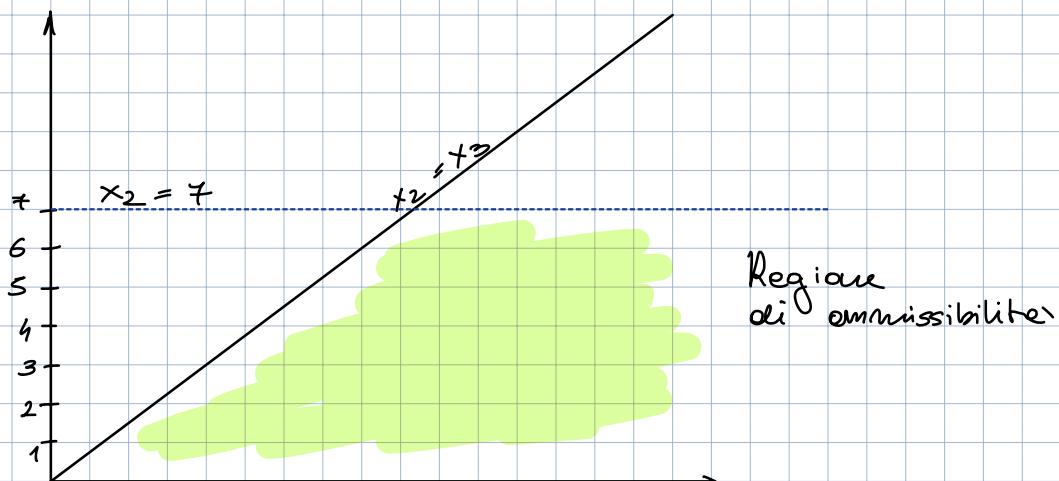
16/04/2020

Disequazioni della regione di ammissibilità

$$x_2 - x_3 \leq 0$$

$$x_2 \leq \mu_3 \quad \mu_3 = \text{f vi Motriche}$$

$x_2 = y$
$x_3 = X$



3/09/2020

$$\begin{aligned} \text{Max } & 24x_1 + 17x_2 + 12x_3 + 6x_4 \\ & 10x_1 + 8x_2 + 6x_3 + 5x_4 \leq 15 \\ & x_i \geq 0 \quad x_i \in \mathbb{Z} \end{aligned}$$

$$\begin{aligned} \text{Min } & -24x_1 - 17x_2 - 12x_3 - 6x_4 - z \\ & 10x_1 + 8x_2 + 6x_3 + 5x_4 + s_1 \leq 15 \\ & x_i \geq 0, s_1 \geq 0 \end{aligned}$$

## I TABLEAU

$x_1$	$x_2$	$x_3$	$x_4$	$s_1$	$z$	$b$
-24	-17	-12	-6	0	-1	0
$s_1$	10	8	6	5	1	0

$s_1$  10 8 6 5 1 0 15

Enter  $s_1$  into  $x_1$

$$R_0' = -24 + 10n = 0, n = \frac{-24}{10} = \frac{12}{5}$$

$$R_0' = R_0 + \frac{12}{5} R_1 = 0 \quad 11/5 \quad 12/5 \quad 6 \quad 12/5 \quad -1 \quad 36$$

$$R_1' = \frac{1}{10} R_1 = 1 \quad 4/5 \quad 3/5 \quad 1/2 \quad 1/10 \quad 0 \quad 3/2$$

## II TABLEAU

$x_1$	$x_2$	$x_3$	$x_4$	$s_1$	$z$	$b$
0	11/5	12/5	6	12/5	-1	36
$x_1$	1	4/5	3/5	1/2	1/10	0

$x_1$  1 4/5 3/5 1/2 1/10 0 3/2

$$X = (x_1, x_2, x_3, x_4, s_1) = (15, 0, 0, 0, 0) \quad z_{\text{MAX}} = -z_{\text{MIN}}$$

$$z_{\text{MAX}} = -(-36)$$

$$z = 36$$

$z = 36$
$x = 1.5$

$$x \leq 1 \quad x \geq 2$$

$$\begin{aligned} \text{Min } & -24x_1 - 17x_2 - 12x_3 - 6x_4 - z \quad \text{N.T.} \\ & 10x_1 + 8x_2 + 6x_3 + 5x_4 + s_1 \leq 15 \\ & x_1 + s_2 \leq 1 \end{aligned}$$

$$x_i \geq 0$$

I TABLEAU

	$x_1$	$x_2$	$x_3$	$x_4$	$s_1$	$s_2$	$z$	b
0	1/1/5	1/2/5	6	1/2/5	0	-1	36	
$x_1$	1	4/5	3/5	1/2	1/10	0	0	3/2
$s_2$	1	0	0	0	0	1	0	1

$$R_2^1 : R_2 - R_1 = 0 \quad -4/5 \quad -3/5 \quad -1/2 \quad -1/10 \quad 1 \quad 0 \quad -1/2$$

II TABLEAU

	$x_1$	$x_2$	$x_3$	$x_4$	$s_1$	$s_2$	$z$	b
0	1/1/5	1/2/5	6	1/2/5	0	-1	36	
$x_1$	1	4/5	3/5	1/2	1/10	0	0	1
$x_2$	0	-4/5	-3/5	-1/2	-1/10	1	0	1/2

Non continuo  
perche' ora  
conseguono  
il resto e'  
per PLIB che  
non abbiamo  
fatto

Quesito 2

$$\begin{aligned} & \text{Max } 5x_1 + x_2 + 3x_3 \\ & 5x_1 - 4x_2 + 3x_3 \leq M_7 \quad \checkmark \\ & -5x_1 + 5x_2 + 3x_3 \leq M_6 \quad \checkmark \\ & x_1 + 5x_2 - 4x_3 \geq -M_5 \quad \checkmark \\ & x_i \geq 0 \end{aligned}$$

$$\begin{aligned} & \text{Min } -5x_1 - x_2 - 3x_3 - z = 0 \\ & 5x_1 - 4x_2 + 3x_3 + s_1 = 3 \\ & -5x_1 + 5x_2 + 3x_3 + s_2 = 7 \\ & x_1, s_i \geq 0 \end{aligned}$$

III TABLEAU

	$x_1$	$x_2$	$x_3$	$s_1$	$s_2$	$s_3$	$z$	b
-5	-1	-3	0	0	0	0	-1	0
$s_1$	5	-4	3	1	0	0	0	3
$s_2$	-5	5	3	0	1	0	0	7
$s_3$	-1	-5	4	0	0	1	0	2

entre

escr      

$$R_0' = R_0 + R_1 = 0 \ -5 \ 0 \ 1 \ 0 \ 0 \ -1 \ 3$$

$$R_1' = \frac{1}{5} R_1 = 1 \ -\frac{4}{5} \ \frac{3}{5} \ \frac{1}{5} \ 0 \ 0 \ 0 \ \frac{3}{5}$$

$$R_2' = R_2 + R_1 = 0 \ 1 \ 6 \ 1 \ 1 \ 0 \ 0 \ 10$$

$$R_3' = -1 + 5k = 0 ; k = \frac{1}{5} ; R_3 = R_3 + \frac{1}{5} R_1 = 0 \ -\frac{24}{5} \ \frac{23}{5} \ \frac{1}{5} \ 0 \ 1 \ 0 \ \frac{18}{5}$$

II TABLEAU

$x_1$	$x_2$	$x_3$	$s_1$	$s_2$	$s_3$	$z$	$b$
0	-5	0	1	0	0	-1	3
$x_1$	1	$-\frac{4}{5}$	$\frac{3}{5}$	$\frac{1}{5}$	0	0	$\frac{3}{5}$
$s_2$	0	(1)	6	1	1	0	10
$s_3$	0	$-\frac{24}{5}$	$\frac{23}{5}$	$\frac{1}{5}$	0	1	$\frac{18}{5}$
	<u>  </u>	<u>  </u>		<u>  </u>	<u>  </u>		
	<u>entre</u>			<u>escr</u>			

$$R_0' = R_0 + 5R_2 = 0 \ 0 \ 30 \ 6 \ 5 \ 0 \ -1 \ 53$$

$$R_1' = -\frac{4}{5} + k = 0 ; k = \frac{4}{5} ; R_1' = R_1 + \frac{4}{5} R_2 = 1 \ 0 \ \frac{27}{5} \ 1 \ \frac{4}{5} \ 0 \ 0 \ \frac{43}{5}$$

$$R_3' = -\frac{24}{5} + k = 0 ; k = \frac{24}{5} ; R_3' = R_3 + \frac{24}{5} R_2 = 0 \ 0 \ \frac{19}{5} \ 6 \ \frac{29}{5} \ 1 \ 0 \ \frac{303}{5}$$

III TABLEAU

$x_1$	$x_2$	$x_3$	$s_1$	$s_2$	$s_3$	$z$	$b$
0	0	30	6	5	0	-1	53
$x_1$	1	0	$\frac{27}{5}$	1	$\frac{4}{5}$	0	$\frac{43}{5}$
$x_2$	0	1	6	1	1	0	10
$s_3$	0	0	$\frac{19}{5}$	6	$\frac{29}{5}$	1	$\frac{303}{5}$

$$X = (x_1, x_2, x_3, s_1, s_2, s_3) = \left( \frac{43}{5}, 10, 0, 0, 0, \frac{303}{5} \right)$$

$$z_{\max} = -z_{\min} \quad z = -(-53) = 53$$

La soluzione e' veritabile per tanto e' corretta

5/02/2024

$$\begin{aligned} \max & 6x_1 + 4x_2 + 8x_3 \\ & 5x_1 + 6x_2 + 5x_3 \geq 5 \\ & 4x_1 + 6x_2 + 6x_3 \leq 2 \\ & 5x_1 + 4x_2 + 8x_3 \leq 4 \end{aligned}$$

$$\begin{aligned} \min & -6x_1 - 4x_2 - 8x_3 - z \\ & 5x_1 + 6x_2 + 5x_3 - s_1 = 5 \\ & 4x_1 + 6x_2 + 6x_3 + s_2 = 2 \\ & 5x_1 + 4x_2 + 8x_3 + s_3 = 4 \end{aligned}$$

I TABLEAU

$x_1$	$x_2$	$x_3$	$s_1$	$s_2$	$s_3$	$-z$	$b$
-6	-4	-8	0	0	0	-1	0
5	(6)	5	-1	0	0	0	5
4	6	6	0	1	0	0	2
5	4	8	0	0	1	0	4

IL PROBLEMA E' NON AMMISSIBILE

ES. \*

$$\begin{aligned} \max & 3x_1 + 2x_2 - 5x_3 \\ & 4x_1 - 2x_2 + 2x_3 \leq 4 \\ & -2x_1 - 2x_2 - x_3 \leq 1 \\ & x_i \geq 0 \end{aligned}$$

$$\begin{aligned} \min & -3x_1 - 2x_2 + 5x_3 - z = 0 \\ & 4x_1 - 2x_2 + 2x_3 + s_1 = 4 \\ & -2x_1 - 2x_2 - x_3 + s_2 = 1 \\ & x_i \geq 0, \quad s_i \geq 0 \end{aligned}$$

I TABLEAU

$x_1$	$x_2$	$x_3$	$s_1$	$s_2$	$-z$	$b$
-3	-2	5	0	0	-1	0
$s_1$ (4)	-2	2	1	0	0	4
$s_2$	-2	-2	-1	0	1	0

ILLIMITATO

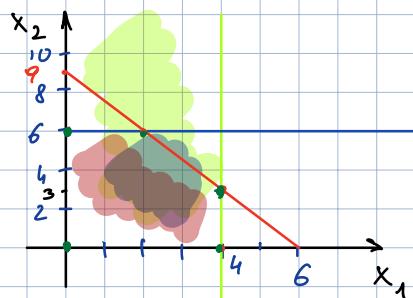
Es. 7

$$\begin{aligned} \text{Max } & 3x_1 + 5x_2 \\ x_1 &\leq 4 \\ 2x_2 &\leq 12 \\ 3x_1 + 2x_2 &\leq 18 \\ x_i &\geq 0 \end{aligned}$$

$$\begin{aligned} \text{Min } & -3x_1 - 5x_2 - z = 0 \\ x_1 + s_1 &= 4 \\ 2x_2 + s_2 &= 12 \\ 3x_1 + 2x_2 + s_3 &= 18 \\ x_i, s_i &\geq 0 \end{aligned}$$

### RAPPRESENTAZIONE GEOMETRICA

Considero  $x_1 = x$ ,  $x_2 = y$



$$\begin{aligned} x &= 4 \\ y &= \frac{12}{2} = 6 \end{aligned}$$

$$\begin{aligned} 3x + 2y &= 18 & 2y &= 18 \cdot y = 9 \\ x = 0 & & 3x &= 18, x = 6 \\ y = 0 & & y &= 0 \end{aligned}$$

Max  $3x_1 + 5x_2$

$$\left. \begin{array}{ll} x_1 = 0 & x_2 = 0 \\ x_1 = 0 & x_2 = 6 \\ x_1 = 2 & x_2 = 6 \\ x_1 = 4 & x_2 = 3 \\ x_1 = 4 & x_2 = 0 \end{array} \right\}$$

$$\begin{aligned} &\text{sostituisco alla funzione obiettivo} \\ &= 0 \\ &= 30 \\ &= 36 \\ &= 24 \\ &= 12 \end{aligned}$$

Determinare le regioni di ammissibilità

I TABLEAU

$$\begin{array}{ccccccc} x_1 & x_2 & s_1 & s_2 & s_3 & z & b \\ -3 & -5 & 0 & 0 & 0 & -1 & 0 \end{array}$$

$$s_1 \ 1 \ 0 \ 1 \ 0 \ 0 \ 0 \ 4$$

$$s_2 \ 0 \ 2 \ 0 \ 1 \ 0 \ 0 \ 12$$

$$s_3 \ 3 \ 2 \ 0 \ 0 \ 1 \ 0 \ 18$$

$$R'_0 = R_0 + \frac{5}{2} R_2 = -3 \ 0 \ 0 \ \frac{5}{2} \ 0 \ -1 \ 30$$

$$R'_2 = \frac{1}{2} R_2 = 0 \ 1 \ 0 \ \frac{1}{2} \ 0 \ 0 \ 6$$

$$R_3^1 = R_3 - R_2 = \begin{matrix} 3 & 0 & 0 & -1 & 1 & 0 & 6 \end{matrix}$$

II TABLEAU

$x_1$	$x_2$	$s_1$	$s_2$	$s_3$	$z$	b
-3	0	0	$\frac{5}{2}$	0	-1	30
$s_1$	1	0	1	0	0	4
$x_2$	0	1	0	$\frac{1}{2}$	0	6
$s_3$	(3)	0	0	-1	1	0

$$R_0^1 = R_0 + R_3 = \begin{matrix} 0 & 0 & 0 & \frac{3}{2} & 1 & -1 & 36 \end{matrix}$$

$$R_1^1 = 1 + 3k = 0 \cdot k = -\frac{1}{3} ; R_1^1 = R_1 - \frac{1}{3} R_3 = \begin{matrix} 0 & 0 & 1 & \frac{1}{3} & -\frac{1}{3} & 0 & 2 \end{matrix}$$

$$R_3^1 = \frac{1}{3} R_3 = \begin{matrix} 1 & 0 & 0 & -\frac{1}{3} & \frac{1}{3} & 0 & 2 \end{matrix}$$

III TABLEAU

$x_1$	$x_2$	$s_1$	$s_2$	$s_3$	$z$	b
0	0	0	$\frac{3}{2}$	1	-1	36
$s_1$	0	0	1	$\frac{1}{3}$	$-\frac{1}{3}$	0
$x_2$	0	1	0	$\frac{1}{2}$	0	6
$x_1$	1	0	0	$-\frac{1}{3}$	$-\frac{1}{3}$	0

$$X = (x_1, x_2, s_1, s_2, s_3) = (2, 6, 2, 0, 0) \quad z_{\text{MAX}} = -z_{\text{MIN}} = 36$$

$$\begin{aligned} \text{Min } & -x_1 - 3x_2 + x_3 \\ & 2x_1 + x_2 \leq 3 \\ & x_1 + x_2 + 3x_3 \leq 6 \\ & 2x_1 + x_2 + 3x_3 \leq 8 \\ & x_i \geq 0 \end{aligned}$$

$$\begin{aligned} \text{Min } & -x_1 - 3x_2 + x_3 - z = 0 \\ & 2x_1 + x_2 + s_1 = 3 \\ & x_1 + x_2 + 3x_3 + s_2 = 6 \\ & 2x_1 + x_2 + 3x_3 + s_3 = 8 \\ & x_i, s_i \geq 0 \end{aligned}$$

I TABLEAU

$x_1$	$x_2$	$x_3$	$s_1$	$s_2$	$s_3$	$z$	b
-1	-3	1	0	0	0	-1	0
$x_1$	2	1	0	1	0	0	3
$s_2$	1	1	3	0	1	0	6
$s_3$	2	1	3	0	0	1	0

$$R_0' = -1 + 2k = 0 \quad ; \quad k = 1/2 \quad ; \quad R_0' = R_0 + \frac{1}{2} R_1 = 0 \quad -\frac{5}{2} \quad 1 \quad \frac{1}{2} \quad 0 \quad \frac{-1}{0} \sqrt{\frac{3}{2}}$$

$$R_1' = \frac{1}{2} R_1 = 1 \quad \frac{1}{2} \quad 0 \quad \frac{1}{2} \quad 0 \quad 0 \quad 0 \quad \frac{3}{2}$$

$$R_2' = 1 + 2k = 0 \quad ; \quad k = -\frac{1}{2} \quad ; \quad R_2' = R_2 - \frac{1}{2} R_1 = 0 \quad \frac{1}{2} \quad 3 \quad -\frac{1}{2} \quad 1 \quad 0 \quad 0 \quad \frac{9}{2}$$

$$R_3' = R_3 - R_1 = 0 \quad 0 \quad 3 \quad -1 \quad 0 \quad 1 \quad 0 \quad 5$$

II TABLEAU

$x_1$	$x_2$	$x_3$	$s_1$	$s_2$	$s_3$	$z$	b
0	$-\frac{5}{2}$	1	$\frac{1}{2}$	0	0	-1	$\frac{3}{2}$
$x_1$	1	$\frac{1}{2}$	0	$\frac{1}{2}$	0	0	$\frac{3}{2}$
$s_2$	0	$\frac{1}{2}$	3	$-\frac{1}{2}$	1	0	0

$$s_3 \quad 0 \quad 0 \quad 3 \quad -1 \quad 0 \quad 1 \quad 0 \quad 5$$

esc pntce

$$R_0' = -\frac{5}{2} + \frac{1}{2} k = 0 \quad ; \quad k = \frac{5}{2} \quad ; \quad R_0' = R_0 + 5R_1 = 5 \quad 0 \quad 1 \quad 3 \quad 0 \quad \frac{-1}{0} \sqrt{\frac{9}{2}}$$

$$R_1' = 2R_1 = 2 \quad 1 \quad 0 \quad 1 \quad 0 \quad 0 \quad 0 \quad 3$$

$$R_2' = R_2 - R_1 = -1 \quad 0 \quad 3 \quad -1 \quad 1 \quad 0 \quad 0 \quad 3$$

$$x = (x_1, x_2, x_3, s_1, s_2, s_3) = (0, 3, 0, 0, 3, 5) \quad z = -9$$

16/04

$$\begin{aligned} \text{Max } & 3x_1 + x_2 - x_3 \\ -x_1 + x_2 - 3x_3 & \leq M_7 \\ -x_1 + 3x_2 - 5x_3 & \leq M_6 \\ 2x_1 - x_2 + 3x_3 & \leq M_5 \\ x_i & \geq 0 \end{aligned}$$

$$\begin{aligned} \text{Min } & -3x_1 - x_2 + x_3 - z = 0 \\ -x_1 + x_2 - 3x_3 + s_1 & = 3 \\ -x_1 + 3x_2 - 5x_3 + s_2 & = 7 \\ 2x_1 - x_2 + 3x_3 + s_3 & = 2 \\ x_i & \geq 0, s_i \geq 0 \end{aligned}$$

## I TABLEAU

$x_1$	$x_2$	$x_3$	$s_1$	$s_2$	$s_3$	$z$	b
-3	-1	1	0	0	0	-1	0
$s_1$	-1	1	-3	1	0	0	3
$s_2$	-1	3	-5	0	1	0	7
$s_3$	2	-1	3	0	0	1	2

$$R_0' = -3 + 2k = 0 \quad k = \frac{3}{2}; \quad R_0' = R_0 + \frac{3}{2} R_3 = 0 \quad -\frac{3}{2} \quad \frac{11}{2} \quad 0 \quad 0 \quad \frac{3}{2} \quad -1$$

$$R_1' = R_1 + \frac{1}{2} R_3 = 0 \quad \frac{1}{2} \quad -\frac{3}{2} \quad 1 \quad 0 \quad \frac{1}{2} \quad 0 \quad 4$$

$$R_2' = R_2 + \frac{1}{2} R_3 = 0 \quad \frac{5}{2} \quad -\frac{7}{2} \quad 0 \quad 1 \quad \frac{1}{2} \quad 0 \quad 8$$

$$R_3' = \frac{1}{2} R_3 = 1 \quad -\frac{1}{2} \quad \frac{3}{2} \quad 0 \quad 0 \quad \frac{1}{2} \quad 0 \quad 1$$

## II TABLEAU

$x_1$	$x_2$	$x_3$	$s_1$	$s_2$	$s_3$	$z$	b
0	$-\frac{3}{2}$	$\frac{11}{2}$	0	0	$\frac{3}{2}$	-1	3
$s_1$	0	$\frac{1}{2}$	$-\frac{3}{2}$	1	0	$\frac{1}{2}$	0
$x_2$	0	$\frac{5}{2}$	$-\frac{7}{2}$	0	1	$\frac{1}{2}$	8
$x_1$	1	$-\frac{1}{2}$	$\frac{3}{2}$	0	0	$\frac{1}{2}$	1

$$h: \frac{1}{2} \mid h \cdot 2 = 8$$

$$8: \frac{5}{2} \mid \frac{8 \cdot 2}{5} \approx 3.2$$

$$R_0' = R_0 + R_2 = 0 \ 0 \ 2 \ 0 \ 1 \ 2 \ -1 \ 11$$

$$R_1' = \frac{1}{2} + \frac{5}{2} n = 0; n = -\frac{1}{2} \cdot \frac{2}{5}; R_1' = R_1 - \frac{1}{5} R_2 = 0 \ 0 \ -\frac{4}{5} \ 1 \ -\frac{1}{5} \ \frac{2}{5} \ 0 \ \frac{12}{5}$$

$$R_2' = \frac{2}{5} R_2 = 0 \ 1 \ -\frac{7}{5} \ 0 \ \frac{2}{5} \ \frac{1}{5} \ 0 \ \frac{16}{5}$$

$$R_3' = R_3 + \frac{1}{5} R_2 = 1 \ 0 \ \frac{8}{5} \ 0 \ \frac{1}{5} \ \frac{3}{5} \ 0 \ \frac{13}{5}$$

## IV TABLEAU

	$x_1$	$x_2$	$x_3$	$s_1$	$s_2$	$s_3$	$z$	$b$
$S_1$	0	0	2	0	1	2	-1	11
$x_1$	0	0	- $\frac{4}{5}$	1	- $\frac{1}{5}$	$\frac{2}{5}$	0	$\frac{12}{5}$
$x_2$	0	1	- $\frac{7}{5}$	0	$\frac{2}{5}$	$\frac{1}{5}$	0	$\frac{16}{5}$
$x_3$	1	0	$\frac{8}{5}$	0	$\frac{1}{5}$	$\frac{3}{5}$	0	$\frac{13}{5}$

$$X = (x_1, x_2, x_3, s_1, s_2, s_3) = \left( \frac{13}{5}, \frac{16}{5}, 0, \frac{12}{5}, 0, 0 \right) \quad z_{\text{MAX}} = -2 \frac{1}{5}$$

$$z_{\text{MAX}} = -(-11)$$

$$z_{\text{MAX}} = 11$$

$$\begin{aligned} \text{Max } & 3x_1 + 5x_2 \\ & x_1 \leq 4 \\ & 2x_2 \leq 12 \\ & 3x_1 + 2x_2 \leq 18 \\ & x_i \geq 0 \end{aligned}$$

$$\begin{aligned} \text{Min } & -3x_1 - 5x_2 - z = 0 \\ & x_1 + s_1 = 4 \\ & 2x_2 + s_2 = 12 \\ & 3x_1 + 2x_2 + s_3 = 18 \\ & x_i \geq 0, s_i \geq 0 \end{aligned}$$

## V TABLEAU

$x_1$	$x_2$	$s_1$	$s_2$	$s_3$	$z$	$b$
-3	-5	0	0	0	-1	0
$x_1$	1	0	1	0	0	4
$s_2$	0	2	0	1	0	12
$s_3$	3	2	0	0	1	0
						18

REGOLA DI BLAND

PER EVITARE CHE IL  
TABLEAU DECRESCA  
SI SCEGLIE SEMPRE LA  
VARIABILE CON INDICE MINORE

$$R_0' = R_0 + 3R_1 = 3 \ -5 \ 3 \ 0 \ 0 \ -1 \ 12$$

$$R_3' = R_3 - 3R_1 = 0 \ 2 \ -3 \ 0 \ 1 \ 0 \ 6$$

III TABLEAU

$x_1$	$x_2$	$s_1$	$s_2$	$s_3$	$z$	b
3	-5	3	0	0	-1	12
$x_1$	1	0	1	0	0	4
$s_2$	0	2	0	1	0	0
$s_3$	0	2	-3	0	1	6

$$R_0' = -s_2 + 2x_1 = 0; k = \frac{5}{2}; R_0' = R_0 + \frac{5}{2}R_3 = 3 \ 0 \ \frac{9}{2} \ 0 \ 1 \ -1 \ 27$$

$$R_2' = R_2 - R_1 = 0 \ 0 \ 3 \ 1 \ -1 \ 0 \ 6$$

$$R_3' = \frac{1}{2}R_3 = 0 \ 1 \ -\frac{3}{2} \ 0 \ \frac{1}{2} \ 0 \ 3$$

IV TABLEAU

$x_1$	$x_2$	$s_1$	$s_2$	$s_3$	$z$	b
3	0	$\frac{9}{2}$	0	1	-1	27
$x_1$	1	0	1	0	0	4
$s_2$	0	0	3	1	-1	0
$x_2$	0	1	$-\frac{3}{2}$	0	$\frac{1}{2}$	6

$$X = (x_1, x_2, s_1, s_2, s_3) = (4, 3, 0, 6, 0) \quad z_{MAX} = -z_{MIN}$$

$$z = 27$$

$$-3(4) - 5(3) + 27 = -12 - 15 + 27 = 0 \quad \checkmark$$

22/01/2021

$$\begin{aligned} \text{min} \quad & -2x_1 - x_2 + x_3 \\ \text{s.t.} \quad & -3x_1 - 3x_2 + x_3 + x_4 \geq 5 \\ & -2x_1 + 2x_2 - 3x_4 \leq 2 \\ & 2x_2 - x_3 = 1 \\ & x_1, x_4 \geq 0 \\ & x_2 \leq 0 \\ & x_3 \text{ libera} \end{aligned}$$

se fosse  
 Stato di  
 massimo  
 e' nei cambiato  
 i segni delle  
 variabili

D:

$$\begin{aligned} \text{max} \quad & s_1 + 2s_2 + s_3 \\ \text{s.t.} \quad & -3s_1 - 2s_2 \leq -2 \\ & -3s_1 + 2s_2 + 2s_3 \geq -1 \\ & s_1 - s_3 = 1 \\ & s_1 - 3s_2 \leq 0 \\ & s_1, s_3 \geq 0 \\ & s_2 \leq 0 \\ & s_3 \text{ libera} \end{aligned}$$

$$\begin{aligned} \text{max} \quad & 5x_1 + 4x_2 + 4x_3 \\ \text{s.t.} \quad & -4x_1 + 5x_2 - x_3 \leq 1 \\ & 2x_1 + 2x_2 - x_3 \leq 2 \\ & 3x_2 + 4x_3 \leq 2 \\ & x_i \geq 0 \end{aligned}$$

$$\begin{aligned} \text{min} \quad & -5x_1 - 4x_2 - 4x_3 - 2 = 0 \checkmark \\ \text{s.t.} \quad & -4x_1 + 5x_2 - x_3 + s_1 = 1 \checkmark \\ & 2x_1 + 2x_2 - x_3 + s_2 = 2 \checkmark \\ & 3x_2 + 4x_3 + s_3 = 2 \checkmark \\ & x_i \geq 0, s_i \geq 0 \end{aligned}$$

II TABLEAU

$x_1$	$x_2$	$x_3$	$s_1$	$s_2$	$s_3$	$-z$	$b$
-5	-4	-4	0	0	0	-1	0
$s_1$	-4	5	-1	1	0	0	1
$s_2$	(2)	2	-1	0	1	0	2
$s_3$	0	3	4	0	0	1	0

$$R_0' = -5 + 2 \cdot 0 = 0 ; h = \frac{5}{2} ; R_0' = R_0 + \frac{5}{2} R_2 = 0 \quad 1 \quad -\frac{13}{2} \quad 0 \quad \frac{5}{2} \quad 0 \quad -1 \quad 5$$

$$R_1' = -4 + 2 \cdot 2 = 0 ; k = \frac{4}{2} = 2 ; R_1' = R_1 + 2R_2 = 0 \quad 9 \quad -3 \quad 1 \quad 2 \quad 0 \quad 0 \quad 5$$

$$R_2' = \frac{1}{2} R_1 = 1 \quad 1 \quad -\frac{1}{2} \quad 0 \quad \frac{1}{2} \quad 0 \quad 0 \quad 1$$

III TABLEAU

$x_1$	$x_2$	$x_3$	$s_1$	$s_2$	$s_3$	$-z$	$b$
0	1	$-\frac{13}{2}$	0	$\frac{5}{2}$	0	-1	5
$s_1$	0	9	-3	1	2	0	0
$x_1$	1	1	$-\frac{1}{2}$	0	$\frac{1}{2}$	0	1
$x_3$	0	3	(4)	0	0	1	0

$$R_0^{-1} = -\frac{1}{2} + \frac{1}{4}h = 0 \quad | \quad h = \frac{1}{2} \cdot \frac{1}{4} \quad | \quad R_0^{-1} = R_0 + \frac{1}{8}R_3 = 0 \quad \begin{matrix} \cancel{-1/8} \\ 0 \end{matrix} \quad 0 \quad 0 \quad \begin{matrix} 1/2 \\ \sqrt{13/8} \end{matrix}$$

$$R_1^{-1} = R_1 + \frac{3}{4}R_3 = 0 \quad \begin{matrix} 1/4 \\ 0 \end{matrix} \quad 1 \quad 2 \quad \begin{matrix} 3/4 \\ 0 \end{matrix} \quad \begin{matrix} 1/2 \\ 0 \end{matrix}$$

$$R_2^{-1} = R_2 + \frac{1}{8}R_3 = 1 \quad \begin{matrix} 11/8 \\ 0 \end{matrix} \quad 0 \quad 0 \quad 1/2 \quad \begin{matrix} 1/8 \\ 0 \end{matrix} \quad \begin{matrix} 5/4 \\ 0 \end{matrix}$$

$$R_3^{-1} = \frac{1}{h}R_3^{-1} = 0 \quad \begin{matrix} 3/4 \\ 1 \end{matrix} \quad 0 \quad 0 \quad 1/4 \quad 0 \quad 1/2$$

$$x = (x_1, x_2, x_3, s_1, s_2, s_3) = (1/4, 0, 1/2, 13/2, 0, 0)$$

$$z_{\max} = -z_{\min}$$

$$z = \frac{33}{h}$$