Neural Operator

Liyao Lyu

Problem Setting

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Deep ONet

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$$\mathcal{G}[a](y) = \sum_{k=1}^{p} \sum_{i=1}^{n} c_i^k \sigma(\sum_{j=1}^{m} w_{ij}^k u(x_j) + b_i^k) \sigma(\omega_k y + \beta)$$

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The full network is done as $\mathcal{G}[u](y) = \sum_{k=1}^{p} b_k t_k(y)$

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For frequency mode $k \in \mathbb{Z}^d$ and $v_l : \Omega \subset \mathbb{R}^d \to \mathbb{R}^{d_{v_l}}$, we have $(\mathcal{F}v_l)(k) \in \mathbb{C}^{d_{v_l}}$ and $R_\theta(k) \in \mathbb{C}^{d_{v_l} \times d_{v_{l+1}}}$

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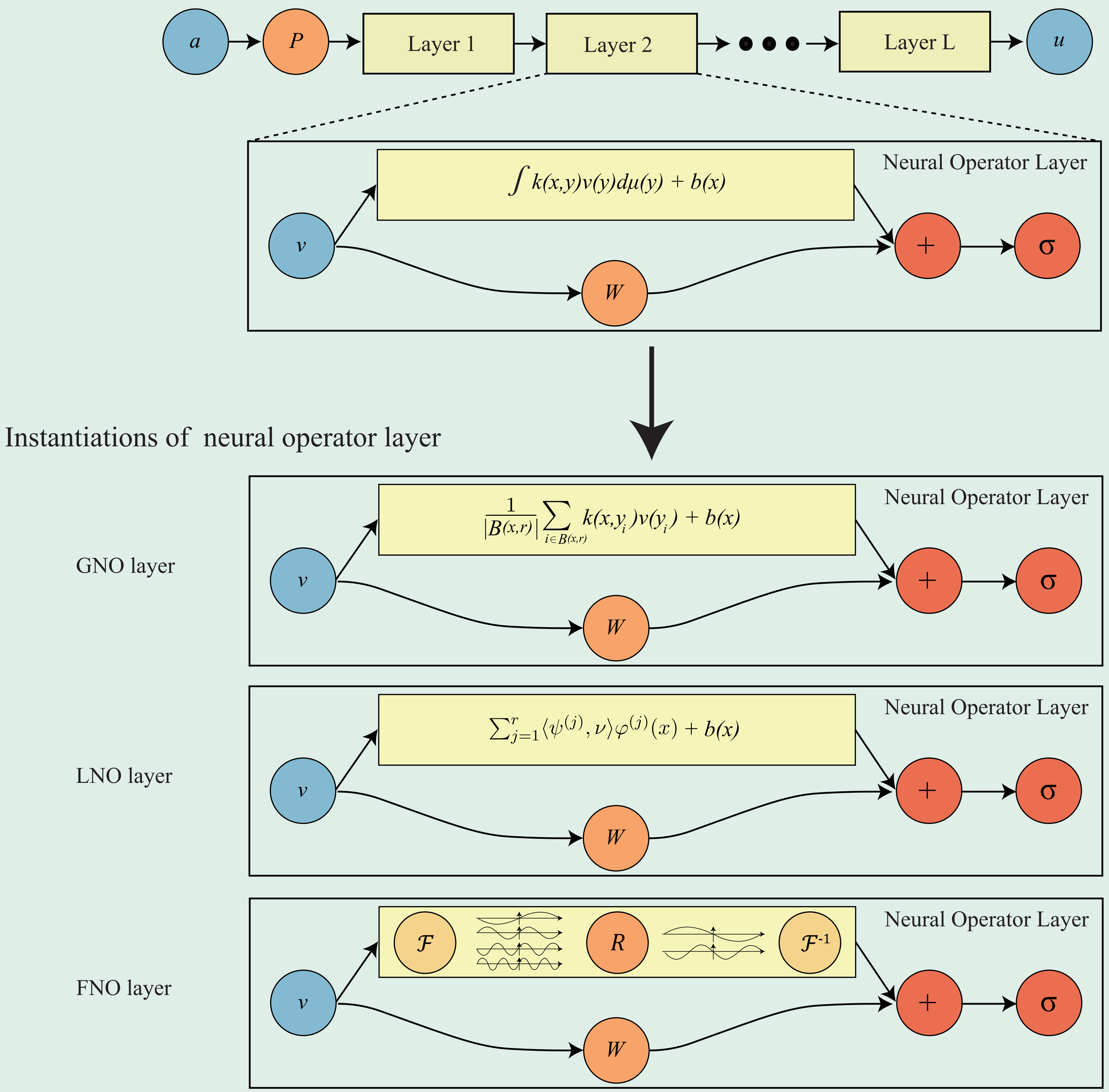

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Layer 2
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Neural Operator Layer
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Instantiations of neural operator layer
GNO layer
Neural Operator Layer
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LNO layer
Neural Operator Layer
Σ_{j=1}^r ⟨ψ^(j), ν⟩φ^(j)(x) + b(x)
FNO layer
Neural Operator Layer
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