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## 1 Foundations

### 1.1 PyMath

```
import math

math.ceil(x) #上高斯
math.floor(x) #下高斯
math.factorial(x) #階乘
math.fabs(x) #絕對值
math.fsum(arr) #求和
math.gcd(x, y)
math.exp(x) # e^x
math.log(x, base)
math.log2(x)
math.log10(x)
math.sqrt(x)
math.pow(x, y, mod)
math.sin(x) # cos, tan, asin, acos, atan,
            atan2, sinh ...
math.hypot(x, y) #歐幾里德範數
math.degrees(x) #x從弧度轉角度
math.radians(x) #x從角度轉弧度
math.gamma(x) #x的gamma函數
math.pi #const
math.e #const
math.inf
```

### 1.2 Java Integer

```
1 // 常量
2 MAX_VALUE, MIN_VALUE, BYTES, SIZE, TYPE
3
4 // 轉換/解析
5 static int parseInt(String s)
6 static int parseInt(String s, int radix)
7 static int parseUnsignedInt(String s)
8 static int parseUnsignedInt(String s, int
   radix)
9 static Integer valueOf(int i)
10 static Integer valueOf(String s)
11 static Integer valueOf(String s, int radix)
12 static String toString(int i)
13 static String toString(int i, int radix)
14 static String toUnsignedString(int i)
15 static String toUnsignedString(int i, int
   radix)
16 static long toUnsignedLong(int x)
17 static Integer decode(String nm)
   // 支援 0x/0/# 前綴
18 static Integer getInteger(String nm[, int
   val]) // 從系統屬性讀取整數
19
20 // 比較/雜湊/聚合
21 static int compare(int x, int y)
22 static int compareUnsigned(int x, int y)
23 static int hashCode(int value)
24 static int min(int a, int b)
25 static int max(int a, int b)
26 static int sum(int a, int b)
27
28 // 位元操作
29 static int bitCount(int i) // 設定位數
30 static int highestOneBit(int i)
31 static int lowestOneBit(int i)
32 static int numberOfLeadingZeros(int i)
33 static int numberOfTrailingZeros(int i)
34 static int rotateLeft(int i, int distance)
35 static int rotateRight(int i, int distance)
36 static int reverse(int i)
37 static int reverseBytes(int i)
38
39 // 無號運算
40 static int divideUnsigned(int dividend, int
   divisor)
```

```
41 static int remainderUnsigned(int dividend,
   int divisor)
```

### 1.3 Java String

```
1 // 查詢
2 int length()
3 boolean isEmpty()
4 boolean isBlank() // (since 11)
5 char charAt(int index)
6 int codePointAt(int index)
7 int codePointBefore(int index)
8 int codePointCount(int beginIndex, int
   endIndex)
9 boolean contains(CharSequence s)
10 boolean startsWith(String prefix[, int
   toffset])
11 boolean endsWith(String suffix)
12 int indexOf(String str[, int fromIndex])
13 int lastIndexOf(String str[, int
   fromIndex])
14
15 // 取子字串/子序列
16 String substring(int beginIndex)
17 String substring(int beginIndex, int
   endIndex)
18 CharSequence subSequence(int beginIndex, int
   endIndex)
19
20 // 比較/等價
21 boolean equals(Object obj)
22 boolean equalsIgnoreCase(String
   anotherString)
23 int compareTo(String anotherString)
24 int compareToIgnoreCase(String str)
25 boolean matches(String regex)
26 boolean regionMatches(int toffset, String
   other, int ooffset, int len)
27 boolean regionMatches(boolean ignoreCase,
   int toffset, String other, int ooffset,
   int len)
28
29 // 建構/轉換/連接
30 String concat(String str)
31 String replace(char oldChar, char newChar)
32 String replace(CharSequence target,
   CharSequence replacement)
33 String replaceAll(String regex, String
   replacement)
34 String replaceFirst(String regex, String
   replacement)
35 String[] split(String regex[, int limit])
36 String toLowerCase()
37 String toUpperCase()
38 String trim()
39 String strip() // (since 11)
40 String stripLeading() // (since 11)
41 String stripTrailing() // (since 11)
42 String repeat(int count) // (since 11)
43 InputStream chars()
44 Stream<String> lines() // (since 11)
45 String intern()
46
47 // 靜態工具
48 static String format(String format,
   Object... args)
49 static String join(CharSequence delimiter,
   CharSequence... elements)
50 static String join(CharSequence delimiter,
   Iterable<? extends CharSequence>
   elements)
51 static String
   valueOf(primitive/char[]/Object)
52 static String copyValueOf(char[] data[, int
   offset, int count])
```

## 1.4 Java String builder

```
1 // 長度/容量
2 int length()
3 int capacity()
4 void ensureCapacity(int minimumCapacity)
5 void trimToSize()
6 void setLength(int newLength)
7
8 // 存取/修改
9 char charAt(int index)
10 void setCharAt(int index, char ch)
11 StringBuilder append(... 各種型別 ...)
12 StringBuilder insert(int offset, ... 各種型別 ...)
13 StringBuilder delete(int start, int end)
14 StringBuilder deleteCharAt(int index)
15 StringBuilder replace(int start, int end, String str)
16 StringBuilder reverse()
17
18 // 子字串/查找
19 String substring(int start)
20 String substring(int start, int end)
21 CharSequence subSequence(int start, int end)
22 int indexOf(String str[, int fromIndex])
23 int lastIndexOf(String str[, int fromIndex])
24
25 // 轉換
26 String toString()
```

## 1.5 Java Math

```
1 // 常量
2 static final double E, PI
3
4 // 絕對值/比較
5 static int/long/float/double abs(x)
6 static T max(a, b)
7 static T min(a, b)
8
9 // 取整/四捨五入
10 static double floor(double a)
11 static double ceil(double a)
12 static double rint(double a) // 最接近整數(偶數優先)
13 static long round(double a) / int round(float a)
14 static int floorDiv(int x, int y)
15 static int floorMod(int x, int y)
16
17 // 溢位保護 (exact 系列, Java 8+)
18 static int/long addExact(a, b)
19 static int/long subtractExact(a, b)
20 static int/long multiplyExact(a, b)
21 static int/long incrementExact(a)
22 static int/long decrementExact(a)
23 static int toIntExact(long value)
24 static int/long negateExact(a)
25
26 // 指數對/冪根
27 static double pow(double a, double b)
28 static double sqrt(double a)
29 static double cbrt(double a)
30 static double exp(double a)
31 static double expm1(double x)
32 static double log(double a)
33 static double log10(double a)
34 static double log1p(double x)
35
36 // 三角/雙曲
37 static double sin/cos/tan(double a)
38 static double asin/acos/atan(double a)
39 static double atan2(double y, double x)
```

```
40 static double sinh/cosh/tanh(double a)
41
42 // 其他實用
43 static double hypot(double x, double y)
44 static double toDegrees(double angrad)
45 static double toRadians(double angdeg)
46 static double copySign(double magnitude, double sign)
47 static double nextUp/nextDown(double a)
48 static double nextAfter(double start, double direction)
49 static double ulp(double d)
50 static double random()
51 static double scalb(double d, int scaleFactor)
52 static double fma(double a, double b, double c) // (since 8)
53 static long multiplyHigh(long x, long y) // (since 9)
54 static long multiplyFull(int x, int y) // (since 9, 回傳 long)
```

# 2 Mathematics & Number Theory

## 2.1 Number Theory

Fermat’s Little Theorem:

$$a^{p-1} \equiv 1 \pmod{p} \quad (\gcd(a, p) = 1, p \text{ prime})$$

Euler’s Theorem:

$$a^{\varphi(n)} \equiv 1 \pmod{n} \quad (\gcd(a, n) = 1)$$

Modular Inverse:

$$a^{-1} \equiv a^{p-2} \pmod{p} \quad (\gcd(a, p) = 1, p \text{ prime})$$

Euler Totient:

$$\varphi(n) = n \prod_{p|n} \left(1 - \frac{1}{p}\right)$$

Fast Modular Exponentiation

```
long long mod_pow(long long a, long long b, long long mod) {
    long long res = 1 % mod;
    while (b > 0) {
        if (b & 1) res = res * a % mod;
        a = a * a % mod;
        b >>= 1;
    }
    return res;
}
```

Counting Coprimes Below  $n$

$$\forall n > 0, \quad \sum_{d|n} \varphi(d) = n, \quad \sum_{d|n} \mu(d) \left\lfloor \frac{n}{d} \right\rfloor = 1,$$

其中  $\mu$  為莫比烏斯函數；常用於反演及計算互質對數量。

### Modulo Arithmetic Quick Facts

$$(a \pm b) \bmod m = ((a \bmod m) \pm (b \bmod m)) \bmod m,$$

$$(ab) \bmod m = ((a \bmod m)(b \bmod m)) \bmod m,$$

$$(a^k) \bmod m = ((a \bmod m)^k) \bmod m,$$

$$(-a) \bmod m = (m - (a \bmod m)) \bmod m.$$

若  $\gcd(a, m) = 1$ ，可計算乘法逆元  $a^{-1}$  並套用  $(a/b) \bmod m \equiv a \cdot b^{-1} \bmod m$ 。

Extended Euclidean algorithm 給定  $a, b, c$ ，求  $ax + by = c$  的解

```
1 ll extgcd(ll a, ll b, ll &x, ll &y){
2     if(b == 0){
3         x = 1 ;
4         y = 0 ;
5         return a ;
6     }
```

```
7     ll d = extgcd(b, a%b, c, x, y), tmp =
8         x ;
9     x = y ;
10    y = tmp - (a/b)*y ;
11    return d ;
12
13    // x, y 為 ax + by = gcd(a, b) 的解
14    x = x * c / d ; // ax + by = c 的解
15    y = y * c / d ; // ax + by = c 的解
```

Modular Inverse Basics 對於  $\gcd(a, m) = 1$ ，存在唯一  $a^{-1} \in [0, m)$  使得  $a \cdot a^{-1} \equiv 1 \pmod{m}$ 。

- 用途：實作模除法、解線性同餘、組合數除法、CRT 等。
- 求法：擴展歐幾里得適用任意模數；若  $m$  為質數可用  $a^{m-2} \bmod m$ 。
- 存在條件：僅當  $\gcd(a, m) = 1$ ；不互質時可除以  $d = \gcd(a, m)$  後再檢查。

```
1 g = gcd(a, m, c, x, y) ;
2 if(g == 1){
3     inv = (x % m + m) % m ;
4 }
```

### Congruence ( $a \equiv b \pmod{m}$ ) Essentials

- $a \equiv b \pmod{m} \iff m \mid (a - b)$ ，同餘類以差整除判斷。
- $a \equiv b \pmod{m} \Rightarrow f(a) \equiv f(b) \pmod{m}$  對所有以整數係數的多項式  $f$  成立。
- 若  $a \equiv b \pmod{m}$  且  $c \equiv d \pmod{m}$ ，則  $a \pm c \equiv b \pm d \pmod{m}$ ， $ac \equiv bd \pmod{m}$ ， $a^n \equiv b^n \pmod{m}$ 。
- 若  $\gcd(k, m) = 1$  且  $ka \equiv kb \pmod{m}$ ，可約去  $k$ ： $a \equiv b \pmod{m}$ 。
- 若  $a \equiv b \pmod{m}$ ，則  $a \equiv b \pmod{d}$  對任何  $d$  整除  $m$  亦成立。
- 若  $d \mid a, b, m$ ，則  $\frac{a}{d} \equiv \frac{b}{d} \pmod{\frac{m}{d}}$

### Bézout’s Identity

若  $a, b \in \mathbb{Z}$ ， $\exists x, y \in \mathbb{Z}$  使得  $ax + by = \gcd(a, b)$ 。

任何方程  $ax + by = c$  有整數解當且僅當  $\gcd(a, b) \mid c$ 。擴展歐幾里得演算法可同時求得  $\gcd$  與一組  $(x, y)$ ，常用於計算模逆與結合 China Remainder。

根據貝祖定理，只有當  $d \mid c$  時， $ax + by = c$  才有整數解。對於  $ax + by = c$  假設有整數解，且  $\gcd(a, b) = 1$  使用  $\text{exgcd}$  求出  $(a_0, b_0)$  的特解後，根據逆模元可得出通解： $a = a_0 + \frac{b}{d}t$ ， $b = b_0 - \frac{a}{d}t$  其中  $t$  可以為任意整數

## Chinese Remainder Theorem

設同餘系統

$$x \equiv a_i \pmod{m_i} \quad (i = 1, \dots, k),$$

其中  $m_i$  兩兩互質，令  $M = \prod_{i=1}^k m_i$ ， $M_i = M/m_i$ ，再取  $t_i \equiv M_i^{-1} \pmod{m_i}$ 。則唯一解 (模  $M$ ) 為

$$x \equiv \sum_{i=1}^k a_i M_i t_i \pmod{M}.$$

## 兩式合併 (允許非互質)

```
1 // solve x a1 (mod m1), x a2 (mod m2)
2 // return {x0, lcm}; if no solution, lcm = -1
3 pair<ll, ll> crt(ll a1, ll m1,
4                ll a2, ll m2) {
5     ll g = std::gcd(m1, m2);
6     if ((a2 - a1) % g != 0) return {0, -1};
7     // no solution
8
9     ll lcm = m1 / g * m2;
10    ll m1_reduced = m1 / g;
11    ll m2_reduced = m2 / g;
```

```
11 ll diff = (a2 - a1) / g % m2_reduced;
12 if (diff < 0) diff += m2_reduced;
13
14
15 ll inv = mod_pow(m1_reduced, m2_reduced
    - 1, m2_reduced);
16 ll step = diff * inv % m2_reduced;
17 ll x0 = (a1 + step * m1) % lcm;
18 if (x0 < 0) x0 += lcm;
19 return {x0, lcm};
20 }
```

遞增地將每個同餘式與當前解做合併即可取得最終答案，也能偵測無解情況。給定  $a, b, c$ ，求  $ax + by = c$  的解

```
1 ll extgcd(ll a, ll b, ll c, ll &x, ll
    &y){
2     if(b == 0){
3         x = c/a ;
4         y = 0 ;
5         return a ;
6     }
7     ll d = extgcd(b, a%b, c, x, y), tmp =
        x ;
8     x = y ;
9     y = tmp - (a/b)*y ;
10    return d ;
11 }
```

## 2.2 Combinatorics

### Binomial Coefficient Identities

$$\binom{n}{k} = \frac{n!}{k!(n-k)!},$$
$$\binom{n}{k} = \binom{n-1}{k} + \binom{n-1}{k-1},$$
$$\sum_{k=0}^n \binom{n}{k} = 2^n, \quad \sum_{k=0}^n k \binom{n}{k} = n2^{n-1}.$$

### Stars and Bars 非負整數解數量：

$$x_1 + x_2 + \cdots + x_k = n \quad \Rightarrow \quad \binom{n+k-1}{k-1}.$$

若各變數至少為 1，將  $x_i = y_i + 1$  轉為非負情況即可。

### Inclusion-Exclusion Principle 對集合 $A_1, \dots, A_k$ ：

$$\left| \bigcup_{i=1}^k A_i \right| = \sum_{i=1}^k |A_i| - \sum_{1 \leq i < j \leq k} |A_i \cap A_j|$$
$$+ \cdots + (-1)^{k-1} |A_1 \cap \cdots \cap A_k|.$$

計算滿足限制的排列或整數解時廣泛使用。

### Catalan Numbers 基本定義：

$$C_n = \frac{1}{n+1} \binom{2n}{n}, \quad C_{n+1} = \frac{4n+2}{n+2} C_n.$$

$$C = 1, 1, 2, 5, 14, 42, 132, 429, 1430, 4862, \dots$$

常見應用包含合法括號、凸多邊形三角剖分、二元樹結構計數等。

## 2.3 Geometry

對於  $V$  個點， $E$  條邊， $F$  個面， $C$  個連通分量

$$V + F = E + 2$$
$$V + F = E + C + 1$$

點座標均是整數或是正方形格子點的簡單多邊形，其面積  $A$  和內部點數量  $i$ ，邊上格點數量  $b$  的關係為

$$A = i + \frac{b}{2} - 1$$

## 3 Data Structure

### 3.1 MST

```
1 struct Edge{
2     int u, v, w ;
3     // 這是最大生成樹，最小生成樹要改成 w < o.w
4     bool operator<(const Edge &o) const
        {return w > o.w ;} ;
5 } ;
6
7 int par[N] ;
8 int sz[N] ;
9 int sum ;
10
11 vector<Edge> edge ;
12
13 void init(){
14     edge.clear() ;
15     for ( int i=0 ; i<N ; i++ ){
16         par[i] = i ;
17         sz[i] = 1 ;
18     }
19     sum = 0 ;
20 }
21
22 int find(int x){
23     if(x == par[x]) return x ;
24     return par[x] = find(par[x]) ;
25 }
26
27 int merge(int x, int y){
28     x = find(x) ;
29     y = find(y) ;
30
31     if(x == y) return 0 ;
32     if(sz[x] > sz[y]) swap(x, y) ;
33     par[x] = y ;
34     sz[y] += sz[x] ;
35
36     return 1 ;
37 }
38
39 void MST(){
40     int cnt = 0 ;
41     for ( int i=0 ; i<edge.size() && cnt < n-1
        ; i++ ){
42         auto [u, v, w] = edge[i] ;
43         if(merge(u, v)){
44             cnt++ ;
45             sum += w ;
46         }
47     }
48 }
49
50 int main(){
51     for ( int i=0 ; i<m ; i++ ){
52         scanf("%d%d%d", &u, &v, &w) ;
53         edge.push_back({u, v, w}) ;
54         sum += w ;
55     }
56
57     sort(edge.begin(), edge.end(),
        greater<Edge>()) ;
58     MST() ;
59 }
```

### 3.2 SegmentTree

```
1 #define lc (id << 1)
2 #define rc ((id << 1) | 1)
3
4 struct LazyTag{
5     // type 0 : increase val
6     // type 1 : set to val
7     // type 1 can overwrite type 0
8 }
```

```
8 int type ;
9 ll val ;
10 } ;
11
12 struct Node{
13     LazyTag tag ;
14     ll sum ;
15     int sz ;
16 }seg[Maxn << 2] ;
17
18 class SegmentTree{
19 private:
20     void pull(int id){
21         seg[id].sum = seg[lc].sum +
            seg[rc].sum ;
22     }
23
24     void AddTag(int id, LazyTag &tag){
25         if(tag.type == 0){
26             seg[id].sum += tag.val *
                seg[id].sz ;
27             seg[id].tag.val += tag.val ;
28         }
29         else{
30             seg[id].sum = tag.val *
                seg[id].sz ;
31             seg[id].tag = {1, tag.val} ;
32         }
33     }
34
35     void push(int id){
36         AddTag(lc, seg[id].tag) ;
37         AddTag(rc, seg[id].tag) ;
38         seg[id].tag = {0, 0} ;
39     }
40
41 public:
42     void build(int L=1, int R=n, int id=1){
43         seg[id].sum = 0 ;
44         seg[id].tag = {0, 0} ;
45         seg[id].sz = 1 ;
46
47         if(L == R){
48             seg[id].sum = arr[L] ;
49             return ;
50         }
51
52         int M = (L + R) >> 1 ;
53         build(L, M, lc) ;
54         build(M+1, R, rc) ;
55
56         pull(id) ;
57         seg[id].sz = seg[lc].sz + seg[rc].sz ;
58     }
59
60     void modify(int l, int r, LazyTag &tag,
        int L=1, int R=n, int id=1){
61         if(l <= L && R <= r){
62             AddTag(id, tag) ;
63             return ;
64         }
65
66         push(id) ;
67         int M = (L + R) >> 1 ;
68         if(r <= M) modify(l, r, tag, L, M,
            lc) ;
69         else if(l > M) modify(l, r, tag, M+1,
            R, rc) ;
70         else{
71             modify(l, r, tag, L, M, lc) ;
72             modify(l, r, tag, M+1, R, rc) ;
73         }
74         pull(id) ;
75     }
76 }
```

```

77 ll query(int l, int r, int L=1, int R=n,
78         int id=1){
79     if(l <= L && R <= r) return
80         seg[id].sum ;
81     push(id) ;
82     int M = (L + R) >> 1 ;
83     if(r <= M) return query(l, r, L, M,
84                             lc) ;
85     else if(l > M) return query(l, r,
86                             M+1, R, rc) ;
87     else return query(l, r, L, M, lc) +
88         query(l, r, M+1, R, rc) ;
89 }
90 }tree ;

```

### 3.3 HLD

```

1  /* HLD */
2  int fa[Maxn], top[Maxn], son[Maxn],
3      sz[Maxn], dep[Maxn] = {0}, dfn[Maxn],
4      rnk[Maxn], dfscnt = 0 ;
5
6  void dfs1(int u, int from){
7      fa[u] = from ;
8      dep[u] = dep[from] + 1 ;
9      sz[u] = 1 ;
10
11      for ( auto v : g[u] ) if(v != from){
12          dfs1(v, u) ;
13          sz[u] += sz[v] ;
14          if(son[u] == -1 || sz[v] > sz[son[u]])
15              son[u] = v ;
16      }
17  }
18
19  void dfs2(int u, int t){
20      top[u] = t ;
21      dfn[u] = ++dfscnt ;
22      rnk[dfscnt] = u ;
23
24      if(son[u] == -1) return ;
25      dfs2(son[u], t) ;
26
27      for ( auto v : g[u] ) if(v != fa[u] && v
28          != son[u]){
29          dfs2(v, v) ;
30      }
31  }
32
33  /* Segment Tree */
34  #define lc (id << 1)
35  #define rc ((id << 1) | 1)
36
37  struct ColorSeg{
38      int left, right, tot ;
39
40      ColorSeg operator+(const ColorSeg &o)
41          const {
42          if(tot == 0) return o ;
43          if(o.tot == 0) return *this ;
44
45          ColorSeg tmp ;
46          tmp.left = left ;
47          tmp.right = o.right ;
48          tmp.tot = tot + o.tot - (right ==
49              o.left) ;
50
51          return tmp ;
52      }
53  } ;
54
55  struct Node{
56      ColorSeg color ;
57      int tag ;
58  }seg[Maxn << 2] ;

```

```

54 class SegmentTree{
55 private:
56     void pull(int id){
57         // normal pull
58     }
59
60     void AddTag(int id, int tag){
61         // normal AddTag
62     }
63
64     void push(int id){
65         // normal push
66     }
67
68     void modify(int l, int r, int tag, int
69         L=1, int R=n, int id=1){
70         // normal modify
71     }
72
73     ColorSeg query(int l, int r, int L=1, int
74         R=n, int id=1){
75         // normal query
76     }
77 public:
78     void build(int L=1, int R=n, int id=1){
79         // normal build
80     }
81
82     // update val from u to v (simple path)
83     void update(int u, int v, int val){
84         while(top[u] != top[v]){
85             if(dep[top[u]] < dep[top[v]]) swap(u,
86                 v) ;
87             modify(dfn[top[u]], dfn[u], val) ;
88             u = fa[top[u]] ;
89         }
90
91         if(dep[u] < dep[v]) swap(u, v) ;
92         modify(dfn[v], dfn[u], val) ;
93     }
94
95     // get sum from u to v (simple path)
96     int get(int u, int v){
97         pair<int, ColorSeg> U, V ;
98         ColorSeg M ;
99         U = {u, {0, 0, 0}} ;
100         V = {v, {0, 0, 0}} ;
101
102         while(top[U.first] != top[V.first]){
103             if(dep[top[U.first]] <
104                 dep[top[V.first]]) swap(U, V) ;
105             U.second = query(dfn[top[U.first]],
106                 dfn[U.first]) + U.second ;
107             U.first = fa[top[U.first]] ;
108         }
109
110         if(dep[U.first] < dep[V.first]) swap(U,
111             V) ;
112         M = query(dfn[V.first], dfn[U.first]) ;
113
114         return (U.second.tot + V.second.tot +
115             M.tot) - (U.second.left == M.right)
116             - (V.second.left == M.left) ;
117     }
118 }tree ;
119
120 void init(){
121     memset(son, -1, sizeof(son)) ;
122 }

```

### 3.4 PST

```

1  // Find range k-th largest number
2  struct Node{
3      int sum, left, right ;

```

```

4  }seg[Maxn + 20 * Maxn] ;
5
6  class PersistentSegmentTree{
7 private:
8     int n ;
9     int cnt ;
10     vector<int> version ;
11
12     int build(int L, int R){
13         int cur_cnt = cnt++ ;
14         if(L == R){
15             seg[cur_cnt] = {0, 0, 0} ;
16             return cur_cnt ;
17         }
18
19         int M = (L + R) >> 1 ;
20         int lc = build(L, M) ;
21         int rc = build(M+1, R) ;
22
23         seg[cur_cnt] = {0, lc, rc} ;
24         return cur_cnt ;
25     }
26 public:
27     PersistentSegmentTree(int _n){
28         n = _n ;
29         cnt = 0 ;
30
31         int root = build(1, n) ;
32         version.push_back(root) ;
33     }
34
35     void update(int ver, int idx){
36         auto upd = [&](auto &&self, const int
37             cur, int L, int R){
38             int cur_cnt = cnt++ ;
39
40             if(L == R){
41                 seg[cur_cnt] = {seg[cur].sum + 1, 0,
42                     0} ;
43                 return cur_cnt ;
44             }
45
46             int M = (L + R) >> 1 ;
47             int lc = seg[cur].left ;
48             int rc = seg[cur].right ;
49
50             if(idx <= M) lc = self(self,
51                 seg[cur].left, L, M) ;
52             else rc = self(self, seg[cur].right,
53                 M+1, R) ;
54
55             seg[cur_cnt] = {seg[lc].sum +
56                 seg[rc].sum, lc, rc} ;
57
58             return cur_cnt ;
59         };
60
61         int root = upd(upd, version[ver], 1, n) ;
62         version.push_back(root) ;
63     }
64
65     int query(int verL, int verR, int k){
66         auto qry = [&](auto &&self, const int
67             cur_old, const int cur_new, int L,
68             int R){
69             if(L == R) return L ;
70
71             int old_l = seg[cur_old].left, old_r =
72                 seg[cur_old].right ;
73             int new_l = seg[cur_new].left, new_r =
74                 seg[cur_new].right ;
75
76             int dl = seg[new_l].sum -
77                 seg[old_l].sum ;
78             int dr = seg[new_r].sum -
79                 seg[old_r].sum ;
80
81             int M = (L + R) >> 1 ;

```

```

71     if(dl >= k) return self(self, old_l,
72         new_l, L, M) ;
73     k -= dl ;
74     return self(self, old_r, new_r, M+1,
75         R) ;
76
77     int idx = qry(qry, version[verL-1],
78         version[verR], 1, n) ;
79     return idx ;
80 }

```

### 3.5 Trie

```

1 class TrieNode{
2 public:
3     set<int> end ;
4     TrieNode *next[26] ;
5
6     TrieNode(){
7         for ( int i=0 ; i<26 ; i++ ) next[i]
8             = nullptr ;
9     };
10
11 class Trie{
12 private:
13     int cnt ;
14     TrieNode *root ;
15 public:
16     Trie() : cnt(0) {
17         root = new TrieNode() ;
18     }
19
20     void insert(string &str, int n){
21         TrieNode* node = root ;
22         for ( auto s : str ){
23             int path = s - 'a' ;
24
25             if(node->next[path] == nullptr)
26                 node->next[path] = new
27                     TrieNode() ;
28             node = node->next[path] ;
29         }
30         node->end.insert(n) ;
31
32     void search(string &str){
33         TrieNode* node = root ;
34         for ( auto s : str ){
35             int path = s - 'a' ;
36             if(node->next[path] == nullptr)
37                 return ;
38             node = node->next[path] ;
39         }
40
41         int flg = 0 ;
42         for ( auto n : node->end ){
43             if(flg) cout << " " ;
44             else flg = 1 ;
45
46             cout << n ;
47         }
48
49     void clear(TrieNode* node) {
50         if (!node) return ;
51         for (int i = 0; i < 26; i++) {
52             if (node->next[i]) {
53                 clear(node->next[i]) ;
54             }
55         }
56         delete node ;
57     }

```

```

58 ~Trie(){
59     clear(root) ;
60 }
61 };

```

### 3.6 BIT 單修區查

```

1 // 單點修改 區間查詢
2 #define lowbit(x) (x & -x)
3
4 int BIT[MAX_SIZE] ;
5 int n ;
6
7 void modify(int idx, int val){
8     for ( ; idx <= n ; idx += lowbit(idx) ){
9         BIT[idx] += val ;
10    }
11 }
12
13 // ans: query(R) - query(L-1)
14 int query(int idx){
15     int sum = 0 ;
16     for ( ; idx ; idx -= lowbit(idx) ){
17         sum += BIT[idx] ;
18     }
19 }
20
21 void init(){
22     memset(BIT, 0, sizeof(BIT)) ;
23 }

```

### 3.7 BIT 區修單查

```

1 // 區間修改, 單點查詢
2 #define lowbit(x) (x & -x)
3
4 int BIT[MAX_SIZE] ;
5 int n ;
6
7 void modify(int idx, int val){
8     for ( ; idx <= n ; idx += lowbit(idx) ){
9         BIT[idx] += val ;
10    }
11 }
12
13 // ans: query(i)
14 int query(int idx){
15     int sum = 0 ;
16     for ( ; idx ; idx -= lowbit(idx) ){
17         sum += BIT[idx] ;
18     }
19     return sum ;
20 }
21
22 void init(){
23     memset(BIT, 0, sizeof(BIT)) ;
24 }
25
26 void build(){
27     arr[0] = 0 ;
28     for ( int i=1 ; i<=n ; i++ ) modify(i,
29         arr[i] - arr[i-1]) ;
30 }
31
32 // usage
33 // add val
34 modify(L, x) ;
35 modify(R+1, -x) ;

```

### 3.8 BIT 區修區查

```

1 // 區間修改 區間查詢
2 #define lowbit(x) x & -x
3
4 ll BIT1[MAXN], BIT2[MAXN] ;
5
6 void update(int idx, ll val){
7     for ( int i=idx ; i<=n ; i+=lowbit(i) ){
8         BIT1[i] += val ;
9         BIT2[i] += idx * val ;
10    }
11 }
12
13 // range query: query(R) - query(X - 1)
14 ll query(int idx){
15     ll sum1 = 0, sum2 = 0 ;
16     for ( int i=idx ; i ; i-=lowbit(i) ){
17         sum1 += BIT1[i] ;
18         sum2 += BIT2[i] ;
19     }
20     return (idx + 1) * sum1 - sum2 ;
21 }
22
23 void build(){
24     for (int i=1 ; i<=n ; i++){
25         update(i, arr[i] - arr[i-1]) ;
26     }
27 }
28
29 void usage(){
30     update(L, x) ;
31     update(R+1, -x) ;
32 }
33
34 void init(){
35     memset(BIT1, 0, sizeof(BIT1)) ;
36     memset(BIT2, 0, sizeof(BIT2)) ;
37 }
38 }

```

## 4 Graph

### 4.1 cut vertex AND bridges

```

1 int dfn[Maxn] = {-1}, low[Maxn] = {-1},
2     dfscnt ;
3
4 void dfs(int u, int fa){
5     dfn[u] = low[u] = ++dfscnt ;
6     int child = 0 ;
7
8     for ( auto v : g[u] ) if(v != fa){
9         if(dfn[v] == -1){
10             child++ ;
11             dfs(v, u) ;
12             low[u] = min(low[u], low[v]) ;
13
14             if(low[v] >= dfn[u]){
15                 // this edge is a bridge
16             }
17
18             if(u != fa && low[v] >= dfn[u]){
19                 // this node v is a articulation point
20             }
21         }
22         else low[u] = min(low[u], dfn[v]) ;
23     }
24
25     if(u == fa && child > 1){
26         // this node u is a articulation point
27     }
28 }

```



## 4.2 SCC - Tarjan

```

1 vector<int> scc[Maxn] ;
2 int dfn[Maxn], low[Maxn], sccId[Maxn],
   dfsCnt = 0, cnt_scc = 0 ;
3 stack<int> st ;
4 bitset<Maxn> inSt, vis ;
5
6 void dfs(int u, int from){
7     dfn[u] = low[u] = ++dfsCnt ;
8     st.push(u) ;
9     inSt[u] = 1 ;
10
11     for ( auto v : g[u] ){
12         if(!inSt[v] && dfn[v] != -1) continue ;
13         if(dfn[v] == -1) dfs(v, u) ;
14         low[u] = min(low[u], low[v]) ;
15     }
16
17     if(dfn[u] == low[u]){
18         cnt_scc++ ;
19         int x ;
20
21         do{
22             x = st.top() ;
23             st.pop() ;
24
25             inSt[x] = 0 ;
26             sccId[x] = cnt_scc ;
27             scc[cnt_scc].push_back(x) ;
28         }
29         while(x != u) ;
30     }
31 }
32
33 // SCC to DAG (after dfs)
34 vector<int> dag[Maxn] ;
35
36 void scc_to_dag(){
37     vector<int> dag[Maxn] ;
38     for ( int u=1 ; u<=n ; u++ ){
39         for ( auto v : g[u] ){
40             if(sccId[u] != sccId[v]){
41                 dag[sccId[u]].push_back(sccId[v]) ;
42             }
43         }
44     }
45 }
46
47 void init(){
48     memset(dfn, -1, sizeof(dfn)) ;
49     memset(low, -1, sizeof(low)) ;
50 }
51
52 int main(){
53     init() ;
54     input() ;
55     for ( int i=1 ; i<=n ; i++ ) if(dfn[i]
        == -1){
56         dfs(i, i) ;
57     }
58 }

```

## 4.3 BCC - Tarjan

```

1 struct Edge{
2     int v, next ;
3 }e[Maxm << 1] ;
4 int head[Maxm], tot = 1 ;
5
6 void add(int u, int v){
7     e[++tot] = {v, head[u]} ;
8     head[u] = tot ;
9     e[++tot] = {u, head[v]} ;
10    head[v] = tot ;
11 }

```

```

12 bitset<Maxm << 1> bz ;
13 vector<vector<int>> > bcc ;
14 int dfn_cnt = 0, dfn[Maxn], low[Maxn],
   vis_bcc[Maxn], bcc_cnt = 0 ;
15
16 void dfs1(int u, int from){
17     dfn[u] = low[u] = ++dfn_cnt ;
18
19     for ( int i=head[u] ; i!=-1 ; i=e[i].next
20         ){
21         int v = e[i].v ;
22
23         if(dfn[v] == -1){
24             dfs1(v, i) ;
25             if(dfn[u] < low[v]) bz[i] = bz[i^1] =
26                 1 ;
27             low[u] = min(low[u], low[v]) ;
28         }
29         else if(i != (from ^ 1)) low[u] =
30             min(low[u], dfn[v]) ;
31     }
32
33 void dfs2(int u, int id){
34     vis_bcc[u] = id ;
35     bcc[id].push_back(u) ;
36
37     for ( int i=head[u] ; i!=-1 ; i=e[i].next
38         ){
39         int v = e[i].v ;
40
41         if(vis_bcc[v] != -1 || bz[i]) continue ;
42         dfs2(v, id) ;
43     }
44 }
45
46 void init(){
47     memset(dfn, -1, sizeof(dfn)) ;
48     memset(head, -1, sizeof(head)) ;
49     memset(vis_bcc, -1, sizeof(vis_bcc)) ;
50 }
51
52 int main(){
53     init() ;
54     input() ;
55     for ( int i=1 ; i<=n ; i++ ) if(dfn[i]
56         == -1){
57         dfs1(i, 0) ;
58     }
59
60     for ( int i=1 ; i<=n ; i++ ) if(vis_bcc[i]
61         == -1){
62         bcc.push_back(vector<int>()) ;
63         dfs2(i, bcc_cnt++) ;
64     }
65 }

```

## 4.4 Convex

```

1 struct Coordinate{
2     long long x, y ;
3
4     friend bool operator<(const Coordinate&a,
5         const Coordinate&b){
6         if(a.x == b.x) return a.y < b.y ;
7         return a.x < b.x ;
8     }
9
10    friend bool operator==(const Coordinate&
11        a, const Coordinate&b){
12        return a.x == b.x && a.y == b.y ;
13    }
14 } ;
15 vector<Coordinate> nodes ;

```

```

16 long long cross(const Coordinate& o, const
   Coordinate& a, const Coordinate& b){
17     return (a.x - o.x) * (b.y - o.y) - (a.y -
   o.y) * (b.x - o.x) ;
18 }
19
20 void input(){
21     nodes.clear() ;
22
23     int n, x, y ;
24     char c ;
25     cin >> n ;
26
27     for ( int i=0 ; i<n ; i++ ){
28         cin >> x >> y >> c ;
29         if(c == 'Y') nodes.push_back({x, y}) ;
30     }
31 }
32
33 void monotone(){
34     sort(nodes.begin(), nodes.end()) ;
35
36     int n = unique(nodes.begin(), nodes.end())
37         - nodes.begin() ;
38
39     vector<Coordinate> ch(n+1) ;
40
41     int m = 0 ;
42
43     for ( int i=0 ; i<n ; i++ ){
44         while(m > 1 && cross(ch[m-2], ch[m-1],
45             nodes[i]) < 0) m-- ;
46         ch[m++] = nodes[i] ;
47     }
48
49     for ( int i=n-2, t=m ; i>=0 ; i-- ){
50         while(m > t && cross(ch[m-2], ch[m-1],
51             nodes[i]) < 0) m-- ;
52         ch[m++] = nodes[i] ;
53     }
54
55     if(n > 1) m-- ;
56     cout << m << endl ;
57
58     for ( int i=0 ; i<m ; i++ ) cout <<
59         ch[i].x << " " << ch[i].y << endl ;
60 }

```

## 4.5 Max Flow

```

1 struct Edge{
2     int v, cap, next ;
3 };
4
5 class MaxFlow{
6 private:
7     int N, S, T ;
8     vector<Edge> e ;
9     vector<int> head, cur, dep ;
10
11     bool bfs(){
12         queue<int> q ;
13         for ( int i=0 ; i<N ; i++ ){
14             cur[i] = head[i] ;
15             dep[i] = -1 ;
16         }
17
18         q.push(S) ;
19         dep[S] = 0 ;
20
21         while(!q.empty()){
22             int u = q.front() ; q.pop() ;
23
24             for ( int i=head[u] ; i!=-1 ;
25                 i=e[i].next ){
26                 int v = e[i].v ;
27                 if(dep[v] == -1 && e[i].cap > 0){
28                     dep[v] = dep[u] + 1 ;
29                 }
30             }
31         }
32     }
33 }

```

```

28     if(v == T) return 1 ;
29     q.push(v) ;
30 }
31 }
32 }
33
34 return 0 ;
35 }
36
37 int dfs(int u, int flow){
38     if(u == T) return flow ;
39     int d, rest = 0 ;
40
41     for ( int &i=cur[u] ; i!=-1 ;
42           i=e[i].next ){
43         int v = e[i].v ;
44         if(dep[v] == dep[u] + 1 && e[i].cap >
45           0){
46             d = dfs(v, min(flow - rest,
47                           e[i].cap)) ;
48
49             if(d > 0){
50                 e[i].cap -= d ;
51                 e[i^1].cap += d ;
52                 rest += d ;
53
54                 if(rest == flow) break ;
55             }
56         }
57
58         if(rest != flow) dep[u] = -1 ;
59         return rest ;
60     }
61
62     public:
63     MaxFlow(int n, int s, int t){
64         N = n ; S = s ; T = t ;
65         e.reserve(n*n) ;
66         head.assign(n+1, -1) ;
67         cur.resize(n+1) ;
68         dep.resize(n+1) ;
69     }
70
71     void AddEdge(int u, int v, int cap){
72         e.push_back({v, cap, head[u]}) ;
73         head[u] = e.size() - 1 ;
74         e.push_back({u, 0, head[v]}) ;
75         head[v] = e.size() - 1 ;
76     }
77
78     int run(){
79         int ans = 0 ;
80         while(bfs()){
81             ans += dfs(S, 0x3f3f3f3f) ;
82         }
83         return ans ;
84     }
85 }
86
87 };

```

## 4.6 min cut max flow

```

1 struct Edge{
2     int v, cap, cost, next ;
3 };
4
5 using pii = pair<int, int> ;
6 class MCMF{
7 private:
8     int N, s, t, tot ;
9     vector<Edge> e ;
10    vector<int> head ;
11 public:
12    MCMF(int n, int _s, int _t){
13        N = n ;
14        s = _s ;
15        t = _t ;
16        e.resize(n*n + 5) ;

```

```

17    head.assign(n+5, -1) ;
18    tot = -1 ;
19 }
20
21 void AddEdge(int u, int v, int cap, int
22             cost){
23     e[++tot] = {v, cap, cost, head[u]} ;
24     head[u] = tot ;
25     e[++tot] = {u, 0, -cost, head[v]} ;
26     head[v] = tot ;
27 }
28
29 int run(){
30     vector<int> dis(N+1), pot(N+1, 0),
31               preE(N+1) ;
32     int flow = 0, cost = 0 ;
33
34     auto dijkstra = [&]() {
35         fill(dis.begin(), dis.end(), INF) ;
36         priority_queue<pii, vector<pii>,
37                       greater<pii>> pq ;
38         dis[s] = 0 ;
39         pq.push({0, s}) ;
40
41         while(!pq.empty()){
42             auto [d, u] = pq.top() ; pq.pop() ;
43             if(d > dis[u]) continue ;
44             for ( int i=head[u] ; i!=-1 ;
45                   i=e[i].next ){
46                 int v = e[i].v, cap = e[i].cap, w =
47                   e[i].cost ;
48                 if(cap && dis[v] > d + w + pot[u] -
49                   pot[v]){
50                     dis[v] = d + w + pot[u] - pot[v] ;
51                     preE[v] = i ;
52                     pq.push({dis[v], v}) ;
53                 }
54             }
55         }
56
57         return dis[t] != INF ;
58     };
59
60     while(dijkstra()){
61         for ( int v=1 ; v<=N ; v++ ) if(dis[v]
62           < INF){
63             pot[v] += dis[v] ;
64         }
65
66         int aug = INT_MAX ;
67         for ( int v=t ; v!=s ;
68               v=e[preE[v]^1].v ){
69             aug = min(aug, e[preE[v]].cap) ;
70         }
71
72         for ( int v=t ; v!=s ;
73               v=e[preE[v]^1].v ){
74             e[preE[v]].cap -= aug ;
75             e[preE[v]^1].cap += aug ;
76             cost += aug * e[preE[v]].cost ;
77         }
78     }
79
80     return cost ;
81 }
82 };

```

## 5 String

### 5.1 KMP

```

1 int Next[N] ;
2 void kmp(string &str){
3     Next[0] = -1 ;
4     if(str.size() <= 1) return ;
5     Next[1] = 0 ;

```

```

6     int cur = 2, check = 0 ;
7
8     while(cur < str.size()){
9         if(str[cur - 1] == str[check])
10             Next[cur++] = ++check ;
11         else if(check > 0) check =
12             Next[check] ;
13         else Next[cur++] = 0 ;
14     }
15
16 int main(){
17     ios::sync_with_stdio(false) ;
18     cin.tie(nullptr) ;
19     cout.tie(nullptr) ;
20
21     string s1, s2 ;
22     while(cin >> s1){
23         s2 = s1 ;
24         reverse(s2.begin(), s2.end()) ;
25         kmp(s2) ;
26
27         int x=0, y=0 ;
28         while(x < s1.size() && y < s2.size()){
29             if(s1[x] == s2[y]){
30                 x++ ;
31                 y++ ;
32             }
33             else if(y > 0) y = Next[y] ;
34             else x++ ;
35         }
36
37         cout << s1 << s2.substr(y) << endl ;
38     }
39
40     return 0 ;
41 }

```

### 5.2 ACAM

```

1 class ACAutomation{
2 private:
3     vector<int> fail, end, order ;
4     vector<vector<int>> tree ;
5
6     int base, alpha ;
7
8     int new_node(){
9         tree.emplace_back(alpha, 0) ;
10        fail.push_back(0) ;
11
12        return tree.size() - 1 ;
13    }
14 public:
15    ACAutomation(int _base='a', int _alpha=26)
16        : base(_base), alpha(_alpha) {
17        clear() ;
18    }
19
20    void clear(){
21        fail.assign(1, 0) ;
22        order.clear() ;
23        end.clear() ;
24        tree.assign(1, vector<int>(alpha, 0)) ;
25    }
26
27    void add_pattern(const string &pattern){
28        int u = 0 ;
29        for ( auto ch : pattern ){
30            int v = ch - base ;
31
32            if(tree[u][v] == 0) tree[u][v] =
33                new_node() ;
34            u = tree[u][v] ;
35        }

```

```

36     end.push_back(u) ;
37 }
38
39 void build(){
40     queue<int> q ;
41     order.clear() ;
42     order.push_back(0) ;
43
44     for ( int i=0 ; i<alpha ; i++ )
45         if(tree[0][i] > 0){
46             q.push(tree[0][i]) ;
47         }
48
49     while(!q.empty()){
50         int u = q.front() ; q.pop() ;
51         order.push_back(u) ;
52
53         for ( int i=0 ; i<alpha ; i++ ){
54             if(tree[u][i] == 0) tree[u][i] =
55                 tree[fail[u]][i] ;
56             else{
57                 fail[tree[u][i]] = tree[fail[u]][i] ;
58                 q.push(tree[u][i]) ;
59             }
60         }
61     }
62
63     vector<int> count_per_pattern(const string
64         &text) const {
65         int u = 0 ;
66         vector<int> vis(tree.size(), 0) ;
67
68         for ( char ch : text ){
69             u = tree[u][ch - base] ;
70             vis[u]++ ;
71         }
72
73         for ( int i=order.size()-1 ; i>=1 ; i-- )
74             {
75                 int x = order[i] ;
76                 vis[fail[x]] += vis[x] ;
77             }
78
79         vector<int> ans(end.size(), 0) ;
80         for ( int id=0 ; id<end.size() ; id++ ){
81             ans[id] = vis[end[id]] ;
82         }
83     };

```

## 6 Techniques

### 6.1 二分搜

```

1 // xxxxxxxx 找最小解
2 bool binary_search(){
3     while(1 < r){
4         int m = (l + r) >> 1 ;
5         if(check(m)) r = m ;
6         else l = m + 1 ;
7     }
8
9     return l ;
10 }
11
12 // oooooxxx 找最大解
13 bool binary_search(){
14     while(1 < r){
15         int m = (l + r) >> 1 ;
16         if(check(m)) l = m ;
17         else r = m - 1 ;
18     }

```

```

19     return l ;
20 }
21
22 // 如果 l & r 太大, m = (l + (r - 1)) >> 1 ;
23

```

### 6.2 倍增 LCA

```

1 #include <bits/stdc++.h>
2
3 using namespace std ;
4
5 const int Maxn = 500005 ;
6
7 vector<int> e[Maxn] ;
8 int depth[Maxn] ;
9 int up[Maxn][40] ;
10 int MaxLog ;
11
12 void dfs(int u, int from, int d){
13     up[u][0] = from ;
14     depth[u] = d ;
15
16     for ( int i=1 ; i<=MaxLog ; i++ ){
17         up[u][i] = up[up[u][i-1]][i-1] ;
18     }
19
20     for ( auto v : e[u] ){
21         if(v == from) continue ;
22         dfs(v, u, d + 1) ;
23     }
24 }
25
26 int lca(int u, int v){
27     if(depth[u] < depth[v]) swap(u, v) ;
28
29     for ( int i=MaxLog ; i>=0 ; i-- )
30         if(depth[u] - (1 << i) >= depth[v]){
31             u = up[u][i] ;
32         }
33
34     if(u == v) return u ;
35
36     for ( int i=MaxLog ; i>=0 ; i-- )
37         if(up[u][i] != up[v][i]){
38             u = up[u][i] ;
39             v = up[v][i] ;
40         }
41
42     return up[u][0] ;
43 }
44
45 int main(){
46     int n, q, root ;
47     scanf("%d%d", &n, &q, &root) ;
48     MaxLog = __lg(n) ;
49
50     for ( int i=0 ; i<n-1 ; i++ ){
51         int u, v ;
52         scanf("%d%d", &u, &v) ;
53         e[u].push_back(v) ;
54         e[v].push_back(u) ;
55     }
56
57     dfs(root, root, 0) ;
58
59     while(q--){
60         int u, v ;
61         scanf("%d%d", &u, &v) ;
62         printf("%d\n", lca(u, v)) ;
63     }
64
65     return 0 ;
66 }

```

### 6.3 SG

```

1 long long SG(long long k){
2
3     if(k % 2 == 0){
4         return k / 2 ;
5     }
6     else{
7         return SG(k / 2) ;
8     }
9 }
10
11 int main(){
12     int cas, n ;
13
14     scanf("%d", &cas) ;
15     while(cas--){
16         scanf("%d", &n) ;
17
18         long long s, v = 0 ;
19
20         for(int i = 0 ; i < n ; i++){
21             scanf("%lld", &s) ;
22             v ^= SG(s) ; //XOR
23         }
24
25         if(v) printf("YES\n") ;
26         else printf("NO\n") ;
27     }
28 }
29
30 int SG[30] ;
31 int vis[Maxn], stone[Maxn] ;
32
33 void build(){
34     SG[0] = 0 ;
35     memset(vis, 0, sizeof(vis)) ;
36
37     for ( int i=1 ; i<30 ; i++ ){
38         int cur = 0 ;
39         for ( int j=0 ; j<i ; j++ ) for ( int
40             k=0 ; k<=j ; k++ ){
41             vis[SG[j] ^ SG[k]] = i ;
42         }
43         while(vis[cur] == i) cur++ ;
44         SG[i] = cur ;
45     }
46 }
47
48 int main(){
49     build() ;
50
51     int T = 0 ;
52     while(~scanf("%d", &n) && n){
53         int ans = 0 ;
54
55         for ( int i=1 ; i<=n ; i++ ) scanf("%d",
56             &stone[i]) ;
57
58         for ( int i=1 ; i<=n ; i++ ) if(stone[i]
59             & 1){
60             ans ^= SG[n-i] ;
61         }
62     }

```

## 7 DP

### 7.1 輪廓線 DP

```

1 #include <bits/stdc++.h>
2
3 using namespace std ;
4 using ll = long long ;
5

```



```

6 ll dp[2][(1 << 10) + 5] ;
7 int n, m ;
8 int cur ;
9
10 void update(int s1, int s2){
11     if(s2 & (1 << m)){
12         dp[cur][s2 ^ (1 << m)] += dp[cur ^
13             1][s1] ;
14     }
15 }
16 int main(){
17     while(~scanf("%d%d", &n, &m)){
18         if(m > n) swap(n, m) ;
19         memset(dp, 0, sizeof(dp)) ;
20         cur = 0 ;
21         dp[cur][(1 << m) - 1] = 1 ;
22         for ( int i=0 ; i<n ; i++ ) for ( int
23             j=0 ; j<m ; j++ ){
24             cur ^= 1 ;
25             memset(dp[cur], 0, sizeof(dp[cur])) ;
26
27             for ( int k=0 ; k<(1 << m) ; k++ ){
28                 update(k, k << 1) ; // not put
29                 if(i && !(k & (1 << (m - 1))))
30                     update(k, (k << 1) | (1 << m) |
31                         1) ; // put up
32                 if(j && !(k & 1)) update(k, (k << 1)
33                     | 3) ; // put left
34             }
35         }
36         printf("%lld\n", dp[cur][(1 << m) - 1]) ;
37     }
38     return 0 ;
39 }

```

## 7.2 數位 DP

```

1 #include <bits/stdc++.h>
2
3 using namespace std ;
4
5 int K ;
6 int dp[20][105][105][2] ;
7 vector<int> dig ;
8
9 int solve(int pos, int sum, int dsum, bool
10     lim){
11     if(pos == -1){
12         if(sum == 0 && dsum == 0) return 1 ;
13         return 0 ;
14     }
15     int &d = dp[pos][sum][dsum][lim] ;
16     if(d != -1) return d ;
17
18     int up = lim ? dig[pos] : 9 ;
19     int res = 0 ;
20     for ( int i=0 ; i<=up ; i++ ){
21         res += solve(pos-1, (sum * 10 + i) %
22             K, (dsum + i) % K, lim && i==up)
23             ;
24     }
25     return d = res ;
26 }
27
28 int count(int n){
29     memset(dp, -1, sizeof(dp)) ;
30     dig.clear() ;
31
32     while(n > 0){
33         dig.push_back(n % 10) ;
34         n /= 10 ;
35     }
36
37     return solve(dig.size() - 1, 0, 0, 1) ;
38 }

```

```

37 }
38
39 int main(){
40     int T ;
41     scanf("%d", &T) ;
42
43     int a, b ;
44     while(T--){
45         scanf("%d%d", &a, &b, &K) ;
46         if(K > 90) printf("0\n") ;
47         else printf("%d\n", count(b) -
48             count(a-1)) ;
49     }
50     return 0 ;
51 }

```

## 7.3 樹 DP

```

1 #include <bits/stdc++.h>
2
3 #define N 505
4 #define INF 0x3f3f3f3f
5
6 using namespace std ;
7
8 struct Edge{
9     int v, w ;
10 } ;
11
12 vector<Edge> edge[N] ;
13 int n ;
14 int cnt[N] ;
15 int dp[N][N][2] ;
16
17 void init(){
18     for ( int i=0 ; i<N ; i++ )
19         edge[i].clear() ;
20     memset(cnt, 0, sizeof(cnt)) ;
21     memset(dp, INF, sizeof(dp)) ;
22 }
23
24 void DFS(int u){
25     cnt[u] = 1 ;
26     for ( auto [v, w] : edge[u] ){
27         DFS(v) ;
28         cnt[u] += cnt[v] ;
29
30         dp[u][1][0] = dp[u][1][1] = 0 ;
31
32         for ( auto [v, w] : edge[u] ){
33             for ( int i=cnt[u] ; i>1 ; i-- ) for (
34                 int j=1 ; j<i && j<=cnt[v] ; j++ ){
35                 dp[u][i][1] = min(dp[u][i][1],
36                     dp[u][i-j][1] + dp[v][j][1] + 2 *
37                     w) ;
38                 dp[u][i][0] = min(dp[u][i][0],
39                     dp[u][i-j][1] + dp[v][j][0] + w) ;
40                 dp[u][i][0] = min(dp[u][i][0],
41                     dp[u][i-j][0] + dp[v][j][1] + 2 *
42                     w) ;
43             }
44         }
45     }
46 }
47
48 int main(){
49     int t = 0 ;
50
51     while(~scanf("%d", &n) && n){
52         init() ;
53         for ( int i=0 ; i<n-1 ; i++ ){
54             int u, v, w ;
55             scanf("%d%d%d", &u, &v, &w) ;
56             edge[u].push_back({v, w}) ;
57         }
58     }
59 }

```

```

52 DFS(0) ;
53 printf("Case %d:\n", ++t) ;
54
55 int q, e ;
56 scanf("%d", &q) ;
57
58 while(q--){
59     scanf("%d", &e) ;
60
61     for ( int i=n ; i>=1 ; i-- )
62         if(dp[0][i][0] <= e){
63             printf("%d\n", i) ;
64             break ;
65         }
66     }
67
68     return 0 ;
69 }

```