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## 1 Basic

### 1.1 PyMath

```

1 import math
2
3     math.ceil(x) #上高斯
4     math.floor(x) #下高斯
5     math.factorial(x) #階乘
6     math.fabs(x) #絕對值
7     math.fsum(arr) #求和
8     math.gcd(x, y)
9     math.exp(x) # e^x
10    math.log(x, base)
11    math.log2(x)
12    math.log10(x)
13    math.sqrt(x)
14    math.pow(x, y, mod)
15    math.sin(x) # cos, tan, asin, acos, atan,
16      atan2, sinh ...
17    math.hypot(x, y) #歐幾里得範數
18    math.degrees(x) #x從弧度轉角度
19    math.radians(x) #x從角度轉弧度
20    math.gamma(x) #x的gamma函數
21    math.pi #const
22    math.e #const
23    math.inf

```

### 1.2 Java Integer

```

1 // 常量
2 MAX_VALUE, MIN_VALUE, BYTES, SIZE, TYPE
3
4 // 轉換/解析
5 static int parseInt(String s)
6 static int parseInt(String s, int radix)
7 static int parseUnsignedInt(String s)
8 static int parseUnsignedInt(String s, int
      radix)
9 static Integer valueOf(int i)
10 static Integer valueOf(String s)
11 static Integer valueOf(String s, int radix)
12 static String toString(int i)
13 static String toString(int i, int radix)
14 static String toUnsignedString(int i)
15 static String toUnsignedString(int i, int
      radix)
16 static long toUnsignedLong(int x)
17 static Integer decode(String nm)
      // 支援 0x/0/# 前綴
18 static Integer getInteger(String nm[, int
      val]) // 從系統屬性讀取整數
19
20 // 比較/雜湊/聚合
21 static int compare(int x, int y)
22 static int compareUnsigned(int x, int y)
23 static int hashCode(int value)
24 static int min(int a, int b)
25 static int max(int a, int b)
26 static int sum(int a, int b)
27
28 // 位元操作
29 static int bitCount(int i) // 設定位數
30 static int highestOneBit(int i)
31 static int lowestOneBit(int i)
32 static int numberOfLeadingZeros(int i)
33 static int numberOfTrailingZeros(int i)
34 static int rotateLeft(int i, int distance)
35 static int rotateRight(int i, int distance)
36 static int reverse(int i)
37 static int reverseBytes(int i)
38
39 // 無號運算
40 static int divideUnsigned(int dividend, int
      divisor)

```

```

41 static int remainderUnsigned(int dividend,
      int divisor)

```

### 1.3 Java String

```

1 // 查詢
2 int length()
3 boolean isEmpty()
4 boolean isBlank() // (since 11)
5 char charAt(int index)
6 int codePointAt(int index)
7 int codePointBefore(int index)
8 int codePointCount(int beginIndex, int
      endIndex)
9 boolean contains(CharSequence s)
10 boolean startsWith(String prefix[, int
      toffset])
11 boolean endsWith(String suffix)
12 int indexOf(String str[, int fromIndex])
13 int lastIndexOf(String str[, int
      fromIndex])
14
15 // 取子字串/子序列
16 String substring(int beginIndex)
17 String substring(int beginIndex, int
      endIndex)
18 CharSequence subSequence(int beginIndex, int
      endIndex)
19
20 // 比較/等價
21 boolean equals(Object obj)
22 boolean equalsIgnoreCase(String
      anotherString)
23 int compareTo(String anotherString)
24 int compareToIgnoreCase(String str)
25 boolean matches(String regex)
26 boolean regionMatches(int toffset, String
      other, int offset, int len)
27 boolean regionMatches(boolean ignoreCase,
      int toffset, String other, int offset,
      int len)
28
29 // 建構/轉換/連接
30 String concat(String str)
31 String replace(char oldChar, char newChar)
32 String replace(CharSequence target,
      CharSequence replacement)
33 String replaceAll(String regex, String
      replacement)
34 String replaceFirst(String regex, String
      replacement)
35 String[] split(String regex[, int limit])
36 String toLowerCase()
37 String toUpperCase()
38 String trim()
39 String strip() // (since 11)
40 String stripLeading() // (since 11)
41 String stripTrailing() // (since 11)
42 String repeat(int count) // (since 11)
43 IntStream chars()
44 Stream<String> lines() // (since 11)
45 String intern()
46
47 // 靜態工具
48 static String format(String format,
      Object... args)
49 static String join(CharSequence delimiter,
      CharSequence... elements)
50 static String join(CharSequence delimiter,
      Iterable<? extends CharSequence>
      elements)
51 static String
      valueOf(primitive/char[]/Object)
52 static String copyValueOf(char[] data[, int
      offset, int count])

```

## 1.4 Java String builder

```
1 // 長度 / 容量
2 int length()
3 int capacity()
4 void ensureCapacity(int minimumCapacity)
5 void trimToSize()
6 void setLength(int newLength)
7
8 // 存取 / 修改
9 char      charAt(int index)
10 void     setCharAt(int index, char ch)
11 StringBuilder append(... 各種型別 ...)
12 StringBuilder insert(int offset, ... 各種型別 ...
   ...)
13 StringBuilder delete(int start, int end)
14 StringBuilder deleteCharAt(int index)
15 StringBuilder replace(int start, int end,
   String str)
16 StringBuilder reverse()
17
18 // 子串 / 查找
19 String      substring(int start)
20 String      substring(int start, int end)
21 CharSequence subSequence(int start, int end)
22 int        indexOf(String str[, int
   fromIndex])
23 int        lastIndexOf(String str[, int
   fromIndex])
24
25 // 轉換
26 String toString()
```

## 1.5 Java Math

```
1 // 常量
2 static final double E, PI
3
4 // 絶對值/比較
5 static int/long/float,double abs(x)
6 static T max(a, b)
7 static T min(a, b)
8
9 // 取整/四捨五入
10 static double floor(double a)
11 static double ceil(double a)
12 static double rint(double a)
13 // 最接近整數(偶數優先)
14 static long round(double a) / int
15 // round(float a)
16 static int floorDiv(int x, int y)
17 static int floorMod(int x, int y)
18
19 // 溢位保護(exact 系列, Java 8+)
20 static int/long addExact(a, b)
21 static int/long subtractExact(a, b)
22 static int/long multiplyExact(a, b)
23 static int/long incrementExact(a)
24 static int/long decrementExact(a)
25 static int/int long value)
26 static int/int long negateExact(a)
27
28 // 指對數/幕根
29 static double pow(double a, double b)
30 static double sqrt(double a)
31 static double cbrt(double a)
32 static double exp(double a)
33 static double expm1(double x)
34 static double log(double a)
35 static double log10(double a)
36 static double log1p(double x)
37
38 // 三角/雙曲
39 static double sin/cos/tan(double a)
40 static double asin/acos/atan(double
41 static double atan2(double y, double
```

```

40 static double sinh/cosh/tanh(double a)    // type 1 can overwrite type 0
41 // 其他實用
42 static double hypot(double x, double y)
43 static double toDegrees(double angrad)
44 static double toRadians(double angdeg)
45 static double copySign(double magnitude,
46                         double sign)
47 static double nextUp/nextDown(double a)
48 static double nextAfter(double start, double
49                         direction)
50 static double ulp(double d)
51 static double random()
52 static double scalb(double d, int
53                         scaleFactor)
54 static double fma(double a, double b, double
55                         c) // (since 8)
56 static long multiplyHigh(long x, long y)
57                         // (since 9)
58 static long multiplyFull(int x, int y)
59                         // (since 9, 回傳 long)

2 Math
2.1 formula
1. Catalan Number

$$C_n = \frac{1}{n} \binom{2n}{n}, C_{n+1} = \frac{2(2n+1)}{n+2} C_n$$


$$C = 1, 1, 2, 5, 14, 42, 132, 429, 1430, 4862, \dots$$

2. Euler's Formula
對於 v 個點, e 條邊, f 個面, c 個連通分量

$$V + F = E + 2$$


$$V + F = E + C + 1$$

3. Pick's Theorem
點座標均是整數或是正方形格子點的簡單多邊形, 其面積 A 和內部點數量 i, 邊上格點數量 b 的關係為

$$A = i + \frac{b}{2} - 1$$

2.2 extended gcd
給定 a, b, c, 求  $ax + by = c$  的解
1 ll extgcd(ll a, ll b, ll c, ll &x, ll
2             &y){
3     if(b == 0){
4         x = c/a;
5         y = 0;
6         return a;
7     }
8     ll d = extgcd(b, a%b, c, x, y), tmp =
9             x;
10    x = y;
11    y = tmp - (a/b)*y;
12    return d;
13 }

3 Tree
3.1 SegmentTree
#define lc (id << 1)
#define rc ((id << 1) | 1)
struct LazyTag{
    // type 0 : increase val
    // type 1 : set to val
    int type;
    ll val;
} seg[Maxn << 2];
class SegmentTree{
private:
    void pull(int id){
        seg[id].sum = seg[lc].sum +
                      seg[rc].sum;
    }
    void AddTag(int id, LazyTag &tag){
        if(tag.type == 0){
            seg[id].sum += tag.val *
                           seg[id].sz;
            seg[id].tag.val += tag.val;
        } else{
            seg[id].sum = tag.val *
                           seg[id].sz;
            seg[id].tag = {1, tag.val};
        }
    }
    void push(int id){
        AddTag(lc, seg[id].tag);
        AddTag(rc, seg[id].tag);
        seg[id].tag = {0, 0};
    }
public:
    void build(int L=1, int R=n, int id=1){
        seg[id].sum = 0;
        seg[id].tag = {0, 0};
        seg[id].sz = 1;
        if(L == R){
            seg[id].sum = arr[L];
            return;
        }
        int M = (L + R) >> 1;
        build(L, M, lc);
        build(M+1, R, rc);

        pull(id);
        seg[id].sz = seg[lc].sz + seg[rc].sz;
    }
    void modify(int l, int r, LazyTag &tag,
               int L=1, int R=n, int id=1){
        if(l <= L && R <= r){
            AddTag(id, tag);
            return;
        }

        push(id);
        int M = (L + R) >> 1;
        if(r <= M) modify(l, r, tag, L, M,
                           lc);
        else if(l > M) modify(l, r, tag, M+1,
                               R, rc);
        else{
            modify(l, r, tag, L, M, lc);
            modify(l, r, tag, M+1, R, rc);
        }
        pull(id);
    }
    ll query(int l, int r, int L=1, int R=n,
             int id=1){
}
}

```

```

78     if(l <= L && R <= r) return
79         seg[id].sum ;
80
81     push(id) ;
82     int M = (L + R) >> 1 ;
83     if(r <= M) return query(l, r, L, M,
84                               lc) ;
85     else if(l > M) return query(l, r,
86                                  M+1, R, rc) ;
87     else return query(l, r, L, M, lc) +
88             query(l, r, M+1, R, rc) ;
89
90 }tree :

```

### 3.2 HLD

```

56 private:
57     void pull(int id){
58         // normal pull
59     }
60
61     void AddTag(int id, int tag){
62         // normal AddTag
63     }
64
65     void push(int id){
66         // normal push
67     }
68
69     void modify(int l, int r, int tag, int
70                 L=1, int R=n, int id=1){
71         // normal modify
72     }
73
74     ColorSeg query(int l, int r, int L=1, in
75                     R=n, int id=1){
76         // normal query
77     }
78 public:
79     void build(int L=1, int R=n, int id=1){
80         // normal build
81     }
82
83     // update val from u to v (simple path)
84     void update(int u, int v, int val){
85         while(top[u] != top[v]){
86             if(dep[top[u]] < dep[top[v]]) swap(u,
87                                         v) ;
88             modify(dfn[top[u]], dfn[u], val) ;
89             u = fa[top[u]] ;
90         }
91
92         if(dep[u] < dep[v]) swap(u, v) ;
93         modify(dfn[v], dfn[u], val) ;
94     }
95
96     // get sum from u to v (simple path)
97     int get(int u, int v){
98         pair<int, ColorSeg> U, V ;
99         ColorSeg M ;
100        U = {u, {0, 0, 0}} ;
101        V = {v, {0, 0, 0}} ;
102
103        while(top[U.first] != top[V.first]){
104            if(dep[top[U.first]] <
105               dep[top[V.first]]) swap(U, V) ;
106            U.second = query(dfn[top[U.first]],
107                             dfn[U.first]) + U.second ;
108            U.first = fa[top[U.first]] ;
109        }
110
111        if(dep[U.first] < dep[V.first]) swap(U,
112                                              V) ;
113
114        M = query(dfn[V.first], dfn[U.first])
115
116        return (U.second.tot + V.second.tot +
117                M.tot) - (U.second.left == M.right
118                           - (V.second.left == M.left) ;
119    }
120 }tree ;
121
122 void init(){
123     memset(son, -1, sizeof(son)) ;
124 }

```

3 3 PST

```

6 class PersistentSegmentTree{
7 private:
8     int n ;
9     int cnt ;
10    vector<int> version ;
11
12    int build(int L, int R){
13        int cur_cnt = cnt++ ;
14        if(L == R){
15            seg[cur_cnt] = {0, 0, 0} ;
16            return cur_cnt ;
17        }
18
19        int M = (L + R) >> 1 ;
20        int lc = build(L, M) ;
21        int rc = build(M+1, R) ;
22
23        seg[cur_cnt] = {0, lc, rc} ;
24        return cur_cnt ;
25    }
26 public:
27    PersistentSegmentTree(int _n){
28        n = _n ;
29        cnt = 0 ;
30
31        int root = build(1, n) ;
32        version.push_back(root) ;
33    }
34
35    void update(int ver, int idx){
36        auto upd = [&](auto &&self, const int
37                        cur, int L, int R){
38            int cur_cnt = cnt++ ;
39
40            if(L == R){
41                seg[cur_cnt] = {seg[cur].sum + 1, 0,
42                                0} ;
43                return cur_cnt ;
44            }
45
46            int M = (L + R) >> 1 ;
47            int lc = seg[cur].left ;
48            int rc = seg[cur].right ;
49
50            if(idx <= M) lc = self(self,
51                                     seg[cur].left, L, M) ;
52            else rc = self(self, seg[cur].right,
53                           M+1, R) ;
54
55            seg[cur_cnt] = {seg[lc].sum +
56                            seg[rc].sum, lc, rc} ;
57
58            return cur_cnt ;
59        };
60
61        int root = upd(upd, version[ver], 1, n) ;
62        version.push_back(root) ;
63    }
64
65    int query(int verL, int verR, int k){
66        auto qry = [&](auto &&self, const int
67                        cur_old, const int cur_new, int L,
68                        int R){
69            if(L == R) return L ;
70
71            int old_l = seg[cur_old].left, old_r =
72                        seg[cur_old].right ;
73            int new_l = seg[cur_new].left, new_r =
74                        seg[cur_new].right ;
75
76            int dl = seg[new_l].sum -
77                     seg[old_l].sum ;
78            int dr = seg[new_r].sum -
79                     seg[old_r].sum ;
80
81            int M = (L + R) >> 1 ;
82
83            if(k <= M) return dl + self(self,
84                                         seg[new_l].left, L, M) ;
85            else return dr + self(self, seg[new_r].right,
86                                  M+1, R) ;
87        };
88
89        int res = qry(qry, version[verL], 1, n) ;
90        for(int i = verL + 1; i < verR; i++)
91            res = qry(res, version[i], 1, n) ;
92
93        return res ;
94    }
95
96    void print(){
97        cout << "version: " << version.size() << endl ;
98        cout << "seg: " << seg.size() << endl ;
99    }
100}
```

```
72     if(dl >= k) return self(self, old_l,  
73         new_l, L, M) ;  
74     k -= dl ;  
75     return self(self, old_r, new_r, M+1,  
76         R) ;  
77 } ;  
78  
79     int idx = qry(qry, version[verL-1],  
80         version[verR], 1, n) ;  
81     return idx ;  
82 } ;
```

4 Graph

#### 4.1 cut vertex AND bridges

```

1 int dfn[Maxn] = {-1}, low[Maxn] = {-1},
2     dfscnt ;
3
4 void dfs(int u, int fa){
5     dfn[u] = low[u] = ++dfscnt ;
6     int child = 0 ;
7
8     for ( auto v : g[u] ) if(v != fa){
9         if(dfn[v] == -1){
10             child++ ;
11             dfs(v, u) ;
12             low[u] = min(low[u], low[v]) ;
13
14             if(low[v] >= dfn[u]){
15                 // this edge is a bridge
16             }
17
18             if(u != fa && low[v] >= dfn[u]){
19                 // this node v is a articulation point
20             }
21             else low[u] = min(low[u], dfn[v]) ;
22         }
23     }
24     if(u == fa && child > 1){
25         // this node u is a articulation point
26     }
27 }
```

## 4.2 SCC - Tarjan

```

1 vector<int> scc[Maxn] ;
2 int dfn[Maxn], low[Maxn], sccId[Maxn],
   dfscnt = 0, cnt_scc = 0 ;
3 stack<int> st ;
4 bitset<Maxn> inSt, vis ;
5
6 void dfs(int u, int from){
7   dfn[u] = low[u] = ++dfscnt ;
8   st.push(u) ;
9   inSt[u] = 1 ;
10
11  for ( auto v : g[u] ){
12    if(!inSt[v] && dfn[v] != -1) continue
13    if(dfn[v] == -1) dfs(v, u) ;
14    low[u] = min(low[u], low[v]) ;
15  }
16
17  if(dfn[u] == low[u]){
18    cnt_scc++ ;
19    int x ;
20
21    do{
22      x = st.top() ;
23      st.pop() ;
24
25      inSt[x] = 0 ;

```

```

26     sccId[x] = cnt_scc ;
27     scc[cnt_scc].push_back(x) ;
28 }
29 while(x != u) ;
30 }
31 }
32
33 // SCC to DAG (after dfs)
34 vector<int> dag[Maxn] ;
35
36 void scc_to_dag(){
37     vector<int> dag[Maxn] ;
38     for ( int u=1 ; u<=n ; u++ ){
39         for ( auto v : g[u] ){
40             if(sccId[u] != sccId[v]){
41                 dag[sccId[u]].push_back(sccId[v])
42             }
43         }
44     }
45 }
46
47 void init(){
48     memset(dfn, -1, sizeof(dfn)) ;
49     memset(low, -1, sizeof(low)) ;
50 }
51
52 int main(){
53     init() ;
54     input() ;
55     for ( int i=1 ; i<=n ; i++ ) if(dfn[i]
56                                     == -1){
57         dfs(i, i) ;
58     }

```

### 4.3 BCC - Tarjam

```

1 struct Edge{
2     int v, next ;
3 }e[Maxm << 1] ;
4 int head[Maxm], tot = 1 ;
5
6 void add(int u, int v){
7     e[++tot] = {v, head[u]} ;
8     head[u] = tot ;
9     e[++tot] = {u, head[v]} ;
10    head[v] = tot ;
11 }
12
13 bitset<Maxm << 1> bz ;
14 vector<vector<int>> bcc ;
15 int dfn_cnt = 0, dfn[Maxn], low[Maxn],
16     vis_bcc[Maxn], bcc_cnt = 0 ;
17
18 void dfs1(int u, int from){
19     dfn[u] = low[u] = ++dfn_cnt ;
20
21     for ( int i=head[u] ; i!=-1 ; i=e[i].next )
22     {
23         int v = e[i].v ;
24
25         if(dfn[v] == -1){
26             dfs1(v, i) ;
27             if(dfn[u] < low[v]) bz[i] = bz[i^1]
28                         1 ;
29             low[u] = min(low[u], low[v]) ;
30         }
31         else if(i != (from ^ 1)) low[u] =
32             min(low[u], dfn[v]) ;
33     }
34 }
35
36 void dfs2(int u, int id){
37     vis_bcc[u] = id ;
38     bcc[id].push_back(u) ;
39 }

```

```

36     for ( int i=head[u] ; i!=-1 ; i=e[i].next
37         ){
38         int v = e[i].v ;
39
40         if(vis_bcc[v] != -1 || bz[i]) continue ;
41         dfs2(v, id) ;
42     }
43 }
44 void init(){
45     memset(dfn, -1, sizeof(dfn)) ;
46     memset(head, -1, sizeof(head)) ;
47     memset(vis_bcc, -1, sizeof(vis_bcc)) ;
48 }
49
50 int main(){
51     init() ;
52     input() ;
53     for ( int i=1 ; i<=n ; i++ ) if(dfn[i]
54         == -1){
55         dfs1(i, 0) ;
56     }
57     for ( int i=1 ; i<=n ; i++ ) if(vis_bcc[i]
58         == -1){
59         bcc.push_back(vector<int>()) ;
60         dfs2(i, bcc_cnt++) ;
61     }
62 }
```

## 4.4 Convex

```

1 struct Coordinate{
2     long long x, y ;
3
4     friend bool operator<(const Coordinate&a,
5         const Coordinate& b){
6         if(a.x == b.x) return a.y < b.y ;
7         return a.x < b.x ;
8     }
9
10    friend bool operator==(const Coordinate&
11        a, const Coordinate& b){
12        return a.x == b.x && a.y == b.y ;
13    }
14 } ;
15
16 vector<Coordinate> nodes ;
17
18 long long cross(const Coordinate& o, const
19     Coordinate& a, const Coordinate& b){
20     return (a.x - o.x) * (b.y - o.y) - (a.y -
21         o.y) * (b.x - o.x) ;
22 }
23
24 void input(){
25     nodes.clear() ;
26
27     int n, x, y ;
28     char c ;
29     cin >> n ;
30
31     for ( int i=0 ; i<n ; i++ ){
32         cin >> x >> y >> c ;
33         if(c == 'Y') nodes.push_back({x, y}) ;
34     }
35 }
36
37 void monotone(){
38     sort(nodes.begin(), nodes.end()) ;
39
40     int n = unique(nodes.begin(), nodes.end())
41             - nodes.begin() ;
42
43     vector<Coordinate> ch(n+1) ;
44
45     int m = 0 ;

```

```

for ( int i=0 ; i<n ; i++ ){
    while(m > 1 && cross(ch[m-2], ch[m-1],
        nodes[i]) < 0) m-- ;
    ch[m++] = nodes[i] ;
}
for ( int i=n-2, t=m ; i>=0 ; i-- ){
    while(m > t && cross(ch[m-2], ch[m-1],
        nodes[i]) < 0) m-- ;
    ch[m++] = nodes[i] ;
}
if(n > 1) m-- ;
cout << m << endl ;

for ( int i=0 ; i<m ; i++ ) cout <<
    ch[i].x << " " << ch[i].y << endl ;
}

```

## 5 String

## 5.1 KMP

```

int Next[N] ;
void kmp(string &str){
    Next[0] = -1 ;
    if(str.size() <= 1) return ;
    Next[1] = 0 ;

    int cur = 2, check = 0 ;

    while(cur < str.size()){
        if(str[cur - 1] == str[check])
            Next[cur++] = ++check ;
        else if(check > 0) check =
            Next[check] ;
        else Next[cur++] = 0 ;
    }
}

int main(){
    ios::sync_with_stdio(false) ;
    cin.tie(nullptr) ;
    cout.tie(nullptr) ;

    string s1, s2 ;
    while(cin >> s1){
        s2 = s1 ;
        reverse(s2.begin(), s2.end()) ;
        kmp(s2) ;

        int x=0, y=0 ;
        while(x < s1.size() && y < s2.size()){
            if(s1[x] == s2[y]){
                x++ ;
                y++ ;
            }
            else if(y > 0) y = Next[y] ;
            else x++ ;
        }
        cout << s1 << s2.substr(y) << endl ;
    }
}

```

## 5.2 Trie

```
class TrieNode{  
public:  
    set<int> end ;  
    TrieNode *next[26] ;
```

```

6   TrieNode(){
7     for ( int i=0 ; i<26 ; i++ ) next[i] = nullptr ;
8   }
9 };
10
11 class Trie{
12 private:
13   int cnt ;
14   TrieNode *root ;
15 public:
16   Trie() : cnt(0) {
17     root = new TrieNode() ;
18   }
19
20   void insert(string &str, int n){
21     TrieNode* node = root ;
22     for ( auto s : str ){
23       int path = s - 'a' ;
24
25       if(node->next[path] == nullptr)
26         node->next[path] = new TrieNode() ;
27       node = node->next[path] ;
28     }
29     node->end.insert(n) ;
30   }
31
32   void search(string &str){
33     TrieNode* node = root ;
34     for ( auto s : str ){
35       int path = s - 'a' ;
36       if(node->next[path] == nullptr)
37         return ;
38       node = node->next[path] ;
39
40       int flg = 0 ;
41       for ( auto n : node->end ){
42         if(flg) cout << " " ;
43         else flg = 1 ;
44
45         cout << n ;
46     }
47
48   void clear(TrieNode* node) {
49     if (!node) return ;
50     for (int i = 0; i < 26; i++) {
51       if (node->next[i]) {
52         clear(node->next[i]) ;
53       }
54     }
55     delete node ;
56   }
57
58 ~Trie(){
59   clear(root) ;
60 }
61 };

```

---

### 5.3 ACAM

```

1 class ACAutomation{
2 private:
3   vector<int> fail, end, order ;
4   vector<vector<int>> tree ;
5
6   int base, alpha ;
7
8   int new_node(){
9     tree.emplace_back(alpha, 0) ;
10    fail.push_back(0) ;
11
12    return tree.size() - 1 ;
13 }
14 public:
15
16   ACAutomation(int _base='a', int _alpha=26)
17     : base(_base), alpha(_alpha) {
18       clear() ;
19   }
20
21   void clear(){
22     fail.assign(1, 0) ;
23     order.clear() ;
24     end.clear() ;
25     tree.assign(1, vector<int>(alpha, 0)) ;
26   }
27
28   void add_pattern(const string &pattern){
29     int u = 0 ;
30     for ( auto ch : pattern ){
31       int v = ch - base ;
32
33       if(tree[u][v] == 0) tree[u][v] =
34         new_node() ;
35       u = tree[u][v] ;
36     }
37
38     end.push_back(u) ;
39   }
40
41   void build(){
42     queue<int> q ;
43     order.clear() ;
44     order.push_back(0) ;
45
46     for ( int i=0 ; i<alpha ; i++ )
47       if(tree[0][i] > 0){
48         q.push(tree[0][i]) ;
49     }
50
51     while(!q.empty()){
52       int u = q.front() ; q.pop() ;
53       order.push_back(u) ;
54
55       for ( int i=0 ; i<alpha ; i++ ){
56         if(tree[u][i] == 0) tree[u][i] =
57           tree[fail[u]][i] ;
58         else{
59           fail[tree[u][i]] = tree[fail[u]][i] ;
60           q.push(tree[u][i]) ;
61         }
62       }
63     }
64
65   vector<int> count_per_pattern(const string
66     &text) const {
67     int u = 0 ;
68     vector<int> vis(tree.size(), 0) ;
69
70     for ( char ch : text ){
71       u = tree[u][ch - base] ;
72       vis[u]++;
73     }
74
75     for ( int i=order.size()-1 ; i>=1 ; i-- )
76       if (int x = order[i] ;
77           vis[fail[x]] += vis[x] ) ;
78
79     vector<int> ans(end.size(), 0) ;
80     for ( int id=0 ; id<end.size() ; id++ ){
81       ans[id] = vis[end[id]] ;
82     }
83
84     return ans ;
85   }
86
87 };

```

## 6 Algorithm

### 6.1 LCA

```

1 #include <bits/stdc++.h>
2
3 using namespace std ;
4
5 const int Maxn = 500005 ;
6
7 vector<int> e[Maxn] ;
8 int depth[Maxn] ;
9 int up[Maxn][40] ;
10 int MaxLog ;
11
12 void dfs(int u, int from, int d){
13     up[u][0] = from ;
14     depth[u] = d ;
15
16     for ( int i=1 ; i<=MaxLog ; i++ ){
17         up[u][i] = up[up[u][i - 1]][i - 1] ;
18     }
19
20     for ( auto v : e[u] ){
21         if(v == from) continue ;
22         dfs(v, u, d + 1) ;
23     }
24 }
25
26 int lca(int u, int v){
27     if(depth[u] < depth[v]) swap(u, v) ;
28
29     for ( int i=MaxLog ; i>=0 ; i-- )
30         if(depth[u] - (1 << i) >= depth[v]){
31             u = up[u][i] ;
32
33         if(u == v) return u ;
34
35         for ( int i=MaxLog ; i>=0 ; i-- )
36             if(up[u][i] != up[v][i]){
37                 u = up[u][i] ;
38                 v = up[v][i] ;
39             }
40
41     return up[u][0] ;
42 }
43
44 int main(){
45     int n, q, root ;
46     scanf( "%d%d%d" , &n, &q, &root ) ;
47     MaxLog = __lg(n) ;
48
49     for ( int i=0 ; i<n-1 ; i++ ){
50         int u, v ;
51         scanf( "%d%d" , &u, &v ) ;
52         e[u].push_back(v) ;
53         e[v].push_back(u) ;
54     }
55
56     dfs(root, root, 0) ;
57
58     while(q--){
59         int u, v ;
60         scanf( "%d%d" , &u, &v ) ;
61         printf( "%d\n" , lca(u, v)) ;
62     }
63
64     return 0 ;
65 }
```

### 6.2 MST

```

1 struct Edge{
2     int u, v, w ;
3     // 這是最大生成樹，最小生成樹要改成 w < o.w
```

```

4     bool operator>(const Edge &o) const
5     {return w > o.w ;} ;
6
7     int par[N] ;
8     int sz[N] ;
9     int sum ;
10    vector<Edge> edge ;
11
12    void init(){
13        edge.clear() ;
14        for ( int i=0 ; i<N ; i++ ){
15            par[i] = i ;
16            sz[i] = 1 ;
17        }
18        sum = 0 ;
19    }
20
21    int find(int x){
22        if(x == par[x]) return x ;
23        return par[x] = find(par[x]) ;
24    }
25
26    int merge(int x, int y){
27        x = find(x) ;
28        y = find(y) ;
29
30        if(x == y) return 0 ;
31        if(sz[x] > sz[y]) swap(x, y) ;
32        par[x] = y ;
33        sz[y] += sz[x] ;
34
35        return 1 ;
36    }
37
38    void MST(){
39        int cnt = 0 ;
40        for ( int i=0 ; i<edge.size() && cnt < n-1
41               ; i++ ){
42            auto [u, v, w] = edge[i] ;
43            if(merge(u, v)){
44                cnt++ ;
45                sum -= w ;
46            }
47        }
48    }
49
50    int main(){
51        for ( int i=0 ; i<m ; i++ ){
52            scanf( "%d%d%d" , &u, &v, &w ) ;
53            edge.push_back({u, v, w}) ;
54            sum += w ;
55        }
56
57        sort(edge.begin(), edge.end(),
58              greater<Edge>()) ;
59        MST() ;
60    }
61 }
```

### 6.3 SG

```

1 long long SG(long long k){
2
3     if(k % 2 == 0){
4         return k / 2 ;
5     }
6     else{
7         return SG(k / 2);
8     }
9
10    }
11
12    int main(){
13        int cas, n;
14        scanf( "%d" , &cas);
```

```

16    while(cas--){
17        scanf( "%d" , &n);
18
19        long long s, v = 0 ;
20
21        for(int i = 0; i < n; i++){
22            scanf( "%lld" , &s);
23            v ^= SG(s); //XOR
24        }
25
26        if(v) printf( "YES\n");
27        else printf( "NO\n");
28    }
29
30    int SG[30] ;
31    int vis[Maxn], stone[Maxn] ;
32
33    void build(){
34        SG[0] = 0 ;
35        memset(vis, 0, sizeof(vis)) ;
36
37        for ( int i=1 ; i<30 ; i++ ){
38            int cur = 0 ;
39            for ( int j=0 ; j<i ; j++ ) for ( int
40                  k=0 ; k<=j ; k++ ){
41                vis[SG[j] ^ SG[k]] = i ;
42            }
43            while(vis[cur] == i) cur++ ;
44            SG[i] = cur ;
45        }
46
47        int main(){
48            build() ;
49
50            int T = 0 ;
51            while(~scanf( "%d" , &n) && n){
52                int ans = 0 ;
53
54                for ( int i=1 ; i<=n ; i++ ) scanf( "%d" ,
55                                              &stone[i] ) ;
56
57                for ( int i=1 ; i<=n ; i++ ) if(stone[i]
58                                              & 1){
59                    ans ^= SG[n-i] ;
60                }
61            }
62        }
63    }
64 }
```

### 6.4 Max Flow

```

1 struct Edge{
2     int v, cap, next ;
3 };
4
5 class MaxFlow{
6 private:
7     int N, S, T ;
8     vector<Edge> e ;
9     vector<int> head, cur, dep ;
10
11    bool bfs(){
12        queue<int> q ;
13        for ( int i=0 ; i<=N ; i++ ){
14            cur[i] = head[i] ;
15            dep[i] = -1 ;
16        }
17
18        q.push(S) ;
19        dep[S] = 0 ;
20
21        while(!q.empty()){
22            int u = q.front() ; q.pop() ;
23
24            for ( int i=head[u] ; i!= -1 ;
25                  i=e[i].next ){
```

```

        int v = e[i].v ;
        if(dep[v] == -1 && e[i].cap > 0){
            dep[v] = dep[u] + 1 ;
            if(v == T) return 1 ;
            q.push(v) ;
        }
    }

    return 0 ;
}

int dfs(int u, int flow){
    if(u == T) return flow ;
    int d, rest = 0 ;

    for ( int &i=cur[u] ; i!=-1 ;
          i=e[i].next ){
        int v = e[i].v ;
        if(dep[v] == dep[u] + 1 && e[i].cap >
           0){
            d = dfs(v, min(flow - rest,
                            e[i].cap)) ;
            if(d > 0){
                e[i].cap -= d ;
                e[i^1].cap += d ;
                rest += d ;
                if(rest == flow) break ;
            }
        }
    }

    if(rest != flow) dep[u] = -1 ;
    return rest ;
}
public:
MaxFlow(int n, int s, int t){
    N = n ; S = s ; T = t ;
    e.reserve(n*n) ;
    head.assign(n+1, -1) ;
    cur.resize(n+1) ;
    dep.resize(n+1) ;
}

void AddEdge(int u, int v, int cap){
    e.push_back({v, cap, head[u]}) ;
    head[u] = e.size() - 1 ;
    e.push_back({u, 0, head[v]}) ;
    head[v] = e.size() - 1 ;
}

int run(){
    int ans = 0 ;
    while(bfs()){
        ans += dfs(S, 0x3f3f3f3f) ;
    }
    return ans ;
}
};

```

## 6.5 min cut max flow

```

struct Edge{
    int v, cap, cost , next ;
};

using pii = pair<int, int> ;
class MCMF{
private:
    int N, s, t, tot ;
    vector<Edge> e ;
    vector<int> head ;
public:
    MCMF(int n, int _s, int _t)
        N = n ;
}

```

```

s = _s ;
t = _t ;
e.resize(n*n + 5) ;
head.assign(n+5, -1) ;
tot = -1 ;
}

void AddEdge(int u, int v, int cap, int
cost){
e[++tot] = {v, cap, cost, head[u]} ;
head[u] = tot ;
e[++tot] = {u, 0, -cost, head[v]} ;
head[v] = tot ;
}

int run(){
vector<int> dis(N+1), pot(N+1, 0),
preE(N+1) ;
int flow = 0, cost = 0 ;

auto dijkstra = [&](){
fill(dis.begin(), dis.end(), INF) ;
priority_queue<pii, vector<pii>,
greater<pii>> pq ;
dis[s] = 0 ;
pq.push({0, s}) ;

while(!pq.empty()){
auto [d, u] = pq.top() ; pq.pop() ;
if(d > dis[u]) continue ;
for (int i=head[u] ; i!=-1 ;
i=e[i].next ){
int v = e[i].v, cap = e[i].cap,
e[i].cost ;
if(cap && dis[v] > d + w + pot[v]){
pot[v] = dis[v] = d + w + pot[u] - pot[v] ;
dis[v] = d + w + pot[u] - pot[v] ;
preE[v] = i ;
pq.push({dis[v], v}) ;
}
}
}
}

return dis[t] != INF ;
}

```

```

3  using namespace std ;
4  using ll = long long ;
5
6  ll dp[2][(1 << 10) + 5] ;
7  int n, m ;
8  int cur ;
9
10 void update(int s1, int s2){
11     if(s2 & (1 << m)){
12         dp[cur][s2 ^ (1 << m)] += dp[cur] ^
13             1][s1] ;
14    }
15}
16
17 int main(){
18     while(~scanf("%d%d", &n, &m)){
19         if(m > n) swap(n, m) ;
20         memset(dp, 0, sizeof(dp)) ;
21         cur = 0 ;
22         dp[cur][(1 << m) - 1] = 1 ;
23         for (int i=0 ; i<n ; i++) for (int
24             j=0 ; j<m ; j++) {
25             cur ^= 1 ;
26             memset(dp[cur], 0, sizeof(dp[cur])) ;
27
28             for (int k=0 ; k<(1 << m) ; k++ ){
29                 update(k, k << 1) ; // not put
30                 if(i && !(k & (1 << (m - 1)))) {
31                     update(k, (k << 1) | (1 << m) |
32                         1) ; // put up
33                 if(j && !(k & 1)) update(k, (k << 1)
34                         | 3) ; // put left
35             }
36         }
37         printf("%lld\n", dp[cur][(1 << m) - 1]) ;
38     }
39     return 0 ;
40 }

```

7.2 數位 DF

```

    return dis[t] != INF ;
};

while(dijkstra()){
    for ( int v=1 ; v<=N ; v++ ) if(dis[v]
        < INF){
        pot[v] += dis[v] ;
    }

    int aug = INT_MAX ;
    for ( int v=t ; v!=s ;
        v=e[preE[v]^1].v ){
        aug = min(aug, e[preE[v]].cap) ;
    }

    for ( int v=t ; v!=s ;
        v=e[preE[v]^1].v ){
        e[preE[v]].cap -= aug ;
        e[preE[v]^1].cap += aug ;
        cost += aug * e[preE[v]].cost ;
    }
}

return cost ;
};


```

---

## 7 DP

### 7.1 輪廓線 DP

```

#include <bits/stdc++.h>

```

```

1 #include <bits/stdc++.h>
2
3 using namespace std ;
4
5 int K ;
6 int dp[20][105][105][2] ;
7 vector<int> dig ;
8
9 int solve(int pos, int sum, int dsum, bool
10    lim){
11    if(pos == -1){
12        if(sum == 0 && dsum == 0) return 1 ;
13        return 0 ;
14    }
15    int &d = dp[pos][sum][dsum][lim] ;
16    if(d != -1) return d ;
17
18    int up = lim ? dig[pos] : 9 ;
19    int res = 0 ;
20    for ( int i=0 ; i<=up ; i++ ){
21        res += solve(pos-1, (sum * 10 + i) %
22                      K, (dsum + i) % K, lim && i==up)
23                      ;
24    }
25
26    return d = res ;
27}
28
29 int count(int n){
30    memset(dp, -1, sizeof(dp)) ;
31    dig.clear() ;
32
33    while(n > 0){
34        dig.push_back(n % 10) ;
35        n /= 10 ;
36    }
37
38    cout << solve(19, 0, 0, 1) << endl ;
39
40    return 0 ;
41}

```

```

34     }
35     edge[u].push_back({v, w}) ;
36   }
37 }
38 int main(){
39   int T ;
40   scanf("%d", &T) ;
41
42   int a, b ;
43   while(T--){
44     scanf(" %d%d%d", &a, &b, &K) ;
45     if(K > 90) printf("0\n") ;
46     else printf("%d\n", count(b) -
47       count(a-1)) ;
48   }
49
50   return 0 ;
51 }
```

```

49   edge[u].push_back({v, w}) ;
50 }
51 DFS(0) ;
52 printf("Case %d:\n", ++t) ;
53
54 int q, e ;
55 scanf(" %d", &q) ;
56
57 while(q--){
58   scanf(" %d", &e) ;
59
60   for ( int i=n ; i>=1 ; i-- )
61     if(dp[0][i][0] <= e){
62       printf("%d\n", i) ;
63       break ;
64     }
65   }
66
67
68 return 0 ;
69 }
```

### 7.3 樹 DP

```

1 #include <bits/stdc++.h>
2
3 #define N 505
4 #define INF 0x3f3f3f3f
5
6 using namespace std ;
7
8 struct Edge{
9   int v, w ;
10 } ;
11
12 vector<Edge> edge[N] ;
13 int n ;
14 int cnt[N] ;
15 int dp[N][N][2] ;
16
17 void init(){
18   for ( int i=0 ; i<N ; i++ )
19     edge[i].clear() ;
20   memset(cnt, 0, sizeof(cnt)) ;
21   memset(dp, INF, sizeof(dp)) ;
22 }
23
24 void DFS(int u){
25   cnt[u] = 1 ;
26   for ( auto [v, w] : edge[u] ){
27     DFS(v) ;
28     cnt[u] += cnt[v] ;
29   }
30
31   dp[u][1][0] = dp[u][1][1] = 0 ;
32
33   for ( auto [v, w] : edge[u] ){
34     for ( int i=cnt[u] ; i>1 ; i-- ) for (
35       int j=1 ; j<i && j<=cnt[v] ; j++ ){
36       dp[u][i][1] = min(dp[u][i][1],
37         dp[u][i-j][1] + dp[v][j][1] + 2 *
38         w) ;
39       dp[u][i][0] = min(dp[u][i][0],
40         dp[u][i-j][1] + dp[v][j][0] + w) ;
41       dp[u][i][0] = min(dp[u][i][0],
42         dp[u][i-j][0] + dp[v][j][1] + 2 *
43         w) ;
44     }
45   }
46 }
47
48 int main(){
49   int t = 0 ;
50
51   while(~scanf(" %d", &n) && n){
52     init() ;
53     for ( int i=0 ; i<n-1 ; i++ ){
54       int u, v, w ;
55       scanf(" %d%d%d", &v, &u, &w) ;
56     }
57   }
58 }
```