Contents

1 Foundations

1.1 PyMath

```python
import math

math.ceil(x) #上高斯
math.floor(x) #下高斯
math.factorial(x) #階乘
math.fabs(x) #絕對值
math.fsum(arr) #求和
math.gcd(x, y)
math.exp(x) # e^x
math.log(x, base)
math.log2(x)
math.log10(x)
math.sqrt(x)
math.pow(x, y, mod)
math.sin(x) # cos, tan, asin, acos, atan, atan2, sinh ...
math.hypot(x, y) #歐幾里德範數
math.degrees(x) #x從弧度轉角度
math.radians(x) #x從角度轉弧度
math.gamma(x) #x的gamma函數
math.pi #const
math.e #const
math.inf
```

1.2 Java Integer

```java
// 常量
MAX_VALUE, MIN_VALUE, BYTES, SIZE, TYPE

// 轉換/解析
static int   parseInt(String s)
static int   parseInt(String s, int radix)
static int   parseUnsignedInt(String s)
static int   parseUnsignedInt(String s, int radix)
static Integer valueOf(int i)
static Integer valueOf(String s)
static Integer valueOf(String s, int radix)
static String toString(int i)
static String toString(int i, int radix)
static String toUnsignedString(int i)
static String toUnsignedString(int i, int radix)
static long   toUnsignedLong(int x)
static Integer decode(String nm)            // 支援 0x/0/# 前綴
static Integer getInteger(String nm[, int val]) // 從系統屬性讀取整數

// 比較/雜湊/聚合
static int compare(int x, int y)
static int compareUnsigned(int x, int y)
static int hashCode(int value)
static int min(int a, int b)
static int max(int a, int b)
static int sum(int a, int b)

// 位元操作
static int bitCount(int i)   // 設定位數
static int highestOneBit(int i)
static int lowestOneBit(int i)
static int numberOfLeadingZeros(int i)
static int numberOfTrailingZeros(int i)
static int rotateLeft(int i, int distance)
static int rotateRight(int i, int distance)
static int reverse(int i)
static int reverseBytes(int i)

// 無號運算
static int divideUnsigned(int dividend, int divisor)
static int remainderUnsigned(int dividend, int divisor)
```

1.3 Java String

```java
// 查詢
int     length()
boolean isEmpty()
boolean isBlank()     // (since 11)
char    charAt(int index)
int     codePointAt(int index)
int     codePointBefore(int index)
int     codePointCount(int beginIndex, int endIndex)
boolean contains(CharSequence s)
boolean startsWith(String prefix[, int toffset])
boolean endsWith(String suffix)
int     indexOf(String str[, int fromIndex])
int     lastIndexOf(String str[, int fromIndex])

// 取子字串/子序列
String      substring(int beginIndex)
String      substring(int beginIndex, int endIndex)
CharSequence subSequence(int beginIndex, int endIndex)

// 比較/等價
boolean equals(Object obj)
boolean equalsIgnoreCase(String anotherString)
int     compareTo(String anotherString)
int     compareToIgnoreCase(String str)
boolean matches(String regex)
boolean regionMatches(int toffset, String other, int ooffset, int len)
boolean regionMatches(boolean ignoreCase, int toffset, String other, int ooffset, int len)

// 建構/轉換/連接
String concat(String str)
String replace(char oldChar, char newChar)
String replace(CharSequence target, CharSequence replacement)
String replaceAll(String regex, String replacement)
String replaceFirst(String regex, String replacement)
String[] split(String regex[, int limit])
String toLowerCase()
String toUpperCase()
String trim()
String strip()          // (since 11)
String stripLeading()   // (since 11)
String stripTrailing()  // (since 11)
String repeat(int count) // (since 11)
IntStream chars()
Stream<String> lines()   // (since 11)
String intern()

// 靜態工具
static String format(String format, Object... args)
static String join(CharSequence delimiter, CharSequence... elements)
static String join(CharSequence delimiter, Iterable<? extends CharSequence> elements)
static String valueOf(primitive/char[]/Object)
static String copyValueOf(char[] data[, int offset, int count])
```

1.4 Java String builder

```java
// 長度/容量
int  length()
int  capacity()
void ensureCapacity(int minimumCapacity)
void trimToSize()
void setLength(int newLength)

// 存取/修改
char          charAt(int index)
void          setCharAt(int index, char ch)
StringBuilder append(... 各種型別 ...)
StringBuilder insert(int offset, ... 各種型別 ...)
StringBuilder delete(int start, int end)
StringBuilder deleteCharAt(int index)
StringBuilder replace(int start, int end, String str)
StringBuilder reverse()

// 子字串/查找
String       substring(int start)
String       substring(int start, int end)
CharSequence subSequence(int start, int end)
int          indexOf(String str[, int fromIndex])
int          lastIndexOf(String str[, int fromIndex])

// 轉換
String toString()
```

1.5 Java Math

```java
// 常量
static final double E, PI

// 絕對值/比較
static int/long/float/double abs(x)
static T max(a, b)
static T min(a, b)

// 取整/四捨五入
static double floor(double a)
static double ceil(double a)
static double rint(double a)          // 最接近整數(偶數優先)
static long  round(double a) / int round(float a)
static int   floorDiv(int x, int y)
static int   floorMod(int x, int y)

// 溢位保護 (exact 系列, Java 8+)
static int/long addExact(a, b)
static int/long subtractExact(a, b)
static int/long multiplyExact(a, b)
static int/long incrementExact(a)
static int/long decrementExact(a)
static int    toIntExact(long value)
static int/long negateExact(a)

// 指對數/冪根
static double pow(double a, double b)
static double sqrt(double a)
static double cbrt(double a)
static double exp(double a)
static double expm1(double x)
static double log(double a)
static double log10(double a)
static double log1p(double x)

// 三角/雙曲
static double sin/cos/tan(double a)
static double asin/acos/atan(double a)
static double atan2(double y, double x)
static double sinh/cosh/tanh(double a)

// 其他實用
static double hypot(double x, double y)
static double toDegrees(double angrad)
static double toRadians(double angdeg)
static double copySign(double magnitude, double sign)
static double nextUp/nextDown(double a)
static double nextAfter(double start, double direction)
static double ulp(double d)
static double random()
static double scalb(double d, int scaleFactor)
static double fma(double a, double b, double c) // (since 8)
static long  multiplyHigh(long x, long y)  // (since 9)
static long  multiplyFull(int x, int y)  // (since 9, 回傳 long)
```

2 Mathematics AND Number Theory

2.1 formula

1. Catalan Number

$$C_n = \frac{1}{n}\binom{2n}{n}, C_{n+1} = \frac{2(2n+1)}{n+2}C_n$$

$$C = 1, 1, 2, 5, 14, 42, 132, 429, 1430, 4862, \ldots$$

2. Euler's Formula

對於 V 個點, E 條邊, F 個面, C 個連通分量

$$V + F = E + 2$$

$$V + F = E + C + 1$$

3. Pick's Theorem

點座標均是整數或是正方形格子點的簡單多邊形, 其面積 A 和內部點數量 i, 邊上格點數量 b 的關係為

$$A = i + \frac{b}{2} - 1$$

2.2 extended gcd

給定 a, b, c, 求 $ax + by = c$ 的解

```cpp
    ll extgcd(ll a, ll b, ll c, ll &x, ll &y){
        if(b == 0){
            x = c/a ;
            y = 0 ;
            return a ;
        }
        ll d = extgcd(b, a%b, c, x, y), tmp = x ;
        x = y ;
        y = tmp - (a/b)*y ;
        return d ;
    }
```

3 Data Structure

3.1 MST

```cpp
struct Edge{
  int u, v, w ;
    // 這是最大生成樹, 最小生成樹要改成 w < o.w
  bool operator>(const Edge &o) const {return w > o.w ;} ;
} ;

int par[N] ;
int sz[N] ;
int sum ;

vector<Edge> edge ;

void init(){
  edge.clear() ;
  for ( int i=0 ; i<N ; i++ ){
    par[i] = i ;
    sz[i] = 1 ;
  }
  sum = 0 ;
}

int find(int x){
  if(x == par[x]) return x ;
  return par[x] = find(par[x]) ;
}

int merge(int x, int y){
  x = find(x) ;
  y = find(y) ;

  if(x == y) return 0 ;
  if(sz[x] > sz[y]) swap(x, y) ;
  par[x] = y ;
  sz[y] += sz[x] ;

  return 1 ;
}

void MST(){
  int cnt = 0 ;
  for ( int i=0 ; i<edge.size() && cnt < n-1 ; i++ ){
    auto [u, v, w] = edge[i] ;
    if(merge(u, v)){
      cnt++ ;
      sum -= w ;
    }
  }
}

int main(){
    for ( int i=0 ; i<m ; i++ ){
        scanf("%d%d%d", &u, &v, &w) ;
        edge.push_back({u, v, w}) ;
        sum += w ;
    }

    sort(edge.begin(), edge.end(), greater<Edge>()) ;
    MST() ;
}
```

3.2 SegmentTree

```cpp
#define lc (id << 1)
#define rc ((id << 1) | 1)

struct LazyTag{
    // type 0 : increase val
    // type 1 : set to val
    // type 1 can overwrite type 0
    int type ;
    ll val ;
} ;

struct Node{
    LazyTag tag ;
    ll sum ;
    int sz ;
}seg[Maxn << 2] ;

class SegmentTree{
private:
    void pull(int id){
        seg[id].sum = seg[lc].sum +
             seg[rc].sum ;
    }

    void AddTag(int id, LazyTag &tag){
        if(tag.type == 0){
            seg[id].sum += tag.val *
                 seg[id].sz ;
            seg[id].tag.val += tag.val ;
        }
        else{
            seg[id].sum = tag.val *
                 seg[id].sz ;
            seg[id].tag = {1, tag.val} ;
        }
    }

    void push(int id){
        AddTag(lc, seg[id].tag) ;
        AddTag(rc, seg[id].tag) ;
        seg[id].tag = {0, 0} ;
    }

public:
    void build(int L=1, int R=n, int id=1){
        seg[id].sum = 0 ;
        seg[id].tag = {0, 0} ;
        seg[id].sz = 1 ;

        if(L == R){
            seg[id].sum = arr[L] ;
            return ;
        }

        int M = (L + R) >> 1 ;
        build(L, M, lc) ;
        build(M+1, R, rc) ;

        pull(id) ;
        seg[id].sz = seg[lc].sz + seg[rc].sz ;
    }

    void modify(int l, int r, LazyTag &tag,
         int L=1, int R=n, int id=1){
        if(l <= L && R <= r){
            AddTag(id, tag) ;
            return ;
        }

        push(id) ;
        int M = (L + R) >> 1 ;
        if(r <= M) modify(l, r, tag, L, M,
             lc) ;
        else if(l > M) modify(l, r, tag, M+1,
             R, rc) ;
        else{
            modify(l, r, tag, L, M, lc) ;
            modify(l, r, tag, M+1, R, rc) ;
        }
        pull(id) ;
    }

    ll query(int l, int r, int L=1, int R=n,
         int id=1){
        if(l <= L && R <= r) return
             seg[id].sum ;

        push(id) ;
        int M = (L + R) >> 1 ;
        if(r <= M) return query(l, r, L, M,
             lc) ;
        else if(l > M) return query(l, r,
             M+1, R, rc) ;
        else return query(l, r, L, M, lc) +
             query(l, r, M+1, R, rc) ;
    }
}tree ;
```

3.3 HLD

```
/* HLD */
int fa[Maxn], top[Maxn], son[Maxn],
     sz[Maxn], dep[Maxn] = {0}, dfn[Maxn],
     rnk[Maxn], dfscnt = 0 ;

void dfs1(int u, int from){
  fa[u] = from ;
  dep[u] = dep[from] + 1 ;
  sz[u] = 1 ;

  for ( auto v : g[u] ) if(v != from){
    dfs1(v, u) ;
    sz[u] += sz[v] ;
    if(son[u] == -1 || sz[v] > sz[son[u]])
         son[u] = v ;
  }
}

void dfs2(int u, int t){
  top[u] = t ;
  dfn[u] = ++dfscnt ;
  rnk[dfscnt] = u ;

  if(son[u] == -1) return ;

  dfs2(son[u], t) ;

  for ( auto v : g[u] ) if(v != fa[u] && v
       != son[u]){
    dfs2(v, v) ;
  }
}

/* Segment Tree */
#define lc (id << 1)
#define rc ((id << 1) | 1)

struct ColorSeg{
  int left, right, tot ;

  ColorSeg operator+(const ColorSeg &o)
       const {
    if(tot == 0) return o ;
    if(o.tot == 0) return *this ;

    ColorSeg tmp ;
    tmp.left = left ;
    tmp.right = o.right ;
    tmp.tot = tot + o.tot - (right ==
         o.left) ;

    return tmp ;
  }
} ;

struct Node{
  ColorSeg color ;
  int tag ;
}seg[Maxn << 2] ;

class SegmentTree{
private:
  void pull(int id){
    // normal pull
  }

  void AddTag(int id, int tag){
    // normal AddTag
  }

  void push(int id){
    // normal push
  }

  void modify(int l, int r, int tag, int
       L=1, int R=n, int id=1){
    // normal modify
  }

  ColorSeg query(int l, int r, int L=1, int
       R=n, int id=1){
    // normal query
  }
public:
  void build(int L=1, int R=n, int id=1){
    // normal build
  }

    // update val from u to v (simple path)
  void update(int u, int v, int val){
    while(top[u] != top[v]){
      if(dep[top[u]] < dep[top[v]]) swap(u,
           v) ;
      modify(dfn[top[u]], dfn[u], val) ;
      u = fa[top[u]] ;
    }

    if(dep[u] < dep[v]) swap(u, v) ;
    modify(dfn[v], dfn[u], val) ;
  }

    // get sum from u to v (simple path)
  int get(int u, int v){
    pair<int, ColorSeg> U, V ;
    ColorSeg M ;
    U = {u, {0, 0, 0}} ;
    V = {v, {0, 0, 0}} ;

    while(top[U.first] != top[V.first]){
      if(dep[top[U.first]] <
           dep[top[V.first]]) swap(U, V) ;
      U.second = query(dfn[top[U.first]],
           dfn[U.first]) + U.second ;
      U.first = fa[top[U.first]] ;
    }

    if(dep[U.first] < dep[V.first]) swap(U,
         V) ;

    M = query(dfn[V.first], dfn[U.first]) ;

    return (U.second.tot + V.second.tot +
         M.tot) - (U.second.left == M.right)
         - (V.second.left == M.left) ;
  }
}tree ;

void init(){
  memset(son, -1, sizeof(son)) ;
}
```

3.4 PST

```
// Find range k-th largest number
struct Node{
  int sum, left, right ;
}seg[Maxn + 20 * Maxn] ;

class PersistentSegmentTree{
private:
  int n ;
  int cnt ;
  vector<int> version ;
```

```cpp

  int build(int L, int R){
    int cur_cnt = cnt++ ;
    if(L == R){
      seg[cur_cnt] = {0, 0, 0} ;
      return cur_cnt ;
    }

    int M = (L + R) >> 1 ;
    int lc = build(L, M) ;
    int rc = build(M+1, R) ;

    seg[cur_cnt] = {0, lc, rc} ;
    return cur_cnt ;
  }
public:
  PersistentSegmentTree(int _n){
    n = _n ;
    cnt = 0 ;

    int root = build(1, n) ;
    version.push_back(root) ;
  }

  void update(int ver, int idx){
    auto upd = [&](auto &&self, const int cur, int L, int R){
      int cur_cnt = cnt++ ;

      if(L == R){
        seg[cur_cnt] = {seg[cur].sum + 1, 0, 0} ;
        return cur_cnt ;
      }

      int M = (L + R) >> 1 ;
      int lc = seg[cur].left ;
      int rc = seg[cur].right ;

      if(idx <= M) lc = self(self, seg[cur].left, L, M) ;
      else rc = self(self, seg[cur].right, M+1, R) ;

      seg[cur_cnt] = {seg[lc].sum + seg[rc].sum, lc, rc} ;

      return cur_cnt ;
    };

    int root = upd(upd, version[ver], 1, n) ;
    version.push_back(root) ;
  }

  int query(int verL, int verR, int k){
    auto qry = [&](auto &&self, const int cur_old, const int cur_new, int L, int R){
      if(L == R) return L ;

      int old_l = seg[cur_old].left, old_r = seg[cur_old].right ;
      int new_l = seg[cur_new].left, new_r = seg[cur_new].right ;

      int dl = seg[new_l].sum - seg[old_l].sum ;
      int dr = seg[new_r].sum - seg[old_r].sum ;

      int M = (L + R) >> 1 ;

      if(dl >= k) return self(self, old_l, new_l, L, M) ;
      k -= dl ;
      return self(self, old_r, new_r, M+1, R) ;
    };

    int idx = qry(qry, version[verL-1], version[verR], 1, n) ;
    return idx ;
  }
};
```

3.5 Trie

```cpp
class TrieNode{
public:
    set<int> end ;
    TrieNode *next[26] ;

    TrieNode(){
        for ( int i=0 ; i<26 ; i++ ) next[i] = nullptr ;
    }
};

class Trie{
private:
    int cnt ;
    TrieNode *root ;
public:
    Trie() : cnt(0) {
        root = new TrieNode() ;
    }

    void insert(string &str, int n){
        TrieNode* node = root ;
        for ( auto s : str ){
            int path = s - 'a' ;

            if(node->next[path] == nullptr)
                node->next[path] = new TrieNode() ;
            node = node->next[path] ;
        }
        node->end.insert(n) ;
    }

    void search(string &str){
        TrieNode* node = root ;
        for ( auto s : str ){
            int path = s - 'a' ;
            if(node->next[path] == nullptr)
                return ;
            node = node->next[path] ;
        }

        int flg = 0 ;
        for ( auto n : node->end ){
            if(flg) cout << " " ;
            else flg = 1 ;

            cout << n ;
        }
    }

    void clear(TrieNode* node) {
        if (!node) return ;
        for (int i = 0; i < 26; i++) {
            if (node->next[i]) {
                clear(node->next[i]) ;
            }
        }
        delete node ;
    }

    ~Trie(){
        clear(root) ;
    }
};
```

3.6 BIT ver1

```cpp
// 單點修改 區間查詢
#define lowbit(x) (x & -x)

int BIT[MAX_SIZE] ;
int n ;

void modify(int idx, int val){
    for ( ; idx <= n ; idx += lowbit(idx) ){
        BIT[idx] += val ;
    }
}

// ans: query(R) - query(L-1)
int query(int idx){
    int sum = 0 ;
    for ( ; idx ; idx -= lowbit(idx) ){
        sum += BIT[idx] ;
    }
}

void init(){
    memset(BIT, 0, sizeof(BIT)) ;
}
```

3.7 BIT ver2

```cpp
// 區間修改, 單點查詢
#define lowbit(x) (x & -x)

int BIT[MAX_SIZE] ;
int n ;

void modify(int idx, int val){
    for ( ; idx <= n ; idx += lowbit(idx) ){
        BIT[idx] += val ;
    }
}

// ans: query(i)
int query(int idx){
    int sum = 0 ;
    for ( ; idx ; idx -= lowbit(idx) ){
        sum += BIT[idx] ;
    }

    return sum ;
}

void init(){
    memset(BIT, 0, sizeof(BIT)) ;
}

void build(){
    arr[0] = 0 ;
    for ( int i=1 ; i<=n ; i++ ) modify(i, arr[i] - arr[i-1]) ;
}

// usage
// add val
modify(L, x) ;
modify(R+1, -x) ;
```

3.8 BIT ver3

```cpp
// 區間修改 區間查詢
#define lowbit(x) x & -x

ll BIT1[MAXN], BIT2[MAXN] ;

void update(int idx, ll val){
    for ( int i=idx ; i<=n ; i+=lowbit(i) ){
        BIT1[i] += val ;
        BIT2[i] += idx * val ;
    }
}

// range query: query(R) - query(X - 1)
ll query(int idx){
    ll sum1 = 0, sum 2 = 0 ;
    for ( int i=idx ; i ; i-=lowbit(i) ){
        sum1 += BIT1[i] ;
        sum2 += BIT2[i] ;
    }

    return (idx + 1) * sum1 - sum2 ;
}

void build(){
    for (int i=1 ; i<=n ; i++ ){
        update(i, arr[i] - arr[i-1]) ;
    }
}

void usage(){
    update(L, x) ;
    update(R+1, -x) ;
}

void init(){
    memset(BIT1, 0, sizeof(BIT1)) ;
    memset(BIT2, 0, sizeof(BIT2)) ;
}
```

4 Graph

4.1 cut vertex AND bridges

```
int dfn[Maxn] = {-1}, low[Maxn] = {-1},
     dfscnt ;

void dfs(int u, int fa){
  dfn[u] = low[u] = ++dfscnt ;
  int child = 0 ;

  for ( auto v : g[u] ) if(v != fa){
    if(dfn[v] == -1){
      child++ ;
      dfs(v, u) ;
      low[u] = min(low[u], low[v]) ;

      if(low[v] >= dfn[u]){
        // this edge is a bridge
      }

      if(u != fa && low[v] >= dfn[u]){
        // this node v is a articulation point
      }
    }
    else low[u] = min(low[u], dfn[v]) ;
  }

  if(u == fa && child > 1){
    // this node u is a articulation point
  }
}
```

4.2 SCC - Tarjan

```
vector<int> scc[Maxn] ;
int dfn[Maxn], low[Maxn], sccId[Maxn],
     dfscnt = 0, cnt_scc = 0 ;
stack<int> st ;
bitset<Maxn> inSt, vis ;

void dfs(int u, int from){
  dfn[u] = low[u] = ++dfscnt ;
  st.push(u) ;
  inSt[u] = 1 ;

  for ( auto v : g[u] ){
    if(!inSt[v] && dfn[v] != -1) continue ;
    if(dfn[v] == -1) dfs(v, u) ;
    low[u] = min(low[u], low[v]) ;
  }

  if(dfn[u] == low[u]){
    cnt_scc++ ;
    int x ;

    do{
      x = st.top() ;
      st.pop() ;

      inSt[x] = 0 ;
      sccId[x] = cnt_scc ;
      scc[cnt_scc].push_back(x) ;
    }
    while(x != u) ;
  }
}

// SCC to DAG (after dfs)
vector<int> dag[Maxn] ;

void scc_to_dag(){
  vector<int> dag[Maxn] ;
  for ( int u=1 ; u<=n ; u++ ){
    for ( auto v : g[u] ){
      if(sccId[u] != sccId[v]){
        dag[sccId[u]].push_back(sccId[v]) ;
      }
    }
  }
}

void init(){
  memset(dfn, -1, sizeof(dfn)) ;
  memset(low, -1, sizeof(low)) ;
}

int main(){
    init() ;
    input() ;
    for ( int i=1 ; i<=n ; i++ ) if(dfn[i]
         == -1){
    dfs(i, i) ;
  }
}
```

4.3 BCC - Tarjan

```
struct Edge{
  int v, next ;
}e[Maxm << 1] ;
int head[Maxm], tot = 1 ;

void add(int u, int v){
  e[++tot] = {v, head[u]} ;
  head[u] = tot ;
  e[++tot] = {u, head[v]} ;
  head[v] = tot ;
}

bitset<Maxm << 1> bz ;
vector<vector<int> > bcc ;
int dfn_cnt = 0, dfn[Maxn], low[Maxn],
     vis_bcc[Maxn], bcc_cnt = 0 ;

void dfs1(int u, int from){
  dfn[u] = low[u] = ++dfn_cnt ;

  for ( int i=head[u] ; i!=-1 ; i=e[i].next
       ){
    int v = e[i].v ;

    if(dfn[v] == -1){
      dfs1(v, i) ;
      if(dfn[u] < low[v]) bz[i] = bz[i^1] =
           1 ;
      low[u] = min(low[u], low[v]) ;
    }
    else if(i != (from ^ 1)) low[u] =
         min(low[u], dfn[v]) ;
  }
}

void dfs2(int u, int id){
  vis_bcc[u] = id ;
  bcc[id].push_back(u) ;

  for ( int i=head[u] ; i!=-1 ; i=e[i].next
       ){
    int v = e[i].v ;

    if(vis_bcc[v] != -1 || bz[i]) continue ;
    dfs2(v, id) ;
  }
}

void init(){
  memset(dfn, -1, sizeof(dfn)) ;
  memset(head, -1, sizeof(head)) ;
  memset(vis_bcc, -1, sizeof(vis_bcc)) ;
}

int main(){
    init() ;
    input() ;
    for ( int i=1 ; i<=n ; i++ ) if(dfn[i]
         == -1){
    dfs1(i, 0) ;
  }

  for ( int i=1 ; i<=n ; i++ ) if(vis_bcc[i]
       == -1){
    bcc.push_back(vector<int>()) ;
    dfs2(i, bcc_cnt++) ;
  }
}
```

4.4 Convex

```
struct Coordinate{
  long long x, y ;

  friend bool operator<(const Coordinate&a,
       const Coordinate& b){
    if(a.x == b.x) return a.y < b.y ;
    return a.x < b.x ;
  }

  friend bool operator==(const Coordinate&
       a, const Coordinate& b){
    return a.x == b.x && a.y == b.y ;
  }
} ;

vector<Coordinate> nodes ;

long long cross(const Coordinate& o, const
     Coordinate& a, const Coordinate& b){
  return (a.x - o.x) * (b.y - o.y) - (a.y -
       o.y) * (b.x - o.x) ;
}

void input(){
  nodes.clear() ;

  int n, x, y ;
  char c ;
  cin >> n ;

  for ( int i=0 ; i<n ; i++ ){
    cin >> x >> y >> c ;
    if(c == 'Y') nodes.push_back({x, y}) ;
  }
}

void monotone(){
  sort(nodes.begin(), nodes.end()) ;

  int n = unique(nodes.begin(), nodes.end()) - nodes.begin() ;

  vector<Coordinate> ch(n+1) ;

  int m = 0 ;

  for ( int i=0 ; i<n ; i++ ){
    while(m > 1 && cross(ch[m-2], ch[m-1], nodes[i]) < 0) m-- ;
    ch[m++] = nodes[i] ;
  }
  for  ( int i=n-2, t=m ; i>=0 ; i-- ){
    while(m > t && cross(ch[m-2], ch[m-1], nodes[i]) < 0) m-- ;
    ch[m++] = nodes[i] ;
  }

  if(n > 1) m-- ;
  cout << m << endl ;

  for ( int i=0 ; i<m ; i++ ) cout << ch[i].x << " " << ch[i].y << endl ;
}
```

4.5 Max Flow

```
struct Edge{
  int v, cap, next ;
};

class MaxFlow{
private:
  int N, S, T ;
  vector<Edge> e ;
  vector<int> head, cur, dep ;

  bool bfs(){
    queue<int> q ;
    for ( int i=0 ; i<=N ; i++ ){
      cur[i] = head[i] ;
      dep[i] = -1 ;
    }

    q.push(S) ;
    dep[S] = 0 ;

    while(!q.empty()){
      int u = q.front() ; q.pop() ;

      for ( int i=head[u] ; i!=-1 ; i=e[i].next ){
        int v = e[i].v ;
        if(dep[v] == -1 && e[i].cap > 0){
          dep[v] = dep[u] + 1 ;
          if(v == T) return 1 ;
          q.push(v) ;
        }
      }
    }

    return 0 ;
  }

  int dfs(int u, int flow){
    if(u == T) return flow ;
    int d, rest = 0 ;

    for ( int &i=cur[u] ; i!=-1 ; i=e[i].next ){
      int v = e[i].v ;
      if(dep[v] == dep[u] + 1 && e[i].cap > 0){
        d = dfs(v, min(flow - rest, e[i].cap)) ;

        if(d > 0){
          e[i].cap -= d ;
          e[i^1].cap += d ;
          rest += d ;

          if(rest == flow) break ;
        }
      }
    }

    if(rest != flow) dep[u] = -1 ;
    return rest ;
  }
public:
  MaxFlow(int n, int s, int t){
    N = n ; S = s ; T = t ;
    e.reserve(n*n) ;
    head.assign(n+1, -1) ;
    cur.resize(n+1) ;
    dep.resize(n+1) ;
  }

  void AddEdge(int u, int v, int cap){
    e.push_back({v, cap, head[u]}) ;
    head[u] = e.size() - 1 ;
    e.push_back({u, 0, head[v]}) ;
    head[v] = e.size() - 1 ;
  }

  int run(){
    int ans = 0 ;
    while(bfs()){
      ans += dfs(S, 0x3f3f3f3f) ;
    }
    return ans ;
  }
};
```

4.6 min cut max flow

```
struct Edge{
  int v, cap, cost , next ;
};

using pii = pair<int, int> ;
class MCMF{
private:
  int N, s, t, tot ;
  vector<Edge> e ;
  vector<int> head ;
public:
  MCMF(int n, int _s, int _t){
    N = n ;
    s = _s ;
    t = _t ;
    e.resize(n*n + 5) ;
    head.assign(n+5, -1) ;
    tot = -1 ;
  }

  void AddEdge(int u, int v, int cap, int cost){
    e[++tot] = {v, cap, cost, head[u]} ;
    head[u] = tot ;
    e[++tot] = {u, 0, -cost, head[v]} ;
    head[v] = tot ;
  }

  int run(){
    vector<int> dis(N+1), pot(N+1, 0), preE(N+1) ;
    int flow = 0, cost = 0 ;

    auto dijkestra = [&](){
      fill(dis.begin(), dis.end(), INF) ;
      priority_queue<pii, vector<pii>, greater<pii>> pq ;
      dis[s] = 0 ;
      pq.push({0, s}) ;

      while(!pq.empty()){
        auto [d, u] = pq.top() ; pq.pop() ;
        if(d > dis[u]) continue ;
        for ( int i=head[u] ; i!=-1 ; i=e[i].next ){
          int v = e[i].v, cap = e[i].cap, w = e[i].cost ;
          if(cap && dis[v] > d + w + pot[u] - pot[v]){
            dis[v] = d + w + pot[u] - pot[v] ;
            preE[v] = i ;
            pq.push({dis[v], v}) ;
          }
        }
      }

      return dis[t] != INF ;
    };

    while(dijkestra()){
      for ( int v=1 ; v<=N ; v++ ) if(dis[v] < INF){
        pot[v] += dis[v] ;
      }

      int aug = INT_MAX ;
      for ( int v=t ; v!=s ; v=e[preE[v]^1].v ){
        aug = min(aug, e[preE[v]].cap) ;
      }

      for ( int v=t ; v!=s ; v=e[preE[v]^1].v ){
        e[preE[v]].cap -= aug ;
        e[preE[v]^1].cap += aug ;
        cost += aug * e[preE[v]].cost ;
      }
    }

    return cost ;
  }
};
```

5 String

5.1 KMP

```
int Next[N] ;
void kmp(string &str){
    Next[0] = -1 ;
    if(str.size() <= 1) return ;
    Next[1] = 0 ;

    int cur = 2, check = 0 ;

    while(cur < str.size()){
        if(str[cur - 1 ] == str[check])
            Next[cur++] = ++check ;
        else if(check > 0) check = Next[check] ;
        else Next[cur++] = 0 ;
```

```cpp
    }
}

int main(){
    ios::sync_with_stdio(false) ;
    cin.tie(nullptr) ;
    cout.tie(nullptr) ;

    string s1, s2 ;
    while(cin >> s1){
        s2 = s1 ;
        reverse(s2.begin(), s2.end()) ;
        kmp(s2) ;

        int x=0, y=0 ;
        while(x < s1.size() && y < s2.size()){
            if(s1[x] == s2[y]){
                x++ ;
                y++ ;
            }
            else if(y > 0) y = Next[y] ;
            else x++ ;
        }

        cout << s1 << s2.substr(y) << endl ;
    }

    return 0 ;
}
```

5.2 ACAM

```cpp
class ACAutomation{
private:
  vector<int> fail, end, order ;
  vector<vector<int>> tree ;

  int base, alpha ;

  int new_node(){
    tree.emplace_back(alpha, 0) ;
    fail.push_back(0) ;

    return tree.size() - 1 ;
  }
public:
  ACAutomation(int _base='a', int _alpha=26)
    : base(_base), alpha(_alpha) {
      clear() ;
    }

  void clear(){
    fail.assign(1, 0) ;
    order.clear() ;
    end.clear() ;
    tree.assign(1, vector<int>(alpha, 0)) ;
  }

  void add_pattern(const string &pattern){
    int u = 0 ;
    for ( auto ch : pattern ){
      int v = ch - base ;

      if(tree[u][v] == 0) tree[u][v] = new_node() ;
      u = tree[u][v] ;
    }

    end.push_back(u) ;
  }

  void build(){
    queue<int> q ;
    order.clear() ;
    order.push_back(0) ;

    for ( int i=0 ; i<alpha ; i++ ) if(tree[0][i] > 0){
      q.push(tree[0][i]) ;
    }

    while(!q.empty()){
      int u = q.front() ; q.pop() ;
      order.push_back(u) ;

      for ( int i=0 ; i<alpha ; i++ ){
        if(tree[u][i] == 0) tree[u][i] = tree[fail[u]][i] ;
        else{
          fail[tree[u][i]] = tree[fail[u]][i] ;
          q.push(tree[u][i]) ;
        }
      }
    }
  }

  vector<int> count_per_pattern(const string &text) const {
    int u = 0 ;
    vector<int> vis(tree.size(), 0) ;

    for ( char ch : text ){
      u = tree[u][ch - base] ;
      vis[u]++ ;
    }

    for ( int i=order.size()-1 ; i>=1 ; i-- ){
      int x = order[i] ;
      vis[fail[x]] += vis[x] ;
    }

    vector<int> ans(end.size(), 0) ;
    for ( int id=0 ; id<end.size() ; id++ ){
      ans[id] = vis[end[id]] ;
    }

    return ans ;
  }
};
```

6 Techniques

6.1 二分搜

```cpp
// xxxxxoooo 找最小解
bool binary_search(){
    while(l < r){
        int m = (l + r) >> 1 ;
        if(check(m)) r = m ;
        else l = m + 1 ;
    }

    return l ;
}

// ooooxxxx 找最大解
bool binary_search(){
    while(l < r){
        int m = (l + r) >> 1 ;
        if(check(m)) l = m ;
        else r = m - 1 ;
    }

    return l ;
}

// 如果l & r 太大, m = (l + (r - l)) >> 1 ;
```

6.2 倍增 LCA

```cpp
#include <bits/stdc++.h>

using namespace std ;

const int Maxn = 500005 ;

vector<int> e[Maxn] ;
int depth[Maxn] ;
int up[Maxn][40] ;
int MaxLog ;

void dfs(int u, int from, int d){
  up[u][0] = from ;
  depth[u] = d ;

  for ( int i=1 ; i<=MaxLog ; i++ ){
    up[u][i] = up[up[u][i - 1]][i - 1] ;
  }

  for ( auto v : e[u] ){
    if(v == from) continue ;
    dfs(v, u, d + 1) ;
  }
}

int lca(int u, int v){
  if(depth[u] < depth[v]) swap(u, v) ;

  for ( int i=MaxLog ; i>=0 ; i-- ) if(depth[u] - (1 << i) >= depth[v]){
    u = up[u][i] ;
  }

  if(u == v) return u ;

  for ( int i=MaxLog ; i>=0 ; i-- ) if(up[u][i] != up[v][i]){
    u = up[u][i] ;
    v = up[v][i] ;
  }

  return up[u][0] ;
}

int main(){
  int n, q, root ;
  scanf("%d%d%d", &n, &q, &root) ;
  MaxLog = __lg(n) ;

  for ( int i=0 ; i<n-1 ; i++ ){
    int u, v ;
    scanf("%d%d", &u, &v) ;
    e[u].push_back(v) ;
    e[v].push_back(u) ;
  }

  dfs(root, root, 0) ;

  while(q--){
    int u, v ;
    scanf("%d%d", &u, &v) ;
    printf("%d\n", lca(u, v)) ;
  }

  return 0 ;
}
```

6.3 SG

```cpp
long long SG(long long k){

  if(k % 2 == 0){
    return k / 2;
  }
  else{
    return SG(k / 2);
  }

}
int main(){
  int cas, n;

  scanf("%d", &cas);
  while(cas--){
    scanf("%d", &n);

    long long s, v = 0;

    for(int i = 0; i < n; i++){
      scanf("%lld", &s);
      v ^= SG(s); //XOR
    }

    if(v) printf("YES\n");
    else printf("NO\n");
  }
}

int SG[30] ;
int vis[Maxn], stone[Maxn] ;

void build(){
  SG[0] = 0 ;
  memset(vis, 0, sizeof(vis)) ;

  for ( int i=1 ; i<30 ; i++ ){
    int cur = 0 ;
    for ( int j=0 ; j<i ; j++ ) for ( int k=0 ; k<=j ; k++ ){
      vis[SG[j] ^ SG[k]] = i ;
    }
    while(vis[cur] == i) cur++ ;
    SG[i] = cur ;
  }
}

int main(){
  build() ;

  int T = 0 ;
  while(~scanf("%d", &n) && n){
    int ans = 0 ;

    for ( int i=1 ; i<=n ; i++ ) scanf("%d", &stone[i]) ;

    for ( int i=1 ; i<=n ; i++ ) if(stone[i] & 1){
      ans ^= SG[n-i] ;
    }
  }
}
```

7 DP

7.1 輪廓線 DP

```
#include <bits/stdc++.h>

using namespace std ;
using ll = long long ;

ll dp[2][(1 << 10) + 5] ;
int n, m ;
int cur ;

void update(int s1, int s2){
  if(s2 & (1 << m)){
    dp[cur][s2 ^ (1 << m)] += dp[cur ^ 1][s1] ;
  }
}

int main(){
  while(~scanf("%d%d", &n, &m)){
    if(m > n) swap(n, m) ;
    memset(dp, 0, sizeof(dp)) ;
    cur = 0 ;
    dp[cur][(1 << m) - 1] = 1 ;
    for ( int i=0 ; i<n ; i++ ) for ( int j=0 ; j<m ; j++ ){
      cur ^= 1 ;
      memset(dp[cur], 0, sizeof(dp[cur])) ;

      for ( int k=0 ; k<(1 << m) ; k++ ){
        update(k, k << 1) ; // not put
        if(i && !(k & (1 << (m - 1)))) update(k, (k << 1) | (1 << m) | 1) ; // put up
        if(j && !(k & 1)) update(k, (k << 1) | 3) ; // put left
      }
    }
    printf("%lld\n", dp[cur][(1 << m) - 1]) ;
  }
  return 0 ;
}
```

7.2 數位 DP

```
#include <bits/stdc++.h>

using namespace std ;

int K ;
int dp[20][105][105][2] ;
vector<int> dig ;

int solve(int pos, int sum, int dsum, bool lim){
    if(pos == -1){
        if(sum == 0 && dsum == 0) return 1 ;
        return 0 ;
    }

    int &d = dp[pos][sum][dsum][lim] ;
    if(d != -1) return d ;

    int up = lim ? dig[pos] : 9 ;
    int res = 0 ;
    for ( int i=0 ; i<=up ; i++ ){
        res += solve(pos-1, (sum * 10 + i) % K, (dsum + i) % K, lim && i==up) ;
    }

    return d = res ;
}

int count(int n){
    memset(dp, -1, sizeof(dp)) ;
    dig.clear() ;

    while(n > 0){
        dig.push_back(n % 10) ;
        n /= 10 ;
    }

    return solve(dig.size() - 1, 0, 0, 1) ;
}

int main(){
    int T ;
    scanf("%d", &T) ;

    int a, b ;
    while(T--){
        scanf("%d%d%d", &a, &b, &K) ;
        if(K > 90) printf("0\n") ;
        else printf("%d\n", count(b) - count(a-1)) ;
    }

    return 0 ;
}
```

7.3 樹 DP

```
#include <bits/stdc++.h>

#define N 505
#define INF 0x3f3f3f3f

using namespace std ;

struct Edge{
  int v, w ;
} ;

vector<Edge> edge[N] ;
int n ;
int cnt[N] ;
int dp[N][N][2] ;

void init(){
  for ( int i=0 ; i<N ; i++ ) edge[i].clear() ;
  memset(cnt, 0, sizeof(cnt)) ;
  memset(dp, INF, sizeof(dp)) ;
}

void DFS(int u){
  cnt[u] = 1 ;
  for ( auto [v, w] : edge[u] ){
    DFS(v) ;
    cnt[u] += cnt[v] ;
  }

  dp[u][1][0] = dp[u][1][1] = 0 ;

  for ( auto [v, w] : edge[u] ){
    for ( int i=cnt[u] ; i>1 ; i-- ) for ( int j=1 ; j<i && j<=cnt[v] ; j++ ){
      dp[u][i][1] = min(dp[u][i][1], dp[u][i-j][1] + dp[v][j][1] + 2 * w) ;
      dp[u][i][0] = min(dp[u][i][0], dp[u][i-j][1] + dp[v][j][0] + w) ;
      dp[u][i][0] = min(dp[u][i][0], dp[u][i-j][0] + dp[v][j][1] + 2 * w) ;
    }
  }
}

int main(){
  int t = 0 ;

  while(~scanf("%d", &n) && n){
    init() ;
    for ( int i=0 ; i<n-1 ; i++ ){
      int u, v, w ;
      scanf("%d%d%d", &v, &u, &w) ;
      edge[u].push_back({v, w}) ;
    }

    DFS(0) ;
    printf("Case %d:\n", ++t) ;

    int q, e ;
    scanf("%d", &q) ;

    while(q--){
      scanf("%d", &e) ;

      for ( int i=n ; i>=1 ; i-- )
           if(dp[0][i][0] <= e){
        printf("%d\n", i) ;
        break ;
      }
    }
  }

  return 0 ;
}
```