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1 Foundations

1.1 PyMath

```

1 import math
2
3     math.ceil(x) #上高斯
4     math.floor(x) #下高斯
5     math.factorial(x) #階乘
6     math.fabs(x) #絕對值
7     math.fsum(arr) #求和
8     math.gcd(x, y)
9     math.exp(x) # e^x
10    math.log(x, base)
11    math.log2(x)
12    math.log10(x)
13    math.sqrt(x)
14    math.pow(x, y, mod)
15    math.sin(x) # cos, tan, asin, acos, atan,
16      atan2, sinh ...
17    math.hypot(x, y) #歐幾里德範數
18    math.degrees(x) #x從弧度轉角度
19    math.radians(x) #x從角度轉弧度
20    math.gamma(x) #x的gamma函數
21    math.pi #const
22    math.e #const
23    math.inf

```

1.2 Java Integer

```

1 // 常量
2 MAX_VALUE, MIN_VALUE, BYTES, SIZE, TYPE
3
4 // 轉換/解析
5 static int parseInt(String s)
6 static int parseInt(String s, int radix)
7 static int parseUnsignedInt(String s)
8 static int parseUnsignedInt(String s, int
9   radix)
10 static Integer valueOf(int i)
11 static Integer valueOf(String s)
12 static Integer valueOf(String s, int radix)
13 static String toString(int i)
14 static String toString(int i, int radix)
15 static String toUnsignedString(int i)
16 static String toUnsignedString(int i, int
17   radix)
18 static long toUnsignedLong(int x)
19 static Integer decode(String nm)
20   // 支援 0x/0/# 前綴
21 static Integer getInteger(String nm[, int
22   val]) // 從系統屬性讀取整數
23
24 // 比較/雜湊/聚合
25 static int compare(int x, int y)
26 static int compareUnsigned(int x, int y)
27 static int hashCode(int value)
28 static int min(int a, int b)
29 static int max(int a, int b)
30 static int sum(int a, int b)
31
32 // 位元操作
33 static int bitCount(int i) // 設定位數
34 static int highestOneBit(int i)
35 static int lowestOneBit(int i)
36 static int numberOfLeadingZeros(int i)
37 static int numberOfTrailingZeros(int i)
38 static int rotateLeft(int i, int distance)
39 static int rotateRight(int i, int distance)
40 static int reverse(int i)
41 static int reverseBytes(int i)
42
43 // 無號運算
44 static int divideUnsigned(int dividend, int
45   divisor)

```

```

41 static int remainderUnsigned(int dividend,
42   int divisor)

```

1.3 Java String

```

1 // 查詢
2 int length()
3 boolean isEmpty()
4 boolean isBlank() // (since 11)
5 char charAt(int index)
6 int codePointAt(int index)
7 int codePointBefore(int index)
8 int codePointCount(int beginIndex, int
9   endIndex)
10 boolean contains(CharSequence s)
11 boolean startsWith(String prefix[, int
12   toffset])
13 boolean endsWith(String suffix)
14 int indexOf(String str[, int fromIndex])
15 int lastIndexOf(String str[, int
16   fromIndex])
17 // 取子字串/子序列
18 String substring(int beginIndex)
19 String substring(int beginIndex, int
20   endIndex)
21 CharSequence subSequence(int beginIndex, int
22   endIndex)
23
24 // 比較/等價
25 boolean equals(Object obj)
26 boolean equalsIgnoreCase(String
27   anotherString)
28 int compareTo(String anotherString)
29 int compareToIgnoreCase(String str)
30 boolean matches(String regex)
31 boolean regionMatches(int toffset, String
32   other, int offset, int len)
33 boolean regionMatches(boolean ignoreCase,
34   int toffset, String other, int offset,
35   int len)
36
37 // 建構/轉換/連接
38 String concat(String str)
39 String replace(char oldChar, char newChar)
40 String replace(CharSequence target,
41   CharSequence replacement)
42 String replaceAll(String regex, String
43   replacement)
44 String replaceFirst(String regex, String
45   replacement)
46 String[] split(String regex[, int limit])
47 String toLowerCase()
48 String toUpperCase()
49 String trim()
50 String strip() // (since 11)
51 String stripLeading() // (since 11)
52 String stripTrailing() // (since 11)
53 String repeat(int count) // (since 11)
54 IntStream chars()
55 Stream<String> lines() // (since 11)
56 String intern()
57
58 // 靜態工具
59 static String format(String format,
60   Object... args)
61 static String join(CharSequence delimiter,
62   CharSequence... elements)
63 static String join(CharSequence delimiter,
64   Iterable<? extends CharSequence>
65   elements)
66 static String
67   valueOf(primitive/char[]/Object)
68 static String copyValueOf(char[] data[, int
69   offset, int count])

```

1.4 Java String builder

```

1 // 長度/容量
2 int length()
3 int capacity()
4 void ensureCapacity(int minimumCapacity)
5 void trimToSize()
6 void setLength(int newLength)
7
8 // 存取/修改
9 char      charAt(int index)
10 void     setCharAt(int index, char ch)
11 StringBuilder append(... 各種型別 ...)
12 StringBuilder insert(int offset, ... 各種型別
... )
13 StringBuilder delete(int start, int end)
14 StringBuilder deleteCharAt(int index)
15 StringBuilder replace(int start, int end,
    String str)
16 StringBuilder reverse()
17
18 // 子字串/查找
19 String      substring(int start)
20 String      substring(int start, int end)
21 CharSequence subSequence(int start, int end)
22 int         indexOf(String str[], int
    fromIndex])
23 int         lastIndexOf(String str[], int
    fromIndex])
24
25 // 轉換
26 String toString()

```

1.5 Java Math

```

1 // 常量
2 static final double E, PI
3
4 // 絶對值/比較
5 static int/long/float,double abs(x)
6 static T max(a, b)
7 static T min(a, b)
8
9 // 取整/四捨五入
10 static double floor(double a)
11 static double ceil(double a)
12 static double rint(double a)          // 最接近整數(偶數優先)
13 static long round(double a) / int
    round(float a)
14 static int   floorDiv(int x, int y)
15 static int   floorMod(int x, int y)
16
17 // 溢位保護(exact 系列, Java 8+)
18 static int/long addExact(a, b)
19 static int/long subtractExact(a, b)
20 static int/long multiplyExact(a, b)
21 static int/long incrementExact(a)
22 static int/long decrementExact(a)
23 static int   toIntExact(long value)
24 static int/long negateExact(a)
25
26 // 指對數/冪根
27 static double pow(double a, double b)
28 static double sqrt(double a)
29 static double cbrt(double a)
30 static double exp(double a)
31 static double expm1(double x)
32 static double log(double a)
33 static double log10(double a)
34 static double log1p(double x)
35
36 // 三角/雙曲
37 static double sin/cos/tan(double a)
38 static double asin/acos/atan(double a)
39 static double atan2(double y, double x)

```

```

40 static double sinh/cosh/tanh(double a)
41
42 // 其他實用
43 static double hypot(double x, double y)
44 static double toDegrees(double angrad)
45 static double toRadians(double angdeg)
46 static double copySign(double magnitude,
    double sign)
47 static double nextUp/nextDown(double a)
48 static double nextAfter(double start, double
    direction)
49 static double ulp(double d)
50 static double random()
51 static double scalb(double d, int
    scaleFactor)
52 static double fma(double a, double b, double
    c) // (since 8)
53 static long multiplyHigh(long x, long y)
    // (since 9)
54 static long multiplyFull(int x, int y)
    // (since 9, 回傳 long)

```

2 Mathematics & Number Theory

2.1 Number Theory

Fermat's Little Theorem:

$$a^{p-1} \equiv 1 \pmod{p} \quad (\gcd(a, p) = 1, p \text{ prime})$$

Euler's Theorem:

$$a^{\varphi(n)} \equiv 1 \pmod{n} \quad (\gcd(a, n) = 1)$$

Modular Inverse:

$$a^{-1} \equiv a^{p-2} \pmod{p} \quad (\gcd(a, p) = 1, p \text{ prime})$$

Euler Totient:

$$\varphi(n) = n \prod_{p|n} \left(1 - \frac{1}{p}\right)$$

Fast Modular Exponentiation

```

long long mod_pow(long long a, long long b,
    long long mod) {
    long long res = 1 % mod;
    while (b > 0) {
        if (b & 1) res = res * a % mod;
        a = a * a % mod;
        b >>= 1;
    }
    return res;
}

```

Counting Coprimes Below n

$$\forall n > 0, \sum_{d|n} \varphi(d) = n, \quad \sum_{d|n} \mu(d) \left\lfloor \frac{n}{d} \right\rfloor = 1,$$

其中 μ 為莫比烏斯函數；常用於反演及計算互質對數量。

Modulo Arithmetic Quick Facts

$$\begin{aligned} (a \pm b) \bmod m &= ((a \bmod m) \pm (b \bmod m)) \bmod m, \\ (ab) \bmod m &= ((a \bmod m)(b \bmod m)) \bmod m, \\ (a^k) \bmod m &= ((a \bmod m)^k) \bmod m, \\ (-a) \bmod m &= (m - (a \bmod m)) \bmod m. \end{aligned}$$

若 $\gcd(a, m) = 1$ ，可計算乘法逆元 a^{-1} 並套用 $(a/b) \bmod m \equiv a \cdot b^{-1} \bmod m$ 。

Congruence ($a \equiv b \pmod{m}$) Essentials

- $a \equiv b \pmod{m} \iff m | (a - b)$ ，同餘類以差除判斷。
- $a \equiv b \pmod{m} \Rightarrow f(a) \equiv f(b) \pmod{m}$ 對所有以整數係數的多項式 f 成立。
- 若 $a \equiv b \pmod{m}$ 且 $c \equiv d \pmod{m}$ ，則 $a \pm c \equiv b \pm d \pmod{m}$, $ac \equiv bd \pmod{m}$, $a^n \equiv b^n \pmod{m}$ 。

- 若 $\gcd(k, m) = 1$ 且 $ka \equiv kb \pmod{m}$ ，可約去 $k: a \equiv b \pmod{m}$ 。
- 若 $a \equiv b \pmod{m}$ ，則 $a \equiv b \pmod{d}$ 對任何 d 整除 m 亦成立。
- 若 $d | a, b, m$ ，則 $\frac{a}{d} \equiv \frac{b}{d} \pmod{\frac{m}{d}}$

Bézout's Identity

若 $a, b \in \mathbb{Z}, \exists x, y \in \mathbb{Z}$ 使得 $ax + by = \gcd(a, b)$.

任何方程 $ax + by = c$ 有整數解當且僅當 $\gcd(a, b) | c$ 。擴展歐幾里得演算法可同時求得 \gcd 與一組 (x, y) ，常用於計算模逆與結合 China Remainder。

// Extended Euclidean algorithm 給定 a, b, c ，求 $ax + by = c$ 的解

```

1  ll extgcd(ll a, ll b, ll c, ll &x, ll
    &y){
2      if(b == 0){
3          x = c/a ;
4          y = 0 ;
5          return a ;
6      }
7      ll d = extgcd(b, a%b, c, x, y), tmp =
8          x ;
9      x = y ;
10     y = tmp - (a/b)*y ;
11     return d ;
}

```

根據貝祖定理，只有當 $d | c$ 時， $ax + by = c$ 才有整數解

Chinese Remainder Theorem

設同餘系統

$$x \equiv a_i \pmod{m_i} \quad (i = 1, \dots, k),$$

其中 m_i 兩兩互質，令 $M = \prod_{i=1}^k m_i$, $M_i = M/m_i$ ，再取 $t_i \equiv M_i^{-1} \pmod{m_i}$ 。則唯一解 (模 M) 為

$$x \equiv \sum_{i=1}^k a_i M_i t_i \pmod{M}.$$

兩式合併 (允許非互質)

```

1 // solve x a1 (mod m1), x a2 (mod m2)
2 // return {x0, lcm}; if no solution, lcm = -1
3 pair<ll, ll> crt(ll a1, ll m1,
4                   ll a2, ll m2) {
5     ll g = std::gcd(m1, m2);
6     if ((a2 - a1) % g != 0) return {0, -1};
    // no solution
7
8     ll lcm = m1 / g * m2;
9     ll m1_reduced = m1 / g;
10    ll m2_reduced = m2 / g;
11
12    ll diff = (a2 - a1) / g % m2_reduced;
13    if (diff < 0) diff += m2_reduced;
14
15    ll inv = mod_pow(m1_reduced, m2_reduced
    - 1, m2_reduced);
16    ll step = diff * inv % m2_reduced;
17    ll x0 = (a1 + step * m1) % lcm;
18    if (x0 < 0) x0 += lcm;
19    return {x0, lcm};
}

```

遞增地將每個同餘式與當前解做合併即可取得最終答案，也能偵測無解情況。給定 a, b, c ，求 $ax + by = c$ 的解

```

1  ll extgcd(ll a, ll b, ll c, ll &x, ll
    &y){
2      if(b == 0){
3          x = c/a ;
4          y = 0 ;
5          return a ;
6      }
}

```

```

7     ll d = extgcd(b, a%b, c, x, y), tmp =
8         x ;
9     x = y ;
10    y = tmp - (a/b)*y ;
11    return d ;
}

```

2.2 Combinatorics

Binomial Coefficient Identities

$$\begin{aligned} \binom{n}{k} &= \frac{n!}{k!(n-k)!}, \\ \binom{n}{k} &= \binom{n-1}{k} + \binom{n-1}{k-1}, \\ \sum_{k=0}^n \binom{n}{k} &= 2^n, \quad \sum_{k=0}^n k \binom{n}{k} = n2^{n-1}. \end{aligned}$$

Stars and Bars 非負整數解數量：

$$x_1 + x_2 + \dots + x_k = n \Rightarrow \binom{n+k-1}{k-1}.$$

若各變數至少為 1，將 $x_i = y_i + 1$ 轉為非負情況即可。

Inclusion-Exclusion Principle 對集合 A_1, \dots, A_k :

$$\left| \bigcup_{i=1}^k A_i \right| = \sum_{i=1}^k |A_i| - \sum_{1 \leq i < j \leq k} |A_i \cap A_j| - \dots + (-1)^{k-1} |A_1 \cap \dots \cap A_k|.$$

計算滿足限制的排列或整數解時廣泛使用。

Catalan Numbers 基本定義：

$$C_n = \frac{1}{n+1} \binom{2n}{n}, \quad C_{n+1} = \frac{4n+2}{n+2} C_n.$$

$$C = 1, 1, 2, 5, 14, 42, 132, 429, 1430, 4862, \dots$$

常見應用包含合法括號、凸多邊形三角剖分、二元樹結構計數等。

2.3 Geometry

對於 V 個點，E 條邊，F 個面，C 個連通分量

$$V + F = E + 2$$

$$V + F = E + C + 1$$

點座標均是整數或是正方形格子點的簡單多邊形，其面積 A 和內部點數量 i，邊上格點數量 b 的關係為

$$A = i + \frac{b}{2} - 1$$

3 Data Structure

3.1 MST

```

1 struct Edge{
2     int u, v, w ;
3     // 這是最大生成樹，最小生成樹要改成 w < o.w
4     bool operator>(const Edge &o) const
5     {return w > o.w ;} ;
6 }
7 int par[N] ;
8 int sz[N] ;
9 int sum ;

```

```

10
11 vector<Edge> edge ;
12
13 void init(){
14     edge.clear() ;
15     for ( int i=0 ; i<N ; i++ ){
16         par[i] = i ;
17         sz[i] = 1 ;
18     }
19     sum = 0 ;
20 }
21
22 int find(int x){
23     if(x == par[x]) return x ;
24     return par[x] = find(par[x]) ;
25 }
26
27 int merge(int x, int y){
28     x = find(x) ;
29     y = find(y) ;
30
31     if(x == y) return 0 ;
32     if(sz[x] > sz[y]) swap(x, y) ;
33     par[x] = y ;
34     sz[y] += sz[x] ;
35
36     return 1 ;
37 }
38
39 void MST(){
40     int cnt = 0 ;
41     for ( int i=0 ; i<edge.size() && cnt < n-1
42             ; i++ ){
43         auto [u, v, w] = edge[i] ;
44         if(merge(u, v)){
45             cnt++ ;
46             sum -= w ;
47         }
48     }
49
50     int main(){
51         for ( int i=0 ; i<m ; i++ ){
52             scanf("%d%d%d", &u, &v, &w) ;
53             edge.push_back({u, v, w}) ;
54             sum += w ;
55         }
56
57         sort(edge.begin(), edge.end(),
58              greater<Edge>()) ;
59         MST() ;
60     }
61
62     for ( int i=0 ; i<n ; i++ ){
63         printf("%d %d %d\n", i, par[i], sz[i]) ;
64     }
65
66     cout << "Sum: " << sum ;
67 }
68
69
70
71 #define lc (id << 1)
72 #define rc ((id << 1) | 1)
73
74 struct LazyTag{
75     // type 0 : increase val
76     // type 1 : set to val
77     // type 1 can overwrite type 0
78     int type ;
79     ll val ;
80 } ;
81
82 struct Node{
83     LazyTag tag ;
84     ll sum ;
85     int sz ;
86 }seg[Maxn << 2] ;
87
88 class SegmentTree{
89 private:
90     void pull(int id){
91         seg[id].sum = seg[lc].sum +
92                     seg[rc].sum ;
93     }
94
95     void AddTag(int id, LazyTag &tag){
96         if(tag.type == 0){
97             seg[id].sum += tag.val *
98                         seg[id].sz ;
99             seg[id].tag.val += tag.val ;
100        }
101        else{
102            seg[id].sum = tag.val *
103                         seg[id].sz ;
104            seg[id].tag = {1, tag.val} ;
105        }
106    }
107
108    void push(int id){
109        AddTag(lc, seg[id].tag) ;
110        AddTag(rc, seg[id].tag) ;
111        seg[id].tag = {0, 0} ;
112    }
113
114 public:
115     void build(int L=1, int R=n, int id=1){
116         seg[id].sum = 0 ;
117         seg[id].tag = {0, 0} ;
118         seg[id].sz = 1 ;
119
120         if(L == R){
121             seg[id].sum = arr[L] ;
122             return ;
123         }
124
125         int M = (L + R) >> 1 ;
126         build(L, M, lc) ;
127         build(M+1, R, rc) ;
128
129         pull(id) ;
130         seg[id].sz = seg[lc].sz + seg[rc].sz ;
131     }
132
133     void modify(int l, int r, LazyTag &tag,
134                int L=1, int R=n, int id=1){
135         if(l <= L && R <= r){
136             AddTag(id, tag) ;
137             return ;
138         }
139
140         push(id) ;
141         int M = (L + R) >> 1 ;
142         if(r <= M) modify(l, r, tag, L, M,
143                           lc) ;
144         else if(l > M) modify(l, r, tag, M+1,
145                               R, rc) ;
146         else{
147             modify(l, r, tag, L, M, lc) ;
148             modify(l, r, tag, M+1, R, rc) ;
149         }
150         pull(id) ;
151     }
152
153     ll query(int l, int r, int L=1, int R=n,
154              int id=1){
155         if(l <= L && R <= r) return
156             seg[id].sum ;
157
158         push(id) ;
159         int M = (L + R) >> 1 ;
160         if(r <= M) return query(l, r, L, M,
161                                 lc) ;
162         else if(l > M) return query(l, r,
163                                     M+1, R, rc) ;
164         else return query(l, r, L, M, lc) +
165             query(l, r, M+1, R, rc) ;
166     }
167 }
168 }
```

3.3 HLD

```

70 // normal modify
71 }
72 ColorSeg query(int l, int r, int L=1, int
73 R=n, int id=1){
74 // normal query
75 }
76 public:
77 void build(int L=1, int R=n, int id=1){
78 // normal build
79 }
80
81 // update val from u to v (simple path)
82 void update(int u, int v, int val){
83 while(top[u] != top[v]){
84 if(dep[top[u]] < dep[top[v]]) swap(u,
85 v) ;
86 modify(dfn[top[u]], dfn[u], val) ;
87 u = fa[top[u]] ;
88 }
89 if(dep[u] < dep[v]) swap(u, v) ;
90 modify(dfn[v], dfn[u], val) ;
91 }
92
93 // get sum from u to v (simple path)
94 int get(int u, int v){
95 pair<int, ColorSeg> U, V ;
96 ColorSeg M ;
97 U = {u, {0, 0, 0}} ;
98 V = {v, {0, 0, 0}} ;
99
100 while(top[U.first] != top[V.first]){
101 if(dep[top[U.first]] <
102 dep[top[V.first]]) swap(U, V) ;
103 U.second = query(dfn[top[U.first]],
104 dfn[U.first]) + U.second ;
105 U.first = fa[top[U.first]] ;
106 }
107 if(dep[U.first] < dep[V.first]) swap(U,
108 V) ;
109
110 M = query(dfn[V.first], dfn[U.first]) ;
111
112 return (U.second.tot + V.second.tot +
113 M.tot) - (U.second.left == M.right)
114 - (V.second.left == M.left) ;
115 }
116 }tree ;
117
118 void init(){
119 memset(son, -1, sizeof(son)) ;
120 }

```

3.4 PST

3.5 Trie

```

1 class TrieNode{
2 public:
3     set<int> end ;
4     TrieNode *next[26] ;
5 
6     TrieNode(){
7         for ( int i=0 ; i<26 ; i++ ) next[i]
8             = nullptr ;
9     }
10 }
11 class Trie{
12 private:
13     int cnt ;
14     TrieNode *root ;
15 public:
16     Trie() : cnt(0) {
17         root = new TrieNode() ;
18     }
19 
20     void insert(string &str, int n){
21         TrieNode* node = root ;
22         for ( auto s : str ){
23             int path = s - 'a' ;
24 
25             if(node->next[path] == nullptr)
26                 node->next[path] = new
27                     TrieNode() ;
28             node = node->next[path] ;
29         }
30         node->end.insert(n) ;
31     }
32 
33     void search(string &str){
34         TrieNode* node = root ;
35         for ( auto s : str ){
36             int path = s - 'a' ;
37             if(node->next[path] == nullptr)
38                 return ;
39             node = node->next[path] ;
40         }
41 
42         int flg = 0 ;
43         for ( auto n : node->end ){
44             if(flg) cout << " " ;
45             else flg = 1 ;
46 
47             cout << n ;
48         }
49 
50         void clear(TrieNode* node) {
51             if (!node) return ;
52             for ( int i = 0; i < 26; i++ ) {
53                 if (node->next[i]) {
54                     clear(node->next[i]) ;
55                 }
56             }
57             delete node ;
58         }
59 
60         ~Trie(){
61             clear(root) ;
62         }
63     }

```

3.6 BIT 單修區查

```

1 // 單點修改 區間查詢
2 #define lowbit(x) (x & -x)
3 
4 int BIT[MAX_SIZE] ;
5 int n ;
6 
7 void modify(int idx, int val){
8     for ( ; idx <= n ; idx += lowbit(idx) ){
9         BIT[idx] += val ;
10    }
11 }
12 
13 // range query: query(R) - query(X - 1)
14 ll query(int idx){
15     ll sum1 = 0, sum2 = 0 ;
16     for ( int i=idx ; i ; i-=lowbit(i) ){
17         sum1 += BIT[i] ;
18         sum2 += BIT2[i] ;
19    }
20 }
21 
22 void update(int idx, ll val){
23     for ( int i=idx ; i<=n ; i+=lowbit(i) ){
24         BIT[i] += val ;
25         BIT2[i] += idx * val ;
26     }
27 }
28 
29 void init(){
30     memset(BIT, 0, sizeof(BIT)) ;
31 }
32 
```

```

10    }
11 }
12 
13 // ans: query(R) - query(L-1)
14 int query(int idx){
15     int sum = 0 ;
16     for ( ; idx ; idx -= lowbit(idx) ){
17         sum += BIT[idx] ;
18     }
19 }
20 
21 void init(){
22     memset(BIT, 0, sizeof(BIT)) ;
23 }
24 
25 void build(){
26     for ( int i=1 ; i<=n ; i++ ){
27         update(i, arr[i] - arr[i-1]) ;
28     }
29 }
30 
31 void usage(){
32     update(L, x) ;
33     update(R+1, -x) ;
34 }
35 
36 void init(){
37     memset(BIT1, 0, sizeof(BIT1)) ;
38     memset(BIT2, 0, sizeof(BIT2)) ;
39 }
40 
```

3.7 BIT 區修單查

```

1 // 區間修改，單點查詢
2 #define lowbit(x) (x & -x)
3 
4 int BIT[MAX_SIZE] ;
5 int n ;
6 
7 void modify(int idx, int val){
8     for ( ; idx <= n ; idx += lowbit(idx) ){
9         BIT[idx] += val ;
10    }
11 }
12 
13 // ans: query(i)
14 int query(int idx){
15     int sum = 0 ;
16     for ( ; idx ; idx -= lowbit(idx) ){
17         sum += BIT[idx] ;
18     }
19 
20     return sum ;
21 }
22 
23 void init(){
24     memset(BIT, 0, sizeof(BIT)) ;
25 }
26 
27 void build(){
28     arr[0] = 0 ;
29     for ( int i=1 ; i<=n ; i++ ) modify(i,
30             arr[i] - arr[i-1]) ;
31 }
32 
33 // usage
34 // add val
35 modify(L, x) ;
36 modify(R+1, -x) ;
37 
```

3.8 BIT 區修區查

```

1 // 區間修改 區間查詢
2 #define lowbit(x) (x & -x)
3 
4 ll BIT1[MAXN], BIT2[MAXN] ;
5 
6 void update(int idx, ll val){
7     for ( int i=idx ; i<=n ; i+=lowbit(i) ){
8         BIT1[i] += val ;
8         BIT2[i] += idx * val ;
9     }
10 }
11 
12 
13 // range query: query(R) - query(X - 1)
14 ll query(int idx){
15     ll sum1 = 0, sum2 = 0 ;
16     for ( int i=idx ; i ; i-=lowbit(i) ){
17         sum1 += BIT1[i] ;
18         sum2 += BIT2[i] ;
19    }
20 }
21 
```

4 Graph

4.1 cut vertex AND bridges

```

1 int dfn[Maxn] = {-1}, low[Maxn] = {-1},
2     dfscnt ;
3 
4 void dfs(int u, int fa){
5     dfn[u] = low[u] = ++dfscnt ;
6     int child = 0 ;
7 
8     for ( auto v : g[u] ) if(v != fa){
9         if(dfn[v] == -1){
10             child++ ;
11             dfs(v, u) ;
12             low[u] = min(low[u], low[v]) ;
13 
14             if(low[v] >= dfn[u]){
15                 // this edge is a bridge
16             }
17 
18             if(u != fa && low[v] >= dfn[u]){
19                 // this node v is a articulation point
20             }
21             else low[u] = min(low[u], dfn[v]) ;
22         }
23 
24         if(u == fa && child > 1){
25             // this node u is a articulation point
26         }
27     }
28 }
```

4.2 SCC - Tarjan

```

1 vector<int> scc[Maxn] ;
2 int dfn[Maxn], low[Maxn], sccId[Maxn],
3     dfscnt = 0, cnt_scc = 0 ;
4 stack<int> st ;
5 bitset<Maxn> inSt, vis ;
6 
7 void dfs(int u, int from){
8     dfn[u] = low[u] = ++dfscnt ;
8     st.push(u) ;
9     inSt[u] = 1 ;
10 
11     for ( auto v : g[u] ){
12         if(!inSt[v] && dfn[v] != -1) continue ;
13         if(dfn[v] == -1) dfs(v, u) ;
14         low[u] = min(low[u], low[v]) ;
15     }
16 
17     if(dfn[u] == low[u]){
18 
```

```

18     cnt_scc++ ;
19     int x ;
20
21     do{
22         x = st.top() ;
23         st.pop() ;
24
25         inSt[x] = 0 ;
26         sccId[x] = cnt_scc ;
27         scc[cnt_scc].push_back(x) ;
28     }
29     while(x != u) ;
30 }
31 }
32 // SCC to DAG (after dfs)
33 vector<int> dag[Maxn] ;
34
35 void scc_to_dag(){
36     vector<int> dag[Maxn] ;
37     for ( int u=1 ; u<=n ; u++ ){
38         for ( auto v : g[u] ){
39             if(sccId[u] != sccId[v]){
40                 dag[sccId[u]].push_back(sccId[v]) ;
41             }
42         }
43     }
44 }
45
46 void init(){
47     memset(dfn, -1, sizeof(dfn)) ;
48     memset(low, -1, sizeof(low)) ;
49 }
50
51 int main(){
52     init() ;
53     input() ;
54     for ( int i=1 ; i<=n ; i++ ) if(dfn[i]
55         == -1){
56         dfs(i, i) ;
57     }
58 }
```

4.3 BCC - Tarjan

```

1 struct Edge{
2     int v, next ;
3 }e[Maxm << 1] ;
4 int head[Maxm], tot = 1 ;
5
6 void add(int u, int v){
7     e[++tot] = {v, head[u]} ;
8     head[u] = tot ;
9     e[++tot] = {u, head[v]} ;
10    head[v] = tot ;
11 }
12
13 bitset<Maxm << 1> bz ;
14 vector<vector<int>> bcc ;
15 int dfn_cnt = 0, dfn[Maxn], low[Maxn],
16     vis_bcc[Maxn], bcc_cnt = 0 ;
17
18 void dfs1(int u, int from){
19     dfn[u] = low[u] = ++dfn_cnt ;
20
21     for ( int i=head[u] ; i!= -1 ; i=e[i].next
22         ){
23         int v = e[i].v ;
24
25         if(dfn[v] == -1){
26             dfs1(v, i) ;
27             if(dfn[u] < low[v]) bz[i] = bz[i^1] =
28                 1 ;
29             low[u] = min(low[u], low[v]) ;
30         }
31     else if(i != (from ^ 1)) low[u] =
32         min(low[u], dfn[v]) ;
33 }
```

4.4 Convex

```

1 struct Coordinate{
2     long long x, y ;
3
4     friend bool operator<(const Coordinate&a,
5                             const Coordinate& b){
6         if(a.x == b.x) return a.y < b.y ;
7         return a.x < b.x ;
8     }
9     friend bool operator==(const Coordinate&
10                            a, const Coordinate& b){
11         return a.x == b.x && a.y == b.y ;
12     }
13     vector<Coordinate> nodes ;
14
15     long long cross(const Coordinate& o, const
16                      Coordinate& a, const Coordinate& b){
17         return (a.x - o.x) * (b.y - o.y) - (a.y -
18             o.y) * (b.x - o.x) ;
19     }
20     void input(){
21         nodes.clear() ;
22
23         int n, x, y ;
24         char c ;
25         cin >> n ;
26
27         for ( int i=0 ; i<n ; i++ ){
28             cin >> x >> y >> c ;
29             if(c == 'Y') nodes.push_back({x, y}) ;
30         }
31     }
32
33     void monotone(){
34         sort(nodes.begin(), nodes.end()) ;
35 }
```

```

35
36     int n = unique(nodes.begin(), nodes.end())
37         - nodes.begin() ;
38     vector<Coordinate> ch(n+1) ;
39
40     int m = 0 ;
41
42     for ( int i=0 ; i<n ; i++ ){
43         while(m > 1 && cross(ch[m-2], ch[m-1],
44             nodes[i]) < 0) m-- ;
45         ch[m++] = nodes[i] ;
46
47     for ( int i=n-2, t=m ; i>=0 ; i-- ){
48         while(m > t && cross(ch[m-2], ch[m-1],
49             nodes[i]) < 0) m-- ;
50         ch[m++] = nodes[i] ;
51
52     if(n > 1) m-- ;
53     cout << m << endl ;
54
55 }
```

4.5 Max Flow

```

1 struct Edge{
2     int v, cap, next ;
3 };
4
5 class MaxFlow{
6 private:
7     int N, S, T ;
8     vector<Edge> e ;
9     vector<int> head, cur, dep ;
10
11 bool bfs(){
12     queue<int> q ;
13     for ( int i=0 ; i<=N ; i++ ){
14         cur[i] = head[i] ;
15         dep[i] = -1 ;
16     }
17
18     q.push(S) ;
19     dep[S] = 0 ;
20
21     while(!q.empty()){
22         int u = q.front() ; q.pop() ;
23
24         for ( int i=head[u] ; i!= -1 ;
25             i=e[i].next ){
26             int v = e[i].v ;
27             if(dep[v] == -1 && e[i].cap > 0){
28                 dep[v] = dep[u] + 1 ;
29                 if(v == T) return 1 ;
30                 q.push(v) ;
31             }
32         }
33
34     }
35
36     return 0 ;
37 }
38
39 int dfs(int u, int flow){
40     if(u == T) return flow ;
41     int d, rest = 0 ;
42
43     for ( int &i=cur[u] ; i!= -1 ;
44         i=e[i].next ){
45         int v = e[i].v ;
46         if(dep[v] == dep[u] + 1 && e[i].cap >
47             0){
48             d = dfs(v, min(flow - rest,
49                     e[i].cap)) ;
50         }
51     }
52 }
```

```

    if(d > 0){
        e[i].cap -= d ;
        e[i+1].cap += d ;
        rest += d ;
    }

    if(rest == flow) break ;
}

}

if(rest != flow) dep[u] = -1 ;
return rest ;
}

public:
MaxFlow(int n, int s, int t){
    N = n ; S = s ; T = t ;
    e.reserve(n*n) ;
    head.assign(n+1, -1) ;
    cur.resize(n+1) ;
    dep.resize(n+1) ;
}

void AddEdge(int u, int v, int cap){
    e.push_back({v, cap, head[u]}) ;
    head[u] = e.size() - 1 ;
    e.push_back({u, 0, head[v]}) ;
    head[v] = e.size() - 1 ;
}

int run(){
    int ans = 0 ;
    while(bfs()){
        ans += dfs(S, 0x3f3f3f3f) ;
    }
    return ans ;
}
};

```

4.6 min cut max flow

```

dis[s] = 0 ;                                25    kmp(s2) ;
pq.push({0, s}) ;                           26
                                              27
while(!pq.empty()){                      28    int x=0, y=0 ;
    auto [d, u] = pq.top() ; pq.pop() ;      29    while(x < s1.size() && y < s2.size()){
        if(d > dis[u]) continue ;           30        if(s1[x] == s2[y]){
        for ( int i=head[u] ; i!= -1 ;       31            x++ ;
            i=e[i].next ){                  32            y++ ;
                }                           33        }
        int v = e[i].v, cap = e[i].cap, w =      34        else if(y > 0) y = Next[y] ;
            e[i].cost ;                   35        else x++ ;
        if(cap && dis[v] > d + w + pot[u] -      36        }
            pot[v]){
            dis[v] = d + w + pot[u] - pot[v] ; 37        cout << s1 << s2.substr(y) << endl ;
            preE[v] = i ;                     38        }
            pq.push({dis[v], v}) ;           39
        }                               40    return 0 ;
}                                         41 }
}

```

5.2 ACAM

```

while(dijkstra()){

    for ( int v=1 ; v<=N ; v++ ) if(dis[v] < INF){
        pot[v] += dis[v] ;
    }

    int aug = INT_MAX ;
    for ( int v=t ; v!=s ;
           v=e[preE[v]^1].v ){
        aug = min(aug, e[preE[v]].cap) ;
    }

    for ( int v=t ; v!=s ;
           v=e[preE[v]^1].v ){
        e[preE[v]].cap -= aug ;
        e[preE[v]^1].cap += aug ;
        cost += aug * e[preE[v]].cost ;
    }
}

return cost ;
}

;

5 String

5.1 KMP

int Next[N] ;
void kmp(string &str){
    Next[0] = -1 ;
    if(str.size() <= 1) return ;
    Next[1] = 0 ;

    int cur = 2, check = 0 ;

    while(cur < str.size()){
        if(str[cur - 1] == str[check])
            Next[cur++] = ++check ;
        else if(check > 0) check =
            Next[check] ;
        else Next[cur++] = 0 ;
    }

    main(){
        ios::sync_with_stdio(false) ;
        cin.tie(nullptr) ;
        cout.tie(nullptr) ;

        string s1, s2 ;
        while(cin >> s1){
            s2 = s1 ;
            reverse(s2.begin(), s2.end()) ;
        }
    }
}

class ACAutomation{
private:
    vector<int> fail, end, order ;
    vector<vector<int>> tree ;
    int base, alpha ;

    int new_node(){
        tree.emplace_back(alpha, 0) ;
        fail.push_back(0) ;
    }

    return tree.size() - 1 ;
}

public:
ACAutomation(int _base='a', int _alpha=26)
    : base(_base), alpha(_alpha) {
    clear() ;
}

void clear(){
    fail.assign(1, 0) ;
    order.clear() ;
    end.clear() ;
    tree.assign(1, vector<int>(alpha, 0)) ;
}

void add_pattern(const string &pattern){
    int u = 0 ;
    for ( auto ch : pattern ){
        int v = ch - base ;

        if(tree[u][v] == 0) tree[u][v] =
            new_node() ;
        u = tree[u][v] ;
    }

    end.push_back(u) ;
}

void build(){
    queue<int> q ;
    order.clear() ;
    order.push_back(0) ;

    for ( int i=0 ; i<alpha ; i++ )
        if(tree[0][i] > 0){
            q.push(tree[0][i]) ;
        }

    while(!q.empty()){
        int u = q.front() ; q.pop() ;
        order.push_back(u) ;

        for ( int i=0 ; i<alpha ; i++ ){
            if(tree[u][i] == 0) tree[u][i] =
                tree[fail[u]][i] ;
            else{

```

```

55     fail[tree[u][i]] = tree[fail[u]][i];
56     ;
57     q.push(tree[u][i]);
58   }
59 }
60 }
61
62 vector<int> count_per_pattern(const string
63   &text) const {
64   int u = 0;
65   vector<int> vis(tree.size(), 0);
66
67   for (char ch : text) {
68     u = tree[u][ch - base];
69     vis[u]++;
70   }
71
72   for (int i=order.size()-1; i>=1; i--) {
73     int x = order[i];
74     vis[fail[x]] += vis[x];
75   }
76
77   vector<int> ans(end.size(), 0);
78   for (int id=0; id<end.size(); id++) {
79     ans[id] = vis[end[id]];
80   }
81
82   return ans;
83 };

```

6 Techniques

6.1 二分搜

```

1 // xxxxxxxx 找最小解
2 bool binary_search(){
3   while(l < r){
4     int m = (l + r) >> 1;
5     if(check(m)) r = m;
6     else l = m + 1;
7   }
8
9   return l;
10}
11
12 // oooooxxx 找最大解
13 bool binary_search(){
14   while(l < r){
15     int m = (l + r) >> 1;
16     if(check(m)) l = m;
17     else r = m - 1;
18   }
19
20   return l;
21}
22
23 // 如果l & r 太大, m = (l + (r - 1)) >> 1;

```

6.2 倍增 LCA

```

1 #include <bits/stdc++.h>
2
3 using namespace std;
4
5 const int Maxn = 500005;
6
7 vector<int> e[Maxn];
8 int depth[Maxn];
9 int up[Maxn][40];
10 int MaxLog;
11

```

```

12 void dfs(int u, int from, int d){
13   up[u][0] = from;
14   depth[u] = d;
15
16   for (int i=1; i<=MaxLog; i++) {
17     up[u][i] = up[up[u][i-1]][i-1];
18   }
19
20   for (auto v : e[u]) {
21     if(v == from) continue;
22     dfs(v, u, d+1);
23   }
24 }
25
26 int lca(int u, int v){
27   if(depth[u] < depth[v]) swap(u, v);
28
29   for (int i=MaxLog; i>=0; i--) {
30     if(depth[u] - (1 << i) >= depth[v]){
31       u = up[u][i];
32     }
33
34     if(u == v) return u;
35
36     for (int i=MaxLog; i>=0; i--) {
37       if(up[u][i] != up[v][i]){
38         u = up[u][i];
39         v = up[v][i];
40       }
41
42     return up[u][0];
43   }
44
45   int main(){
46     int n, q, root;
47     scanf("%d%d", &n, &q, &root);
48     MaxLog = __lg(n);
49
50     for (int i=0; i<n-1; i++) {
51       int u, v;
52       scanf("%d%d", &u, &v);
53       e[u].push_back(v);
54       e[v].push_back(u);
55     }
56
57     dfs(root, root, 0);
58
59     while(q--){
60       int u, v;
61       scanf("%d%d", &u, &v);
62       printf("%d\n", lca(u, v));
63     }
64   }

```

6.3 SG

```

1 long long SG(long long k){
2
3   if(k % 2 == 0){
4     return k / 2;
5   }
6   else{
7     return SG(k / 2);
8   }
9
10 }
11
12 int main(){
13   int cas, n;
14
15   scanf("%d", &cas);
16   while(cas--){
17     scanf("%d", &n);
18
19     long long s, v = 0;

```

```

20
21   for(int i = 0; i < n; i++){
22     scanf("%lld", &s);
23     v ^= SG(s); //XOR
24   }
25
26   if(v) printf("YES\n");
27   else printf("NO\n");
28 }
29
30
31 int SG[30];
32 int vis[Maxn], stone[Maxn];
33
34 void build(){
35   SG[0] = 0;
36   memset(vis, 0, sizeof(vis));
37
38   for (int i=1; i<30; i++) {
39     int cur = 0;
40     for (int j=0; j<i; j++) for (int
41       k=0; k<=j; k++) {
42       vis[SG[j] ^ SG[k]] = i;
43     }
44     while(vis[cur] == i) cur++;
45     SG[i] = cur;
46   }
47
48 int main(){
49   build();
50
51   int T = 0;
52   while(~scanf("%d", &n) && n){
53     int ans = 0;
54
55     for (int i=1; i<=n; i++) scanf("%d",
56       &stone[i]);
57
58     for (int i=1; i<=n; i++) if(stone[i]
59       & 1){
60       ans ^= SG[n-i];
61     }
62   }

```

7 DP

7.1 輪廓線 DP

```

1 #include <bits/stdc++.h>
2
3 using namespace std;
4 using ll = long long;
5
6 ll dp[2][(1 << 10) + 5];
7 int n, m;
8 int cur;
9
10 void update(int s1, int s2){
11   if(s2 & (1 << m)){
12     dp[cur][s2 ^ (1 << m)] += dp[cur] ^
13     [s1];
14   }
15
16 int main(){
17   while(~scanf("%d%d", &n, &m)){
18     if(m > n) swap(n, m);
19     memset(dp, 0, sizeof(dp));
20     cur = 0;
21     dp[cur][(1 << m) - 1] = 1;
22     for (int i=0; i<n; i++) for (int
23       j=0; j<m; j++) {
24       cur ^= 1;
25       memset(dp[cur], 0, sizeof(dp[cur]));

```

```

25     for ( int k=0 ; k<(1 << m) ; k++ ){
26         update(k, k << 1) ; // not put
27         if(i && !(k & (1 << (m - 1)))){
28             update(k, (k << 1) | (1 << m) |
29                 1) ; // put up
30             if(j && !(k & 1)) update(k, (k << 1)
31                 | 3) ; // put left
32         }
33     }
34     printf("%lld\n", dp[cur][(1 << m) - 1]) ;
35 }

```

7.2 數位 DP

```

1 #include <bits/stdc++.h>
2
3 using namespace std ;
4
5 int K ;
6 int dp[20][105][105][2] ;
7 vector<int> dig ;
8
9 int solve(int pos, int sum, int dsum, bool
10    lim){
11    if(pos == -1){
12        if(sum == 0 && dsum == 0) return 1 ;
13        return 0 ;
14    }
15    int &d = dp[pos][sum][dsum][lim] ;
16    if(d != -1) return d ;
17
18    int up = lim ? dig[pos] : 9 ;
19    int res = 0 ;
20    for ( int i=0 ; i<=up ; i++ ){
21        res += solve(pos-1, (sum * 10 + i) %
22                      K, (dsum + i) % K, lim && i==up)
23                      ;
24    }
25    return d = res ;
26}
27 int count(int n){
28    memset(dp, -1, sizeof(dp)) ;
29    dig.clear() ;
30
31    while(n > 0){
32        dig.push_back(n % 10) ;
33        n /= 10 ;
34    }
35
36    return solve(dig.size() - 1, 0, 0, 1) ;
37}
38
39 int main(){
40    int T ;
41    scanf("%d", &T) ;
42
43    int a, b ;
44    while(T--){
45        scanf("%d%d%d", &a, &b, &K) ;
46        if(K > 90) printf("0\n") ;
47        else printf("%d\n", count(b) -
48                    count(a-1)) ;
49    }
50
51    return 0 ;
52}

```

7.3 樹 DP

```

1 #include <bits/stdc++.h>
2
3 #define N 505
4 #define INF 0x3f3f3f3f
5
6 using namespace std ;
7
8 struct Edge{
9     int v, w ;
10}
11
12 vector<Edge> edge[N] ;
13 int n ;
14 int cnt[N] ;
15 int dp[N][N][2] ;
16
17 void init(){
18    for ( int i=0 ; i<N ; i++ )
19        edge[i].clear() ;
20    memset(cnt, 0, sizeof(cnt)) ;
21    memset(dp, INF, sizeof(dp)) ;
22}
23
24 void DFS(int u){
25    cnt[u] = 1 ;
26    for ( auto [v, w] : edge[u] ){
27        DFS(v) ;
28        cnt[u] += cnt[v] ;
29    }
30
31    dp[u][1][0] = dp[u][1][1] = 0 ;
32
33    for ( auto [v, w] : edge[u] ){
34        for ( int i=cnt[u] ; i>1 ; i-- ) for (
35            int j=1 ; j<i && j<=cnt[v] ; j++ ){
36            dp[u][i][1] = min(dp[u][i][1],
37                               dp[u][i-j][1] + dp[v][j][1] + 2 *
38                               w) ;
39            dp[u][i][0] = min(dp[u][i][0],
40                               dp[u][i-j][1] + dp[v][j][0] + w) ;
41            dp[u][i][0] = min(dp[u][i][0],
42                               dp[u][i-j][0] + dp[v][j][1] + 2 *
43                               w) ;
44        }
45    }
46}
47
48 int main(){
49    int t = 0 ;
50
51    while(~scanf("%d", &n) && n){
52        init() ;
53        for ( int i=0 ; i<n-1 ; i++ ){
54            int u, v, w ;
55            scanf("%d%d%d", &v, &u, &w) ;
56            edge[u].push_back({v, w}) ;
57        }
58
59        DFS(0) ;
60        printf("Case %d:\n", ++t) ;
61
62        int q, e ;
63        scanf("%d", &q) ;
64
65        while(q--){
66            scanf("%d", &e) ;
67
68            for ( int i=n ; i>=1 ; i-- )
69                if(dp[0][i][0] <= e){
70                    printf("%d\n", i) ;
71                    break ;
72                }
73        }
74    }
75
76    return 0 ;
77}

```