MATH69122 Stochastic Control

Exercise Sheet, Week 3: Markov Decision Processes

Assignment

Attempt the exercises and submit your work by Monday 20th February 11:00. You will find the submission link on the Course Content page on Blackboard.

Exercise 1 (Non-Markovian Strategy): Consider the example from the first lecture.

10	1	12	END
11	14	4	8
3	2	5	9
START	13	7	6

Every number in a cell is the cost of being in that cell (costs at START and END are 0). We go up or right (except at END where we stay). If up and right are possible, we choose one of two coins and flip the chosen coin (head means up, and tail means right). The coins have switched probabilities with 1/4 and 3/4 for head and tail, respectively. We want to minimise the cost.

(a) Is this a Markov Decision Problem (i.e. can you apply dynamic programming)?

(b) Find the optimal Markovian strategy (i.e. all actions depend only on the current state).

(c) Find an optimal non-Markovian strategy (i.e. some actions depend on previous states).

Exercise 2 (Only terminal reward): A sequence of N coin tosses have probability of heads $p_1, \dots, p_N$. We can bet a proportion of our current wealth on the next coin landing heads. If we win, our bet doubles. If we lose, our bet is gone. Let X_N be our final and $X_0 = 1$ our initial wealth.

(a) What should you bet at each coin flip to maximise $\mathbb{E}[\log(X_N)]$? Try $V_t(x) = \varphi_t + \log(x)$.

(b) What should you bet at each coin flip to maximise $\mathbb{E}[X_N]$? Try the ansatz from the first tutorial.

Additional Practice Exercises

The following questions do not count towards your submission.

Exercise 3 (Interchanging $\mathbb{E}$ and inf; Bayes Estimator): Let Θ be a random $\mathbb{R}$-valued parameter. Let X be a sample. Consider the conditional density $f(\theta|x)$ of the parameter given the data. Consider the estimator d_X depending on the data with minimal squared error, i.e.

$$\inf_{d_X} \mathbb{E}[(\Theta - d_X)^2].$$

(a) Interchange $\mathbb{E}$ and inf by looking at $\inf_{d\in\mathbb{R}} \int_{\mathbb{R}} (\theta - d)^2 f(\theta|X)\, \mathrm{d}\theta$. Notice that $\inf_{d_X}$ looks at all functions depending on X while $\inf_d$ looks at a single $\mathbb{R}$-valued parameter.

(b) Compute d_X using a derivative argument.

Exercise 4 (Calculating the Value Function): We take cards sequentially from a 52-card deck. We can bet at any point that the next card is red. If correct, we win £1000. If incorrect, we win nothing, and the game stops. What is the value function of the maximum average winnings?