COMP 125 Programming with Python
Systems of Linear Equations

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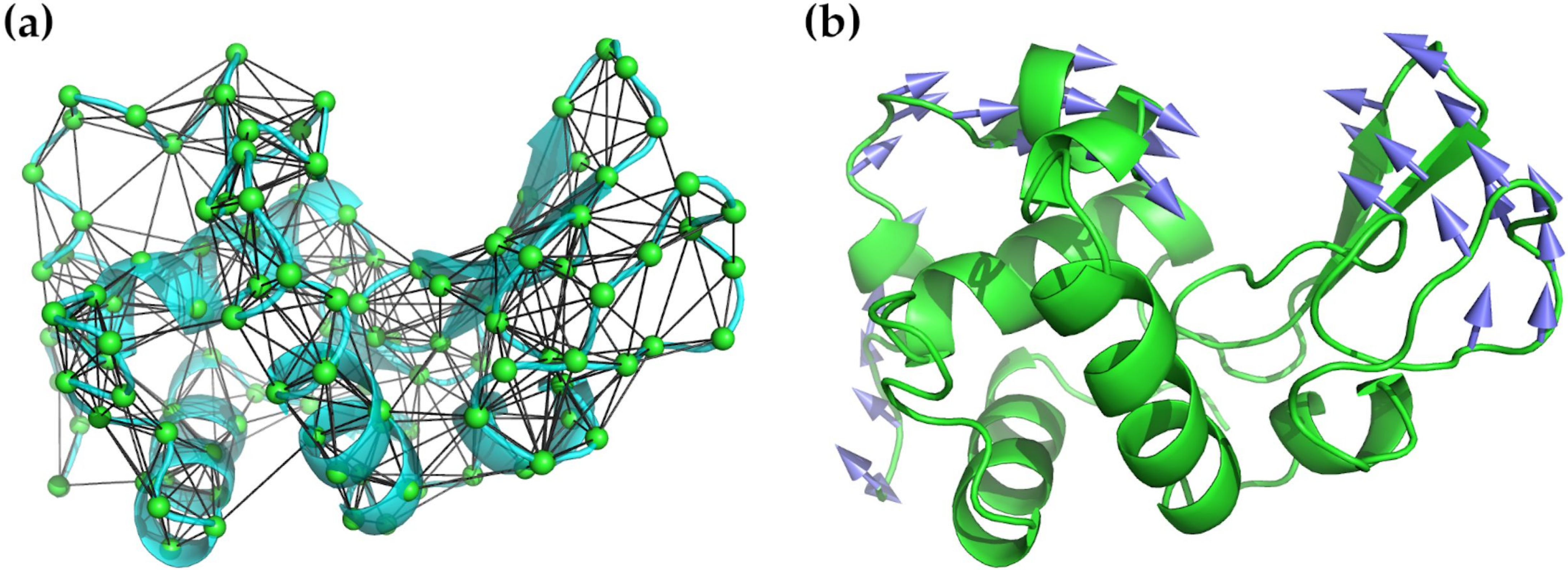

(a)
(b)

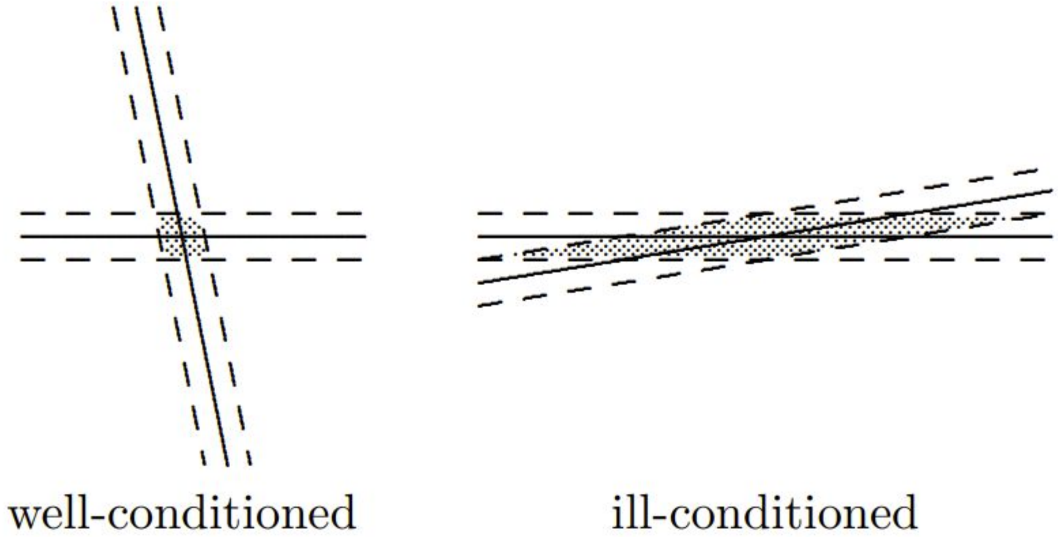


Figure 2.2: Well-conditioned and ill-conditioned linear systems.

Systems of Linear Equations.

$$az + by = c$$

$$dz + ey = f$$

$$\boxed{A_{n\times n}\, x_{n\times 1} = b_{n\times 1}}$$

Matrix formulation.

$$\underbrace{\begin{bmatrix} a & b \\ d & e \end{bmatrix}_{2\times 2}}_{A} \underbrace{\begin{Bmatrix} z \\ y \end{Bmatrix}}_{x} = \underbrace{\begin{Bmatrix} c \\ f \end{Bmatrix}}_{b}$$

Method 1

Use matrix inverse.

Inverse of a matrix

$$A_{n\times n} A^{-1}_{n\times n} = I_{n\times n} \quad \begin{bmatrix} 1 & & 0 \\ & \ddots & \\ 0 & & 1 \end{bmatrix}_{n\times n}$$

inverse of A

identity matrix

$$A x = b$$

$$A^{-1} A x = A^{-1} b$$

$$I x = A^{-1} b$$

$$x = A^{-1} b$$

How to obtain it with Numpy?

$$A = \begin{bmatrix} 0.1 & 0.2 & 0.3 \\ 0.4 & 0.5 & 0.6 \\ 0.7 & 0.8 & 0.9 \end{bmatrix}$$

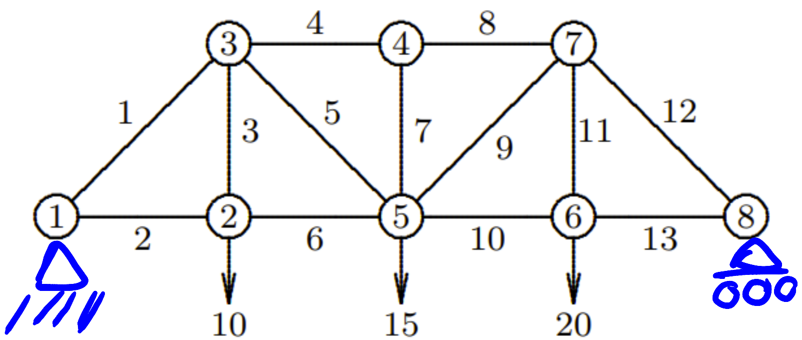
$$\det(A) = 0.1 \times 0.5 \times 0.9 + 0.3 \times 0.4 \times 0.8$$
$$+ 0.2 \times 0.6 \times 0.7$$
$$- 0.3 \times 0.5 \times 0.7 - 0.2 \times 0.4 \times 0.9$$
$$- 0.1 \times 0.6 \times 0.8$$

$$\det(A) = 0.045 + 0.036 + 0.084 - 0.105 - 0.072 - 0.048 = 0$$

Determinant zero,

Hence no matrix inverse !!!

2.3 The following diagram depicts a plane truss having 13 members (the numbered lines) connected by 10 joints (the numbered circles). The indicated loads, in tons, are applied at joints 2, 5, and 6, and we wish to determine the resulting force on each member of the truss.



For the truss to be in static equilibrium, there must be no net force, horizontally or vertically, at any joint. Thus, we can determine the member forces by equating the horizontal forces to the left and right at each joint, and similarly equating the vertical forces upward and downward at each joint. For the eight joints, this would give 16 equations, which is more than the 13 unknown forces to be determined. For the truss to be statically determinate, that is, for there to be a unique solution, we assume that joint 1 is rigidly fixed both horizontally and vertically, and that joint 8 is fixed vertically. Resolving the member forces into horizontal and vertical components and defining $\alpha = \sqrt{2}/2$, we obtain the following system of equations for the member forces f_i:

$$\text{Joint } 2: \begin{cases} f_2 = f_6 \\ f_3 = 10 \end{cases}$$
$$\text{Joint } 3: \begin{cases} \alpha f_1 = f_4 + \alpha f_5 \\ \alpha f_1 + f_3 + \alpha f_5 = 0 \end{cases}$$
$$\text{Joint } 4: \begin{cases} f_4 = f_8 \\ f_7 = 0 \end{cases}$$
$$\text{Joint } 5: \begin{cases} \alpha f_5 + f_6 = \alpha f_9 + f_{10} \\ \alpha f_5 + f_7 + \alpha f_9 = 15 \end{cases}$$
$$\text{Joint } 6: \begin{cases} f_{10} = f_{13} \\ f_{11} = 20 \end{cases}$$
$$\text{Joint } 7: \begin{cases} f_8 + \alpha f_9 = \alpha f_{12} \\ \alpha f_9 + f_{11} + \alpha f_{12} = 0 \end{cases}$$
$$\text{Joint } 8: \begin{cases} f_{13} + \alpha f_{12} = 0 \end{cases}$$

Use a library routine to solve this system of linear equations for the vector $\boldsymbol{f}$ of member forces. Note that the matrix of this system is quite sparse, so you may wish to experiment with a band solver or more general sparse solver, although this particular problem instance is too small for these to offer significant advantage over a general solver.

$$A f = x$$

$$\begin{bmatrix} 0 & 1 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ & & & & & & & & & & & \end{bmatrix} \begin{Bmatrix} f_1 \\ f_2 \\ f_3 \\ f_4 \\ f_5 \\ f_6 \\ f_7 \\ f_8 \\ f_9 \\ f_{10} \\ f_{11} \\ f_{12} \end{Bmatrix} = \begin{bmatrix} 0 \\ -10 \\ \\ \end{bmatrix}$$