28/07/2021

UNIT-II The Discrete Fourier Transform

Analysis of LTI system using Z-transform

Z-transform is a mathematical tool for the analysis of linear time invariant discrete time systems in the freq. domain

$x(n)$ - incoming i/p signal

$Z[x(n)] = X(Z)$

Z - complex variable

$$\boxed{X(Z) = \sum_{n=-\infty}^{\infty} x(n) Z^{-n}} \longrightarrow \text{Z-transform}$$

$Z = re^{jw}$ // r - radius of the circle.

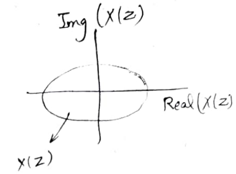
$$X(re^{jw}) = \sum_{n=-\infty}^{\infty} x(n)\, r^{-n} e^{-jwn} \longrightarrow \text{Z-transform}$$

When $r = 1$:

$$X(e^{jw}) = \sum_{n=-\infty}^{\infty} x(n)\, e^{-jwn} \longrightarrow \text{Discrete Time Fourier Transform (DTFT)}$$

Relation b/w Z-transform & DTFT — X.

ROC: Region of Convergence



The ROC of $X(z)$ is the set of all values of z for which $X(z)$ exists.

$x(Z) \neq \infty$

Z transform & ROC of finite duration sequence

(i) Right hand sequence:

Eg: Q: Find the Z transform & ROC of the causal sequence

$n > 0$

$x(n) = \{1, 0, 3, 0, -1, 2\}$
          ↑

$x(n) = \{1, 0, 3, 0, -1, 2\}$
          ↑

$\Rightarrow x(n)$

$$\Rightarrow \sum_{n=-\infty}^{\infty} x(n) z^{-n}$$

$$\sum_{n=0}^{\infty} x(n) z^{-n}$$

$= x(0) + x(1) z^{-1} + x(2) z^{-2} + x(3) z^{-3} + x(4) z^{-4} + x(5) z^{-5}$

$x(Z) = 1 + 0 + 3Z^{-2} + 0 - 1Z^{-4} + 2Z^{-5}$

$x(z) = 1 + 3z^{-2} - z^{-4} + 2z^{-5} \Rightarrow 1 + \frac{3}{z^2} - \frac{1}{z^4} + \frac{2}{z^5}$

When $z = 0$

$x(z) = \infty$

$X(z)$ converges for all values of z except at $z = 0$

ROC exists for all values except at $z = 0$

(ii) Left hand sequence: (~~Non~~ Anti causal sequences)

~~Exists~~ on n>0 & n<0

x(n

Q: Find the Z transform & ROC of the anti-causal sequence

$$x(n) = \{ \overset{-4}{-3}, \overset{-3}{-2}, \overset{-2}{-1}, \overset{-1}{0}, \underset{\uparrow}{\overset{0}{1}} \}$$

$$X(z) = \sum_{n=-\infty}^{\infty} x(n) z^{-n}$$

$$= x(0) z^{0} + x(-1) z^{1} + x(-2) z^{2} + x(-3) z^{3} + x(-4) z^{4}$$

$$X(z) = 1 + 0 - 1z^{2} - 2z^{3} - 3z^{4}$$

$$X(z) = 1 - z^{2} - 2z^{3} - 3z^{4}$$

X(z) converges for all values of z except at $z = \infty$

(iii) Two-sided sequence (Bilateral sequences)

A signal that has finite duration on both the left & right side is known as 2 sided sequence.

Q: Find the Z transform & ROC of the sequence.

$$x(n) = \{ 2, -1, 3, 2, \underset{\uparrow}{1}, 0, 2, 3, -1 \}$$

$$X(z) = \sum_{n=-\infty}^{\infty} x(n) z^{-n}$$

$$X(z) = 2z^{4} - z^{3} + 3z^{2} + 2z^{1} + 1 + 2z^{-2} + 3z^{-3} - z^{-4}$$

⇒ X(z) converges for all values of z except at

$z = 0$ & $z = \infty$

Z-transform & ROC of infinite duration sequence.

(i) Right side sequence

Q: Determine the Z transform & ROC of the signal

$$x(n) = a^{n} u(n) \qquad u(n) \Rightarrow n \geq 0 \qquad X(z) = \sum_{n=-\infty}^{\infty} x(n) z^{-n}$$

$$\Rightarrow X(z) = \sum_{n=0}^{\infty} x(n) z^{-n}$$

$$= \sum_{n=0}^{\infty} a^{n} u(n) . z^{-n}$$

$$= \sum_{n=0}^{\infty} a^{n} z^{-n} \Rightarrow \sum_{n=0}^{\infty} (a z^{-1})^{n}$$

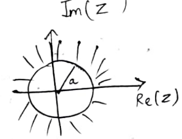
$$\boxed{\sum_{k=0}^{\infty} a^{k} = \frac{1}{1-a}}$$ } Infinite summation formula

$$\boxed{X(z) = \frac{1}{1-(a z^{-1})}}$$

ROC:

$$|a z^{-1}| < 1$$

$$\left| \frac{a}{z} \right| < 1 \Rightarrow \boxed{|z| > |a|}$$



X(z) exist for all the values outside the circle of radius = a

ROC

(ii) Left side sequence:

Q: Determine the Z transform & ROC of the signal

$$x(n) = -b^{n} u(-n-1)$$

$\rightarrow \quad u(-n-1) = \begin{cases} 0, & n \geq 0 \\ 1, & n < 0 \end{cases}$

$u(-n) = \begin{cases} 0, & n \geq 0 \\ 1, & n < 0 \end{cases}$

$u(\cancel{\alpha}(n+1)) = \begin{cases} 1, & n \geq -1 \\ 0, & n < -1 \end{cases}$

$u(-(n+1)) = \{$

$X(z) = \sum_{n=-\infty}^{\infty} x(n) z^{-n}$

Left-side sequence

$\Rightarrow X(z) = \sum_{n=-\infty}^{-1} x(n) z^{-n}$

$= \sum_{n=-\infty}^{-1} -b^{n} z^{-n} \Rightarrow -\sum_{n=-\infty}^{-1} (b^{-1}z)^{-n}$

$m = -n$

$n = -\infty \Rightarrow m = \infty$
$n = -1 \Rightarrow m = 1$

$\Rightarrow$ ~~$X(z) = -\sum_{m=1}^{\infty} (b^{-1}z)$~~

$X(z) = -\sum_{m=1}^{m=\infty} (b^{-1}z)^{m}$

At $m = 0$

$(b^{-1}z)^{0} = \boxed{1}$

$X(z) = -\left[\sum_{m=0}^{\infty} (b^{-1}z)^{m} - 1 \right]$

$X(z) = -\left[\frac{1}{1-(b^{-1}z)} - 1 \right]$

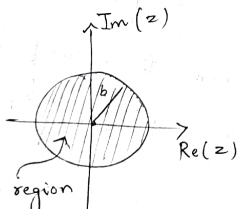
$$\boxed{X(z) = 1 - \frac{1}{1-b^{-1}z}}$$

ROC:

$|b^{-1}z| < 1 \Rightarrow \left|\frac{z}{b}\right| < 1$

$|z| < b$

ROC is now in the interior of the circle having radius 'b'



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(iii) Two sides sequences - Bilateral [Infinite duration]

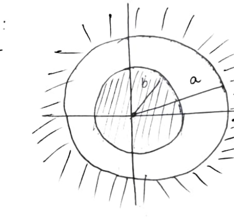
$x(n) = a^{n}u(n) + b^{n}u(-n-1)$

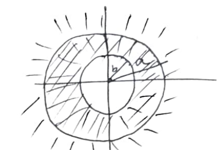
I^{st} series conververges if $z > a$

II^{nd} series converges if $z < b$

ROC:

$a \in b \qquad a > b$





$a < |z| < b$

Properties of ROC

(i) The ROC is a ring / disk in the z-plane centered @ the origin.

(ii) If $x(n)$ is a finite causal seq. then the ROC is the entire z-plane except at $z = 0$.

(iii) If $x(n)$ is a finite non-causal seq. then the ROC is the entire z-plane except at $z = \infty$.

(iv) If $x(n)$ is a finite 2 sided seq. then the ROC is the entire z-plane except at $z = 0$ & $z = \infty$.

(v) If $x(n)$ is infinite, causal seq. then the ROC is at outside of unit circle.

(vi) If $x(n)$ is a infinite, non-causal seq. then the Roc is at inside the unit circle.

(vii) If $x(n)$ is a ∞ 2 sided seq.. the ROC is ring in the z-plane.

(viii) The ROC of LTI stable system contains the unit circle

Q:

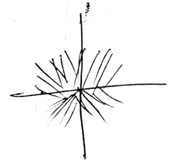
1. Find the z transform of the signal

$x(n) = \delta(n)$

$\rightarrow$ $\delta(n) = \begin{cases} 1 & , n = 0 \\ 0 & , \text{otherwise} \end{cases}$

$X(z) = \sum_{n=-\infty}^{\infty} x(n) z^{-n} = \sum_{n=-\infty}^{\infty} \delta(n) z^{-n}$

$\boxed{X(z) = 1}$



Roc is entire z-plane

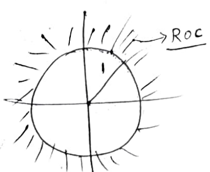
2. Find the z transform of the signal

$x(n) = u(n)$

$\rightarrow$ $u(n) = \begin{cases} 1 & , n \geq 0 \\ 0 & , \text{otherwise} \end{cases}$

$X(z) = \sum_{n=-\infty}^{\infty} x(n) . z^{-n}$

$= \sum_{n=0}^{\infty} u(n) . z^{-n} = \sum_{n=0}^{\infty} z^{-n}$



$\boxed{X(z) = \cancel{} \dfrac{1}{1 - z^{-1}}}$

ROC: $|z^{-1}| < 1 \Rightarrow \left|\frac{1}{z}\right| < 1$

$\Rightarrow |z| > 1$

3. Find the z transform of the signal

$x(n) = \left[3(3)^n - 4(2)^n\right] u(n)$

$\rightarrow$ $u(n) = \begin{cases} 1 & , n \geq 0 \\ 0 & , \text{otherwise} \end{cases}$

$X(z) = \sum_{n=-\infty}^{\infty} x(n) . z^{-n}$

$= \sum_{n=0}^{\infty} x(n) . z^{-n}$

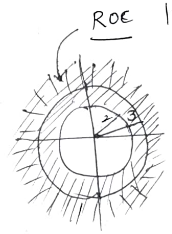
$= \sum_{n=0}^{\infty} \left[3(3)^n - 4(2)^n\right] z^{-n}$

$= 3 \sum_{n=0}^{\infty} 3^n z^{-n} - 4 \sum_{n=0}^{\infty} (2)^n z^{-n}$

$= 3 \sum_{n=0}^{\infty} (3z^{-1})^n - 4 (2z^{-1})^n$

Z transform, $X(z) = \dfrac{3}{1 - 3z^{-1}} - \dfrac{4}{1 - 2z^{-1}}$

ROC $|z| > 3$



ROC,

$\lvert 3z^{-1} \rvert < 1$	$\lvert 2z^{-1} \rvert < 1$
$\lvert z \rvert > 3$	$\lvert z \rvert > 2$

$\Rightarrow$ Common area $\Rightarrow |z| > 3$

4. Find the z transform of the signal

$x(n) = 0.8^n u(n) + 1.3^n u(n)$

$\rightarrow x(n) = (0.8)^n u(n) + (1.3)^n u(n)$

$$X(z) = \sum_{n=\infty}^{\infty} x(n) z^{-n}$$

$$= \sum_{n=0}^{\infty} \left[(0.8)^n + (1.3)^n\right] z^{-n}$$

$$= \sum_{n=0}^{\infty} (0.8 z^{-1})^n + \sum_{n=0}^{\infty} (1.3 z^{-1})^n$$

$$\boxed{X(z) = \frac{1}{1-0.8z^{-1}} + \frac{1}{1-1.3z^{-1}}}$$

ROC:

$\lvert 0.8z^{-1}\rvert > 1$	$\lvert 1.3z^{-1}\rvert > 1$
$\lvert z\rvert > 0.8$	$\lvert z\rvert > 1.3$

$\Rightarrow$ ROC : $|z| > 1.3$ //.

5. Find the z transform of the signal

$x(n) = (0.8)^n u(n) - 1.3^n u(-n-1)$

$$\rightarrow X(z) = \sum_{n=0}^{\infty} ((0.8)z^{-1})^n - \sum_{n=-\infty}^{-1} (1.3)^n z^{-n}$$

$$= \frac{1}{1-0.8z^{-1}} - \sum_{n=-\infty}^{-1} ((1.3)^{-1} z)^{-n}$$

Put $m = -n$

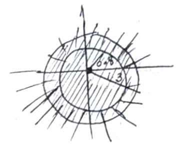
$$= \frac{1}{1-0.8z^{-1}} - \sum_{m=1}^{\infty} ((1.3)^{-1} z)^{m}$$

$$= \frac{1}{1-0.8z^{-1}} - \left\{\sum_{m=0}^{\infty} \left(\left((1.3)^{-1} z\right)\right)^{m} - 1\right\}$$

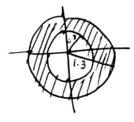
$$\boxed{X(z) = \frac{1}{1-0.8z^{-1}} - \frac{1}{1-(1.3)^{-1}z} + 1}$$

ROC:

$\lvert z\rvert > 0.8$	$\lvert (1.3)^{-1} z\rvert < 1$
	$\lvert z\rvert < 1.3$



$\boxed{\text{ROC: } 0.8 < |z| < 1.3}$



6. Find the z transform:

$x(n) = -0.8^n u(-n-1) - 1.3^n u(-n-1)$

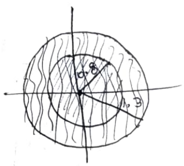
$$\rightarrow X(z) = -\sum_{n=-\infty}^{-1} (0.8)^n z^{-n} - \sum_{n=-\infty}^{-1} (1.3)^n z^{-n}$$

$$X(z) = -\frac{1}{1-(0.8)^{-1}z} + 1 - \frac{1}{1-(1.3)^{-1}z} + 1$$

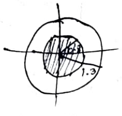
$$\boxed{X(z) = -\left(\left(\frac{1}{1-(0.8)^{-1}z}\right) + \left(\frac{1}{1-(1.3)^{-1}z}\right)\right) + 2}$$

ROC:

$\lvert z\rvert < 0.8$	$\lvert z\rvert < 1.3$



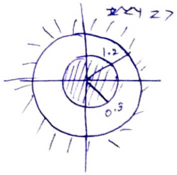
ROC: $\boxed{|z| < 0.8}$



7. Find the z transform of the signal:

$x(n) = 1.2^n u(n) - 0.3^n u(-n-1)$

$\rightarrow$ ROC: $z > 1.2$; $z < 0.3$



$\Rightarrow$ ROC not exist

02/08/2021

Stability & ROC

A linear time invariant system with the system function $H(z)$ is BIBO stable if & only if the ROC for $H(z)$ contains unit circle.

Q: ① Determine the stability of the system whose impulse response is $h(n) = 2^n u(n)$.

$$\rightarrow X(z) = \sum_{n=-\infty}^{\infty} x(n) z^{-n}$$

$$X(z) = \sum_{n=-\infty}^{\infty} 2^n u(n) z^{-n}$$

$$= \sum_{n=0}^{\infty} (2z^{-1})^n \qquad \sum_{k=0}^{\infty} a^k = \frac{1}{1-a}$$

$$\Rightarrow X(z) = \frac{1}{1-2z^{-1}}$$

ROC: $\Rightarrow |2z^{-1}| < 1$

$2 < z$

$\Rightarrow |z| > 2 \Rightarrow$ It does not contain unit circle.

$\therefore$ system is unstable.

Properties of z-transform:

(i) Time shifting property / Sample shifting:

$$Z[x(n-n_0)] = z^{-n_0} X(z)$$

$z(x(n)) = X(z)$

① Find z transform of the signal:

$x(n) = \frac{1}{2}\delta(n) + \delta(n-1) + \frac{1}{3}\delta(n-2)$

$\rightarrow$ Using z transform,

$\frac{1}{2}\delta(n) = \frac{1}{2} Z[\delta(n)]$

$= \frac{1}{2} \cdot 1 = \boxed{\frac{1}{2}}$

$\Rightarrow \frac{1}{2} + Z[\delta(n-1)] + \frac{1}{3} Z[\delta(n-2)]$

$= \frac{1}{2} + z^{-1} \cdot 1 + \frac{1}{3} z^{-2} \cdot 1$

$\cancel{X(z)} = \frac{1}{2} + z^{-1} + \frac{z^{-2}}{3}$

ROC exists for all values of z except at $z=0$

② Find z transform of the signal:

$x(n) = u(n-2)$

$\rightarrow z(u(n-2)) = z^{-2} \cdot z(u(n))$

$= z^{-2}\left[\frac{z}{z-1}\right]$

$= \frac{1}{z^2} \cdot \frac{z}{z-1} \Rightarrow \frac{z^{-1}}{z\left(1-\frac{1}{z}\right)}$

$$\cancel{X}(z) = \frac{z^{-2}}{1-\frac{1}{z}}$$

ROC: $\frac{1}{z} < 1$

$$\boxed{|z| > 1}$$

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(3) Find the z transform:

$$x(n) = \left(\frac{1}{3}\right)^{n-1} u(n-1)$$

$\rightarrow$ $$z\left[\left(\frac{1}{3}\right)^n u(n)\right] = \frac{1}{1-\frac{1}{3}z^{-1}} \Rightarrow \frac{z}{z-\frac{1}{3}}$$

$$z[x(n-1)] = z^{-1} X(z)$$

$$= z^{-1}\left(\frac{z}{z-\frac{1}{3}}\right)$$

$$= \frac{1}{z\left(1-\frac{1}{3z}\right)}$$

ROC: $\left(\frac{1}{3z}\right) < 1$

$\Rightarrow$ $\boxed{z > 1/3}$

(ii) Differentiation of the z-domain:

$$z[n\,x(n)] = -z \frac{d}{dz} X(z)$$

Q: (1) Find the z-transform of the signal: $n a^n u(n)$.

$\rightarrow$ $x(n) = a^n u(n)$

$$\cancel{X(z) = a^z u(z)}$$

$$X(z) = \sum_{n=-\infty}^{\infty} x(n) \cdot z^{-n}$$

$\Rightarrow$ $$X(z) = \frac{1}{1-az^{-1}}$$

$$z[n\,x(n)] = -z \frac{d}{dz} X(z)$$

$$= -z\left[\frac{d}{dz}\left(\frac{1}{1-az^{-1}}\right)\right]$$

$$= -z\left[\frac{1.(0-az^{-2}) - (1-az^{-1}).0}{(1-az^{-1})^2}\right]$$

$$\boxed{z[n\,x(n)] = \cancel{az}\left\{\frac{az^{-1}}{(1-az^{-1})^2}\right.}$$

(iii) Time Reversal property:

$$\left.\begin{aligned} z[x(n)] &= X(z) \\ z[x(-n)] &= X(1/z) \end{aligned}\right\} \text{(NT)}$$

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Discrete Fourier Transform: (DFT)

DFT: $$\boxed{X(k) = \sum_{n=0}^{N-1} x(n)\, e^{-j\frac{2\pi}{N}kn}}$$

where n & k = 0, 1, ..., N-1

N - No. of sample pts.

$$\boxed{W_N = e^{-j\frac{2\pi}{N}}}$$

W_N - weighting factor

$$\boxed{X(k) = \sum_{n=0}^{N-1} x(n)\, W_N^{kn}}$$

n, k = 0, 1, ... N-1

DTFT: (Discrete Time Fourier Transform)

$$X(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x(n)\, e^{-j\omega n}$$

z-transform:

$$X(z) = \sum_{n=-\infty}^{\infty} x(n)\, z^{-n}$$

↳ z-transform

$z = re^{-j\omega}$

$r = 1$

$$\Rightarrow X(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x(n)\, e^{-j\omega}$$

↳ DTFT

Purpose of z-transform

DTFT: Time domain to Freq. domain signal

Inverse Discrete Fourier Transform: FD to TD (IDFT)

$$\boxed{x(n) = \frac{1}{N}\sum_{k=0}^{N-1} X(k)\, e^{j\frac{2\pi}{N}kn}}$$

$W_n = e^{-j\frac{2\pi}{N}}$

$$\Rightarrow \boxed{x(n) = \frac{1}{N}\sum_{k=0}^{N-1} W_N^{-nk}}$$

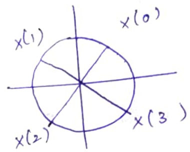
Q. (i) Find the DFT of a sequence {1, 1, 0, 0}

→ $x(n) = \{1, 1, 0, 0\}$ → 4 pt. DFT ⇒ N = 4

no. of samples = 4

DFT:

$$X(k) = \sum_{n=0}^{N-1} x(n)\, e^{-j\frac{2\pi}{N}kn}$$



~~k&n~~ k, n = 0, 1, ... N-1

Here N = 4 ⇒ k, n = 0, 1, 2, 3

When X(0) i.e. $\boxed{k=0}$

$$X(0) = \sum_{n=0}^{3} x(n)\, e^{-j\frac{2\pi}{4}\cdot 0\cdot n}$$

$$= \sum_{n=0}^{3} x(n)\cdot 1$$

$$\Rightarrow x(0) + x(1) + x(2) + x(3)$$

$$= 1 + 1 + 0 + 0 = \boxed{2}$$

$$\boxed{X(0) = 2}$$

When X(1) $\boxed{k=1}$

$$X(1) = \sum_{n=0}^{3} x(n)\, e^{-j\frac{2\pi}{4}\cdot 1\cdot n}$$

$$= \sum_{n=0}^{3} x(n)\cdot e^{-j\frac{\pi}{2}n}$$

$$\Rightarrow x(0)\cdot e^{0} + x(1)\, e^{-j\pi/2} + x(2)\, e^{-j\pi} + x(3)\, e^{-j\frac{\pi}{2}\times 3}$$

$$\Rightarrow (1\cdot 1) + (1\cdot e^{-j\pi/2}) + 0 + 0$$

$$\Rightarrow 1 + e^{-j\pi/2}$$

$$\Rightarrow \boxed{1 + [\cos\pi/2 - j\sin\pi/2]}$$

$\Rightarrow 1 + (0 - j.1) \Rightarrow 1 - j$

$\Rightarrow \boxed{X(1) = 1 - j}$

When $\boxed{k = 2}$:

$$X(2) = \sum_{n=0}^{3} x(n) e^{-j\frac{2\pi}{4}(2).n}$$

$$= \sum_{n=0}^{3} x(n) e^{-j\pi n}$$

$$\Rightarrow x(0)e^{0} + x(1)e^{-j\pi} + x(2)e^{-j2\pi} + x(3)e^{-j3\pi}$$

$$= 1.1 + 1.e^{-j\pi} + 0 + 0$$

$$= 1 + e^{-j\pi}$$

$$= 1 + [\cos\pi - j\sin\pi] \Rightarrow 1 + [-1 - j.0]$$

$$\Rightarrow 1 - 1 = 0$$

$\Rightarrow \boxed{X(2) = 0}$

When $\boxed{k = 3}$:

$$X(3) = \sum_{n=0}^{3} x(n) e^{-j\frac{2\pi}{4}(3)n}$$

$$= \sum_{n=0}^{3} x(n) e^{-j\pi/2 (3)n}$$

$$\Rightarrow x(0).e^{0} + x(1).e^{-j3\pi/2} + x(2)e^{-j3\pi} + x(3)e^{-j\pi/2}$$

$$= 1.1 + 1.e^{-j3\pi/2}$$

$$= 1 + \cos\frac{3\pi}{2} - j\sin\frac{3\pi}{2}$$

$$= 1 + [0 - j(-1)]$$

$\Rightarrow \boxed{X(3) = 1 + j}$

$\Rightarrow \boxed{X(k) = \{2, 1-j, 0, 1+j\}}$

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(2) Find the IDFT of $X(k) = \{1, 0, 1, 0\}$

$$\rightarrow x(n) = \frac{1}{N} \sum_{k=0}^{N-1} X(k) e^{j\frac{2\pi}{N}kn}$$

$\boxed{N = 4}$

When $n = 0$:

$$x(0) = \frac{1}{4} \sum_{k=0}^{3} X(k) e^{j.\frac{2\pi}{4}.0.k}$$

$$= \frac{1}{4}[1 + 0 + 1 + 0] = \boxed{1/2}$$

When $n = 1$:

$$x(1) = \frac{1}{4} \sum_{k=0}^{3} X(k) e^{j.\frac{2\pi}{4}.1.k}$$

$$= \frac{1}{4} \sum_{k=0}^{3} X(k) e^{j\frac{\pi}{2}k}$$

$$= \frac{1}{4}\left[1.e^{0} + 0 + 1.e^{j\pi/2 . 2}\right]$$

$$= \frac{1}{4}[1 + e^{j\pi}] \Rightarrow \frac{1}{4}[1 + \cos\pi + j\sin\pi]$$

$$= \frac{1}{4}(1 - 1) = \boxed{0}$$

When n=2:

$$x(2) = \frac{1}{4}\sum_{k=0}^{3} x(k)\, e^{j\frac{2\pi}{4}\cdot 2k}$$

$$= \frac{1}{4}\sum_{k=0}^{3} x(k)\, e^{j\pi k}$$

$$= \frac{1}{4}\left[1\cdot e^{0} + 0 + 1\cdot e^{j\pi\times 2}\right] \Rightarrow \frac{1}{4}\left[1 + e^{j2\pi}\right]$$

$$= \frac{1}{4}\left[1 + \cos 2\pi + j\sin 2\pi\right]$$

$$= \frac{1}{4}\left[1+0+1+0\right] = \frac{1}{4}(2) = \boxed{\frac{1}{2}}$$

When n=3:

$$x(3) = \frac{1}{4}\sum_{n=0}^{3} x(k)\, e^{j\frac{2\pi}{4}\cdot 3\cdot k}$$

$$= \frac{1}{4}\sum_{n=0}^{3} x(k)\cdot e^{j\frac{3\pi}{2}k} \Rightarrow \frac{1}{4}\left[1\cdot e^{0} + 0 + 1\cdot e^{j\frac{3\pi}{2}\times 2}\right]$$

$$= \frac{1}{4}\left[1+0+e^{j3\pi}\right] \Rightarrow \frac{1}{4}\left[1 + (\cos 3\pi + j\sin 3\pi)\right]$$

$$= \frac{1}{4}(1-1-0) = \boxed{0}$$

$$\Rightarrow x(n) = \left\{\frac{1}{2}, 0, \frac{1}{2}, 0\right\}$$

(3) Determine the 8-pt. DFT of the sequence

$$x(n) = \{1, 1, 1, 1, 1, 1, 0, 0\}$$

→ N=8

$$X(k) = \sum_{n=0}^{N-1} x(n)\, e^{-j\frac{2\pi}{N}kn}$$

When K=0

$$X(0) = \sum_{n=0}^{7} x(n)\, e^{-j\frac{2\pi}{8}\cdot 0} \Rightarrow \sum_{n=0}^{7} x(n)\cdot 1$$

$$= x(0) + x(1) + \dots + x(7) = \boxed{6}$$

When K=1

$$X(1) = \sum_{n=0}^{7} x(n)\cdot e^{-j\frac{2\pi}{8}\cdot 1}$$

$$= x(0)e^{-j\frac{2\pi}{8}\cdot 0} + x(1)\cdot e^{-j\frac{2\pi}{8}\cdot 1} + x(2)e^{-j\frac{2\pi}{8}(2)} + x(3)e^{-j\frac{2\pi}{8}(3)}$$

$$+ x(4)e^{-j\frac{2\pi}{8}\cdot 4} + x(5)e^{-j\frac{2\pi}{8}\cdot 5} + 0 + 0$$

$$= 1 + e^{-j\pi/4} + e^{-j\pi/2} + e^{-j3\pi/4} + e^{-j\pi} + e^{-j5\pi/4}$$

$$= 1 + \left[\cos\frac{\pi}{4} - j\sin\frac{\pi}{4}\right] + \left[\cos\frac{\pi}{2} - j\sin\frac{\pi}{2}\right] + \left[\cos\frac{3\pi}{4} - j\sin\frac{3\pi}{4}\right]$$

$$+ \left[\cos\pi - j\sin\pi\right] + \left[\cos\frac{5\pi}{4} - j\sin\frac{5\pi}{4}\right]$$

$$= 1 + \left[\frac{1}{\sqrt{2}} - j\frac{1}{\sqrt{2}}\right] + \left[0 - j(1)\right] + \left[\frac{-1}{\sqrt{2}} - j\frac{1}{\sqrt{2}}\right] + (-1+0)$$

$$+ \left(\frac{-1}{\sqrt{2}} + j\frac{1}{\sqrt{2}}\right)$$

$$= \frac{-1}{\sqrt{2}} - j\frac{1}{\sqrt{2}} - j$$

$$\Rightarrow \boxed{X(1) = \frac{-1}{\sqrt{2}} - j\left(\frac{1}{\sqrt{2}} + 1\right)}$$

When k=2

$$X(2) = \sum_{n=0}^{7} x(n)\, e^{-j\frac{2\pi}{8}\cdot 2n}$$

$$= x(0)e^{-j\frac{2\pi}{8}\cdot 2\cdot 0} + x(1)e^{-j\frac{2\pi}{8}\cdot 2\cdot 1} + x(2)e^{-j\frac{2\pi}{8}\cdot 2\cdot 2}$$

$$+ x(3)e^{-j\frac{2\pi}{8}\cdot 2\cdot 3} + x(4)e^{-j\frac{2\pi}{8}\cdot 2\cdot 4} + x(5)e^{-j\frac{2\pi}{8}\cdot 2\cdot 5}$$

$$= 1 + e^{-j\frac{\pi}{2}} + e^{-j\pi} + e^{-j\frac{3\pi}{4}} + e^{-j2\pi} + e^{-j\frac{5\pi}{2}}$$

$$= 1 + \left[\cos\frac{\pi}{2} - j\sin\frac{\pi}{2}\right] + \left[\cos\pi - j\sin\pi\right] + \left[\cos\frac{3\pi}{4} - j\sin\frac{3\pi}{4}\right]$$

$$+ \left[\cos 2\pi - j\sin 2\pi\right] + \left[\cos\frac{5\pi}{2} - j\sin\frac{5\pi}{2}\right]$$

$$= 1 + [0 - j(1)] + [-1 - j(0)] + \left[-\frac{1}{\sqrt{2}} - j\frac{1}{\sqrt{2}}\right]$$

$$+ [1 - j(0)] + [0 - j(1)]$$

$$= 1 + (-j) - 1 - \frac{1}{\sqrt{2}} - j\frac{1}{\sqrt{2}} + 1 - j$$

$$= \boxed{1 - \frac{1}{\sqrt{2}} - j\left(\frac{1}{\sqrt{2}} + 2\right)}$$

When k=3

$$X(3) = \sum_{n=0}^{7} x(n)\, e^{-j\frac{2\pi}{8}\cdot 3\cdot n}$$

$$= \sum_{n=0}^{7} x(n)\, e^{-j\frac{3\pi}{4}\cdot n}$$

$$= x(0)e^{-j\frac{3\pi}{4}\cdot 0} + x(1)e^{-j\frac{3\pi}{4}} + x(2)e^{-j\frac{3\pi}{4}\times 2} + x(3)e^{-j\frac{3\pi}{4}\times 3}$$

$$+ x(4)e^{-j\frac{3\pi}{4}\cdot 4} + x(5)e^{-j\frac{3\pi}{4}\cdot 5}$$

$$= 1 + e^{-j\frac{3\pi}{4}} + e^{-j\frac{3\pi}{2}} + e^{-j\frac{9\pi}{4}} + e^{-j3\pi} + e^{-j\frac{15\pi}{4}}$$

$$= 1 + \left[\cos\frac{3\pi}{4} - j\sin\frac{3\pi}{4}\right] + \left[\cos\frac{3\pi}{2} - j\sin\frac{3\pi}{2}\right] + \left[\cos\frac{9\pi}{4} - j\sin\frac{9\pi}{4}\right]$$

$$+ \left[\cos 3\pi - j\sin 3\pi\right] + \left[\cos\frac{15\pi}{4} - j\sin\frac{15\pi}{4}\right]$$

$$= 1 + \left[\frac{-1}{\sqrt{2}} - j\frac{1}{\sqrt{2}}\right] + [0 - j(-1)] + \left[\frac{1}{\sqrt{2}} - j\frac{1}{\sqrt{2}}\right] + [-1 - j(0)] + \left[\frac{1}{\sqrt{2}} - j\left(-\frac{1}{\sqrt{2}}\right)\right]$$

$$= 1 + j + \frac{1}{\sqrt{2}} - j\frac{1}{\sqrt{2}} - 1$$

$$= \boxed{\frac{1}{\sqrt{2}} + j\left(1 - \frac{1}{\sqrt{2}}\right)}$$

When k=4

$$X(4) = \sum_{n=0}^{7} x(n)\, e^{-j\frac{2\pi}{8}\cdot 4\cdot n}$$

$$= \sum_{n=0}^{7} x(n)\, e^{-j\pi n}$$

$$= x(0)e^{-j\pi\cdot 0} + x(1)e^{-j\pi\cdot 1} + x(2)\cdot e^{-j\pi\cdot 2} + x(3)\cdot e^{-j\pi\cdot 3}$$

$$+ x(4)e^{-j\pi\cdot 4} + x(5)\cdot e^{-j\pi 5}$$

$$= 1.1 + 1\left[\cos\pi - j\sin\pi\right] + \left[\cos 2\pi - j\sin 2\pi\right]$$

$$+ \left[\cos 3\pi - j\sin 3\pi\right] + \left[\cos 4\pi - j\sin 4\pi\right] +$$

$$\left[\cos 5\pi - j\sin 5\pi\right]$$

$$= 1 - 1 + 1 - 1 + 1 - 1$$

$$= \boxed{0}$$

When $K=5$:

$$X(5) = \sum_{n=0}^{7} x(n) e^{-j\frac{2\pi}{8}.5.n} \Rightarrow \sum_{n=0}^{7} x(n) e^{-j.\frac{5\pi}{4}.n}$$

$$= x(0) e^{-j\frac{5\pi}{4}.0} + x(1) e^{-j\frac{5\pi}{4}.1} + x(2) e^{-j\frac{5\pi}{4}.2} + x(3) e^{-j\frac{5\pi}{4}.3} + x(4) e^{-j\frac{5\pi}{4}.4} + x(5) e^{-j\frac{5\pi}{4}.5}$$

$$= 1.1 + e^{-j\frac{5\pi}{4}} + e^{-j\frac{5\pi}{2}} + e^{-j\frac{15\pi}{4}} + e^{-j5\pi} + e^{-j\frac{25\pi}{4}}$$

$$= 1 + \left[\cos\frac{5\pi}{4} - j\sin\frac{5\pi}{4}\right] + \left[\cos\frac{5\pi}{2} - j\sin\frac{5\pi}{2}\right] + \left[\cos\frac{15\pi}{4} - j\sin\frac{15\pi}{4}\right] + \left[\cos 5\pi - j\sin 5\pi\right] + \left[\cos\frac{25\pi}{4} - j\sin\frac{25\pi}{4}\right]$$

$$\boxed{X(5) = \frac{1}{\sqrt{2}} - j\left(1 - \frac{1}{\sqrt{2}}\right)}$$

When $k=6$:

$$\boxed{X(6) = 1 + j}$$

When $K=7$:

$$\boxed{X(7) = -\frac{1}{\sqrt{2}} + j\left(1 + \frac{1}{\sqrt{2}}\right)}$$

$$\Rightarrow X(K) = \left\{6, \frac{-1}{\sqrt{2}} - j\left(\frac{1}{\sqrt{2}}+1\right), 1-\frac{1}{\sqrt{2}} - j\left(\frac{1}{\sqrt{2}}+2\right), \frac{1}{\sqrt{2}} + j\left(1-\frac{1}{\sqrt{2}}\right), 0, \frac{1}{\sqrt{2}} - j\left(1-\frac{1}{\sqrt{2}}\right), 1+j, \frac{-1}{\sqrt{2}} + j\left(1+\frac{1}{\sqrt{2}}\right)\right\}$$

23/08/2021

DFT: $X(k) = \sum_{n=-N}^{N} x(n) e^{-j\frac{2\pi}{N}kn}$

Limitation of DFT : → Computation speed is very slow

⇒ To compute all the N no. of pts., we need to perform N^2 complex multiplication & $N(N-1)$ addition ⇒ Time consuming operation

Eg: $N = 1024$

⇒ DFT requires,

$N^2 = (1024)^2 = 10^6$ multiplication

$N(N-1) \Rightarrow (1024)(1023)$ addition

} operations

∴ we go to FFT (Fast Fourier transform)

No. of operations require ⇒ $\frac{N}{2}\log_2 N$.

Eg: $N = 1024$

⇒ FFT requires, $\frac{1024}{2}\log_2 1024 = 5120$

Speed improvement } $= \frac{10^6}{5000} = 200$ times faster than DFT

2 types of FFT algorithm

→ DIT → Decimation in time (✓) (Butterfly diagram)

→ ~~OR~~ DIF → Decimation in freq.

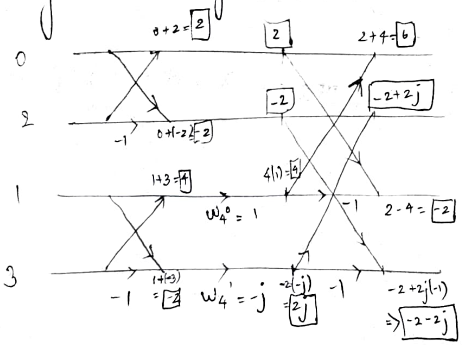
DFT vs FFT Speed Comparison

No. of stages M	No. of pts. N	No. of complex multiplication using DFT: N^2	FFT: $(N/2)\log_2 N$	Speed improvement factor: $\frac{N^2}{(N/2)\log_2 N}$

Q:

(1) Compute the 4-pt. DFT of a sequence $x(n) = \{0, 1, 2, 3\}$ using DIT-FFT algorithm.

⇒



$W_4^0 \rightarrow$ Twiddle factor

$$\boxed{W_N = e^{\frac{-j2\pi}{N}}}$$

$$\Rightarrow W_4^0 = W_4 = e^{-\left(\frac{j2\pi}{4}\right)^0} = 1$$

$$\Rightarrow W_4^1 = e^{-\left(\frac{j2\pi}{4}\right)^1}$$

$$W_4^1 = e^{-j\pi/2}$$

$$\Rightarrow \cos\pi/2 - j\sin\pi/2 \Rightarrow 0 - j = \boxed{-j}$$

$$x(n) = \{0, 1, 2, 3\}$$

Binary pattern

00 → 0
01 → 1
10 → 2
11 → 3

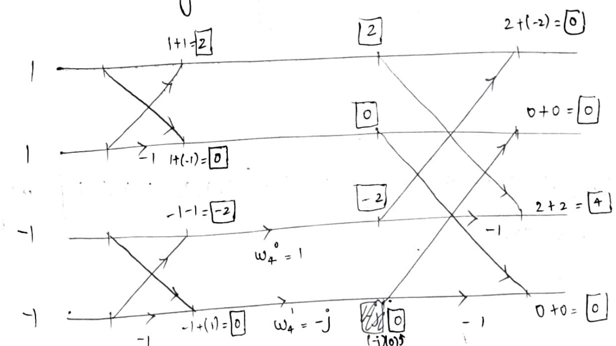
Reverse binary pattern:
(Bit reversal)

00 → 0
10 → 2
01 → 1
11 → 3

$$\Rightarrow \boxed{X(K) = \{6, -2+2j, -2, -2-2j\}}$$

(2) Compute the 4 pt. DFT of a sequence $x(n) = \{1, -1, 1, -1\}$ using DIT-FFT algorithm.

⇒



00 → 1
01 → -1
10 → 1
11 → -1

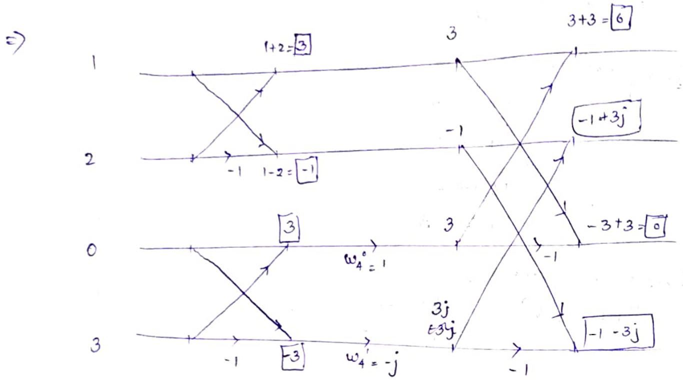
00 → 1
10 → 1
01 → -1
11 → -1

Bit reversal

$$\Rightarrow \boxed{X(K) = \{0, 0, 4, 0\}}$$

(3) Compute the 4 pt. DFT of a sequence $x(n) = \{1, 0, 2, 3\}$ using DIT-FFT algorithm.

⇒



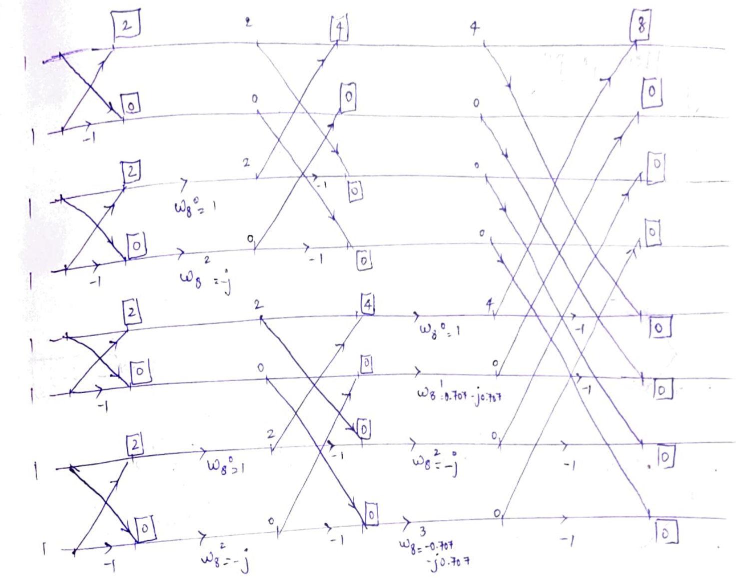
$00 \rightarrow 1$	$00 \rightarrow 1$
$01 \rightarrow 0$	$10 \rightarrow 2$
$10 \rightarrow 2$	$01 \rightarrow 0$
$11 \rightarrow 3$	$11 \rightarrow 3$

Bit reversal

$$\boxed{X(K) = \{6, -1+3j, 0, -1-3j\}}$$

(4) Compute the 8 pt. DFT of a seq. $x(n) = \{1, 1, 1, 1, 1, 1, 1, 1\}$ using DIT-FFT algorithm.

⇒ Here 3 iterations ∵ $N = 8$ //.



$000 \rightarrow 1$	$000 \rightarrow 1$
$001 \rightarrow 1$	$100 \rightarrow 1$
$010 \rightarrow 1$	$010 \rightarrow 1$
$011 \rightarrow 1$	$110 \rightarrow 1$
$100 \rightarrow 1$	$001 \rightarrow 1$
$101 \rightarrow 1$	$101 \rightarrow 1$
$110 \rightarrow 1$	$011 \rightarrow 1$
$111 \rightarrow 1$	$111 \rightarrow 1$

Bit reversal

$W_8^0 = e^{\left(-j\frac{2\pi}{8}\right)\cdot 0} = \boxed{1}$

$W_8^1 = e^{\left(-j\frac{2\pi}{8}\right)\cdot 1} \Rightarrow e^{-j\pi/4}$

$= \cos\frac{\pi}{4} - j\sin\pi/4 \Rightarrow \boxed{0.707 - j0.707}$

$W_8^2 = e^{\left(-j\frac{2\pi}{8}\right)\cdot 2} \Rightarrow e^{-j\frac{4\pi}{8}} \Rightarrow e^{-j\pi/2}$

$= \cos\pi/2 - j\sin\pi/2 = \boxed{-j}$

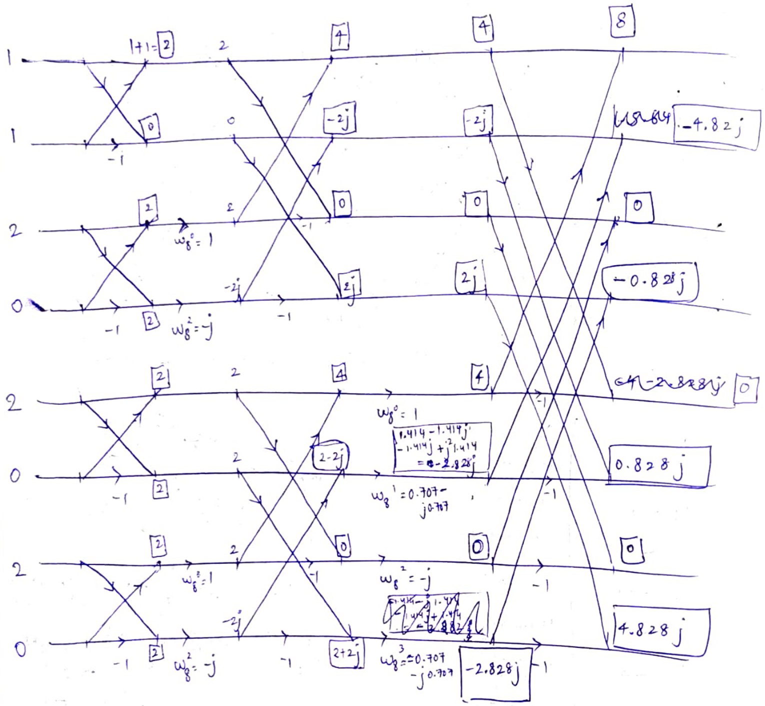
$W_8^3 = e^{\left(-j\frac{6\pi}{8}\right)} \Rightarrow e^{-j3\pi/4}$

$= \cos\frac{3\pi}{4} - j\sin\frac{3\pi}{4} \Rightarrow \boxed{-0.707 - j0.707}$

$$\Rightarrow \boxed{X(K) = \{8, 0, 0, 0, 0, 0, 0, 0\}}$$

25/08/2021

① Using flow graph, determine the 8 pt. DSP of a sequence $x(n)=\{1,2,2,2,1,0,0,0\}$ using DIT-FFT algorithm.



000 → 1	000 → 1
001 → 2	100 → 1
010 → 2	010 → 2
011 → 2	110 → 0
100 → 1	001 → 2
101 → 0	101 → 0
110 → 0	011 → 2
111 → 0	111 → 0

Bit reversal

$W_8^0 = \boxed{1}$

$W_8^1 = \boxed{0.707 - j0.707}$

$W_8^2 = \boxed{-j}$

$W_8^3 = \boxed{-0.707 - j0.707}$

$X(K) = \{8, -4.828j, 0, -0.828j, 0, 0.828j, 0, 4.828j\}$

Discrete Fourier Transform (DFT)

⇒ DFT - is a powerful tool & used to compute Fourier Transform $X(e^{j\omega})$ on a digital computer / specially designed hardware.

⇒ DTFT/FT ⟶ Finite/infinite sequence

DFT ⟶ defined only for finite duration sequence.

⇒ Since $X(e^{j\omega})$ is periodic with period 2π ∴ within one period of (0 to 2π), we take N no. of samples; then we shld take samples @ every $2\pi/N$ i.e. range 2π is divided into N equal intervals.

DFT,

$$X(K) = \sum_{n=0}^{N-1} x(n)\, e^{-j\frac{2\pi}{N}kn}$$

Twiddle factor

$$W_N = e^{-j\frac{2\pi}{N}}$$

$$\boxed{X(K) = \sum_{n=0}^{N-1} x(n) W_N^{kn}}$$

IDFT,

$$x(n) = \frac{1}{N} \sum_{K=0}^{N-1} X(k)\, e^{j\frac{2\pi}{N}kn}$$

$$\Rightarrow \boxed{x(n) = \frac{1}{N} \sum_{K=0}^{N-1} X(k)\, W_N^{-nk}}$$

$\boxed{N=4}$, W_4 $k, n = 0, \dots N-1$

$$\Rightarrow X(0) = \sum_{n=0}^{N-1} x(n).1 = x(0) + x(1) + x(2) + x(3)$$

$$\Rightarrow X(1) = \sum_{n=0}^{N-1} x(n) W_4^{1.n} = \sum_{n=0}^{3} x(n) W_4^{n}$$

$$X(1) = x(0) \cancel{W_4^0}.1 + x(1) W_4^1 + x(2) W_4^2 + x(3) W_4^3$$

$$\Rightarrow X(2) = \sum_{n=0}^{3} x(n) W_4^{2.n}$$

$$X(2) = x(0).1 + x(1) W_4^2 + x(2) W_4^4 + x(3) W_4^6$$

$$\Rightarrow X(3) = \sum_{n=0}^{3} x(n) W_4^{3.n}$$

$$X(3) = x(0) + x(1) W_4^3 + x(2) W_4^6 + x(3) W_4^9$$

(Twiddle factor matrix for DFT)

$$\Rightarrow W_4 = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & W_4^1 & W_4^2 & W_4^3 \\ 1 & W_4^2 & W_4^4 & W_4^6 \\ 1 & W_4^3 & W_4^6 & W_4^9 \end{bmatrix}$$

cont... next page

Matrix multiplication method : $X = W_N x$

N=4:

$$\begin{bmatrix} X(0) \\ X(1) \\ X(2) \\ X(3) \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & W_4^1 & W_4^2 & W_4^3 \\ 1 & W_4^2 & W_4^4 & W_4^6 \\ 1 & W_4^3 & W_4^6 & W_4^9 \end{bmatrix} \begin{bmatrix} x(0) \\ x(1) \\ x(2) \\ x(3) \end{bmatrix}$$

$$W_N = e^{-j\frac{2\pi}{N}}$$

$$W_4 = e^{-j\frac{2\pi}{4}} = e^{-j\pi/2}$$

$$\Rightarrow W_4^1 = e^{\left(-j\frac{2\pi}{4}\right).1} = e^{-j\pi/2} \Rightarrow \cos\frac{\pi}{2} - j\sin\frac{\pi}{2} \Rightarrow 0 - j.1 = \boxed{-j}$$

$$\Rightarrow W_4^2 = e^{\left(-j\frac{2\pi}{4}.2\right)} = e^{-j\pi} \Rightarrow \cos\pi - j\sin\pi \Rightarrow -1 - j.0 = \boxed{-1}$$

$$\Rightarrow W_4^3 = e^{\left(-j\frac{2\pi}{4}\right).3} = e^{-j\frac{3\pi}{2}} \Rightarrow \cos\frac{3\pi}{2} - j\sin\frac{3\pi}{2} \Rightarrow 0 - j(-1) = \boxed{j}$$

Twiddle factor matrix for DFT

$$W_4 = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix}$$

IDFT :

$$x = W_N^{-1} X \quad \text{— (2)}$$

$$x = \frac{1}{4} W_N^{*} X \quad \text{— (3)}$$

$$x = \frac{1}{4} \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & j & -1 & -j \\ 1 & -1 & 1 & -1 \\ 1 & -j & -1 & j \end{bmatrix} . X \Rightarrow x = \frac{1}{4} W_N^{*} X$$

Subs. (2) in (3)

$$W_N^{-1} X = \frac{1}{N} W_N^{*} X \Rightarrow \text{Multiply both sides by } W_N$$

$$\Rightarrow I_N = \frac{1}{N} W_N^{*} W_N$$

$$\boxed{W_N^{*} W_N = I_N . N}$$

DFT as a linear transformation

If We can able to convert the signal from time domain $x(n)$ to freq. domain $X(k)$ ~~as~~ & freq. domain $X(k)$ .. to time domain $x(n)$ ⟹ Linear transformation algorithm.

26/08/2021

Circular Convolution : ~~the~~ DFT / IDFT

(or) Fast convolution

$$X_1(K)\,X_2(K) = DFT\{x_1(n) \circledast x_2(n)\}$$

Q:

① Perform circular convolution of the 2 sequences
- Graphical method
- Matrix method
- DFT & IDFT

$x_1(n) = \{2, 1, 2, 1\}$

$x_2(n) = \{1, 2, 3, 4\}$

$$\rightarrow \begin{bmatrix} 2 & 1 & 2 & 1 \\ 1 & 2 & 1 & 2 \\ 2 & 1 & 2 & 1 \\ 1 & 2 & 1 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix} = \begin{bmatrix} 2+2+6+4 \\ 1+4+3+8 \\ 2+2+6+4 \\ 1+4+3+8 \end{bmatrix} \Rightarrow \underbrace{\begin{bmatrix} 14, 16, 14, 16 \end{bmatrix}}_{x_1(n) \circledast x_2(n)}$$

1st column ⟹ $x_1(n)$

2nd column ⟹ 1 left bit of $x(n)$

3rd column ⟹ 1 left bit

4th column ⟹ 1 left bit

② ~~By using DFT-IDFT~~ Perform circular convolution of the foll. data sequences

$x_1(n) = \{1, 1, 2, 1\}$

$x_2(n) = \{1, 2, 3, 4\}$

$$\begin{bmatrix} 1 & 1 & 2 & 1 \\ 1 & 1 & 1 & 2 \\ 2 & 1 & 1 & 1 \\ 1 & 2 & 1 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 1+2+6+4 \\ 1+2+3+8 \\ 2+2+3+4 \\ 1+4+3+4 \end{bmatrix} = \begin{bmatrix} 13, 14, 11, 12 \end{bmatrix}$$

③ Find the circular convolution of the 2 sequences:

$x_1(n) = \{1, 2, 3, 1\}$

$x_2(n) = \{4, 3, 2, 2\}$

$$\begin{bmatrix} 1 & 1 & 3 & 2 \\ 2 & 1 & 1 & 3 \\ 3 & 2 & 1 & 1 \\ 1 & 3 & 2 & 1 \end{bmatrix} \begin{bmatrix} 4 \\ 3 \\ 2 \\ 2 \end{bmatrix} = \begin{bmatrix} 4+3+6+4 \\ 8+3+2+6 \\ 12+6+2+2 \\ 4+9+4+2 \end{bmatrix}$$

$$= \begin{bmatrix} 17, 19, 22, 19 \end{bmatrix}$$

4) By using DFT-IDFT, determine the output response $y(n)$ if $h(n) = \{1,1,1\}$ & $x(n) = \{1,2,3,1\}$ by using

a) Linear convolution

b) Circular convolution

a) Linear convolution - Tabulation method

$h(n)$ \ $x(n)$	1	2	3	1
1	1	2	3	1
1	1	2	3	1
1	1	2	3	1

Length of $x(n)$ = 4

Length of $h(n)$ = 3

Response length $\Rightarrow N_1 + N_2 - 1$
$= 4+3-1$
$= \boxed{6}$

$$\Rightarrow y(n) = \{1, 3, \cancel{4}6, \cancel{4}6, 4, 1\}$$

b) Circular convolution:

But for circular convolution, length of the i/p signal must be same.

$x(n) = \{1,2,3,1\}$

$h(n) = \{1,1,1\}$

To make them equal length,

$\Rightarrow x(n) = \{1,2,3,1\}$

$h(n) = \{1,1,1,0\}$

$$\begin{bmatrix} 1 & 1 & 3 & 2 \\ 2 & 1 & 1 & 3 \\ 3 & 2 & 1 & 1 \\ 1 & 3 & 2 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 1+1+3+0 \\ 2+1+1+0 \\ 3+2+1+0 \\ 1+3+2+0 \end{bmatrix}$$

$$= \{[5, 4, 6, 6]\}$$

$$\Rightarrow y(n) = \{5, 4, 6, 6\}$$

cont. & next page

Linear filtering:

$$x(n) \longrightarrow \boxed{h(n)} \longrightarrow y(n)$$

$$y(n) = x(n) * h(n)$$

Purpose of linear filtering $\Rightarrow$ To find the response of the system

How to get proper result for circular convolution?

$\Rightarrow$ Circular convolution with zero padding.

$x(n) = \underbrace{\{1,2,3,1\}}_{N_1 = 4}$; $h(n) = \underbrace{\{1,1,1\}}_{N_2 = 3}$

Length of $y(n) = N_1 + N_2 - 1 \Rightarrow 4+3-1$
$= \boxed{6}$

If you want to increase the length of ~~the~~ $y(n)$, then you have to increase the size of $x(n)$ & $h(n)$ by adding zeroes.

4th sum cont..

$\Rightarrow x(n) = \{1, 2, 3, 1, 0, 0\}$

$h(n) = \{1, 1, 1, 0, 0, 0\}$

$$\begin{bmatrix} 1 & 0 & 0 & 1 & 3 & 2 \\ 2 & 1 & 0 & 0 & 1 & 3 \\ 3 & 2 & 1 & 0 & 0 & 1 \\ 1 & 3 & 2 & 1 & 0 & 0 \\ 0 & 1 & 3 & 2 & 1 & 0 \\ 0 & 0 & 1 & 3 & 2 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} 1+0+0+0+0+0 \\ 2+1+0+0+0+0 \\ 3+2+1+0+0+0 \\ 1+3+2+0+0+0 \\ 0+1+3+0+0+0 \\ 0+0+1+0+0+0 \end{bmatrix} = \begin{bmatrix} 1, 3, 6, 6, 4, 1 \end{bmatrix}$$

$\Rightarrow y(n) = \{1, 3, 6, 6, 4, 1\}$

Now ~~by~~ the o/p are same for both linear convolution & circular convolution.

---

5 ~~By using DFT - IDFT determine the o/p response $y(n)$ if $h(n) = \{1, 1,$~~

Steps involved in finding the response $y(n)$ by using DFT - IDFT:

(i) Find $X(K)$ using DFT,

$$x(n) \xrightarrow{DFT} X(k)$$

(ii) Find $H(K)$ using DFT,

$$h(n) \xrightarrow{DFT} H(K)$$

(iii) Compute $X(K).H(K) \longrightarrow$ Product (or) $X(K).H(K)$

(iv) Take IDFT of $[X(K).H(K)] \Rightarrow y(n)$

---

1 ~~By using DFT - IDFT, determine the o/p response $y(n)$ if $h(n) = \{1, 1, 1\}$ & $x(n) = \{1, 2, 3, 1\}$~~

~~$\Rightarrow h(n) = \{1, 1, 1, 0\}$; $x(n) = \{1, 2, 3, 1\}$~~

$\rightarrow$ DFT:

~~When $X(0)$ $K=0$: $\sum_{n=0}^{3} x(n) \cdot e^{-j\frac{2\pi}{4} \cdot 0 \cdot n}$~~

~~$= \sum_{n=0}^{3} x(n) \cdot 1$~~

~~$\Rightarrow 1+2+3+1 = \boxed{7}$~~

27/08/2021

① Find the convolution of $x(n) = \{1,1,1\}$ & $h(n) = \{-1,-1\}$

Linear convolution:

h(n) \ x(n)	1	1	1
-1	-1	-1	-1
-1	-1	-1	-1

$\Rightarrow y(n) = \{-1,-2,-2,-1\}$

② Find the response using circular convolution of $x(n) = \{1,1,1\}$ & $h(n) = \{-1,-1\}$

$\Rightarrow$ Response length $= 3+2-1$
$= \boxed{4}$

$x(n) = \{1,1,1,0\}$ $\qquad h(n) = \{-1,-1,0,0\}$

$$\begin{bmatrix} 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} -1 \\ -1 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} -1+0+0+0 \\ -1-1+0+0 \\ -1-1+0+0 \\ 0-1+0+0 \end{bmatrix}$$

$= [-1,-2,-2,-1]$

~~⑧ $x_1(n) = \{2,1,2,1\}$~~

~~$x_2(n) =$~~

③ Find the response of LTI system when the impulse response is $h(n) = \{-1,-1\}$ & input signal $x(n) = \{1,1,1\}$

$\rightarrow$ $x(n) = [1,1,1] \longrightarrow \boxed{h(n) = [-1,-1]} \longrightarrow y(n) = x(n) * h(n)$

x(n) \ h(n)	-1	-1
1	-1	-1
1	-1	-1
1	-1	-1

$\Rightarrow [-1,-2,-2,-1]$
↳ $y(n)$

④ Find the response of LTI system when the impulse response is $h(n) = \{-1,-1\}$ & i/p signal $x(n) = \{1,1,1\}$ using circular convolution.

$\Rightarrow$ $x(n) = [1,1,1] \longrightarrow \boxed{h(n) = (-1,-1)} \longrightarrow y(n)$
↳ circular convolution

Length of response of LFT $\Big\}$ $N_1 + N_2 - 1$
$\Rightarrow 3+2-1$
$= \boxed{4}$

$y(n) = x(n) \circledast h(n)$

$\Rightarrow x(n) = [1,1,1,0]$

$h(n) = [-1,-1,0,0]$

$$\begin{bmatrix} 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} -1 \\ -1 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} \text{ans} -1, -2, -2, -1 \end{bmatrix}$$

(4)

---

(5) Determine the o/p response $y(n)$ if

$h(n) = \{1, 1, 1\}$

$x(n) = \{1, 2, 3, 1\}$

$\Rightarrow$ Length of response, $y(n) = N_1 + N_2 - 1$

$\Rightarrow 4 + 3 - 1 = \boxed{6}$

$x(n) = \{1, 2, 3, 1, 0, 0\}$

$h(n) = \{1, 1, 1, 0, 0, 0\}$

$$\begin{bmatrix} 1 & 0 & 0 & 1 & 3 & 2 \\ 2 & 1 & 0 & 0 & 1 & 3 \\ 3 & 2 & 1 & 0 & 0 & 1 \\ 1 & 3 & 2 & 1 & 0 & 0 \\ 0 & 1 & 3 & 2 & 1 & 0 \\ 0 & 0 & 1 & 3 & 2 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} \\ \\ \\ \\ \\ \\ \end{bmatrix}$$

S. No.	Date	Title	Page No.	Teacher's Sign/Remarks

27/08/2021

UNIT-II

⑥ Find the response of the LTI system when the impulse response is $h(n) = \{-1,-1\}$ & input signal $x(n) = \{1,1,1\}$ using DFT-IDFT method

$\Rightarrow$ $[1,1,1]$ $x(n) \rightarrow \boxed{h(n)} \xrightarrow{y(n)}$; $h(n) = [-1,-1]$

Length $\Rightarrow y(n) = N_1 + N_2 - 1 \Rightarrow 3+2-1 = \boxed{4}$

$\Rightarrow x(n) = [1,1,1,0]$; $h(n) = [-1,-1,0,0]$

— Using DFT- IDFT method —

(i) Find $X(k) \Rightarrow DFT[x(n)]$

(ii) Find $H(k) \Rightarrow DFT[h(n)]$

(iii) Find $Y(k) \Rightarrow X(k).H(k)$

(iv) Find IDFT $[Y(k)] = y(n)$

$x(n) = \{1,1,1,0\}$ $\boxed{X = W_N \cdot x}$

$\underline{N=4}$

$X_4 = W_4 x$

$$= \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \\ 0 \end{bmatrix}$$

DFT:

$$X(k) = \sum_{n=0}^{N-1} x(n) e^{-j\frac{2\pi}{N} nk}$$

$\downarrow$

$x_t \cdot W_N$

$$X(K) = \begin{bmatrix} 3 \\ -j \\ 1 \\ j \end{bmatrix}$$

$X(K) = \{3, -j, 1, j\}$

$$H_4 = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix} \begin{bmatrix} -1 \\ -1 \\ 0 \\ 0 \end{bmatrix}$$

$$H_K = \begin{bmatrix} -2 \\ -1+j \\ 0 \\ -1-j \end{bmatrix}$$

$\Rightarrow Y(K) = X(K) \cdot H(K)$

$= \{3, -j, 1, j\} \{-2, -1+j, 0, -1-j\}$

$Y(K) = \{-6, j+1, 0, -j+1\}$

IDFT $[y(k)]$

$$y(n) = \frac{1}{4} \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & j & -1 & -j \\ 1 & -1 & 1 & -1 \\ 1 & -j & -1 & j \end{bmatrix} \begin{bmatrix} -6 \\ 1+j \\ 0 \\ 1-j \end{bmatrix}$$

$$y(n) = \frac{1}{4} \begin{bmatrix} -6+1+j+1-j \\ -6+j-1-j+1 \\ -6-1-j-1+j \\ -6-j+1+j+1 \end{bmatrix} = \begin{bmatrix} -4 \\ -8 \\ -8 \\ -4 \end{bmatrix}$$

$y(n) = \frac{1}{4}\{-4, -8, -8, -4\} \Rightarrow y(n) = [-1, -2, -2, -1]$

---

(7) By using DFT - IDFT, determine the o/p response $y(n)$ if $h(n) = \{1, 1, 1\}$ & $x(n) = \{1, 2, 3, 1\}$

$\rightarrow$ $x(n) = \{1, 1, 1, 0\}$

$X_4 = W_4 \, x_4$

$$= \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \\ 0 \end{bmatrix}$$

$$X_4 = \begin{bmatrix} 3 \\ -j \\ 1 \\ j \end{bmatrix}$$

$\Rightarrow X(K) = \{3, -j, 1, j\}$

IDFT is,

$$x_N = \frac{1}{N}[W_N]^* X_N$$

01/09/2021

Properties of DFT - Circular convolution

Convolution property:

DFT of:
$x_1(n) = X_1(k)$
$x_2(n) = X_2(k)$

$$X_1(k)\, X_2(k) = DFT\{x_1(n) \circledast x_2(n)\}$$

$$\Rightarrow x_1(n) \circledast x_2(n) = IDFT[X_1(k)\, X_2(k)]$$

Convolution ~~of~~ property of DFT $\Rightarrow$ multiplication of DFT of 2 sequences is equivalent to DFT of circular convolution of 2 sequences.

Circular convolution: [CC]

(i) CC of 2 seq. requires that one of seq. shld be periodic.

(ii) CC can be performed only if both the seq. consists of ~~sample~~ same no. of samples.

(iii) If the seq. have diff no. of samples then convert the smaller size sequence to the size of large size by zero padding.

(iv) Linear convolution: Output length $= N_1 + N_2 - 1$
Circular convolution: Output length $= \max(N_1, N_2)$

Method of Performing CC

(i) Graphical method / concentric circle method (Refer textbook)

(ii) CC using matrices

(iii) CC using DFT & IDFT

①

Perform CC of 2 sequences using matrices method.

$x_1(n) = \{\underbrace{2,1,2,1}_{4}\}$ $x_2(n) = \{\underbrace{1,2,3,4}_{4}\}$

$$\Rightarrow \begin{bmatrix} x_2(0) & x_2(3) & x_2(2) & x_2(1) \\ x_2(1) & x_2(0) & x_2(3) & x_2(2) \\ x_2(2) & x_2(1) & x_2(0) & x_2(3) \\ x_2(3) & x_2(2) & x_2(1) & x_2(0) \end{bmatrix} \begin{bmatrix} x_3(0) \\ x_3(1) \\ x_3(2) \\ x_3(3) \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 4 & 3 & 2 \\ 2 & 1 & 4 & 3 \\ 3 & 2 & 1 & 4 \\ 4 & 3 & 2 & 1 \end{bmatrix} \begin{bmatrix} 2 \\ 1 \\ 2 \\ 1 \end{bmatrix} = \begin{bmatrix} 14 \\ 16 \\ 14 \\ 16 \end{bmatrix}$$

CC using DFT & IDFT:

(i) Take N pt. DFT $x_1(n)$ & $x_2(n)$

$X_1(K) = DFT(x_1(n))$ & $X_2(K) = DFT(x_2(n))$

(ii) Product of $X_1(K)$ & $X_2(K)$

$X_3(K) = X_1(K)\, X_2(K)$

(iii) $x_3(n) = IDFT[X_3(K)]$

CASE 2.

① Find the response of the system whose impulse response & i/p seq. are $h(n) = \{0.5, 1\}$ & $x(n) = \{1, 0.5\}$ using ~~DFT - IDFT~~ circular convolution (linear convolution)

$\rightarrow$ $N_1 = 2$ & $N_2 = 2$

$N = N_1 + N_2 - 1 \Rightarrow 2+2-1$

$N = 3$

N shld be power of 2..

$\Rightarrow h(n) = \{0.5, 1, 0\}$; $x(n) = \{1, 0.5, 0\}$

~~But N pt. DFT, Here N = 3~~

~~$\Rightarrow h(n) = \{0.5, 1, 0, 0\}$; $x(n) = \{1, 0.5, 0, 0\}$~~

$$\begin{bmatrix} 1 & 0 & 0.5 \\ 0.5 & 1 & 0 \\ 0 & 0.5 & 1 \end{bmatrix} \begin{bmatrix} 0.5 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 0.5 \\ 1.25 \\ 0.5 \end{bmatrix} \Leftarrow y(n)$$

$\boxed{\text{CASE 1:}}$

Circular convolution $\longrightarrow$ length of $x_1(n)$ = length of $h(n) = N$

$\boxed{\text{CASE 2:}}$

Response using CC $\longrightarrow$ $N_1 + N_2 - 1 = N$

CASE 1:

② Compute cc :

$x(n) = \{1, 0.5\}$

$h(n) = \{0.5, 1\}$

$$\Rightarrow \begin{bmatrix} 1 & 0.5 \\ 0.5 & 1 \end{bmatrix} \begin{bmatrix} 0.5 \\ 1 \end{bmatrix} \Rightarrow y(n) = [1, 1.25]$$

CASE 3: Response using linear convolution.

CASE 4:

③ Find respone: $x(n) = \{1, 0.5\}$, $h(n) = \{0.5, 1\}$ using DFT - IDFT.

$\rightarrow N_1 = 2 \; ; N_2 = 2$

$N = N_1 + N_2 - 1$

$N = 2 + 2 - 1$

$\Rightarrow \boxed{N = 3}$

But N pt. DFT, Here N=3

$\Rightarrow N = 4$

$\Rightarrow x(n) = \{1, 0.5, 0, 0\} \; ; \; h(n) = \{0.5, 1, 0, 0\}$

(i)

* To find X(k)

$X_4 = W_4 x_4$

$$= \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix} \begin{bmatrix} 1 \\ 0.5 \\ 0 \\ 0 \end{bmatrix} \Rightarrow \begin{bmatrix} 1.5 \\ 1 - 0.5j \\ 0.5 \\ 1 + 0.5j \end{bmatrix}$$

$\Rightarrow X(K) = \{1.5, 1 - 0.5j, 0.5, 1 + 0.5j\}$

* To find H(K) :

$H_4 = W_4 h_4$

$$= \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & \cancel{j} -1 \\ 1 & j & -1 & -j \end{bmatrix} \begin{bmatrix} 0.5 \\ 1 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 1.5 \\ 0.5 - j \\ -0.5 \\ 0.5 + j \end{bmatrix}$$

$\Rightarrow H(k) = \{1.5, 0.5 - j, -0.5, 0.5 + j\}$

(ii) $Y(K) = X(K) H(K)$

$\Rightarrow Y(K) = \{2.25, -1.25j, -0.25, 1.25j\}$

(iii) IDFT of Y(k)

IDFT :

$y_n = \frac{1}{N} [W_N]^* Y_N$.

$$y_n = \frac{1}{4} \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & j & -1 & -j \\ 1 & -1 & 1 & -1 \\ 1 & -j & -1 & j \end{bmatrix} \begin{bmatrix} 2.25 \\ -1.25j \\ -0.25 \\ 1.25j \end{bmatrix}$$

$$= \frac{1}{4} \begin{bmatrix} 2.25 - 1.25j - 0.25 + 1.25j \\ 2.25 + 1.25 + 0.25 + 1.25 \\ 2.25 + 1.25j - 0.25 - 1.25j \\ 2.25 - 1.25 + 0.25 - 1.25 \end{bmatrix} \Rightarrow [0.5, 1.25, 0.5, 0]$$

circular convolution

(4) By using ~~DFT-IDFT~~ determine o/p response $y(n)$ if

$h(n) = \{1, 1, 1\}$ & $x(n) = \{1, 2, 3, 1\}$

(5) By using DFT - IDFT, determine o/p response $y(n)$ if $h(n) = \{-1, -1\}$

& $x(n) = \{1, 1, 1\}$

CASE 5 :

Response using DIT - FFT

(1) In a LTI system the input $x(n) = \{1, 1, 1\}$ & the impulse response $h(n) = \{-1, -1\}$. Determine the o/p response $y(n)$ of LTI system by radix-2 DIT-FFT.

→ $N_1 = 3$; $N_2 = 2$

$N = N_1 + N_2 - 1$

$\Rightarrow 3 + 2 - 1 = \boxed{4}$ (Already power of 2)

$\Rightarrow h(n) = \{-1, -1, 0, 0\}$; $x(n) = \{1, 1, 1, 0\}$

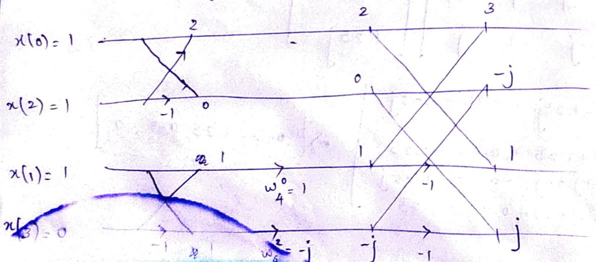
Step 1:

To find $X(k)$

$x(n) = \{1, 1, 1, 0\}$

00 → 1 → x(0)
01 → 1 → x(1)
10 → 1 → x(2)
11 → 0 → x(3)

00 — 1 — x(0)
10 — 1 — x(2)
01 — 1 — x(1)
11 — 0 — x(3)



$\Rightarrow X(k) = \{3, -j, 1, j\}$

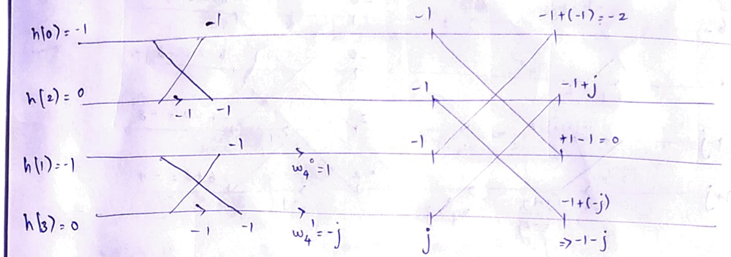
Step 2:

To find $H(k)$

$h(n) = \{-1, -1, 0, 0\}$

00 → -1 → h(0)
01 → -1 → h(1)
10 → 0 → h(2)
11 → 0 → h(3)

00 → -1 → h(0)
10 → 0 → h(2)
01 → -1 → h(1)
11 → 0 → h(3)



$H(k) = \{-2, -1+j, 0, -1-j\}$

Step - 3 :

$Y(k) = X(k) \cdot H(k)$

$X(k) = \{3, -j, 1, j\}$ $H(k) = \{-2, -1+j, 0, -1-j\}$

$\Rightarrow Y(k) = \{-6, 1+j, 0, 1-j\}$

Step - 4 :

To find IDFT of $Y(k)$, first compute the complex conjugate of $Y(k)$.

$Y(k) = \{-6, 1+j, 0, 1-j\}$

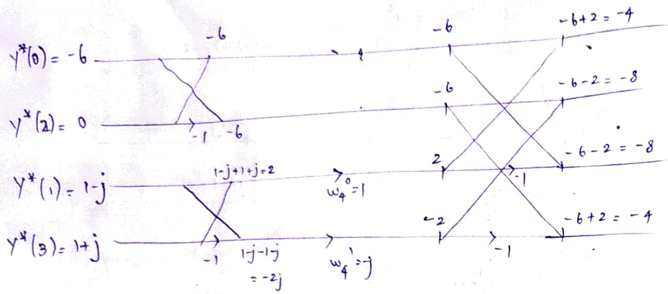
$\Rightarrow Y^*(k) = \{-6, 1-j, 0, 1+j\}$

Now, apply $Y^*(k)$ in DIT - FFT algorithm.

$y^*(k) = \{-6, 1-j, 0, 1+j\}$

$00 \to -6 \to y^*(0)$
$01 \to 1-j \to y^*(1)$
$10 \to 0 \to y^*(2)$
$11 \to 1+j \to y^*(3)$

$00 \to -6 \to y^*(0)$
$10 \to 0 \to y^*(2)$
$01 \to 1-j \to y^*(1)$
$11 \to 1+j \to y^*(3)$



$y(n) = \frac{1}{4}\{-4, -8, -8, -4\}$

$\Rightarrow \boxed{y(n) = \{-1, -2, -2, -1\}}$

HW:

③ Determine the response of LTI system when the I/P seq $x(n) = \{-1, 1, 2, 1, -1\}$ by radix-2 DIT-FFT. The impulse response of the system is $h(n) = \{-1, 1, -1, 1\}$

Hint: $N_1 + N_2 - 1$

$= 5 + 4 - 1 \Rightarrow \boxed{8}$ (Power of 2) $\Rightarrow$ $\boxed{8 \text{ pt. DIT-FFT}}$

02/09/2021

HW:

① By using CC, determine o/p response $y(n)$ if $h(n) = \{1, 1, 1\}$ & $x(n) = \{1, 2, 3, 1\}$

$\rightarrow$ $N_1 = 4$; $N_2 = 3$

$\Rightarrow N = N_1 + N_2 - 1$

$\Rightarrow 4 + 3 - 1 = \boxed{6}$ (not a power of 2)

$\Rightarrow \boxed{N = 8}$

$\Rightarrow x(n) = \{1, 2, 3, 1, 0, 0, 0, 0\}$

$h(n) = \{1, 1, 1, 0, 0, 0, 0, 0\}$

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 1 & 3 & 2 \\ 2 & 1 & 0 & 0 & 0 & 0 & 1 & 3 \\ 3 & 2 & 1 & 0 & 0 & 0 & 0 & 1 \\ 1 & 3 & 2 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 3 & 2 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 3 & 2 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 3 & 2 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 3 & 2 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 \\ 2+1 \\ 3+2+1 \\ 1+3+2 \\ 1+3 \\ 1 \\ 0 \\ 0 \end{bmatrix}$$

$y(n) = [1, 3, 6, 6, 4, 1, 0, 0]$

Linear Convolution

h(n) \ x(n)	1	2	3
1	1	2	3
1	1	2	3
1	1	2	3

$[1, 3, 6, 6, 4, 1]$

② By using DFT - IDFT, determine o/p response $y(n)$ if $h(n) = \{-1, -1\}$ & $x(n) = \{1, 1, 1\}$

→ $N_1 = 3$; $N_2 = 2$

$N = N_1 + N_2 - 1$

$\Rightarrow 3 + 2 - 1 = \boxed{4}$

$\Rightarrow$ $x(n) = \{1, 1, 1, 0\}$; $h(n) = \{-1, -1, 0, 0\}$

(i)
* To find $X(k)$

$X_4 = W_4 x_4$

$$= \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \\ 0 \end{bmatrix} \Rightarrow \begin{bmatrix} 1+1+1 \\ 1-j-1 \\ 1-1+1 \\ 1+j-1 \end{bmatrix} \Rightarrow \begin{bmatrix} 3 \\ -j \\ 1 \\ j \end{bmatrix}$$

$X(k) = \{3, -j, 1, j\}$

* To find $H(k)$

$H_4 = W_4 h_4$

$$= \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix} \begin{bmatrix} -1 \\ -1 \\ 0 \\ 0 \end{bmatrix} \Rightarrow \begin{bmatrix} -1-1 \\ -1+j \\ -1+1 \\ -1-j \end{bmatrix} \Rightarrow \begin{bmatrix} -2 \\ -1+j \\ 0 \\ -1-j \end{bmatrix}$$

$H(k) = \{-2, -1+j, 0, -1-j\}$

(ii) $Y(K) = X(K) \cdot H(K)$

$= \{3, -j, 1, j\}\{-2, -1+j, 0, -1-j\}$

$Y(K) = \{-6, 1+j, 0, 1-j\}$

(iii) IDFT of $Y(k)$

IDFT: $y_n = \frac{1}{N} [W_N]^* Y_N$

$$y_n = \frac{1}{4} \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & j & -1 & -j \\ 1 & -1 & 1 & -1 \\ 1 & -j & -1 & j \end{bmatrix} \begin{bmatrix} -6 \\ 1+j \\ 0 \\ 1-j \end{bmatrix}$$

$$= \frac{1}{4} \begin{bmatrix} -6+1+\cancel{j}+1-\cancel{j} \\ -6+j\cancel{-1}-j\,-1 \\ -6-1-\cancel{j}-1+\cancel{j} \\ -6-\cancel{j}+1+\cancel{j}+1 \end{bmatrix}$$

$$\Rightarrow \frac{1}{4} \begin{bmatrix} -4 \\ -8 \\ -8 \\ -4 \end{bmatrix} \Rightarrow [-1, -2, -2, -1]$$

$y(n) = \{-1, -2, -2, -1\}$

02/09/2021

Filtering Long duration sequences

Long duration data seq → $h(n)$ → $y(n) = x(n) * h(n)$

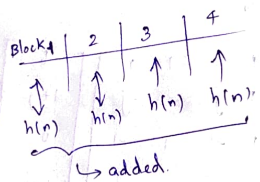
$x[n] = [\quad]$ ↳ $L_s = 10$

$\underbrace{h(n) = [1,1,1]}_{3}$

x(n)

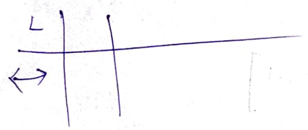
h(n)

Overlap Save method
Overlap Add method



Overlap Save method

(i) L_s = length of $x(n)$; m = length of impulse response

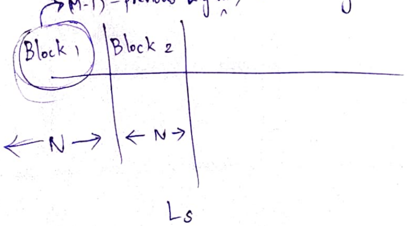


$\boxed{L >> M}$

$M \to h(n)$

$N = L + M - 1$

(M-1) - previous digits ; L data digits (data blocks)



Q:

① Find the o/p $y(n)$ of a filter whose impulse response $h(n) = \{1,1,1\}$ & i/p signal $x(n) = \{3,-1,0,1,3,2,0,1,2,1\}$

Use overlap save method.

→ $L_s = 10$ (length of $x(n)$)

$M = 3$ (length of $h(n)$)

$L >> M$ ⇒ $L = 4$

$N = L + M - 1$
$= 4 + 3 - 1 = 6$

$\boxed{N = 6}$ ⇒ L - info. digits

(M-1) - digits are from previous data blocks.

(M-1) 2   L

| 0 0 | 3 -1 0 1 | ⇒ $x_1(n)$

| 0 1 | 3 2 0 1 | ⇒ $x_2(n)$

| 0 1 | 2 1 0 0 | ⇒ $x_3(n)$

$x_1(n) * h(n) \Rightarrow [0,0,3,-1,0,1] \circledast [1,1,1,0,0,0]$

$x_2(n) * h(n) \Rightarrow [0,1,3,2,0,1] \circledast [1,1,1,0,0,0]$

$x_3(n) * h(n) \Rightarrow [0,1,2,1,0,0] \circledast [1,1,1,0,0,0]$

$x_1(n) \circledast h(n)$

$$\begin{bmatrix} 0 & 1 & 0 & -1 & 3 & 0 \\ 0 & 0 & 1 & 0 & -1 & 3 \\ 3 & 0 & 0 & 1 & 0 & -1 \\ -1 & 3 & 0 & 0 & 1 & 0 \\ 0 & -1 & 3 & 0 & 0 & 1 \\ 1 & 0 & -1 & 3 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 \\ 1 \\ 3 \\ -1+3 \\ -1+3 \\ 1-1 \end{bmatrix}$$

$\Rightarrow y_1(n) = [1, 1, 3, 2, 2, 0]$

$x_2(n) \circledast h(n)$

$$\begin{bmatrix} 0 & 1 & 0 & 2 & 3 & 1 \\ 1 & 0 & 1 & 0 & 2 & 3 \\ 3 & 1 & 0 & 1 & 0 & 2 \\ 2 & 3 & 1 & 0 & 1 & 0 \\ 0 & 2 & 3 & 1 & 0 & 1 \\ 1 & 0 & 2 & 3 & 1 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 \\ 1+1 \\ 3+1 \\ 2+3+1 \\ 2+3 \\ 1+2 \end{bmatrix}$$

$\Rightarrow y_2(n) = [1, 2, 4, 6, 5, 3]$

$x_3(n) \circledast h(n)$

$$\begin{bmatrix} 0 & 0 & 0 & 1 & 2 & 1 \\ 1 & 0 & 0 & 0 & 1 & 2 \\ 2 & 1 & 0 & 0 & 0 & 1 \\ 1 & 2 & 1 & 0 & 0 & 0 \\ 0 & 1 & 2 & 1 & 0 & 0 \\ 0 & 0 & 1 & 2 & 1 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} \Rightarrow \begin{bmatrix} 0 \\ 1 \\ 2+1 \\ 1+2+1 \\ 1+2 \\ 1 \end{bmatrix}$$

$\Rightarrow y_3(n) = [0, 1, 3, 4, 3, 1]$

$\Rightarrow y_1(n) = [1, 1, 3, 2, 2, 0]$

$y_2(n) = [1, 2, 4, 6, 5, 3]$

$y_3(n) = [0, 1, 3, 4, 3, 1]$

} Discard (M-1) pts. from the o/p : $y_1(n), y_2(n), y_3(n)$

$\Rightarrow y(n) = [3, 2, 2, 0, 4, 6, 5, 3, 3, 4, 3, 1]$

② Find the o/p $y(n)$ of a filter whose impulse response $h(n) = \{1, 2\}$ & i/p signal $x(n) = \{1, 2, -1, 2, 3, -2, -3, -1, 1, 1, 2, -1\}$. Use overlap save method.

$\rightarrow$ $L_s = 12$; $\boxed{M = 2}$

$L >> M$

$\Rightarrow \boxed{L = 3}$

$N = L + M - 1$

$\Rightarrow 3 + 2 - 1 = 4$

$\boxed{N = 4}$

$h(n) = \{1, 2, 0, 0\}$

$\boxed{x_1(n)}$ (M-1) L = 3

0	1 2 -1

$\boxed{x_2(n)}$

-1	2 3 -2

$\boxed{x_3(n)}$

-2	-3 -1 1

$\boxed{x_4(n)}$

1	1 2 -1

$x_1(n) \circledast h(n)$

$$\begin{bmatrix} 0 & -1 & 2 & 1 \\ 1 & 0 & -1 & 2 \\ 2 & 1 & 0 & -1 \\ -1 & 2 & 1 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 0 \\ 0 \end{bmatrix} \Rightarrow \begin{bmatrix} -2 \\ 1 \\ 2+2 \\ -1+4 \end{bmatrix}$$

$\Rightarrow y_1(n) = [-2, 1, 4, 3]$

$x_2(n) \circledast h(n)$

$$\begin{bmatrix} -1 & -2 & 3 & 2 \\ 2 & -1 & -2 & 3 \\ 3 & 2 & -1 & -2 \\ -2 & 3 & 2 & -1 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 0 \\ 0 \end{bmatrix} \Rightarrow \begin{bmatrix} -1-4 \\ 2-2 \\ 3+4 \\ -2+6 \end{bmatrix}$$

$y_2(n) = [-5, 0, 7, 4]$

$x_3(n) \circledast h(n)$

$$\begin{bmatrix} -2 & 1 & -1 & -3 \\ -3 & -2 & 1 & -1 \\ -1 & -3 & -2 & 1 \\ 1 & -1 & -3 & -2 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 0 \\ 0 \end{bmatrix} \Rightarrow \begin{bmatrix} -2+2 \\ -3-4 \\ -1-6 \\ 1-2 \end{bmatrix}$$

$y_3(n) = [0, -7, -7, -1]$

$x_4(n) \circledast h(n)$

$$\begin{bmatrix} 1 & -1 & 2 & 1 \\ 1 & 1 & -1 & 2 \\ 2 & 1 & 1 & -1 \\ -1 & 2 & 1 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 0 \\ 0 \end{bmatrix} \Rightarrow \begin{bmatrix} 1-2 \\ 1+2 \\ 2+2 \\ -1+4 \end{bmatrix}$$

$y_4(n) = [-1, 3, 4, 3]$

$\Rightarrow y_1(n) = [\cancel{-2}, 1, 4, 3]$ | $y_3(n) = [\cancel{0}, -7, -7, -1]$

$y_2(n) = [\cancel{-5}, 0, 7, 4]$ | $y_4(n) = [\cancel{-1}, 3, 4, 3]$

} Discard first M-1 pts

Here M = 2

$y(n) = [1, 4, 3, 0, 7, 4, -7, -7, -1, 3, 4, 3]$

Overlap Add Method

In this method the size of the I/P data block is L points & the size of the DFT & IDFT is $N = L+M-1$. To each data block we append M-1 zeroes & compute the N-point DFT.

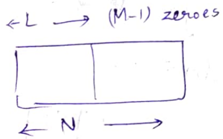
Q) ① Find the O/P $y(n)$ of a filter whose impulse response $h(n) = \{1,1,1\}$ & I/P signal $x(n) = \{3,-1,0,1,3,2,0,1,2,1\}$. Use overlap add method.

→ $L_s = 10$, $M = 3$, $L = 4$

$N = L + M - 1$

$= 4 + 3 - 1$

$\boxed{N = 6}$



$x_1(n)$

3	-1	0	1	0	0

$x_2(n)$

3	2	0	1	0	0

$x_3(n)$

2	1	0	0	0	0

$h(n) = (1,1,1,0,0,0)$

$y_1(n) : x_1(n) \circledast h(n)$

$$\begin{bmatrix} 3 & 0 & 0 & 1 & 0 & -1 \\ -1 & 3 & 0 & 0 & 1 & 0 \\ 0 & -1 & 3 & 0 & 0 & 1 \\ 1 & 0 & -1 & 3 & 0 & 0 \\ 0 & 1 & 0 & -1 & 3 & 0 \\ 0 & 0 & 1 & 0 & -1 & 3 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} \Rightarrow \begin{bmatrix} 3 \\ -1+3 \\ -1+3 \\ 1-1 \\ 1 \\ 1 \end{bmatrix}$$

$\rightarrow y_1(n) = [3, 2, 2, 0, 1, 1]$

$y_2(n) = x_2(n) \circledast h(n)$

$$\begin{bmatrix} 3 & 0 & 0 & 1 & 0 & 2 \\ 2 & 3 & 0 & 0 & 1 & 0 \\ 0 & 2 & 3 & 0 & 0 & 1 \\ 1 & 0 & 2 & 3 & 0 & 0 \\ 0 & 1 & 0 & 2 & 3 & 0 \\ 0 & 0 & 1 & 0 & 2 & 3 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} \Rightarrow \begin{bmatrix} 3 \\ 2+3 \\ 2+3 \\ 1+2 \\ 1 \\ 1 \end{bmatrix}$$

$\Rightarrow y_2(n) = [3, 5, 5, 3, 1, 1]$

$y_3(n) = x_3(n) \circledast h(n)$

$$\begin{bmatrix} 2 & 0 & 0 & 0 & 0 & 1 \\ 1 & 2 & 0 & 0 & 0 & 2 \\ 0 & 1 & 2 & 0 & 0 & 0 \\ 0 & 0 & 1 & 2 & 0 & 0 \\ 0 & 0 & 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} \Rightarrow \begin{bmatrix} 2 \\ 1+2 \\ 1+2 \\ 1 \\ 0 \\ 0 \end{bmatrix}$$

$\Rightarrow y_3(n) = [2, 3, 3, 1, 0, 0]$

$y_1(n) = [3, 2, 2, 0, 1, 1]$

$y_2(n) = [3, 5, 5, 3, 1, 1]$

$y_3(n) = [2, 3, 3, 1, 0, 0]$

$$\begin{array}{rcccccccccccccc} \Rightarrow & 3 & 2 & 2 & 0 & 1 & 1 & & & & & & & & \\ & & & & & 3 & 5 & 5 & 3 & 1 & 1 & & & & \\ (+) & & & & & & & & & 2 & 3 & 3 & 1 & 0 & 0 \\ \hline & 3 & 2 & 2 & 0 & 4 & 6 & 5 & 3 & 3 & 4 & 3 & 1 & & \end{array}$$

$\Rightarrow y(n) = [3, 2, 2, 0, 4, 6, 5, 3, 3, 4, 3, 1]$

② Find the O/P $y(n)$ of a filter whose impulse response $h(n) = \{1, 2\}$ & I/P signal $x(n) = \{1, 2, -1, 2, 3, -2, -3, -1, 1, 1, 2, -1\}$. Use overlap add method.

$\rightarrow L_s = 12$; $\boxed{M = 2}$

$L >> M$

$\Rightarrow \boxed{L = 3}$

$h(n) = \{1, 2, 0, 0\}$

$N = L + M - 1$

$= 3 + 2 - 1 = \boxed{4}$

$x_1(n)$: | 1 2 -1 | 0 |

$x_2(n)$: | 2 3 -2 | 0 |

$x_3(n)$: | -3 -1 1 | 0 |

$x_4(n)$: | 1 2 -1 | 0 |

$y_1(n) = x_1(n) \circledast h(n)$

$$\begin{bmatrix} 1 & 0 & -1 & 2 \\ 2 & 1 & 0 & -1 \\ -1 & 2 & 1 & 0 \\ 0 & -1 & 2 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 0 \\ 0 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 \\ 2+2 \\ -1+4 \\ -2 \end{bmatrix}$$

$\Rightarrow y_1(n) = [1, 4, 3, -2]$

$y_2(n) = x_2(n) \circledast h(n)$

$$\begin{bmatrix} 2 & 0 & -2 & 3 \\ 3 & 2 & 0 & -2 \\ -2 & 3 & 2 & 0 \\ 0 & -2 & 3 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 0 \\ 0 \end{bmatrix} \Rightarrow \begin{bmatrix} 2 \\ 3+4 \\ -2+6 \\ -4 \end{bmatrix}$$

$\Rightarrow y_2(n) = [2, 7, 4, -4]$

$y_3(n) = x_3(n) \circledast h(n)$

$$\begin{bmatrix} -3 & 0 & 1 & -1 \\ -1 & -3 & 0 & 1 \\ 1 & -1 & -3 & 0 \\ 0 & 1 & -1 & -3 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 0 \\ 0 \end{bmatrix} \Rightarrow \begin{bmatrix} -3 \\ -1-6 \\ 1-2 \\ 2 \end{bmatrix}$$

$y_3(n) = [-3, -7, -1, 2]$

$y_4(n) = x_4(n) \circledast h(n)$

$$\begin{bmatrix} 1 & 0 & -1 & 2 \\ 2 & 1 & 0 & -1 \\ -1 & 2 & 1 & 0 \\ 0 & -1 & 2 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 0 \\ 0 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 \\ 2+2 \\ -1+4 \\ -2 \end{bmatrix}$$

$y_4(n) = [1, 4, 3, -2]$

$y_1(n) = [1, 4, 3, -2]$ | $y_3(n) = [-3, -7, -1, 2]$

$y_2(n) = [2, 7, 4, -4]$ | $y_4(n) = [1, 4, 3, -2]$

$\Rightarrow$	1	4	3	-2									
				2	7	4	-4						
(+)							-3	-7	-1	2			
										1	4	3	-2
	1	4	3	0	7	4	-7	-7	-1	3	4	3	-2

$y(n) = [\cancel{1, 4, 3,}\ 0, 7, 4, -7, -7, -1, 3, 4, 3, -2]$

03/09/2021

Properties of DFT:

- Symmetry property
- Periodicity
- Linearity
- Circular convolution
- Time shifting
- Perceval's Theorem

(i) Symmetry Property:

$$x(n) \xleftrightarrow{DFT} X(k)$$

$$X(k) = X^*(N-k)$$

Proof: $X(k) = \sum_{n=0}^{N-1} x(n) e^{-j\frac{2\pi}{N}kn} \qquad k = 0, 1, \dots N-1$

$$X(N-k) = \sum_{n=0}^{N-1} x(n)\, e^{-j\frac{2\pi}{N}n(N-k)}$$

$$= \sum_{n=0}^{N-1} x(n)\, e^{j\frac{2\pi}{N}nk}\, e^{-j\frac{2\pi}{N}\cdot n}$$

$$= \sum_{n=0}^{N-1} x(n)\, e^{j\frac{2\pi}{N}\cdot n\cdot k} \cdot 1 \qquad \left(\because e^{-j\frac{2\pi}{N}\cdot n} = 1\right)$$

$$= \sum_{n=0}^{N-1} \left(x(n)\, e^{-j\frac{2\pi}{N}\cdot nk}\right)^*$$

$$= [X(k)]^* = X^*(k)$$

$$\therefore X(k) = X^*(N-k) \ //$$

Q:

1. Let $X(k)$ be a 14 pt. DFT of a length 14 real sequence $x(n)$.
The 1st 8 samples of $X(k)$ are given by,

$X(0) = 12$	$X(4) = -2+2j$
$X(1) = -1+3j$	$X(5) = 6+3j$
$X(2) = 3+4j$	$X(6) = -2-3j$
$X(3) = 1-5j$	$X(7) = 10$

→ $X(k) = X^*(N-k)$

$\Rightarrow X(8) = X^*(14-8) = X^*(6) \Rightarrow -2+3j$

$X(9) = X^*(14-9) = X^*(5) \Rightarrow 6-3j$

$X(10) = X^*(14-10) = X^*(4) \Rightarrow -2-2j$

$X(11) = X^*(14-11) = X^*(3) \Rightarrow 1+5j$

$X(12) = X^*(14-12) = X^*(2) \Rightarrow 3-4j$

$X(13) = X^*(14-13) = X^*(1) \Rightarrow -1-3j$

2. The 1st five elements of a 8-pt. DFT are $\{0, 1+j, 2, -2-j, 3\}$

$\Rightarrow$ $X(K) = X^*(N-K)$

$X(5) = X^*(8-5) = X^*(3) = -2+j$

$X(6) = X^*(8-6) = X^*(2) = 2$

$X(7) = X^*(8-7) = X^*(1) = 1-j$

$X(0) = 0$

$X(1) = 1+j$

$X(2) = 2$

$X(3) = -2-j$

$X(4) = 3$

(ii) Periodicity:

If $x(n)$ & $X(n)$ are an N-pt. DFT pair,

i.e., $x(n) \xleftrightarrow{DFT} X(k)$

then,

$x(n+N) = x(n) \ \forall n$

$X(k+N) = X(k) \ \forall k$

(iii) Linearity:

If $x_1(n) \xleftrightarrow{DFT} X_1(k)$ & $x_2(n) \xleftrightarrow{DFT} X_2(k)$,

then for any real valued or complex valued constants a_1 & a_2

$a_1 x_1(n) + a_2 x_2(n) \xleftrightarrow[N]{DFT} a_1 X_1(k) + a_2 X_2(k)$

(iv) Circular Convolution:

The convolution property of DFT says that, the multiplication of the DFTs of the 2 sequences is equivalent to the DFT of the circular convolution of the 2 sequences.

Let DFT of $x_1(n) = X_1(k)$; DFT of $x_2(n) = X_2(k)$

then by convolution property,

$X_1(k) X_2(k) = DFT\{x_1(n) \circledast x_2(n)\}$

Hence

$x_1(n) \circledast x_2(n) = IDFT\{X_1(k) X_2(k)\}$