UNIT-I Introduction to Signals & Systems

⇒ Introduction : Classification of signals
⇒ Concept of freq. in cont.-time & discrete time signals
⇒ Sampling theorem
⇒ Discrete-time signals & system :
- Discrete time signals
- Discrete time systems
- Response of LTI systems to arbitary I/P (Convolution Sum)

⇒ Correlation :
- Cross-correlation sequences
- Autocorrelation sequences
- Correlation of periodic sequences

Digital Signal Processing ⟶ Process discrete time signal

SIGNAL : Physical quantity that varies with time, space or any other indept. variables

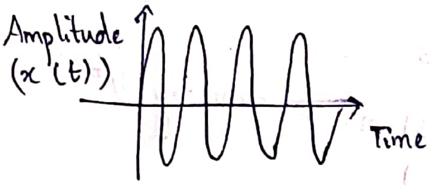
TYPES
→ Cont. time signal [Analog signal]
→ Discrete time signal [Digital signal / Sampled signal]

Out of many signals only some signals.

CONTINUOUS TIME SIGNAL: (Analog signal)	Discrete TIME SIGNAL: (Digital signal/Sampled signal)
Signals can be defined for every value of time (any time interval)	signals can be defined for specific value of time.

$x(t) = \sin 2\pi f t$

$-\alpha < t < \alpha$

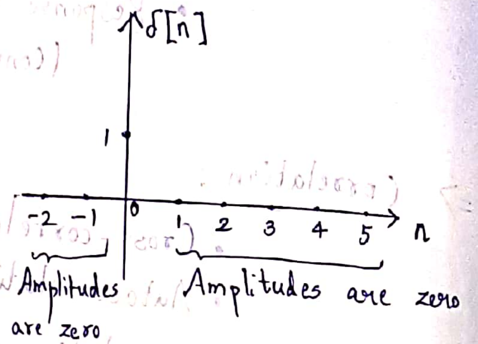


x-axis — Time

y-axis — Amplitude

$$\delta[n] = \begin{cases} 1, & n = 0 \\ 0, & n \neq 0 \end{cases}$$

$\because n \in$ integer

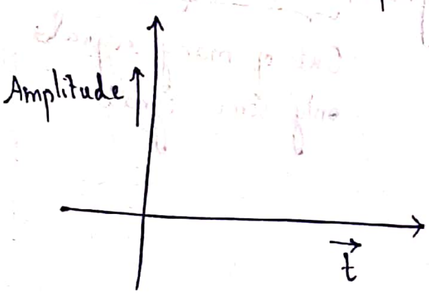


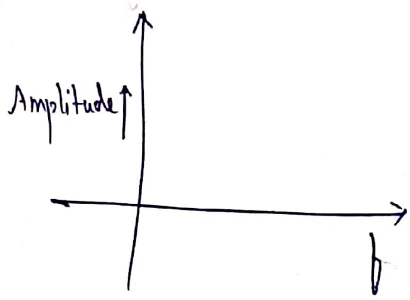
Signal $\delta[n]$ } $\underline{\text{Unit impulse function}}$

Signals are only available ~~only~~ at $n = 0$

$\underline{\text{Amplitudes for } n = 0.5, 1.3, \ldots}$ cannot be defined $\because n \in Z$.

TIME DOMAIN (Time vs Amplitude)	Frequency DOMAIN (Freq. vs Amplitude)





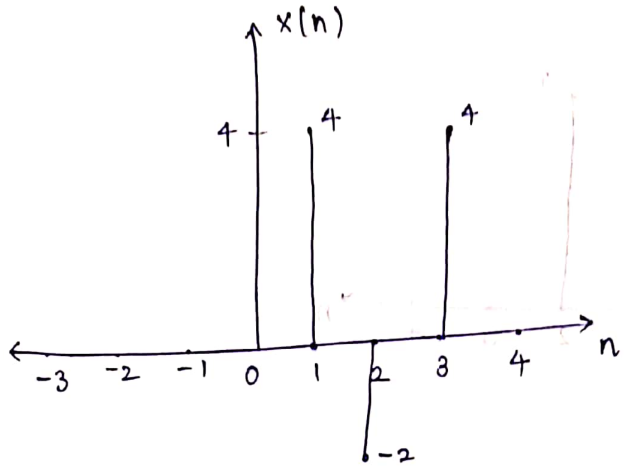
Fourier transform ⇒ Time domain can be converted to frequency domain.

Representation of Discrete Time Signal :

(i) Functional representation :

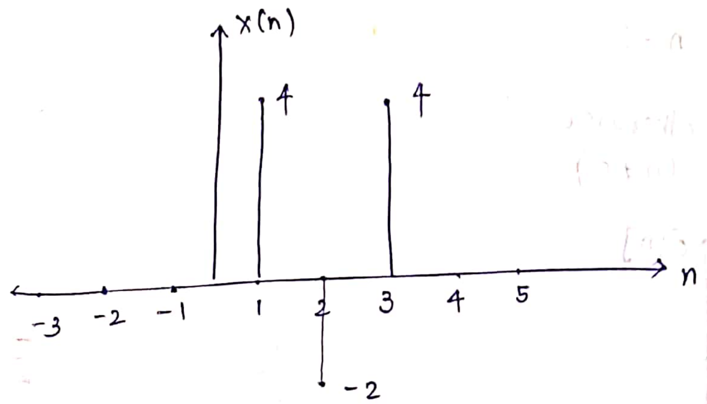
n - Time period
↳ $n \in Z$.

$$x(n) = \begin{cases} 4 & , n = 1, 3 \\ -2 & , n = 2 \\ 0 & , \text{elsewhere} \end{cases}$$



(ii) Tabular method of representation :

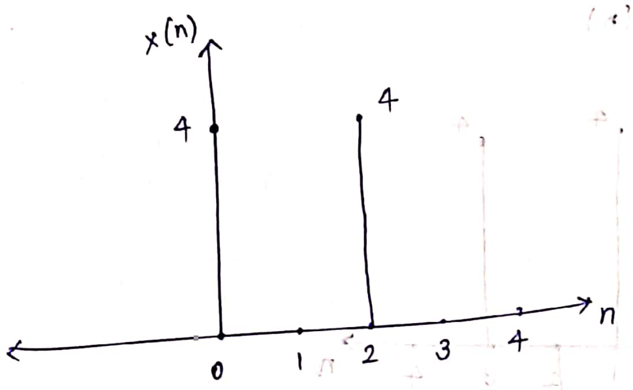
n	-3	-2	-1	0	1	2	3	4	5
x(n)	0	0	0	0	4	-2	4	0	0



(iii) Sequence representation

$$x(n) = \{0, 4, -2, 4, 0, \ldots\}$$
$\uparrow$
$n=0$

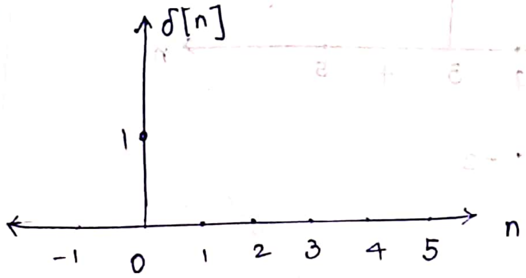
Arrow indicates origin



Some Elementary Signal:

★ Unit Impulse Signal $\delta(n)$ $\rightarrow$ Signal is available only at the origin

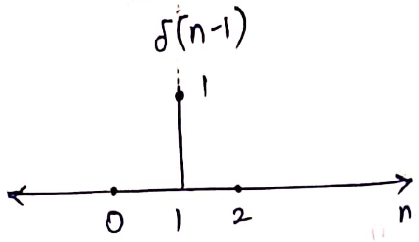
(i) $\delta[n] = \begin{cases} 1, & n=0 \\ 0, & \text{otherwise } (n \neq 0) \end{cases}$



(ii) $\delta[n-1] = \begin{cases} 1, & n=1 \\ 0, & \text{otherwise} \end{cases}$

$\delta[n-1] \rightarrow \delta(0)$

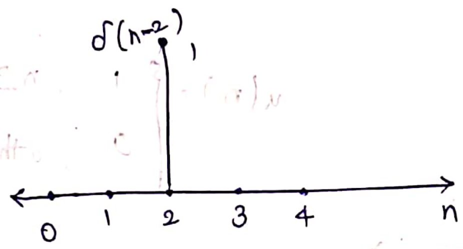
when $n = 1$



(iii) $\delta(n-2) = \begin{cases} 1, & n=2 \\ 0, & \text{otherwise} \end{cases}$

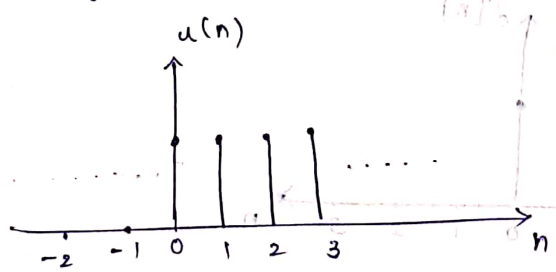
$\delta(n-2) \rightarrow \delta(0)$

when $n = 2$



★ Unit Step Signal $u(n)$

(i) $u(n) = \begin{cases} 1, & n \geq 0 \\ 0, & \text{otherwise} \end{cases}$



Impulse properties :

a) $\int_{-\alpha}^{\alpha} \sin(3.2\pi 1)\, \underbrace{\delta(t-1)}_{t=1}\, dt$

$\Rightarrow \int_{-\alpha}^{\alpha} \sin(3.2\pi.1)\, \delta(t-1)\, dt$

$= \sin(3.2\pi)$

b) $\int_{-\alpha}^{\alpha} e^{-\alpha t^2}\, \underbrace{\delta(t+5)}_{t=-5}\, dt$

$\Rightarrow e^{-\alpha t^2} \Big|_{t=-5} = \boxed{e^{-25\alpha}}$

c) $\int_{0}^{\alpha} e^{-\alpha t^2}\, \underbrace{\delta(t+5)}_{t=-5}\, dt = 0$

located outside range of integration

d) $(t^3+4t^2+3t-6)\, \underbrace{\delta(t)}_{t=0}$

$\Rightarrow (0^3+4.0^2+3.0-6)\, \delta(t)$

$= \boxed{-6\delta(t)}$

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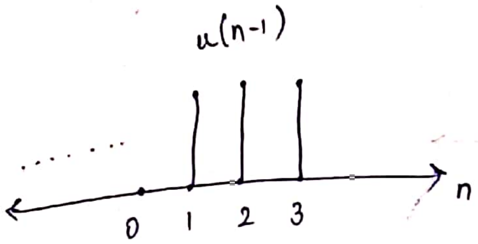
(ii) $u(n-1) = \begin{cases} 1, & n \geq 1 \\ 0, & \text{otherwise} \end{cases}$

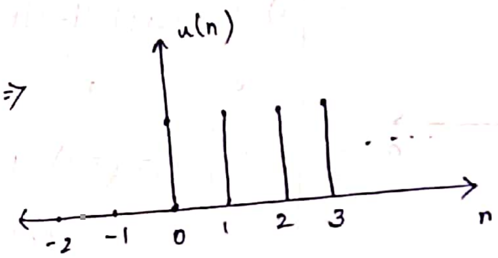
$u(n-1) \rightarrow u(0)$
when $n = 1$



Relation b/w Unit Impulse & Unit Step Function

★ $\delta[n] = u[n] - u[n-1]$

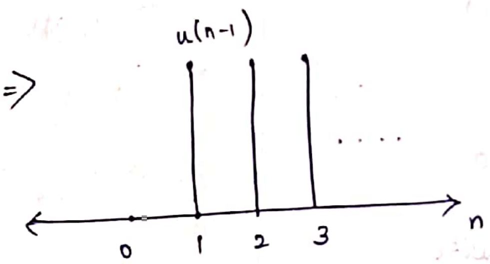
$u[n] \Rightarrow$



$u(n) = \begin{cases} 1, & n \geq 0 \\ 0, & \text{otherwise} \end{cases}$

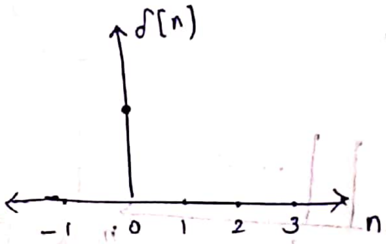
(−)

$u[n-1] \Rightarrow$



$u(n-1) = \begin{cases} 1, & n \geq 1 \\ 0, & \text{otherwise} \end{cases}$

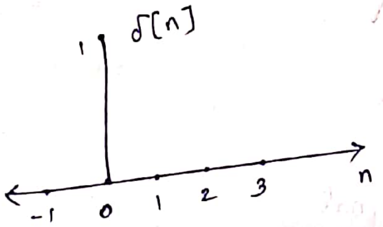
$u[n] - u[n-1] \Rightarrow$
$= \delta[n]$



★ $u[n] = \delta[n] + \delta[n-1] + \delta[n-2] + \dots$

$$u[n] = \sum_{k=0}^{\infty} \delta[n-k]$$

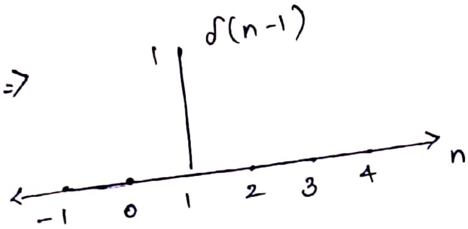
$\delta[n] \Rightarrow$



$$\delta[n] = \begin{cases} 1, & n = 0 \\ 0, & \text{otherwise} \end{cases}$$

(+)

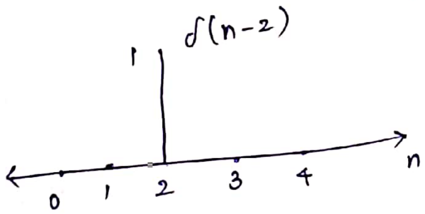
$\delta(n-1) \Rightarrow$



$$\delta(n-1) = \begin{cases} 1, & n = 1 \\ 0, & \text{otherwise} \end{cases}$$

(+)

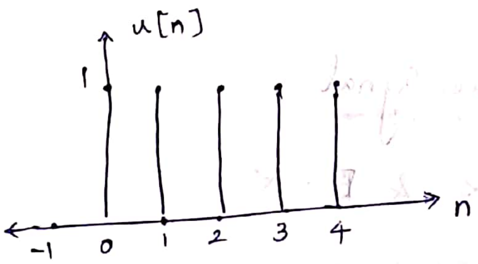
$\delta(n-2) \Rightarrow$



$$\delta(n-2) = \begin{cases} 1, & n = 2 \\ 0, & \text{otherwise} \end{cases}$$

⋮

$\delta[n] + \delta(n-1) + \delta(n-2) + \dots = u[n] \Rightarrow$



Classification of Discrete time signals

(i) Energy signal & power signal
(ii) Periodic signal & Aperiodic signal
(iii) Symmetric (Even) & Assymetric (odd) signal
(iv) Causal & Non-causal signal

14/07/2021

Energy & Power Signal

$$E = \sum_{n=-\infty}^{\infty} |x(n)|^2$$

$$P = \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=-N}^{N} |x(n)|^2$$

Condition for Energy Signal

If $0 < E < \infty$ & $P = 0$

Condition for Power Signal

If $0 < P < \infty$ & $E = \infty$

Finite Summation Identities $(a \neq 1)$

(i) $$\sum_{n=0}^{N} a^n = \frac{1-a^{N+1}}{1-a}$$

(ii) $$\sum_{n=0}^{N} n a^n = \frac{a}{(1-a)^2}\left[1-(N+1)a^N + N a^{N+1}\right]$$

(iii) $$\sum_{n=0}^{N} n^2 a^n = \frac{a}{(1-a)^3}\left[(1+a)-(N+1)^2 a^N + (2N^2+2N-1)a^{N+1} - N^2 a^{N+2}\right]$$

Infinite Summation Identities $(|a|<1)$

(i) $$\sum_{n=0}^{\infty} a^n = \frac{1}{1-a}$$

(ii) $$\sum_{n=0}^{\infty} n a^n = \frac{a}{(1-a)^2}$$

(iii) $$\sum_{n=0}^{\infty} n^2 a^n = \frac{a^2+a}{(1-a)^3}$$

Some Formulae

(i) $$\sum_{n=0}^{N} (u(n))^2 = \sum_{n=0}^{N} u^2(n) = N+1$$

(ii) $$\sum_{n=-N}^{N} u^2(n) = 2N+1$$

(iii) $$\sum_{n=-N}^{N} 1 = 2N+1$$

(i) Energy & Power signals.

1. Determine the power & energy of unit step function

$$x(n) = u(n)$$

→ $$E = \sum_{n=-\alpha}^{\alpha} |x(n)|^2$$ $u(n) \Rightarrow n \geq 0$

$$E = \sum_{n=0}^{\alpha} |u(n)|^2$$

$$= u(0) + u(1) + u(2) + \dots$$

$$= 1+1+1+1 \dots$$

$$\boxed{E = \alpha}$$

$$P = \underset{N\to\alpha}{Lt} \frac{1}{2N+1} \sum_{n=-N}^{N} |x(n)|^2$$

$$P = \underset{N\to\alpha}{Lt} \frac{1}{2N+1} \sum_{n=0}^{N} |u(n)|^2$$

$$\sum_{n=0}^{N} (u(n))^2$$

$$\Rightarrow \sum_{n=0}^{N} u^2(n) = N+1$$

$$P = \underset{N\to\alpha}{Lt} \frac{1}{2N+1} (N+1) \Rightarrow \underset{N\to\alpha}{Lt} \frac{(1+1/N)}{(2+1/N)}$$

$$\boxed{P = \frac{1}{2}}$$

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2. $$x(n) = \begin{cases} 2(-1)^n, & n \geq 0 \\ 0, & \text{otherwise} \end{cases}$$

Determine power

→ $$P = \underset{N\to\alpha}{Lt} \frac{1}{2N+1} \sum_{n=-N}^{N} |x(n)|^2$$

$$= \underset{N\to\alpha}{Lt} \frac{1}{2N+1} \sum_{n=0}^{N} |2(-1)^n|^2$$

$$= \underset{N\to\alpha}{Lt} \frac{1}{2N+1} \cdot 4 \sum_{n=0}^{N} (-1)^{2n}$$

$$\sum_{n=0}^{N} (-1)^{2n} = N+1$$

$$= \underset{N\to\alpha}{Lt} \frac{1}{2N+1} \cdot 4(N+1)$$

$$= \underset{N\to\alpha}{Lt} \frac{4N\left(1+\frac{1}{N}\right)}{N\left(2+\frac{1}{N}\right)}$$

$$\boxed{P = 2}$$

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(3) $x(n) = \left(\frac{1}{3}\right)^n u(n)$

$$\rightarrow E = \sum_{n=-\alpha}^{\alpha} |x(n)|^2$$

$$= \sum_{n=0}^{\infty} \left|\left(\frac{1}{3}\right)^n u(n)\right|^2$$

$$= \sum_{n=0}^{\alpha} \left|\left(\frac{1}{9}\right)^n\right|$$

$$\sum_{n=-\alpha}^{\alpha} a^n = \frac{1}{1-a}$$

$$= \frac{1}{1-\frac{1}{9}} \Rightarrow \frac{9}{8}$$

$$\boxed{E = \frac{9}{8}}$$

$$P = \underset{N\to\alpha}{Lt} \frac{1}{2N+1} \cdot \sum_{n=-N}^{N} |x(n)|^2$$

$$= \underset{N\to\alpha}{Lt} \frac{1}{2N+1} \sum_{n=0}^{N} \left|\left(\frac{1}{3}\right)^n\right|^2$$

$$= \underset{N\to\alpha}{Lt} \frac{1}{2N+1} \sum_{n=0}^{N} \left(\frac{1}{9}\right)^n$$

$$= \underset{N\to\alpha}{Lt} \frac{1}{2N+1} \cdot \frac{1-\left(\frac{1}{9}\right)^{N+1}}{1-\frac{1}{9}}$$

$$\sum_{n=0}^{N} a^n = \frac{1-a^{N+1}}{1-a}$$

$$\boxed{P = 0}$$

Energy is finite & $P = 0$

$\Rightarrow$ $\therefore$ It is an energy signal

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(4) $x(n) = e^{j\left(\frac{\pi}{2}n + \frac{\pi}{4}\right)}$

$\rightarrow$ $E = \sum_{n=-\alpha}^{\alpha} |x(n)|^2 \Rightarrow \sum_{n=-\alpha}^{\alpha} \left|e^{j\left(\frac{\pi}{2}n+\frac{\pi}{4}\right)}\right|^2$

$$E = \sum_{n=-\alpha}^{\alpha} 1 = \alpha \Rightarrow \boxed{E = \alpha}$$

$$\left|e^{j(w+\theta)}\right| = 1$$

$$P = \underset{N\to\alpha}{Lt} \frac{1}{2N+1} \sum_{n=-N}^{N} |x(n)|^2$$

$$= \underset{N\to\alpha}{Lt} \frac{1}{2N+1} \sum_{n=-N}^{N} \left|e^{j\left(\frac{\pi}{2}n+\frac{\pi}{4}\right)}\right|^2$$

$$\sum_{n=-N}^{N} 1 = 2N+1$$

$$= \underset{N\to\alpha}{Lt} \frac{1}{2N+1} (2N+1)$$

$$\boxed{P = 1}$$

Energy is infinite & Power is finite

$\Rightarrow$ It is power signal

5) $x(n) = \sin\left(\frac{\pi}{4} n\right)$

$$\rightarrow E = \sum_{n=-\infty}^{\infty} |x(n)|^2$$

$$= \sum_{n=-\infty}^{\infty} \left|\sin\left(\frac{\pi}{4} n\right)\right|^2 \Rightarrow \sum_{n=-\infty}^{\infty} \left|\sin^2\left(\frac{\pi}{4} n\right)\right|$$

$$= \sum_{n=\infty} \frac{1 - \cos 2\left(\frac{\pi}{4} n\right)}{2}$$

$$= \frac{1}{2} \sum_{n=-\infty}^{\infty} \left(1 - \cos\left(\frac{\pi}{2} n\right)\right)$$

$\sin^2\theta = \frac{1-\cos 2\theta}{2}$

$\cos(\text{even}) = -1$

$\cos(\text{odd}) = 0$

$1 - (-1) =$

$$\boxed{\therefore E = \infty}$$

$$P = \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=-N}^{N} |x(n)|^2$$

$$P = \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=-N}^{N} \left|\sin^2\left(\frac{\pi}{4} n\right)\right|$$

$$P = \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=-N}^{N} \frac{1 - \cos 2\left(\frac{\pi}{4} n\right)}{2}$$

$$= \lim_{N\to\infty} \frac{1}{2N+1} \cdot \frac{1}{2} \sum_{n=-N}^{N} \left[1 - \cos\left(\frac{\pi}{2} n\right)\right]$$

$$= \lim_{N\to\infty} \frac{1}{2N+1} \cdot \frac{1}{2} \cdot (2N+1)$$

$\sum_{n=-N}^{N} \left(1 - \cos\left(\frac{\pi}{2} n\right)\right) = 2N+1$

$$P = \lim_{N\to\infty} \frac{1}{2}$$

$$\boxed{P = \frac{1}{2}}$$

(iii) Periodic & Aperiodic signal

① $x(n) = e^{j6\pi n}$

→ w_0 - fundamental frequency

$w_0 = 6\pi$

(Multiples of π)

⇒ It is periodic signal

Fundamental Period:

$$N = 2\pi\left[\frac{m}{w_0}\right]$$

$$= 2\pi\left[\frac{m}{6\pi}\right]$$

$$\boxed{N = \frac{m}{3}}$$

When $m=3$, then $N \in Z$

$\therefore N = 1$ is a fundamental period.

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② $x(n) = e^{j\frac{3}{5}\left(n+\frac{1}{2}\right)}$

→ $w_0 = \frac{3}{5}$

It is not multiple of π

⇒ It is aperiodic signal

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③ $x(n) = \cos\left(\frac{2\pi}{3}\right) n$

$\rightarrow$ $\omega_0 = \frac{2\pi}{3}$ (Multiple of π)

$\Rightarrow$ It is periodic

$N = 2\pi \left[\frac{m}{\omega_0}\right]$

$= 2\pi \left(\frac{m}{\frac{2\pi}{3}}\right) \Rightarrow 2\pi\left(\frac{3m}{2\pi}\right)$

$\boxed{N = 3m}$

$m = 1$

$\therefore$ Fundamental period $\Rightarrow$ $N = 3$

4/07/2021

Practice Yourself #1

1) Find whether the signal is energy (or) power signals.

$$x(n) = e^{2n} u(n)$$

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$$\rightarrow \quad E = \sum_{n=-\alpha}^{\alpha} |x(n)|^2$$

$$= \sum_{n=0}^{\alpha} |e^{2n}|^2 \Rightarrow \sum_{n=0}^{\alpha} |e^{4n}|$$

$$1 + e^4 + e^8 + \dots = \infty$$

$$\boxed{\therefore E = \alpha}$$

$$P = \underset{N \to \alpha}{Lt} \frac{1}{2N+1} \sum_{n=-N}^{N} \left(e^{2n} u(n)\right)^2$$

$$= \underset{N \to \alpha}{Lt} \frac{1}{2N+1} \sum_{n=0}^{N} e^{4n}$$

$$= \underset{N \to \alpha}{Lt} \frac{1}{2N+1} \cancel{(N+1)} \left[\frac{1 - e^{4N+1}}{1-e^4}\right]$$

$$\sum_{n=0}^{N} e^{4n} = \cancel{N+1} \quad \frac{1 - e^{4N+1}}{1 - e^4}$$

$$= \cancel{\underset{N \to \alpha}{Lt} \frac{N\left(1 + \frac{1}{N}\right)}{N\left(2 + \frac{1}{N}\right)}}$$

$$\Rightarrow \cancel{\boxed{P = \frac{1}{2}}}$$

$$\Rightarrow \boxed{P = \alpha}$$

Both Power ~~is finite~~ & Energy is infinite

$\Rightarrow$ It is neither a power signal nor an energy signal

② Test Periodic / non-periodic signal

(i) $x(n) = e^{j7\pi n}$

$\rightarrow$ $\omega_0 = 7\pi$

It is a multiple of π

$\therefore$ It is periodic signal

Fundamental period :

$$N = 2\pi\left(\frac{m}{\omega_0}\right)$$

$$= 2\pi\left(\frac{m}{7\pi}\right)$$

$$\boxed{N = \frac{2m}{7}}$$

When $m = 7$, $N \in Z$

$\Rightarrow N = 2 \leftarrow$ Fundamental period

(ii) $x(n) = 4e^{j4\pi\frac{(n+1/3)}{5}}$

$\rightarrow$ $\omega_0 = \frac{4\pi}{5}$

It is a multiple of π

$\Rightarrow$ It is a periodic signal

Fundamental period :

$$N = 2\pi\left[\frac{m}{\omega_0}\right]$$

$$= 2\pi\left(\frac{m}{\frac{4\pi}{5}}\right)$$

$$\boxed{N = \frac{5m}{2}}$$

When $m = 2$, $N \in Z$

$\Rightarrow N = 5 \leftarrow$ Fundamental period

All periodic signals are power signals

A discrete time signal $x(n)$ is periodic with period N $(N > 0)$ if & only if,

$$\boxed{x(n+N) = x(n)} \quad \forall n$$

→ Condition for periodic

$\underline{N=0}$

$x(n+0) = x(n)$

$\boxed{x(n) = x(n)}$

$\underline{N=1}$ $\boxed{x(n+1) = x(n)}$

$\underline{N=2}$

$\boxed{x(n+2) = x(n)}$

We are getting same $x(n) \rightarrow$ periodic signal

w_0 - Multiple of π ⇒ Periodic signal

(iii) Periodic & Aperiodic signals

single sinusoidal signal

③ $x(n) = \cos\left(\frac{2\pi}{3}\right)n$. Determine whether periodic or aperiodic. If periodic, compute fundamental period.

→ Fundamental freq., $w_0 = \frac{2\pi}{3}$

Multiples of π

⇒ Periodic.

Fundamental period, $N = 2\pi\left(\frac{m}{w_0}\right) \Rightarrow 2\pi\left(\frac{m}{\frac{2\pi}{3}}\right)$

$$\boxed{N = 3m}$$

$m = 1$

$$\boxed{N = 3}$$

4) $x(n) = \underbrace{\cos\left(\frac{\pi}{3}\right)n}_{1} + \underbrace{\cos\left(\frac{3\pi}{4}\right)n}_{2}$

$\rightarrow$ $\underbrace{\omega_{01} = \frac{\pi}{3} \quad ; \quad \omega_{02} = \frac{3\pi}{4}}$

Multiples of π

$\Rightarrow$ Both are periodic

$N_1 = 2\pi\left(\frac{m_1}{\omega_{01}}\right)$

$= 2\pi\left(\frac{m_1}{\pi/3}\right)$

$N_1 = 6m_1$

$m_1 = 1$

$\Rightarrow \boxed{N_1 = 6}$

$N_2 = 2\pi\left(\frac{m_2}{\omega_{02}}\right)$

$= 2\pi\left(\frac{m_2}{3\pi/4}\right)$

$N_2 = 8m_2/3$

$m_2 = 3$

$\Rightarrow \boxed{N_2 = 8}$

$\Rightarrow$ LCM (N_1, N_2)

LCM $(6, 8) = \boxed{24}$

Fundamental period, $\underline{\underline{N = 24}}$

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5) $x(n) = \sin\left(\frac{2\pi}{3}\right)n$

$\rightarrow$ $\omega_0 = \frac{2\pi}{3}$

Multiples of π

$\Rightarrow$ Periodic

$N = 2\pi\left(\frac{m}{\omega_0}\right) \Rightarrow 2\pi\left(\frac{m}{\frac{2\pi}{3}}\right)$

$N = 3m \Rightarrow m = 1$ & $\underline{\underline{N = 3}} \leftarrow$ Fundamental period

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(6) $x(n) = \cos^2\left(\frac{\pi}{8}\right)n$

$\rightarrow x(n) = \dfrac{1+\cos 2\left(\frac{\pi}{8}\right)n}{2}$

$\cos^2 = \dfrac{1+\cos 2\theta}{2}$

$= \frac{1}{2}\left[1+\cos\left(\frac{\pi}{4}\right)n\right]$

$\Rightarrow \omega_0 = \frac{\pi}{4}$

Multiples of π

$\Rightarrow$ Periodic

$N = 2\pi\left(\frac{m}{\omega_0}\right) \Rightarrow 2\pi\left(\frac{m}{\pi/4}\right)$

$N = 8m$

$m = 1 \Rightarrow \boxed{N = 8}$

(iii) Even (Symmetric) & Odd (Assymetric signal)

Even signal:

A real valued signal $x(n)$ is called symmetric if,

$$\boxed{x(-n) = x(n)}$$

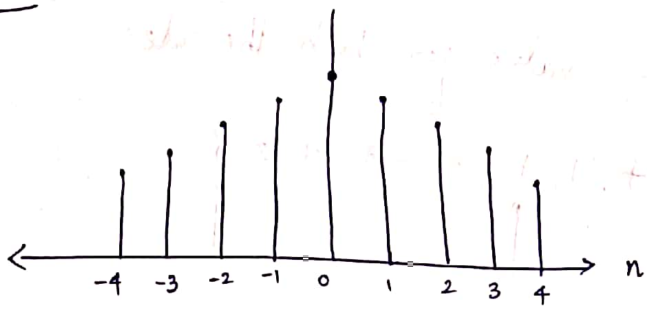
Odd signal:

A signal $x(n)$ is called asymmetric / Antisymmetric if,

$$\boxed{x(-n) = -x(n)}$$

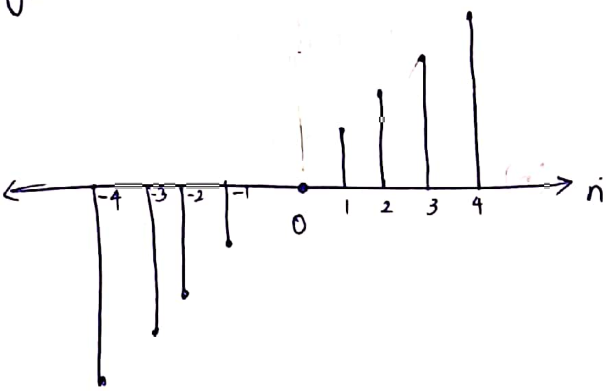
If $x(n)$ is odd, then $x(0) = 0$

Even signal:



$x(n) = x(-n)$

Odd signal:



$x(n) = -x(n)$

An arbitary signal $x(n)$ can be expressed as sum of 2 signal component

One is even

Other is odd

$x(n) = x_e(n) + x_o(n)$

Even signal $\Rightarrow x_e(n) = \frac{1}{2}\left[x(n) + x(-n)\right]$

Odd signal $\Rightarrow x_o(n) = \frac{1}{2}\left[x(n) - x(-n)\right]$

(iv) Causal & Non-causal signal

Causal:

A signal $x(n)$ is said to be causal if its value is zero for $n < 0$

Eg: (i) $x(n) = \{\underset{\uparrow}{1}, 2, -3, -1, 2\}$

$n = 0$

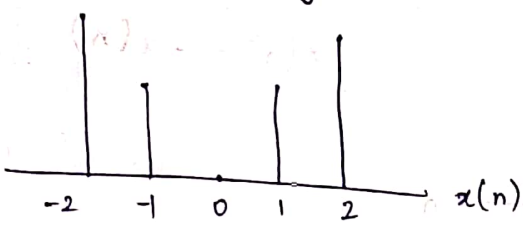
(ii) $u(n)$ is a causal signal

Non-causal :

The signal contains value for both the sides

Eg: $x(n) = \{2, -3, 4, 1, \underset{\uparrow}{1}, 2, -3, -1, 2\}$

① Check Even or odd signal :



→ Even : $x(-n) = x(n)$

$y(n) = x(-n)$

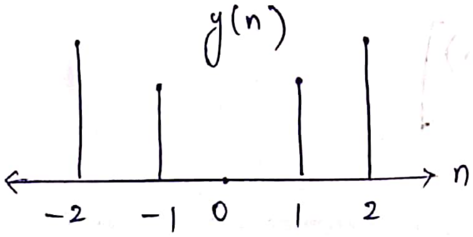
$y(0) = x(-0) = x(0)$

$y(1) = x(-1)$

$y(2) = x(-2)$

$y(-1) = x(1)$

$y(-2) = x(2)$



$x(n) = y(n)$

⇒ Even signal.

Simple Manipulation of Discrete time signals

(I) Transformation of the indept. variables (time):

A signal $x(n)$ may be shifted in time by replacing the indept. variable 'n' by $n-k$, $k \in Z$

$x(n) \rightarrow x(n-k)$

$x(n) \rightarrow x(n+k)$

If k is +ve integer, time shift results in a delay of the signal by 'k' units of time

If k is -ve integer, time shift results in advance of the signal by 'k' units in time

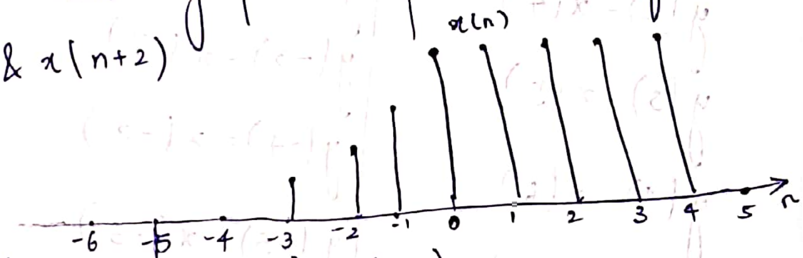
$x(n-k) \rightarrow$ Right shifted

$x(n+k) \rightarrow$ Left shifted

$x(-n) \rightarrow$ Time reversal operation

---

(1) Given a signal $x(n)$. Show a graphical representation of signal: $x(n-3)$ & $x(n+2)$



→ ☆ $y(n) = x(n-3)$

$y(0) = x(0-3) = x(-3)$

$y(1) = x(1-3) = x(-2)$

$y(2) = x(2-3) = x(-1)$

$y(3) = x(0)$

$y(4) = x(1)$

$y(5) = x(2)$

$y(6) = x(3)$

$y(7) = x(4)$

$y(-1) = x(-1-3) = x(-4)$

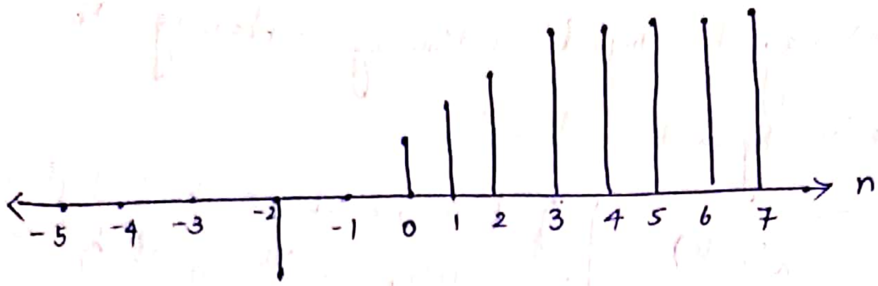
$y(-2) = x(-5)$

$y(-3) = x(-6)$

$y(-4) = x(-7)$

$y(-5) = x(-8)$

$x(n-3)$ → The signal shifted towards right by 3 units



☆ $y(n) = x(n+2)$

$y(0) = x(0+2) = x(2)$

$y(1) = x(3)$

$y(2) = x(4)$

$y(3) = x(5)$

$y(4) = x(6)$

$y(5) = x(7)$

$y(-1) = x(+1)$

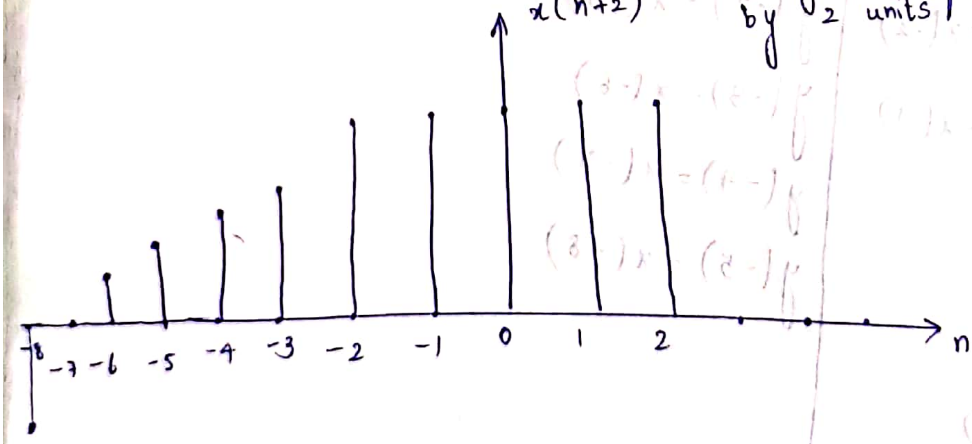
$y(-2) = x(0)$

$y(-3) = x(-1)$

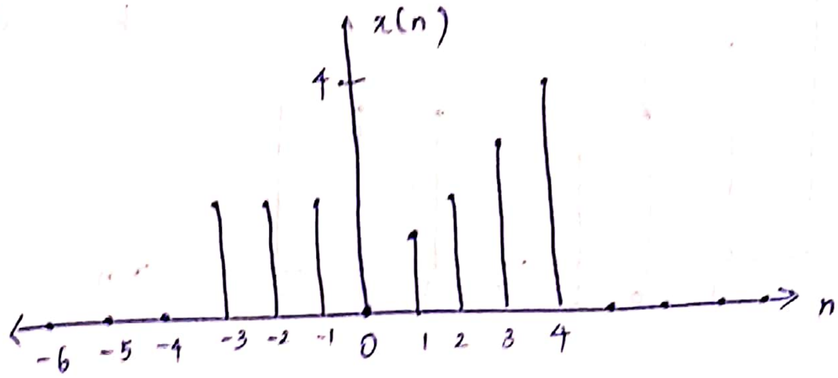
$y(-4) = x(-2)$

$y(-5) = x(-3)$

$x(n+2)$ → The signal shifted towards left by 2 units

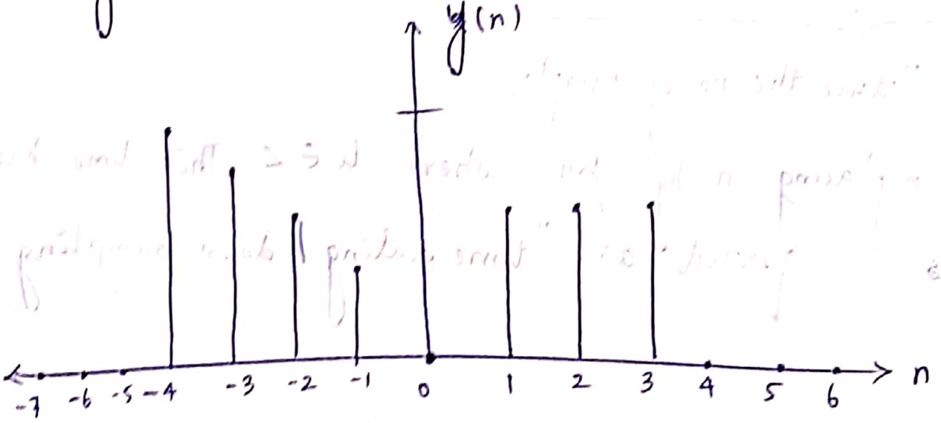


2. Given a signal $x(n)$. Show a graphical representation of signal: $x(-n)$ & $x(-n+2)$



→ ★ $y(n) = x(-n)$

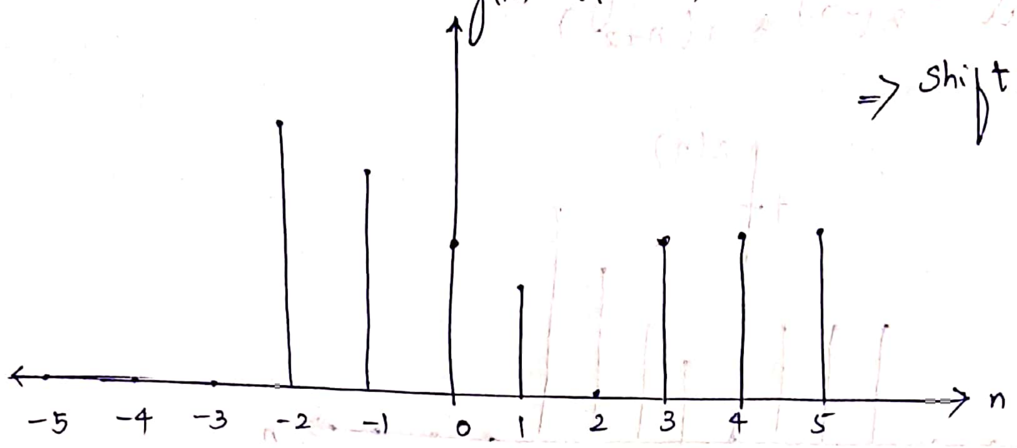
$y(0) = x(0)$	$y(3) = x(-3)$
$y(1) = x(-1)$	$y(4) = x(-4)$
$y(2) = x(-2)$	$y(-3) = x(3)$
$y(-1) = x(1)$	$y(-4) = x(4)$
$y(-2) = x(2)$	



★ $y(n) = x(-n+2)$

$y(0) = x(2)$	$y(5) = x(-3)$	$y(-5) = x(7)$
$y(1) = x(1)$	$y(-1) = x(3)$	
$y(2) = x(0)$	$y(-2) = x(4)$	
$y(3) = x(-1)$	$y(-3) = x(5)$	
$y(4) = x(-2)$	$y(-4) = x(6)$	

$y(n) = x(-n+2)$



$\Rightarrow$ Shifted right.

$x(n+k) \rightarrow$ Left shifted

$x(n-k) \rightarrow$ Right shifted

$x(-n) \rightarrow$ Time reversal

$x(-n+k) \rightarrow$ Right shifted

$x(-n-k) \rightarrow$ Left shifted.

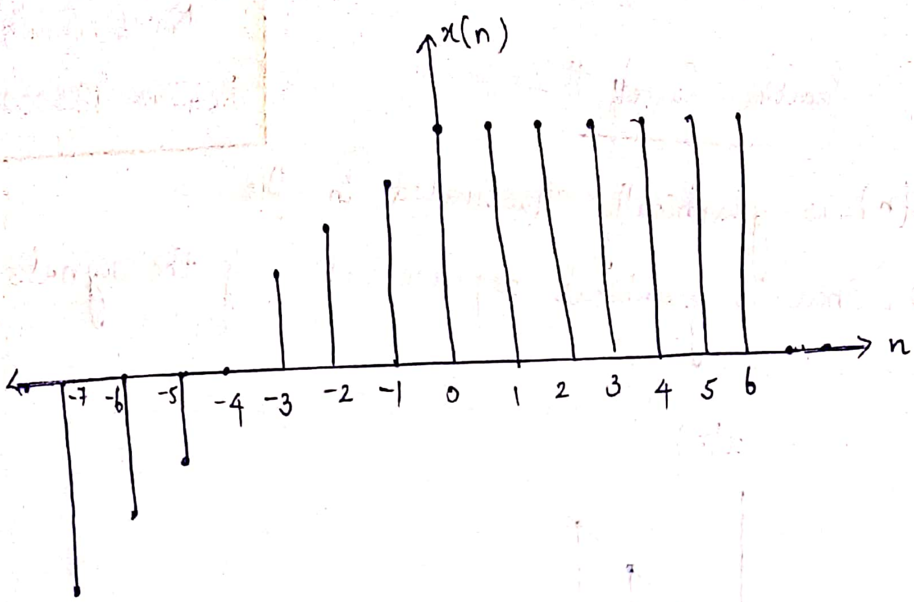
(II) Time Scaling / Down sampling operation :

$\underbrace{\hspace{6em}}_{\text{Reduce the no. of samples}}$

If $x(n)$ replacing n by μn, where $\mu \in Z$. This time-base modifications is referred as "time scaling // down-sampling operation"

---

(1) Given a signal $x(n)$. Show a graphical representation of signals $x(2n)$.



→ $y(n) = x(2n)$

$y(0) = x(2.0) = x(0)$

$y(1) = x(2)$

$y(2) = x(4)$

$y(3) = x(6)$

$y(4) = x(8)$

$y(5) = x(10)$

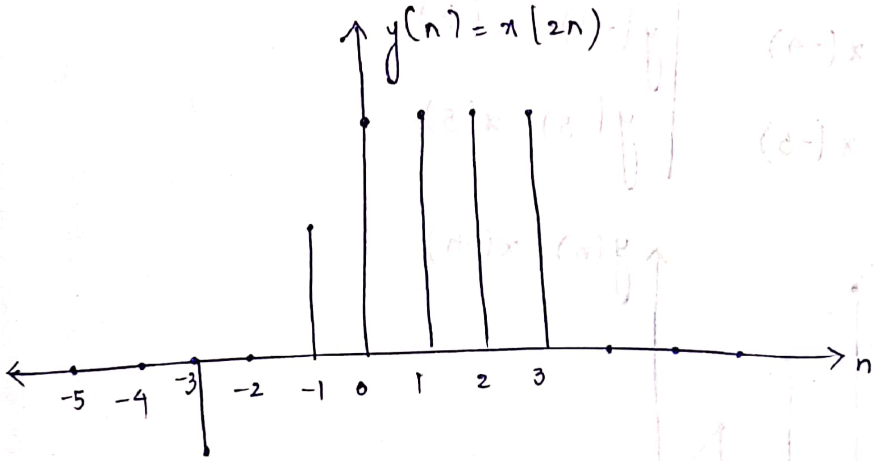
$y(-1) = x(-2)$

$y(-2) = x(-4)$

$y(-3) = x(-6)$

$y(-4) = x(-8)$

$y(-5) = x(-10)$



15/07/2021

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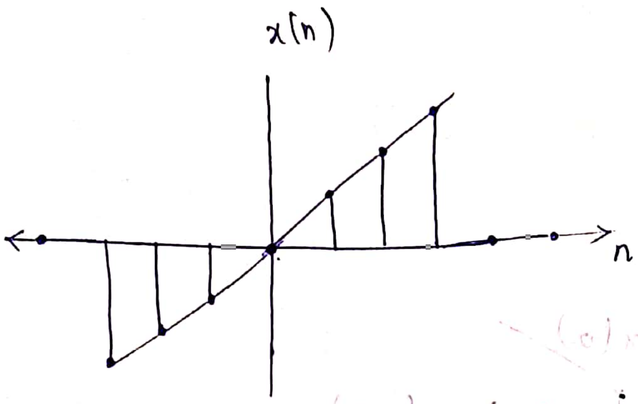
Practice - Yourself #2

① A signal $x(n)$ is graphically illustrated in the figure below. Show a graphical representation of the signals $x(-n)$



→ $y(n) = x(-n)$

$y(0) = x(0)$

$y(1) = x(-1)$

$y(2) = x(-2)$

$y(3) = x(-3)$

$y(4) = x(-4)$

$y(5) = x(-5)$

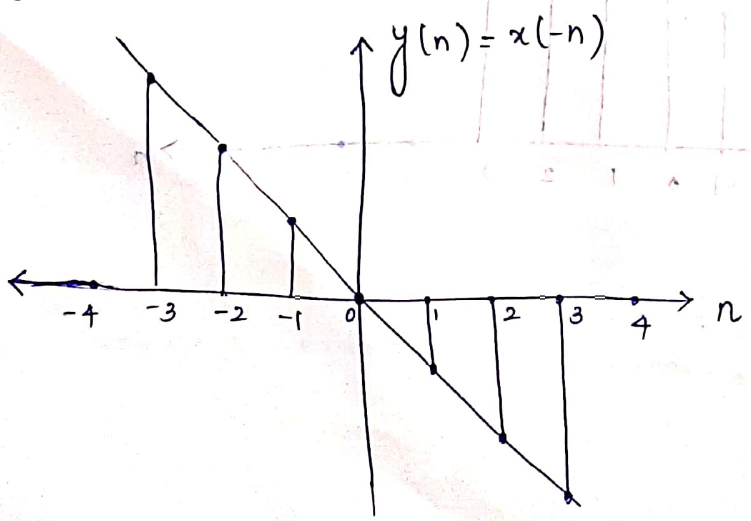
$y(-1) = x(1)$

$y(-2) = x(2)$

$y(-3) = x(3)$

$y(-4) = x(4)$

$y(-5) = x(5)$



② A discrete time signal $x(n)$ is defined as follows:

$$x(n) = \begin{cases} 1+\frac{n}{3}, & -3 \le n \le -1 \\ 1, & 0 \le n \le 3 \\ 0, & \text{elsewhere.} \end{cases}$$

Determine its values & sketch the signal $x(n)$.

→ $x(-3) \Rightarrow 1-1 = 0$

$x(-2) \Rightarrow 1-2/3 = 0.33$

$x(-1) \Rightarrow 1-\frac{1}{3} = 0.66$

$x(0) \Rightarrow 1$

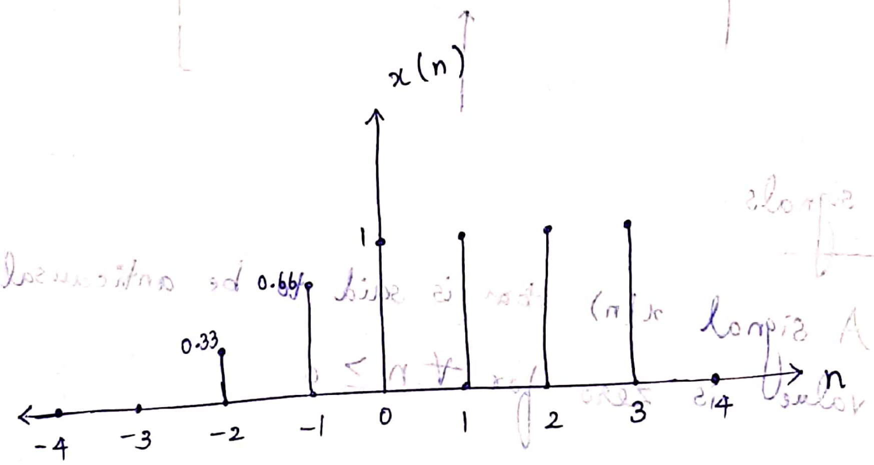
$x(1) = 1$

$x(2) = 1$

$x(3) = 1$

$x(-4) = 0$

$x(4) = 0$



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(iv) Causal & Non-Causal signal

Causal signals :

A signal $x(n)$ is said to be causal if its value is zero for $n < 0$

$$x(n) = \{\underset{\uparrow}{1}, 2, -3, -1, 2\}$$

Non-causal signals :

The signal contains value for both the sides

$$x(n) = \{2, -3, 4, 1, \underset{\uparrow}{1}, 2, -3, -1, 2\}$$

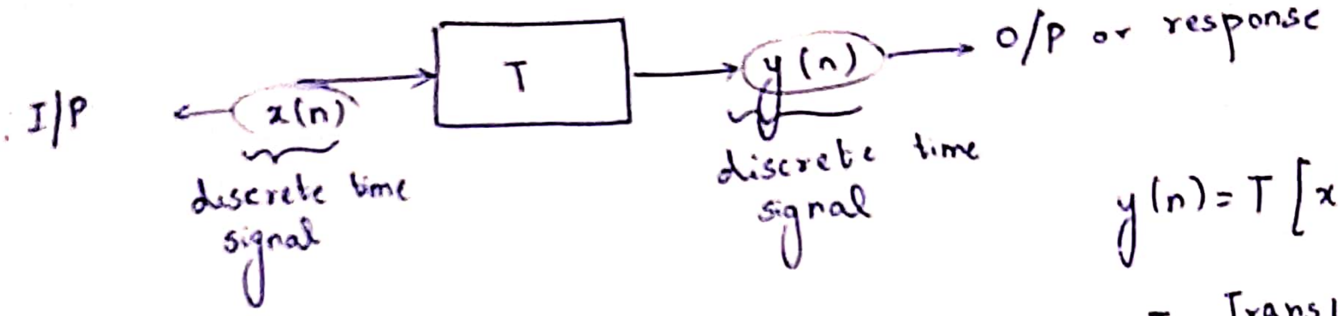
Anticausal signals :

A signal $x(n)$ ~~than~~ is said ~~to~~ to be anticausal if its value is zero for $\forall n \geq 0$.

Eg: $u(-n)$.

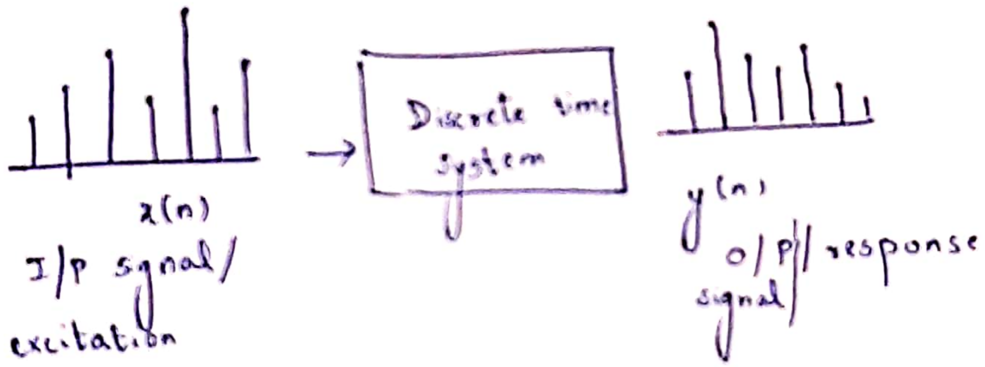
What is Discrete time systems ? (Digital signals)

It is a device/algorithm that operates on discrete time signal called input/excitation, to produce another discrete time signal called output/response of the system.



$$y(n) = T[x(n)]$$

T - Transformation/processing performed by the system on x(n) to produce y(n)



Classification of Discrete time systems

(i) Static vs Dynamic system :

Static / Memory less system :

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(I) Static system vs Dynamic system.

Static system (Memory less system):
O/P @ any time depends on the present I/P.

Dynamic system: (Memory based system)
O/P @ any time depends on the future/past I/P.

(II) Time invariant vs time variant system

When the I/P is delayed by k samples = O/P delayed by k samp

$\Rightarrow$ Time invariant.

I/P delayed by k samples $\neq$ O/P delayed by k samples

$\Rightarrow$ Time variant.

(III) Linear vs Non-linear system.

Linear system:
The one that satisfies the "superposition principle".

Non-linear system:
The one that does not satisfies the "superposition principle".

Eg: $y(n) = x(n)$

$y_3(n) = a_1 y_1(n) + a_2 y_2(n)$

Let us take I/P $x_1(n)$

$y_1(n) = x_1(n)$ —— ①

Let us take I/P $x_2(n)$

$y_2(n) = x_2(n)$ —— ②

$x_3(n) = a_1 x_1(n) + a_2 x_2(n)$

$y_3(n) = x_3(n)$

$= a_1 x_1(n) + a_2 x_2(n) \rightarrow$ LHS

RHS: $a_1 y_1(n) + a_2 y_2(n)$

$\Rightarrow a_1(x_1(n)) + a_2 x_2(n)$

LHS = RHS.

$\Rightarrow$ Linear system.

(IV) Causal vs Non-causal system:

Causal system: Depends on the present / past I/P

Non-causal system: Depends on the present / past / future I/P

(V) ~~Static~~ Stable vs Unstable system.

Q: Check the foll. system are linear, causal, time invariant & static:

(i) $y(n) = x\left(\frac{1}{2n}\right)$

→ Static / dynamic:

$$y(0) = x\left(\frac{1}{0}\right)$$

$$\boxed{y(0) = x(\alpha)}$$

⇒ dynamic //.

TV / TIV:

$$y(n) = x\left(\frac{1}{2n}\right)$$

When the I/P is delayed by k samples

$$y(n,k) = x\left(\frac{1}{2n-k}\right) \quad \text{— (1)}$$

When the o/p is delayed by k samples

$$y(n-k) = x\left(\frac{1}{2(n-k)}\right) \quad \text{— (2)}$$

$$(1) \neq (2)$$

⇒ TV //.

Linear / Non-linear:

$$y(n) = x\left(\frac{1}{2n}\right)$$

a_1, a_2 - arbitary constants

To prove:

$$y_3(n) = a_1 y_1(n) + a_2 y_2(n)$$

When the I/P is $x_1(n)$

$$\Rightarrow y_1(n) = x_1\left(\frac{1}{2n}\right)$$

When the I/P is $x_2(n)$

$$y_2(n) = x_2\left(\frac{1}{2n}\right)$$

$$\Rightarrow x_3\left(\frac{1}{2n}\right) = a_1 x_1\left(\frac{1}{2n}\right) + a_2 x_2\left(\frac{1}{2n}\right)$$

$$y_3(n) = x_3\left(\frac{1}{2n}\right)$$

$$\Rightarrow y_3(n) = a_1 x_1\left(\frac{1}{2n}\right) + a_2 x_2\left(\frac{1}{2n}\right) \rightarrow \underline{LHS}$$

RHS :

$$a_1 y_1(n) + a_2 y_2(n)$$

$$a_1 x_1\left(\frac{1}{2n}\right) + a_2 x_2\left(\frac{1}{2n}\right)$$

LHS = RHS

$\Rightarrow$ Linear system //..

Causal / Non-causal :

$$y(n) = x\left(\frac{1}{2n}\right)$$

$$\boxed{y(0) = x(\infty)}$$

$\Rightarrow$ Non-causal system //..

(ii) $y(n) = \sin(x(n))$

$\rightarrow$ static/dynamic :

$$y(n) = \sin(x(n))$$

$$\boxed{y(0) = \sin(x(0))}$$

$\Rightarrow$ Static //..

TV / TIV:

$$y(n) = \sin(x(n))$$

I/P delayed by k units

$$y(n,k) = \sin(x(n) - k)$$

o/p delayed by k units

$$y(n-k) = \sin(x(n) - k)$$

$\Rightarrow$ TIV //..

Linear / Non-linear:

$$y(n) = \sin(x(n))$$

To prove: $y_3(n) = a_1 y_1(n) + a_2 y_2(n)$

When the I/P is $x_1(n)$

$$y_1(n) = \sin(x_1(n))$$

When the I/P is $x_2(n)$

$$y_2(n) = \sin(x_2(n))$$

$$y_3(n) = \sin(x_3(n))$$

$\Rightarrow \sin(x_3(n)) = \cancel{a_1 \sin(x_1(n)) + a_2 \sin(x_2(n))}$, $a_1 x_1(n) + a_2 x_2(n)$

$\cancel{\Rightarrow \sin(x_3(n)) = a_1 \sin(x_1(n)) + a_2 \sin(x_2(n))}$ $\cancel{\rightarrow LHS}$

$\Rightarrow \cancel{y_3(n) = a_1 \sin(x_1(n)) + a_2 \sin(x_2(n))}$

$$y_3(n) = \sin[a_1 x_1(n) + a_2 x_2(n)] \rightarrow LHS$$

RHS:

$$a_1 y_1(n) + a_2 y_2(n)$$

$$a_1 \sin(x_1(n)) + a_2 \sin(x_2(n))$$

$$LHS \neq RHS$$

$\Rightarrow$ Non-linear //..

Causal / Non-causal :

$$y(n) = \sin(x(n))$$

$$\boxed{y(0) = \sin(x(0))}$$

$\Rightarrow$ Causal //..

(iii) $y(n) = x(n)\cos(x(n))$

$\rightarrow$ Static / dynamic :

$$y(n) = x(n)\cos(x(n))$$

$$\boxed{y(0) = x(0)\cos(x(0))}$$

$\Rightarrow$ Static //..

TV/TIV:

$$y(n) = x(n)\cos(x(n))$$

I/P delayed by k units,

$$y(n,k) = (x(n)-k)(\cos(x(n)-k))$$

o/p delayed by k units,

$$y(n-k) = (x(n)-k)(\cos(x(n)-k))$$

$\Rightarrow$ TIV //..

Linear / Non-linear:

$$y(n) = x(n)\cos(x(n))$$

To prove: $y_3(n) = a_1 y_1(n) + a_2 y_2(n)$

When the I/P is $x_1(n)$,

$$y_1(n) = x_1(n)\cos(x_1(n))$$

When the I/P is $x_2(n)$,

$$y_2(n) = x_2(n)\cos(x_2(n))$$

$$y_3(n) = x_3(n)\cos(x_3(n))$$

$$x_3(n) = a_1 x_1(n) + a_2 x_2(n)$$

$$\Rightarrow y_3(n) = (a_1 x_1(n) + a_2 x_2(n))[\cos(a_1 x_1(n) + a_2 x_2(n))]$$

LHS.

RHS:

$$a_1 y_1(n) + a_2 y_2(n)$$

$$a_1 x_1(n)\cos(x_1(n)) + a_2 x_2(n)\cos(x_2(n))$$

$$LHS \neq RHS$$

$\Rightarrow$ Non-linear //..

Causal / Non-causal:

$$y(n) = x(n)\cos(x(n))$$

$$\boxed{y(0) = x(0)\cos(x(0))}$$

$\Rightarrow$ causal //..

(iv) $y(n) = x(-n+5)$

$\rightarrow$ Static / dynamic:

$$y(0) = x(0+5)$$

$$\boxed{y(0) = x(5)}$$

$\Rightarrow$ Dynamic //..

TV/TIV:

$y(n) = x(-n+5)$

I/P delayed by k units,

$y(n,k) = x(-n-k+5)$

O/P delayed by k units,

$y(n-k) = x(-(n-k)+5)$

$y(n-k) = x(-n+k+5)$

$\Rightarrow$ TV //..

Linear/Non-linear:

$y(n) = x(-n+5)$

To prove: $y_3(n) = a_1 y_1(n) + a_2 y_2(n)$

When the I/P is $x_1(n)$,

$y_1(n) = x_1(-n+5)$

When the I/P is $x_2(n)$,

$y_2(n) = x_2(-n+5)$

$y_3(n) = x_3(-n+5)$

$x_3(-n+5) = a_1 x_1(-n+5) + a_2 x_2(-n+5)$

$\Rightarrow y_3 = a_1 x_1(-n+5) + a_2 x_2(-n+5)$ ← LHS

RHS: $a_1 y_1(n) + a_2 y_2(n)$

$a_1 x_1(-n+5) + a_2 x_2(-n+5)$

LHS = RHS

$\Rightarrow$ Linear //..

Causal/Non-causal:

$y(n) = x(-n+5)$

$\boxed{y(0) = x(5)}$

$\Rightarrow$ Non-causal //..

(v) $y(n) = x(n) + n\,x(n+2)$

→ <u>Static / Dynamic</u>:

$$y(0) = x(0) + 0$$

$$\boxed{y(0) = x(0)}$$

$$\boxed{y(1) = x(1) + x(3)}$$

⇒ Dynamic //..

<u>TV / TIV</u>:

$$y(n) = x(n) + n\,x(n+2)$$

I/P is delayed by k,

$$y(n,k) = x(n-k) + n\,x(n-k+2)$$

O/P is delayed by k,

$$y(n-k) = x(n-k) + (n-k)\,x(n-k+2)$$

⇒ ~~TIV~~ ⇒ TV //..

<u>Linear / Non-linear</u>:

$$y(n) = x(n) + n\,x(n+2)$$

To prove:

$$y_3(n) = a_1 y_1(n) + a_2 y_2(n)$$

When the I/P is $x_1(n)$,

$$y_1(n) = x_1(n) + n\,x_1(n+2)$$

When the I/P is $x_2(n)$,

$$y_2(n) = x_2(n) + n\,x_2(n+2)$$

Scanned with CamScanner

$y_3(n) = x_3(n) + n x_3(n+2)$

$x_3(n) = a_1 x_1(n) + a_2 x_2(n)$

$x_3(n+2) = a_1 x_1(n+2) + a_2 x_2(n+2)$

$\Rightarrow y_3(n) = a_1 x_1(n) + a_2 x_1(n) + n\left[a_1 x_1(n+2) + a_2 x_2(n+2)\right] \rightarrow \underline{LHS}$.

RHS:

$a_1 y_1(n) + a_2 y_2(n)$

$a_1\left[x_1(n) + n x_1(n+2)\right] + a_2\left[x_2(n) + n x_2(n+2)\right]$

$a_1 x_1(n) + a_2 x_2(n) + n\left[a x_1(n+2) + a_2 x_2(n+2)\right]$

LHS = RHS

$\Rightarrow$ Linear //.

Causal / Non-causal:

$y(n) = x(n) + n x(n+2)$

$y(0) = x(0) + 0$

$y(1) = x(1) + x(3)$

$\Rightarrow$ Non-causal //.

Analysis of Discrete time Linear Time Invariant Systems

$\rightarrow$ Response of the system when an unknown I/P signal is applied.

(i) Convolution Sum:

$$\xrightarrow[x(n)]{\text{Input}} \boxed{\begin{matrix}\text{LTI system}\\ h(n)\end{matrix}} \xrightarrow{\text{Output}} y(n) = x(n) * h(n)$$

$\hookrightarrow$ Convolution operation.

$x(n)$ - I/P sequence

$h(n)$ - impulse response of the system

$y(n)$ - response / output

$$\xrightarrow{\delta(n)} \boxed{\text{LTI}} \rightarrow y(n) = T[\delta(n)]$$

$\delta(n)$: Unit impulse signal

$T[\delta(n)]$: Unit impulse response

Convolution sum :

$$y(n) = \underbrace{x(n) * h(n)}_{x(n) \text{ convolved with } h(n)}$$

$$\boxed{y(n) = \sum_{k=-\infty}^{\infty} x(k)\, h(n-k)}$$

No. of elements in $y(n) = n_1 + n_2 - 1$ → n_1 - length of $x(n)$, n_2 - length of $h(n)$

Q:

1) $x[n] = \{1, 2, 3\}$ & $h[n] = \{-1, 2, 2\}$

Find the convolution of 2 sequences.

h[n] \ x[n]	1	2	3
-1	-1	-2	-3
2	2	4	6
2	2	4	6

$\Rightarrow y[n] = \{-1, 0, 3, 10, 6\}$

Length of $x[n] = 3 \Rightarrow n_1$

Length of $h[n] = 3 \Rightarrow n_2$

$n_1 + n_2 - 1 = 5$

No. of elements in $y(n) = \boxed{5}$

② $x(n) = \{-1, 1, -1, 1\}$ & $h(n) = \{-1, 0, 2, 3, 4\}$.

What is the signal obtained when $x(n)$ is linearly convolved with $h(n)$?

→ $y(n) = x(n) * h(n)$

$h(n)$ \ $x(n)$	-1	1	-1	1
-1	1	-1	1	-1
0	0	0	0	0
2	-2	2	-2	2
3	-3	3	-3	3
4	-4	4	-4	4

$\Rightarrow y(n) = \{1, -1, -1, -2, -3, 3, -1, 4\}$ ← Convolved O/P

No. of elements in $y(n) = n_1 + n_2 - 1$

$= 4 + 5 - 1$

$= \boxed{8}$

---

③ $x_1(n) = \{-1, \underset{\uparrow}{2}, 0, 1\}$; $x_2(n) = \{3, 1, \underset{\uparrow}{0}, -1\}$. What is the signal obtained when $x_1(n)$ is linearly convolved with $x_2(n)$?

$x_2(n)$ \ $x_1(n)$	-1	2	0	1
3	-3	6	0	3
1	-1	2	0	1
0	0	0	0	0
-1	1	-2	0	-1

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$y(n) = \{-3, 5, 2, 4, -1, 0, -1\}$

$$\left.\begin{array}{l}\text{Initial value} \\ \text{of } n\end{array}\right\} = \underset{(n_1)}{\text{Origin of } x(n)} + \underset{(n_2)}{\text{Origin of } h(n)}$$

$= -1 - 2$

$= \boxed{-3}$

$\Rightarrow y(n) = \{-3, 5, 2, \underset{\uparrow}{4}, -1, 0, -1\}$

No. of samples in $y(n) = n_1 + n_2 - 1 \Rightarrow 4 + 4 - 1 = \boxed{7}$ samples.

---

(4) The impulse response of LTI system is $h(n) = \{1, \underset{\uparrow}{2}, 1, -1\}$. Determine the response of the system to the I/P. signal $x(n) = \{\underset{\uparrow}{1}, 2, 3, 1\}$

$h(n) \backslash x(n)$	1	2	3	1
1	1	2	3	1
2	2	4	6	2
1	1	2	3	1
-1	-1	-2	-3	-1

$$\left.\begin{array}{l}\text{Starting value} \\ \text{of } n\end{array}\right\} = n_1 + n_2 = -1 + 0 = \boxed{-1}$$

$$\left.\begin{array}{l}\text{No. of samples} \\ \text{present in} \\ y(n)\end{array}\right\} = n_1 + n_2 - 1 = 4 + 4 - 1 = \boxed{7}$$

$\Rightarrow y(n) = \{1, \underset{\uparrow}{4}, 8, 8, 3, -2, -1\}$

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(5) Find the response of the system for the I/P signal :

$x(n) = \{1, 2, 2, 3\}$ & $h(n) = \{1, 0, 3, 2\}$

h(n) \ x(n)	1	2	2	3
1	1	2	2	3
0	0	0	0	0
3	3	6	6	9
2	2	4	4	6

Initial value of n $\Big\} = 0 + 0$

$= \boxed{0}$

No. of samples $\Rightarrow n_1 + n_2 - 1$

$= 4 + 4 - 1$

$= \boxed{7}$

$y(n) = \{\underset{\uparrow}{1}, 2, 5, 11, 10, 13, 6\}$

---

(6) Find the response of the system for the I/P signal $x(n) = \{3, 2, 1, 2\}$

& $h(n) = \{1, \underset{\uparrow}{2}, 1, 2\}$

h(n) \ x(n)	3	2	1	2
1	3	2	1	2
2	6	4	2	4
1	3	2	1	2
2	6	4	2	4

Initial value of n : $0 - 1$

$= \boxed{-1}$

No. of samples : $4 + 4 - 1$

$= \boxed{7}$

$$y(n) = \{ 3, \underset{\uparrow}{8}, 8, 12, 9, 4, 4 \}$$

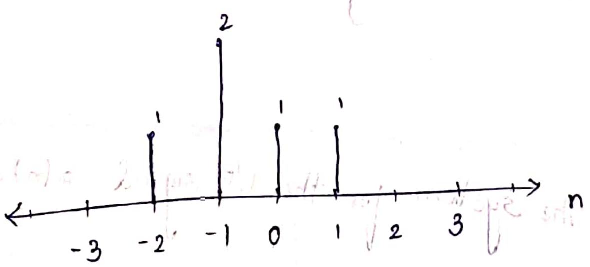
---

(7) Find the convolution of the signals

$$x(n) = \begin{cases} 1 & , n = -2, 0, 1 \\ 2 & , n = -1 \\ 0 & , \text{otherwise} \end{cases}$$

$$h(n) = \delta(n) - \delta(n-1) + \delta(n-2) - \delta(n-3)$$

$\rightarrow$ $y(n) = x(n) * h(n)$



$$x(n) = \{ 1, 2, \underset{\uparrow}{1}, 1 \}$$

$$\delta(n) = \begin{cases} 1 & , n = 0 \\ 0 & , \text{otherwise} \end{cases}$$

$$h(0) = \delta(0) - \delta(-1) + \delta(-2) - \delta(-3) =$$

$$\boxed{h(0) = 1}$$

$h(1) = \delta(1) - \delta(0) + \delta(-1) - \delta(-2)$

$\boxed{h(1) = -1}$

$h(2) = \delta(2) - \delta(1) + \delta(0) - \delta(-1)$

$\boxed{h(2) = 1}$

$\boxed{h(3) = -1}$

$\boxed{h(4) = 0}$

$\boxed{h(5) = 0}$

⋮

$x(n) = \{1, 2, \underset{\uparrow}{1}, 1\}$ $h(n) = \{\underset{\uparrow}{1}, -1, 1, -1\}$

h(n) \ x(n)	1	2	1	1
1	1	2	1	1
-1	-1	-2	-1	-1
1	1	2	1	1
-1	-1	-2	-1	-1

Starting value of $n = -2 + 0 = \boxed{-2}$

No. of samples $= \boxed{7}$

$y(n) = \{1, 1, \underset{\uparrow}{0}, 1, -2, 0, -1\}$

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Using Formula : To compute convolution sum

$$y(n) = \sum_{k=-\infty}^{\infty} x(k)\,h(n-k)$$

① Find convolution of 2 finite duration seq.

$h(n) = a^n u(n) \quad \forall n$

$x(n) = b^n u(n) \quad \forall n$

(i) $a \neq b$

(ii) $a = b$

$\rightarrow$ $$y(n) = \sum_{k=0}^{n} x(k)\,h(n-k)$$

$u(n) = \begin{cases} 1, & n \geq 0 \\ 0, & \text{otherwise} \end{cases}$

$k = 0$ to n

$\therefore x(n)$ & $h(n) \Rightarrow u(n)$

$x(n) = b^n u(n)$

$x(k) = b^k u(k)$

$= b^k . 1$

$\boxed{x(k) = b^k}$

$h(n) = a^n u(n)$

$h(n-k) = a^{n-k} u(n-k)$

$\boxed{h(n-k) = a^{n-k}}$

$$\Rightarrow y(n) = \sum_{k=0}^{n} b^k a^{n-k}$$

$$= a^n \sum_{k=0}^{n} b^k a^{-k}$$

$$= a^n \sum_{k=0}^{n} \left(\frac{b}{a}\right)^k$$

$$\sum_{n=0}^{N} a^n = \frac{1 - a^{N+1}}{1 - a}$$

(i) When $a \neq b$

$$y(n) = a^n \left[\frac{1-\left(\frac{b}{a}\right)^{n+1}}{1-(b/a)} \right] \quad \text{when } a \neq b$$

(ii) When $a = b$

$$y(n) = a^n \sum_{k=0}^{n} \left(b/b\right)^k$$

$$= a^n \sum_{k=0}^{n} 1$$

$$y(n) = a^n (n+1)$$

---

(2) Determine the response of the system

$$h(n) = \left(\frac{1}{2}\right)^n u(n) \quad ; \quad x(n) = 2^n u(n)$$

$$\rightarrow \quad y(n) = \sum_{k=0}^{n} x(k)\, h(n-k)$$

$x(n) = 2^n u(n)$

$x(k) = 2^k u(k)$

$\boxed{x(k) = 2^k}$

$h(n) = \left(\frac{1}{2}\right)^n u(n)$

$h(n-k) = \left(\frac{1}{2}\right)^{n-k} u(n-k)$

$\boxed{h(n-k) = \left(\frac{1}{2}\right)^{n-k}}$

$$y(n) = \sum_{k=0}^{n} (2)^k \left(\frac{1}{2}\right)^{n-k}$$

$$= \left(\frac{1}{2}\right)^n \sum_{k=0}^{n} 2^k \left(\frac{1}{2}\right)^{-k} \Rightarrow \left(\frac{1}{2}\right)^n \sum_{k=0}^{n} (4)^k$$

$$y(n) = \left(\frac{1}{2}\right)^n \left[\frac{1-4^{n+1}}{1-4} \right]$$

$$= \left(\frac{1}{2}\right)^n \left[\frac{1-4^{n+1}}{-3}\right]$$

$$y(n) = \left(\frac{1}{2}\right)^n \left[\frac{4^{n+1}-1}{3}\right]$$

③ Find convolution of 2 finite duration sequence :

$$h(n) = a^n u(n)$$

$$x(n) = u(n)$$

→

$$y(n) = \sum_{k=0}^{n} x(k)\, h(n-k)$$

$x(n) = u(n)$	$h(n) = a^n u(n)$
$x(k) = u(k)$	$h(n-k) = a^{n-k} u(k)$
$\boxed{x(k) = 1}$	$\boxed{h(n-k) = a^{n-k}}$

$$y(n) = \sum_{k=0}^{n} 1\,(a^{n-k})$$

$$= a^n \sum_{k=0}^{n} \left(\frac{1}{a}\right)^k$$

$$= a^n \left[\frac{1-(1/a)^{n+1}}{1-1/a}\right]$$

$$\boxed{y(n) = a^n \left[\frac{1-(1/a)^{n+1}}{1-(1/a)}\right]}$$

(4) Find the convolution of 2 finite duration sequence:

$h(n) = u(n)$

$x(n) = u(n)$

$\rightarrow$ $y(n) = \sum_{k=0}^{n} x(k)\, h(n-k)$

$x(n) = u(n)$

$x(k) = u(k)$

$\boxed{x(k) = 1}$

$h(n) = u(n)$

$h(n-k) = u(n-k)$

$\boxed{h(n-k) = 1}$

$y(n) = \sum_{k=0}^{n} 1$

$\boxed{y(n) = n+1}$

---

(5) Determine the response of the system

$h(n) = \left(\frac{1}{3}\right)^n u(n)$; $x(n) = 2^n u(n)$

$\rightarrow$ $y(n) = \sum_{k=0}^{n} x(k)\, h(n-k)$

$x(k) = 2^k u(k)$

$\boxed{x(k) = 2^k}$

$h(n-k) = \left(\frac{1}{3}\right)^{n-k}$

$y(n) = \sum_{k=0}^{n} 2^k \left(\frac{1}{3}\right)^{n-k}$

$y(n) = \left(\frac{1}{3}\right)^n \sum_{k=0}^{n} 6^k \Rightarrow \left(\frac{1}{3}\right)^n \left[\frac{1-6^{n+1}}{-5}\right]$

$y(n) = \left(\frac{1}{3}\right)^n \left[\frac{6^{n+1}-1}{5}\right]$ //

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(6) $h(n) = a^n u(n) \;\forall n$

$x(n) = n+2 \quad , \quad 0 \leq n \leq 3$

Determine the response of the system $y(n)$

→ $$y(n) = \sum_{k=0}^{3} x(k)h(n-k)$$

$x(n) = n+2$	$h(n-k) = a^{n-k} u(n-k)$
$x(k) = k+2$	$h(n-k) = a^{n-k}$

$$y(n) = \sum_{k=0}^{3} (k+2) a^{n-k}$$

$$y(n) = a^n \sum_{k=0}^{3} (k+2)\left(\frac{1}{a}\right)^k$$

$$y(n) = a^n \left[(2)\left(\frac{1}{a}\right)^0 + 3\left(\frac{1}{a}\right)^1 + 4\left(\frac{1}{a}\right)^2 + 5\left(\frac{1}{a}\right)^3\right]$$

$$y(n) = a^n \left[2 + \frac{3}{a} + \frac{4}{a^2} + \frac{5}{a^3}\right]$$ //.

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(ii) Analysis of LTI system using correlation

Relation b/w 2 signal

Correlation is a measure of degree to which 2 signals are similar

Correlation

Cross-correlation

Compute ~~comp~~ correlation b/w 2 diff. signal

Eg: $x(n)$ & $y(n)$

$$r_{xy}(l) = \sum_{n=-\infty}^{\infty} x(n)\, y(n-l)$$

$l = 0, \pm 1, \pm 2, \ldots$

$$r_{yx}(l) = \sum_{n=-\infty}^{\infty} y(n+l)\, x(n)$$

Autocorrelation

Compute correlation b/w 2 same signal

Eg: $x(n)$ & $x(n-l)$

i.e. @ diff. time intervals

$$r_{xx}(l) = \sum_{n=-\infty}^{\infty} x(n)\, x(n-l)$$

$$r_{xx}(l) = \sum_{n=-\infty}^{\infty} x(n+l)\, x(n)$$

Application:

(i) Speech recognition system

(ii) Radar.

Q:

1. Find cross correlation of the sequences

$x(n) = \{1, 2, 3, 4\}$ $h(n) = \{2, 4, 6\}$

→ Ans. $r_{xy}(l) = x(l) * y(-l)$

$x(l) = x(n) = \{1, 2, 3, 4\}$

$h(-l) = \{6, 4, 2\}$

$x(l)$ \ $h(-l)$	6	4	2
1	6	4	2
2	12	8	4
3	18	12	6
4	24	16	8

$r_{xy}(l) = [6, 16, 28, 40, 22, 8]$

---

(2) Find the cross-correlation of the sequences

$x(n) = \{1, 2, 1, 1\}$ $\quad y(n) = \{1, 1, 2, 1\}$

$\rightarrow$ $x(l) = \{1, 2, 1, 1\}$ $\quad y(-l) = \{1, 2, 1, 1\}$

$x(l)$ \ $y(-l)$	1	2	1	1
1	1	2	1	1
2	2	4	2	2
1	1	2	1	1
1	1	2	1	1

$r_{xy} = \{1, 4, 6, 6, 5, 2, 1\}$

Textbook prblm

(3) $x(n) = \{\ldots, 0, 0, 2, \underset{\uparrow}{-1}, 3, 7, 1, 2, -3, 0, 0, \ldots\}$

$y(n) = \{\ldots 0, 0, 1, -1, 2, \underset{\uparrow}{-2}, 4, 1, -2, 5, 0, 0, \ldots\}$

Find the cross correlation of the sequences.

	5	-2	1	4	-2	2	-1	1
2	10	-4	2	8	-4	4	-2	2
-1	-5	2	-1	-4	2	-2	1	-1
3	15	-6	3	12	-6	6	-3	3
7	35	-14	7	28	-14	14	-7	7
1	5	-2	1	4	-2	2	-1	1
2	10	-4	2	8	-4	4	-2	2
-3	-15	6	-3	-12	6	-6	3	-3

Starting value of n $\Big\}$ $= n_1 + n_2$

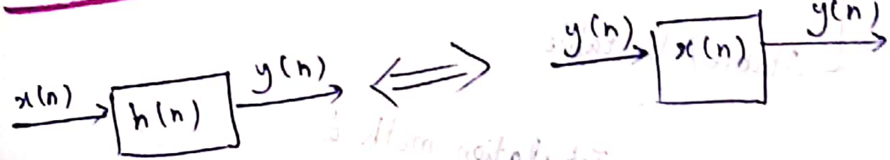
$= (-1) + (-3)$

$= \boxed{-4}$

$$r_{xy}(l) = \{10, -9, 19, 36, \underset{\uparrow}{-14}, 33, 0, 7, 13, -18, 16, -7, 5, -3\}$$

Properties of convolution

(i) Commutative property: $x(n) * h(n) = h(n) * x(n)$



(ii) Associative property:

$$[x(n) * h_1(n)] * h_2(n) = x(n) * [h_1(n) * h_2(n)]$$

(iii) Distributive property:

$$x(n) * [h_1(n) + h_2(n)] = [x(n) * h_1(n)] + [x(n) * h_2(n)]$$

Summary : UNIT 1 : Intro to signals & systems

⇒ What is signal ?

⇒ Classification of signal
- Analog – CT
- Digital – DT
  - Energy, power
  - Even, odd
  - Periodic, Aperiodic
  - Causal, Non-causal

⇒ Elementary signals
- Unit impulse $\delta(n)$
- Unit step $u(n)$

(Fundamental signals)

- Ramp
- Dular

⇒ System
- Discrete time system $x \rightarrow \boxed{h} \rightarrow y$. $y = T[x]$

⇒ Types of DT system
- Static / dynamic
- TV / TIV
- Linear / Non-linear
- Causal / Non-causal
- Stable / Unstable

⇒ LTIV :
- Convolution sum $x(n) * h(n)$
  - Tabulation method
  - Formula $\sum_{k=-\infty}^{\infty} x(k)h(n-k)$
- Correlation
  - Cross correlation ⇒ $x(l) * h(-l)$
  - Auto correlation

⇒ Properties of convolution
- Commutative property
- Associative law
- Distributive law.

⇒ Sampling Theorem:

Analog signal → contain infinite no. of signals

CT → $\boxed{\text{samples}}$ → DT

$\boxed{A \to D}$ → { Sampling, Quantization, Encoding }

$$\boxed{f_s \geq 2 f_h}$$

f_s - sampling freq.

f_h - highest freq. of the I/P signal (Max. freq.)

$$\boxed{T_s = \frac{1}{f_s}}$$

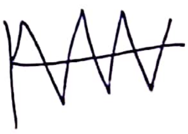
$f_h = 2\,kHz$

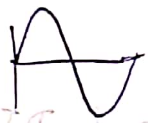
↳ $f_s = 4\,kHz$

$T_s = \frac{1}{f_s} = \frac{1}{4\,kHz} = \boxed{0.25}$

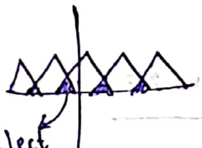
Sampling
- Under sampling: $f_s < 2f_h$
- Critical sampling: $f_s = 2f_h$
- Over sampling: $f_s > 2f_h$

freq.





When sampling theorem is not followed



Aliasing effect

Samples are disturbed ~~distributed~~ by the neighbouring samples

⇓

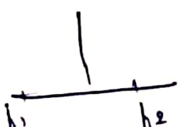
So we'll not be getting the original signal @ the receiving end.

Correctness: Using low pass filter

Fourier transform:

Time domain $x(n)$ $\xrightarrow{FT}$ Freq. domain $x(f)$

Bandwidth:



$\boxed{BW = f_2 - f_1}$ $f_2 > f_1$

BW = highest freq. - lowest freq.