CHAPTER 5

Inner product Spaces

Length and Dot product in R^n

If $\vec{v} = (v_1, v_2)$ is a vector in the plane, the length or magnitude of $\vec{v}$, denoted by $\|\vec{v}\|$ is defined as

$$\|\vec{v}\| = \sqrt{v_1^2 + v_2^2}$$

The length or magnitude of vector $\vec{v} = (v_1, v_2, \dots v_n)$ in R^n is given by

$$\|\vec{v}\| = \sqrt{v_1^2 + v_2^2 + \dots + v_n^2}$$

e.g $\vec{v} = (0, -2, 1, 4, -2)$

$$\|\vec{v}\| = \sqrt{0^2 + (-2)^2 + 1^2 + 4^2 + (-2)^2} = \sqrt{25} = 5$$

Unit Vector in the Direction of $\vec{v}$

If $\vec{v}$ is a nonzero vector in R^n, then the vector

$$\hat{u} = \frac{\vec{v}}{\|\vec{v}\|}$$

has length 1 and has the same direction as $\vec{v}$. This vector $\hat{u}$ is called the unit vector in the direction of $\vec{v}$.

Standard unit Vectors

$\hat{i} = (1, 0)$, $\hat{j} = (0, 1)$ standard unit vectors in R^2

$\hat{i} = (1, 0, 0)$, $\hat{j} = (0, 1, 0)$

$\hat{k} = (0, 0, 1)$ standard unit vectors in R^3.

Example (1) Find the unit vector in the direction of $\vec{v} = (3,-1,2)$ and verify that this vector has length 1.

The unit vector in the direction of $\vec{v}$ is

$$\hat{u} = \frac{\vec{v}}{\|\vec{v}\|} = \frac{(3,-1,2)}{\sqrt{3^2+(-1)^2+2^2}} = \frac{(3,-1,2)}{\sqrt{14}}$$

$$= \left(\frac{3}{\sqrt{14}}, \frac{-1}{\sqrt{14}}, \frac{2}{\sqrt{14}}\right)$$

$$\|\hat{u}\| = \sqrt{\left(\frac{3}{\sqrt{14}}\right)^2 + \left(\frac{-1}{\sqrt{14}}\right)^2 + \left(\frac{2}{\sqrt{14}}\right)^2}$$

$$= \sqrt{\frac{9}{14} + \frac{1}{14} + \frac{4}{14}} = 1$$

Dot product The dot product of $\vec{u} = (u_1, u_2, \ldots u_n)$ and $\vec{v} = (v_1, v_2, \ldots v_n)$ is the scalar quantity

$$\vec{u}\cdot\vec{v} = u_1v_1 + u_2v_2 + \cdots + u_nv_n = \langle \vec{u}, \vec{v}\rangle$$

e.g. the dot product of $\vec{u} = (1,2,0,-3)$, $\vec{v} = (3,-2,4,2)$ is

$$\vec{u}\cdot\vec{v} = 1(3) + 2(-2) + 0(4) + (-3)(2) = -7$$

The Angle Between Two Vectors

The angle θ between two nonzero vectors in R^n is given by

$$\cos\theta = \frac{\vec{u}\cdot\vec{v}}{\|\vec{u}\|\cdot\|\vec{v}\|}$$

The angle between $\vec{u} = (-4,0,2,-2)$ and $\vec{v} = (2,0,-1,1)$ is

$$\cos\theta = \frac{-12}{\sqrt{24}\sqrt{6}} = -1 \Rightarrow \theta = \pi.$$

Orthogonal Vectors Two vectors $\vec{u}$ and $\vec{v}$ in R^n are orthogonal if

$$\vec{u}\cdot\vec{v} = 0.$$

Example ① (a) The vectors $\vec{u} = (1,0,0)$ and $\vec{v} = (0,1,0)$ are orthogonal

$$\vec{u}\cdot\vec{v} = 1(0) + 0(1) + 0(0) = 0$$

(b) The vectors $\vec{u} = (3,2,-1,4)$ and $\vec{v} = (1,-1,1,0)$ are orthogonal

$$\vec{u}\cdot\vec{v} = 3(1) + 2(-1) + (-1)(1) + 4(0) = 0$$

Orthogonal and Orthonormal Sets

A set S of vectors is called orthogonal if every pair of vectors in S is orthogonal.

If each vector in the set is a unit vector, then S is called orthonormal.

For set $S = \{\vec{v}_1, \vec{v}_2, \ldots \vec{v}_n\}$.

orthogonal	orthonormal
$\vec{v}_i\cdot\vec{v}_j = 0, i \neq j$	$\vec{v}_i\cdot\vec{v}_j = 0, \; i\neq j$
	$\lVert\vec{v}_i\rVert = 1, \; i = 1,2,\ldots n.$

The set $S = \{(1,0,0),(0,1,0),(0,0,1)\}$ is orthogonal and orthonormal

$$\vec{v}_1 = (1,0,0), \quad \vec{v}_2 = (0,1,0) \quad \vec{v}_3 = (0,0,1)$$

$\vec{v}_1\cdot\vec{v}_2 = 0 \qquad \lVert\vec{v}_1\rVert = 1$

$\vec{v}_1\cdot\vec{v}_3 = 0 \qquad \lVert\vec{v}_2\rVert = 1$

$\vec{v}_2\cdot\vec{v}_3 = 0 \qquad \lVert\vec{v}_3\rVert = 1$

Example (3) Show that the vectors $\vec{v}_1 = (0,1,0)$, $\vec{v}_2 = (1,0,1)$ and $\vec{v}_3 = (1,0,-1)$ are orthogonal

$$\vec{v}_1 \cdot \vec{v}_2 = 0 \quad , \vec{v}_1 \cdot \vec{v}_3 = 0, \quad \vec{v}_2 \cdot \vec{v}_3 = 0$$

Example (4) Show that the set S given by

$$S = \left\{ \left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, 0\right), \left(\frac{-\sqrt{2}}{6}, \frac{\sqrt{2}}{6}, \frac{2\sqrt{2}}{3}\right), \left(\frac{2}{3}, \frac{-2}{3}, \frac{1}{3}\right) \right\}$$

is orthonormal.

$$\vec{v}_1 = \left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, 0\right), \ \vec{v}_2 = \left(\frac{-\sqrt{2}}{6}, \frac{\sqrt{2}}{6}, \frac{2\sqrt{2}}{3}\right), \ \vec{v}_3 = \left(\frac{2}{3}, -\frac{2}{3}, \frac{1}{3}\right)$$

$$\vec{v}_1 \cdot \vec{v}_2 = -\frac{1}{6} + \frac{1}{6} + 0 = 0$$

$$\vec{v}_1 \cdot \vec{v}_3 = \frac{2}{3\sqrt{2}} - \frac{2}{3\sqrt{2}} + 0 = 0$$

$$\vec{v}_2 \cdot \vec{v}_3 = \frac{-\sqrt{2}}{9} + \left(-\frac{\sqrt{2}}{9}\right) + \frac{2\sqrt{2}}{9} = 0$$

$$\|\vec{v}_1\| = \sqrt{\left(\frac{1}{\sqrt{2}}\right)^2 + \left(\frac{1}{\sqrt{2}}\right)^2 + 0} = 1$$

$$\|\vec{v}_2\| = \sqrt{\left(\frac{-\sqrt{2}}{6}\right)^2 + \left(\frac{\sqrt{2}}{6}\right)^2 + \left(\frac{2\sqrt{2}}{3}\right)^2} = 1$$

$$\|\vec{v}_3\| = \sqrt{\left(\frac{2}{3}\right)^2 + \left(-\frac{2}{3}\right)^2 + \left(\frac{1}{3}\right)^2} = 1$$

Orthogonal Sets are Linearly Independent

If $S = \{\vec{v}_1, \vec{v}_2, \ldots \vec{v}_n\}$ is an orthogonal set of non zero vectors, then S is linearly independent.

Corollary If V is a vector space of dimension n, then any orthogonal set of n vectors is a basis for V.

Orthogonal and orthonormal Basis

1) A basis consisting of orthogonal vectors is called an orthogonal basis

2) A basis consisting of orthonormal vectors is called an orthonormal basis.

Example 5 (a) Show that the set

$$S = \left\{ (0,1,0), \left(\frac{1}{\sqrt{2}}, 0, \frac{1}{\sqrt{2}} \right), \left(\frac{1}{\sqrt{2}}, 0, \frac{-1}{\sqrt{2}} \right) \right\}$$

form an orthonormal basis for R^3.

(b) Show that the set S given by

$$S = \{ (2,3,2,-2), (1,0,0,1), (-1,0,2,1), (-1,2,-1,1) \}$$

form an orthogonal basis for R^4

(a) $\vec{v}_1 = (0,1,0)$, $\vec{v}_2 = \left(\frac{1}{\sqrt{2}}, 0, \frac{1}{\sqrt{2}} \right)$, $\vec{v}_3 = \left(\frac{1}{\sqrt{2}}, 0, -\frac{1}{\sqrt{2}} \right)$

$\vec{v}_1 \cdot \vec{v}_2 = 0$, $\vec{v}_1 \cdot \vec{v}_3 = 0$, $\vec{v}_2 \cdot \vec{v}_3 = 0$

$\|\vec{v}_1\| = 1$, $\|\vec{v}_2\| = 1$, $\|\vec{v}_3\| = 1$

(b) $\vec{v}_1 \cdot \vec{v}_2 = 0$, $\vec{v}_1 \cdot \vec{v}_3 = 0$, $\vec{v}_1 \cdot \vec{v}_4 = 0$

$\vec{v}_2 \cdot \vec{v}_3 = 0$, $\vec{v}_2 \cdot \vec{v}_4 = 0$, $\vec{v}_3 \cdot \vec{v}_4 = 0$

Theorem Every nonzero finite dimensional vector space has an orthonormal basis.

Gram - Schmidt Orthonormalization process

To convert a basis $\{\vec{v}_1, \vec{v}_2, --- \vec{v}_n\}$ into an orthonormal basis $\{\vec{u}_1, \vec{u}_2, --- \vec{u}_n\}$, perform the following steps.

$$\vec{w}_1 = \vec{v}_1$$

$$\vec{u}_1 = \frac{\vec{w}_1}{\|\vec{w}_1\|}$$

$$\vec{w}_2 = \vec{v}_2 - \langle \vec{v}_2, \vec{u}_1 \rangle \vec{u}_1$$

$$\vec{u}_2 = \frac{\vec{w}_2}{\|\vec{w}_2\|}$$

$$\vec{w}_3 = \vec{v}_3 - \langle \vec{v}_3, \vec{u}_1 \rangle \vec{u}_1 - \langle \vec{v}_3, \vec{u}_2 \rangle \vec{u}_2$$

$$\vec{u}_3 = \frac{\vec{w}_3}{\|\vec{w}_3\|}$$

$$\vdots$$

$$\vec{w}_n = \vec{v}_n - \langle \vec{v}_n, \vec{u}_1 \rangle \vec{u}_1 - \langle \vec{v}_n, \vec{u}_2 \rangle \vec{u}_2 - --- \langle \vec{v}_n, \vec{u}_{n-1} \rangle \vec{u}_{n-1}$$

$$\vec{u}_n = \frac{\vec{w}_n}{\|\vec{w}_n\|}.$$

Example (6) Apply the Gram-Schmidt process to the basis of R^2 given

$$B = \{(1,1), (0,1)\}$$

$$\vec{v}_1 = (1,1), \vec{v}_2 = (0,1)$$

$$\vec{w}_1 = \vec{v}_1 = (1,1)$$

$$\vec{u}_1 = \frac{\vec{w}_1}{\|\vec{w}_1\|} = \frac{(1,1)}{\sqrt{1^2+1^2}} = \left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$$

$$\vec{w}_2 = \vec{v}_2 - \langle \vec{v}_2, \vec{u}_1 \rangle \vec{u}_1$$
$$= (0,1) - \left[(0,1)\cdot\left(\tfrac{1}{\sqrt{2}}, \tfrac{1}{\sqrt{2}}\right)\right]\left(\tfrac{1}{\sqrt{2}}, \tfrac{1}{\sqrt{2}}\right)$$
$$= (0,1) - \tfrac{1}{\sqrt{2}}\left(\tfrac{1}{\sqrt{2}}, \tfrac{1}{\sqrt{2}}\right)$$
$$= \left(-\tfrac{1}{2}, \tfrac{1}{2}\right)$$
$$\vec{u}_2 = \frac{\vec{w}_2}{\|\vec{w}_2\|} = \frac{\left(-\tfrac{1}{2}, \tfrac{1}{2}\right)}{\tfrac{1}{\sqrt{2}}} = \left(-\tfrac{\sqrt{2}}{2}, \tfrac{\sqrt{2}}{2}\right)$$

Example 7 Apply Gram-Schmidt process to the basis for R^3

$$B = \{(1,1,0), (1,2,0), (0,1,2)\}$$

$\vec{v}_1 = (1,1,0)$, $\vec{v}_2 = (1,2,0)$, $\vec{v}_3 = (0,1,2)$

$$\vec{w}_1 = \vec{v}_1 = (1,1,0)$$
$$\vec{u}_1 = \frac{\vec{w}_1}{\|\vec{w}_1\|} = \frac{(1,1,0)}{\sqrt{2}} = \left(\tfrac{1}{\sqrt{2}}, \tfrac{1}{\sqrt{2}}, 0\right)$$
$$\vec{w}_2 = \vec{v}_2 - \langle \vec{v}_2, \vec{u}_1 \rangle \vec{u}_1$$
$$= (1,2,0) - \left[(1,2,0)\cdot\left(\tfrac{1}{\sqrt{2}}, \tfrac{1}{\sqrt{2}}, 0\right)\right]\left(\tfrac{1}{\sqrt{2}}, \tfrac{1}{\sqrt{2}}, 0\right)$$
$$= (1,2,0) - \tfrac{3}{\sqrt{2}}\left(\tfrac{1}{\sqrt{2}}, \tfrac{1}{\sqrt{2}}, 0\right)$$
$$= \left(-\tfrac{1}{2}, \tfrac{1}{2}, 0\right)$$
$$\vec{u}_2 = \frac{\vec{w}_2}{\|\vec{w}_2\|} = \frac{\left(-\tfrac{1}{2}, \tfrac{1}{2}, 0\right)}{\tfrac{1}{\sqrt{2}}} = \left(-\tfrac{\sqrt{2}}{2}, \tfrac{\sqrt{2}}{2}, 0\right)$$
$$\vec{w}_3 = \vec{v}_3 - \langle \vec{v}_3, \vec{u}_1 \rangle \vec{u}_1 - \langle \vec{v}_3, \vec{u}_2 \rangle \vec{u}_2$$
$$= (0,1,2) - \left[(0,1,2)\cdot\left(\tfrac{1}{\sqrt{2}}, \tfrac{1}{\sqrt{2}}, 0\right)\right]\left(\tfrac{1}{\sqrt{2}}, \tfrac{1}{\sqrt{2}}, 0\right)$$
$$- \left[(0,1,2)\cdot\left(-\tfrac{\sqrt{2}}{2}, \tfrac{\sqrt{2}}{2}, 0\right)\right]\left(-\tfrac{\sqrt{2}}{2}, \tfrac{\sqrt{2}}{2}, 0\right)$$

$$\vec{w}_3 = (0,1,2) - \frac{1}{\sqrt{2}}\left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, 0\right) - \frac{\sqrt{2}}{2}\left(-\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}, 0\right)$$

$$= (0,1,2) - (1/2, 1/2, 0) - \left(-\frac{1}{2}, \frac{1}{2}, 0\right)$$

$$= (0, 0, 2)$$

$$\vec{U}_3 = \frac{\vec{w}_3}{\|\vec{w}_3\|} = \frac{(0,0,2)}{2} = (0, 0, 1)$$

Example 8 Find an orthonormal basis for the solution space of the homogenous system of linear equations.

$$x_1 + x_2 \qquad + 7x_4 = 0$$

$$2x_1 + x_2 + 2x_3 + 6x_4 = 0$$

The augmented matrix for this system is

$$\begin{bmatrix} 1 & 1 & 0 & 7 & 0 \\ 2 & 1 & 2 & 6 & 0 \end{bmatrix}$$

The reduced row echelon form of the matrix is

$$\begin{bmatrix} 1 & 0 & 2 & -1 & 0 \\ 0 & 1 & -2 & 8 & 0 \end{bmatrix}$$

$x_1 + 2x_3 - x_4 = 0$ $\qquad$ $x_3 = \text{free} = s$

$x_2 - 2x_3 + 8x_4 = 0$ $\qquad$ $x_4 = \text{free} = t$

$x_1 = -2s + t$ $\qquad$ $x_2 = 2s - 8t$

$x_3 = s$ $\qquad$ $x_4 = t$

$$(x_1, x_2, x_3, x_4) = (-2s+t, 2s-8t, s, t)$$

$$(x_1, x_2, x_3, x_4) = s(-2, 2, 1, 0) + t(1, -8, 0, 1)$$

$$B = \{(-2,2,1,0), (1,-8,0,1)\}.$$

The set B is a basis for the solution space. To form an orthonormal basis, we use Gram-Schmidt orthonormalization process

$$\vec{v_1} = (-2,2,1,0), \quad \vec{V_2} = (1,-8,0,1)$$

$$\vec{w_1} = \vec{v_1} = (-2,2,1,0)$$

$$\vec{u_1} = \frac{\vec{w_1}}{\|\vec{w_1}\|} = \frac{(-2,2,1,0)}{3} = \left(-\tfrac{2}{3}, \tfrac{2}{3}, \tfrac{1}{3}, 0\right)$$

$$\vec{w_2} = \vec{v_2} - \langle \vec{v_2}, \vec{u_1} \rangle \vec{u_1}$$

$$= (1,-8,0,1) - \left[(1,-8,0,1)\cdot\left(-\tfrac{2}{3}, \tfrac{2}{3}, \tfrac{1}{3}, 0\right)\right]\left(-\tfrac{2}{3}, \tfrac{2}{3}, \tfrac{1}{3}, 0\right)$$

$$= (1,-8,0,1) - 6\left(-\tfrac{2}{3}, \tfrac{2}{3}, \tfrac{1}{3}, 0\right)$$

$$= (-3,-4,2,1)$$

$$\vec{u_2} = \frac{\vec{w_2}}{\|\vec{w_2}\|} = \frac{(3,-4,2,1)}{\sqrt{30}} = \left(-\frac{3}{\sqrt{30}}, \frac{-4}{\sqrt{30}}, \frac{2}{\sqrt{30}}, \frac{1}{\sqrt{30}}\right)$$

Exercises
Section 5.3
Q 27 – 42 (odd)
Q 49 – 54 (odd)

Least Squares Solutions of Linear Systems

$$A\vec{x} = \vec{b}$$
$$A^T A\vec{x} = A^T\vec{b}$$

This is called the normal equations associated with $A\vec{x}=\vec{b}$.

The solution of the system is an approximate solution. The error vector and error is given by

$$\text{error vector} = \vec{b} - A\vec{x} \qquad \text{error} = \|\vec{b} - A\vec{x}\|.$$

Example (9) Find the least squares solution of the system

$$x_1 + x_2 = 0$$
$$x_1 + 2x_2 = 1$$
$$x_1 + 3x_2 = 3$$

$$\begin{bmatrix} 1 & 1 \\ 1 & 2 \\ 1 & 3 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 3 \end{bmatrix}$$

$$A\vec{x} = \vec{b}$$
$$A^T A\vec{x} = A^T\vec{b}$$

$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \end{bmatrix}\begin{bmatrix} 1 & 1 \\ 1 & 2 \\ 1 & 3 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \end{bmatrix}\begin{bmatrix} 0 \\ 1 \\ 3 \end{bmatrix}$$

$$\begin{bmatrix} 1+1+1 & 1+2+3 \\ 1+2+3 & 1+4+9 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0+1+3 \\ 0+2+9 \end{bmatrix}$$

$$\begin{bmatrix} 3 & 6 \\ 6 & 14 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 4 \\ 11 \end{bmatrix}$$

These are normal equations. The augmented matrix of the system is

$$\begin{bmatrix} 3 & 6 & 4 \\ 6 & 14 & 11 \end{bmatrix} \quad \tfrac{1}{3}R_1$$

$$\sim \begin{bmatrix} 1 & 2 & 4/3 \\ 6 & 14 & 11 \end{bmatrix} \quad -6R_1+R_2$$

$$\sim \begin{bmatrix} 1 & 2 & 4/3 \\ 0 & 2 & 3 \end{bmatrix} \quad \tfrac{1}{2}R_2$$

$$\sim \begin{bmatrix} 1 & 2 & 4/3 \\ 0 & 1 & 3/2 \end{bmatrix} \quad -2R_2+R_1$$

$$\sim \begin{bmatrix} 1 & 0 & -5/3 \\ 0 & 1 & 3/2 \end{bmatrix}$$

$$x_1 = -5/3 \qquad x_2 = 3/2$$

$$\vec{x} = \begin{bmatrix} -5/3 \\ 3/2 \end{bmatrix}$$

The error is given by $= \vec{b} - A\vec{x}$

$$= \begin{bmatrix} 0 \\ 1 \\ 3 \end{bmatrix} - \begin{bmatrix} 1 & 1 \\ 1 & 2 \\ 1 & 3 \end{bmatrix}\begin{bmatrix} -5/3 \\ 3/2 \end{bmatrix}$$

$$= \begin{bmatrix} 0 \\ 1 \\ 3 \end{bmatrix} - \begin{bmatrix} -5/3 + 3/2 \\ -5/3 + 3 \\ -5/3 + 9/2 \end{bmatrix}$$

$$= \begin{bmatrix} 0 \\ 1 \\ 3 \end{bmatrix} - \begin{bmatrix} -1/6 \\ 4/3 \\ 17/6 \end{bmatrix} = \begin{bmatrix} 1/6 \\ -1/3 \\ 1/6 \end{bmatrix}$$

$$\text{error} = \|\vec{b} - A\vec{x}\| = \sqrt{(1/6)^2 + (-1/3)^2 + (1/6)^2} = 0.41$$

Example 10 Find the least squares solution of the system $A\vec{x}=\vec{b}$.

$$A=\begin{bmatrix}3&1&-1\\1&2&0\\0&1&2\\1&1&-1\end{bmatrix}\qquad \vec{b}=\begin{bmatrix}-3\\-3\\8\\9\end{bmatrix}$$

$$A\vec{x}=\vec{b}$$

$$A^TA\vec{x}=A^T\vec{b}$$

$$A^TA=\begin{bmatrix}3&1&0&1\\1&2&1&1\\-1&0&2&-1\end{bmatrix}\begin{bmatrix}3&1&-1\\1&2&0\\0&1&2\\1&1&-1\end{bmatrix}$$

$$=\begin{bmatrix}11&6&-4\\6&7&0\\-4&0&6\end{bmatrix}$$

$$A^T\vec{b}=\begin{bmatrix}3&1&0&1\\1&2&1&1\\-1&0&2&-1\end{bmatrix}\begin{bmatrix}-3\\-3\\8\\9\end{bmatrix}=\begin{bmatrix}-3\\8\\10\end{bmatrix}$$

$$A^TA\vec{x}=A^T\vec{b}$$

$$\begin{bmatrix}11&6&-4\\6&7&0\\-4&0&6\end{bmatrix}\begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix}=\begin{bmatrix}-3\\8\\10\end{bmatrix}$$

The augmented matrix of the system is

$$\begin{bmatrix}11&6&-4&-3\\6&7&0&8\\-4&0&6&10\end{bmatrix}$$

The reduced echelon form is

$$\begin{bmatrix} 1 & 0 & 0 & -1 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 1 \end{bmatrix}$$

$x_1 = 1 \qquad x_2 = 2 \qquad x_3 = 1$

Exercises Section 5.4

Q 21 – 26 (odd)