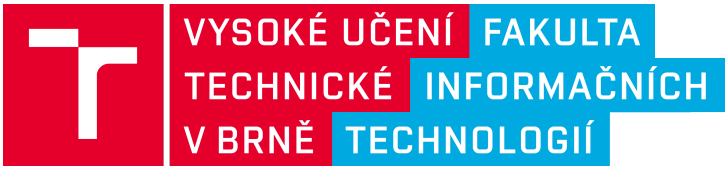


IEL – protokol k projektu

Hugo, Bohácsek
xbohach00

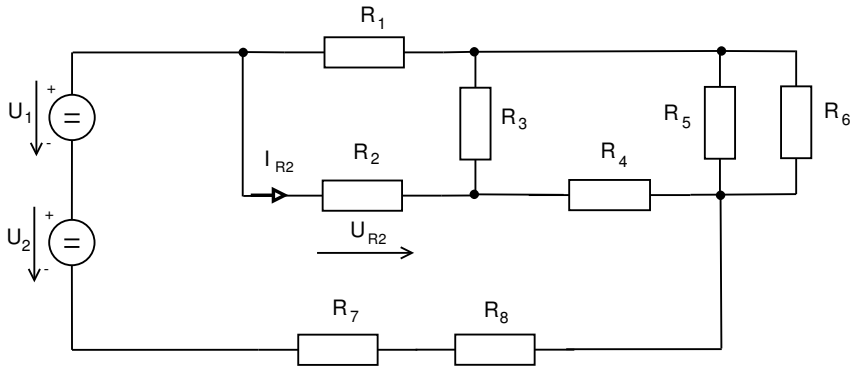
17. decembra 2023

Obsah

Příklad 1

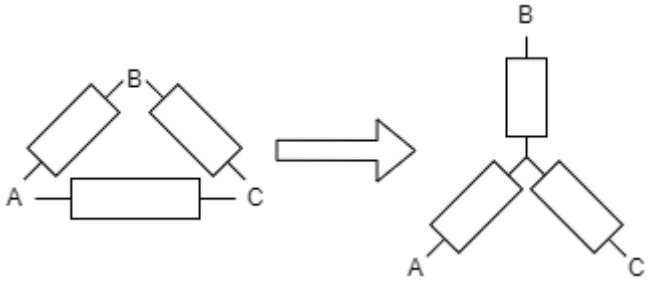
Stanovte napětí U_{R2} a proud I_{R2}. Použijte metodu postupného zjednodušování obvodu.

sk.	U_1 [V]	U_2 [V]	R_1 [Ω]	R_2 [Ω]	R_3 [Ω]	R_4 [Ω]	R_5 [Ω]	R_6 [Ω]	R_7 [Ω]	R_8 [Ω]
E	115	55	485	660	100	340	575	815	255	225

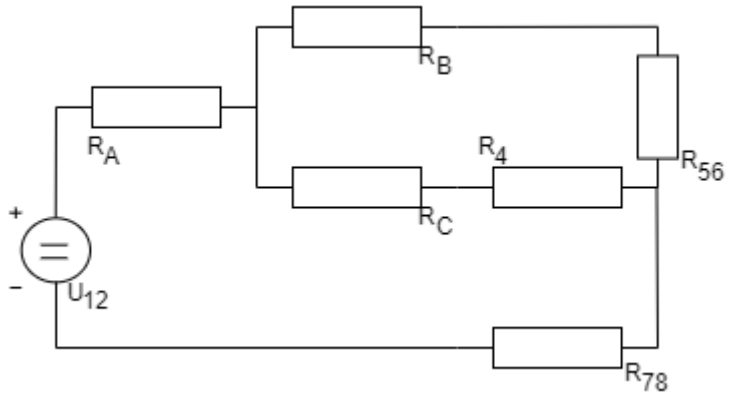


Obr. 1: Pôvodný obvod

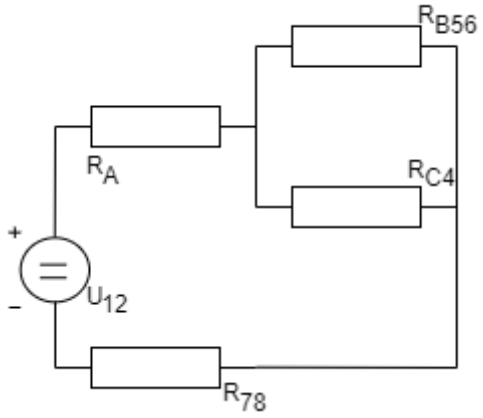
Poďme zjednodušovať!



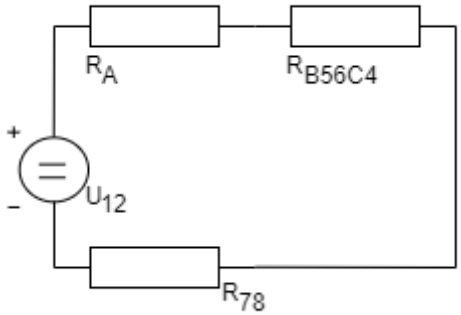
Obr. 2: Spravíme hviezdu



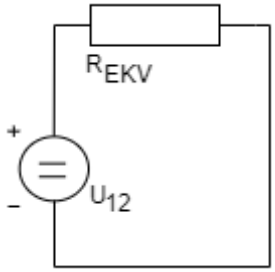
Obr. 3: Vymeníme R_1, R_2, R_3 za hviezdu a spojíme zdroje



Obr. 4: Spájame rezistory



Obr. 5: Pokračujeme v ich spájaní



Obr. 6: A máme hotovo

Poďme rátať!

Po úprave na hviezdu:

$$U_{12} = U_1 + U_2 \qquad R_{56} = \frac{R_5 \times R_6}{R_5 + R_6} \qquad R_A = \frac{R_1 \times R_2}{R_1 + R_2 + R_3}$$
$$R_B = \frac{R_1 \times R_3}{R_1 + R_2 + R_3} \qquad R_C = \frac{R_2 \times R_3}{R_1 + R_2 + R_3} \qquad R_{78} = R_7 + R_8$$

Po 1. spojení rezistorov (obrázok č. 4)

$$R_{B56} = R_B + R_{56} = \frac{R_1 \times R_3}{R_1 + R_2 + R_3} + \frac{R_5 \times R_6}{R_5 + R_6}$$

$$R_{C4} = R_C + R_4 = \frac{R_2 \times R_3}{R_1 + R_2 + R_3} + R_4$$

Po 2. spojení rezistorov (obrázok č. 5)

$$R_{B56C4} = \frac{R_{B56} \times R_{C4}}{R_{B56} + R_{C4}} = \frac{(\frac{R_1 \times R_3}{R_1 + R_2 + R_3} + \frac{R_5 \times R_6}{R_5 + R_6}) \times (\frac{R_2 \times R_3}{R_1 + R_2 + R_3} + R_4)}{(\frac{R_1 \times R_3}{R_1 + R_2 + R_3} + \frac{R_5 \times R_6}{R_5 + R_6}) + (\frac{R_2 \times R_3}{R_1 + R_2 + R_3} + R_4)}$$

Po získaní R_{EKV} (obrázok č. 6)

$$R_{EKV} = R_{B56C4} + R_{78} + R_A = \frac{(\frac{R_1 \times R_3}{R_1 + R_2 + R_3} + \frac{R_5 \times R_6}{R_5 + R_6}) \times (\frac{R_2 \times R_3}{R_1 + R_2 + R_3} + R_4)}{(\frac{R_1 \times R_3}{R_1 + R_2 + R_3} + \frac{R_5 \times R_6}{R_5 + R_6}) + (\frac{R_2 \times R_3}{R_1 + R_2 + R_3} + R_4)} + R_7 + R_8 + \frac{R_1 \times R_2}{R_1 + R_2 + R_3}$$

Postupne dosadíme a vyrátame I_{R2} a U_{R2}!

$$U_{12} = 115 + 55 = 170V$$

$$R_{EKV} = \frac{(\frac{485 \times 100}{485+660+100} + \frac{575 \times 815}{575+815}) \times (\frac{660 \times 100}{485+660+100} + 340)}{(\frac{485 \times 100}{485+660+100} + \frac{575 \times 815}{575+815}) + (\frac{660 \times 100}{485+660+100} + 340)} + 255 + 225 + \frac{485 \times 660}{485+660+100} = \frac{820\,238\,487\,880}{882\,847\,843}\Omega$$

$$I = \frac{U_{12}}{R_{EKV}} = \frac{170}{\frac{820\,238\,487\,880}{882\,847\,843}} = \frac{15\,008\,413\,331}{82\,023\,848\,788}A$$

$$U_{R_A} = R_A \times I = \frac{485 \times 660}{485+660+100} \times \frac{15\,008\,413\,331}{82\,023\,848\,788} = \frac{964\,697\,411\,095}{20\,505\,962\,197}V$$

$$U_{R_{B56C4}} = R_{B56C4} \times I = \frac{(\frac{485 \times 100}{485+660+100} + \frac{575 \times 815}{575+815}) \times (\frac{660 \times 100}{485+660+100} + 340)}{(\frac{485 \times 100}{485+660+100} + \frac{575 \times 815}{575+815}) + (\frac{660 \times 100}{485+660+100} + 340)} \times \frac{15\,008\,413\,331}{82\,023\,848\,788} = \frac{102\,900\,937\,525}{2\,929\,423\,171}V$$

$$I_{R_{C4}} = \frac{U_{R_{B56C4}}}{R_{B56}} = \frac{\frac{102\,900\,937\,525}{2\,929\,423\,171}}{\frac{660 \times 100}{485+660+100} + 340} = \frac{7\,331\,139\,755}{82\,023\,848\,788}A$$

$$U_{R_C} = I_{R_{C4}} \times R_C = \frac{7\,331\,139\,755}{82\,023\,848\,788} \times \frac{660 \times 100}{485+660+100} = \frac{8\,064\,253\,730\,500}{2\,112\,114\,106\,291}V$$

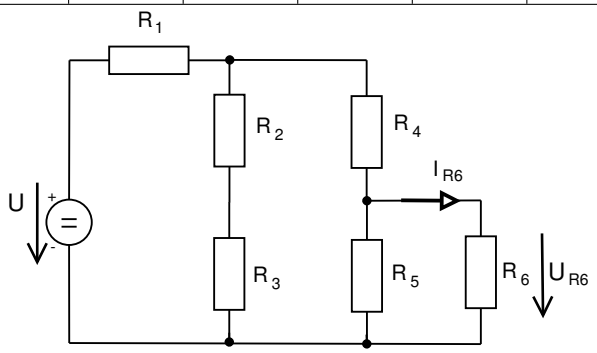
$$U_{R_2} = U_{R_A} + U_{R_C} = \frac{964\,697\,411\,095}{20\,505\,962\,197} + \frac{8\,064\,253\,730\,500}{2\,112\,114\,106\,291} = \frac{107\,428\,087\,073\,285}{2\,112\,114\,106\,291}V \approx \underline{\underline{50.8628V}}$$

$$I_{R_2} = \frac{U_{R_2}}{R_2} = \frac{\frac{107\,428\,087\,073\,285}{2\,112\,114\,106\,291}}{660} = \frac{1\,953\,237\,946\,789}{25\,345\,369\,275\,492}A \approx \underline{\underline{0.0771A}}$$

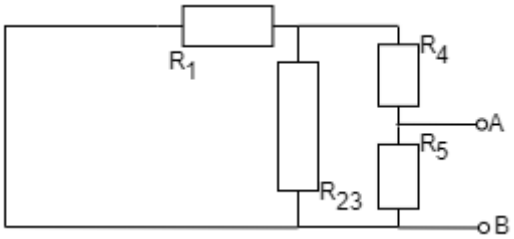
Příklad 2

Stanovte napětí U_{R6} a proud I_{R6}. Použijte metodu Théveninovy věty.

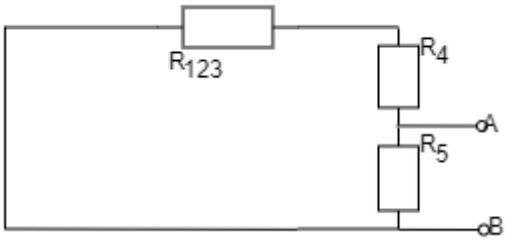
sk.	U [V]	R_1 [Ω]	R_2 [Ω]	R_3 [Ω]	R_4 [Ω]	R_5 [Ω]	R_6 [Ω]
H	220	190	360	580	205	560	250



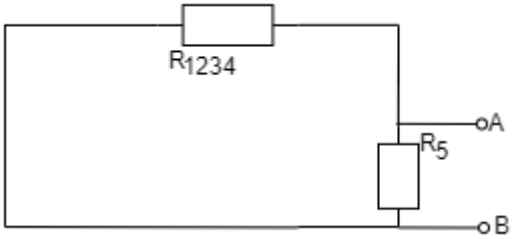
Poďme zjednodušovať!



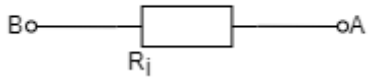
Obr. 1: Odstránime R_6 a zdroj



Obr. 2: Spojíme paralelné rezistory R_1 a R_{23}



Obr. 3: Spojíme sériové rezistory R_{123} a R_4



Obr. 4: Máme hľadaný odpor R_i

Poďme rátať!

Odpory:

$$R_{23} = R_2 + R_3 \qquad R_{123} = \frac{R_{23} \times R_1}{R_{23} + R_1} \qquad R_{1234} = R_{123} + R_4$$

$$R_i = \frac{R_{1234} \times R_5}{R_{1234} + R_5} = \frac{(\frac{(R_2+R_3)\times R_1}{R_2+R_3+R_1} + R_4) \times R_5}{\frac{(R_2+R_3)\times R_1}{R_2+R_3+R_1} + R_4 + R_5} = \frac{(\frac{(360+580)\times 190}{360+580+190} + 205) \times 560}{\frac{(360+580)\times 190}{360+580+190} + 205 + 560} = \frac{4\,594\,800}{20\,861}\Omega$$

A ideme na to:

$$R_{EKV} = \frac{(R_2+R_3)\times(R_4+R_5)}{R_2+R_3+R_4+R_5} + R_1 = \frac{(360+580)\times(205+560)}{360+580+205+560} + 190 = \frac{208\,610}{341}\Omega$$

$$I = \frac{U}{R_{EKV}} = \frac{220}{\frac{208\,610}{341}} = \frac{7502}{20\,861}A$$

$$U_{R45} = U - (I \times R_1) = 220 - (\frac{7502}{20\,861} \times 190) = \frac{3\,164\,040}{20\,861}V$$

$$I_{R45} = \frac{U_{R45}}{R_4+R_5} = \frac{\frac{3\,164\,040}{20\,861}}{205+560} = \frac{4136}{20\,861}A$$

$$U_i = I_{R45} \times R_5 = \frac{4136}{20\,861} \times 560 = \frac{2\,316\,160}{20\,861}V$$

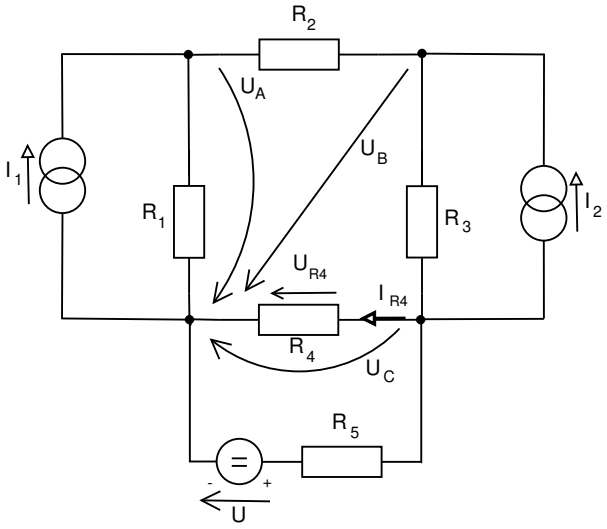
$$I_{R6} = \frac{U_i}{R_i+R_6} = \frac{\frac{2\,316\,160}{20\,861}}{\frac{4\,594\,800}{20\,861}+250} = \frac{231\,616}{981\,005}A \approx \underline{0.2361A}$$

$$U_{R6} = R_6 \times I_{R6} = 250 \times \frac{231\,616}{981\,005} = \frac{11\,580\,800}{196\,201}V \approx \underline{\underline{59.0252V}}$$

Příklad 3

Stanovte napětí U_{R4} a proud I_{R4}. Použijte metodu uzlových napětí (U_A, U_B, U_C).

sk.	U [V]	I_1 [A]	I_2 [A]	R_1 [Ω]	R_2 [Ω]	R_3 [Ω]	R_4 [Ω]	R_5 [Ω]
A	120	0.9	0.7	53	49	65	39	32



Poďme rátať!

Zostavíme matice:

$$\begin{pmatrix} -\frac{1}{R_1}-\frac{1}{R_2} & \frac{1}{R_2} & 0 \\ \frac{1}{R_2} & -\frac{1}{R_2}-\frac{1}{R_3} & \frac{1}{R_3} \\ 0 & \frac{1}{R_3} & -\frac{1}{R_3}-\frac{1}{R_4}-\frac{1}{R_5} \end{pmatrix}^{-1} \begin{pmatrix} -I_1 \\ -I_2 \\ I_2-\frac{U}{R_5} \end{pmatrix} = \begin{pmatrix} U_a \\ U_b \\ U_c \end{pmatrix}$$

Dosadíme do matice:

$$\begin{pmatrix} -\frac{1}{53}-\frac{1}{49} & \frac{1}{49} & 0 \\ \frac{1}{49} & -\frac{1}{49}-\frac{1}{65} & \frac{1}{65} \\ 0 & \frac{1}{65} & -\frac{1}{65}-\frac{1}{39}-\frac{1}{32} \end{pmatrix}^{-1} \begin{pmatrix} -0.9 \\ -0.7 \\ 0.7-\frac{120}{32} \end{pmatrix} = \begin{pmatrix} U_a \\ U_b \\ U_c \end{pmatrix}$$

Spravíme inverziu pomocou determinantu:

$$\begin{pmatrix} -\frac{495\,126}{13\,105} & -\frac{310\,739}{13\,105} & -\frac{66\,144}{13\,105} \\ -\frac{310\,739}{13\,105} & -\frac{598\,026}{13\,105} & -\frac{127\,296}{13\,105} \\ -\frac{66144}{13\,105} & -\frac{127296}{13\,105} & -\frac{208416}{13\,105} \end{pmatrix} \begin{pmatrix} -0.9 \\ -0.7 \\ -3.05 \end{pmatrix} = \begin{pmatrix} U_a \\ U_b \\ U_c \end{pmatrix}$$

Vynásobíme matice:

$$\begin{pmatrix} \frac{8\,648\,699}{131\,050} \\ \frac{10\,865\,361}{131\,050} \\ \frac{3\,921\,528}{65\,525} \end{pmatrix} = \begin{pmatrix} U_a \\ U_b \\ U_c \end{pmatrix}$$

Vyčítame z tejto matice hodnoty prúdov

$$U_a = \frac{8\,648\,699}{131\,050}V \qquad U_b = \frac{10\,865\,361}{131\,050}V \qquad U_c = \frac{3\,921\,528}{65\,525}V$$

Prúd U_{R_4} je vlastne U_c, dorátejme si teda I_{R_4}

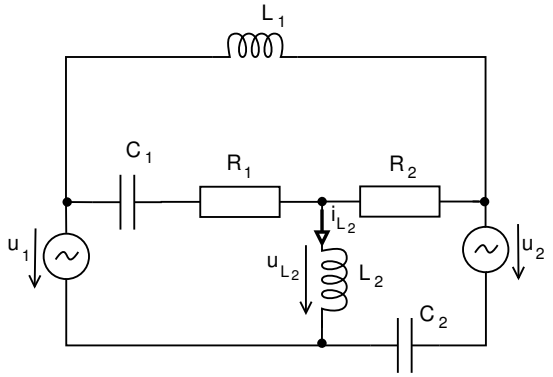
$$U_{R_4} = U_c = \frac{3\,921\,528}{65\,525}V \approx \underline{\underline{59.8478V}}$$

$$I_{R_4} = \frac{U_{R_4}}{R_4} = \frac{\frac{3\,921\,528}{65\,525}}{39} = \frac{100\,552}{65\,525}A \approx \underline{\underline{1.5346A}}$$

Příklad 4

Pro napájecí napětí platí: $u_1 = U_1 \cdot \sin(2\pi ft)$, $u_2 = U_2 \cdot \sin(2\pi ft)$.
Ve vztahu pro napětí $u_{L_2} = U_{L_2} \cdot \sin(2\pi ft + \varphi_{L_2})$ určete $|U_{L_2}|$ a φ_{L_2}. Použijte metodu smyčkových proudů.
Pozn: Pomocné směry šipek napájecích zdrojů platí pro speciální časový okamžik ($t = \frac{\pi}{2\omega}$).

sk.	U_1 [V]	U_2 [V]	R_1 [Ω]	R_2 [Ω]	L_1 [mH]	L_2 [mH]	C_1 [µF]	C_2 [µF]	f [Hz]
E	5	3	14	13	130	60	100	65	90



Poďme rátať!
Pripravme si základné výpočty:

$$\omega = 2 \times \pi \times f = 180\pi$$
$$Z_{L_1} = j \times \omega \times L_1 = j \times 180\pi \times 130 = 23\,400 j\pi\Omega$$
$$Z_{L_2} = j \times \omega \times L_2 = j \times 180\pi \times 60 = 10\,800 j\pi\Omega$$
$$Z_{C_1} = -\frac{j}{\omega \times C_1} = \frac{j}{180\pi \times 100} = \frac{j}{18\,000\pi}\Omega$$
$$Z_{C_2} = -\frac{j}{\omega \times C_2} = \frac{j}{180\pi \times 65} = \frac{j}{11\,700\pi}\Omega$$

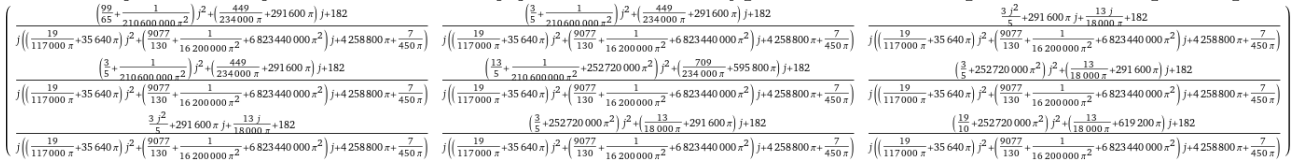
Poďme zostaviť matice:

$$\begin{pmatrix} R_2 + R_1 + Z_{C_1} + Z_{L_1} & -Z_{C_1} - R_1 & -R_2 \\ -R_1 - Z_{C_1} & Z_{C_1} + R_1 + Z_{L_2} & -Z_{L_2} \\ -R_2 & -Z_{L_2} & Z_{L_2} + R_2 + Z_{C_2} \end{pmatrix}^{-1} \begin{pmatrix} 0 \\ U_1 \\ -U_2 \end{pmatrix} = \begin{pmatrix} I_a \\ I_b \\ I_c \end{pmatrix}$$

Čas na dosadenie:

$$\begin{pmatrix} 14 + 13 + \frac{j}{18\,000\pi} + 23\,400 j\pi & -\frac{j}{18\,000\pi} - 14 & -13 \\ -14 - \frac{j}{18\,000\pi} & \frac{j}{18\,000\pi} + 14 + 10\,800 j\pi & -10\,800 j\pi \\ -13 & -10\,800 j\pi & 10\,800 j\pi + 13 + \frac{j}{11\,700\pi} \end{pmatrix}^{-1} \begin{pmatrix} 0 \\ 5 \\ -3 \end{pmatrix} = \begin{pmatrix} I_a \\ I_b \\ I_c \end{pmatrix}$$

Inverzia tejto matice je ťažká. Priložím jej obrázok ale s výpočtami budeme pokračovať až pre napätia.



Po inverzii a vynásobení týchto matíc nám vyjde matica o rozmeroch 1×3

$$I_a = -0.007 + 0.0126jA \qquad I_b = 0.0326 - 0.0625jA$$
$$I_c = -0.019 + 0.1077jA$$

Poďme to dotiahnuť do konca:

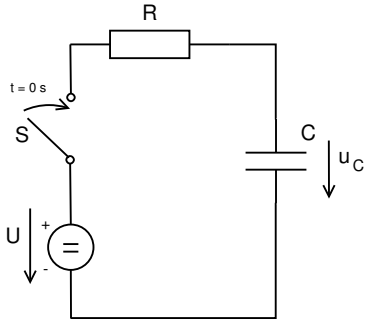
$$U_{L_2} = |Z_{L_2} \times (I_b - I_c)| = 10\,800j\pi \times ((0.0326 - 0.0625j) - (-0.019 + 0.1077j)) \approx \underline{6.0334V}$$

$$\varphi = atan(\frac{\text{imaginárna zložka } U_{L_2}}{\text{reálna zložka } U_{L_2}}) \times \frac{180}{\pi} = atan(\frac{1.7535}{5.773}) \times \frac{180}{\pi} \approx \underline{\underline{16.8957°}}$$

Příklad 5

V obvodu na obrázku níže v čase $t = 0$ [s] sepne spínač S. Sestavte diferenciální rovnici popisující chování obvodu na obrázku, dále ji upravte dosazením hodnot parametrů. Vypočítejte analytické řešení $u_C = f(t)$. Proveďte kontrolu výpočtu dosazením do sestavené diferenciální rovnice.

sk.	U [V]	C [F]	R [Ω]	$u_C(0)$ [V]
H	18	2	200	3



Začneme vytvorením základnej rovnice pomocou II. Kirchhoffovho zákona:

$$U_R + U_C - U = 0$$

$$R \times I + U_C - U = 0$$

$$I = \frac{U - U_C}{R}$$

Teraz si vieme spraviť rovnicu pre u_c' a vytvoriť jej deriváciu:

$$u_c' = \frac{1}{C} \times I$$

$$u_c' = \frac{U - U_c}{R \times C} \Rightarrow \frac{18 - U_c}{200 \times 2}$$

$$u_c' + U_c \times \frac{1}{400} = \frac{9}{200}$$

Čas na λ:

$$\lambda + \frac{1}{R \times C} = 0$$

$$\lambda + \frac{1}{400} = 0$$

$$\lambda = -\frac{1}{400}$$

Zostavme očakávaný tvar rovnice:

$$u_c(t) = k(t)e^{\lambda \times t}$$

$$u_c(t) = k(t)e^{-\frac{1}{400}}$$

Dosadíme a derivujeme:

$$u_c'(t) = k'(t)e^{-\frac{t}{400}} + k(t)(-\frac{1}{400})e^{-\frac{t}{400}}$$

Dosaďme do očakávaného tvaru rovnice:

$$u_c' + U_c\frac{1}{400} = \frac{9}{200}$$

$$k'(t)e^{-\frac{t}{400}} + k(t)(-\frac{1}{400})e^{-\frac{t}{400}} + k(t)(\frac{1}{400})e^{-\frac{t}{400}} = \frac{9}{200}$$

$$k'(t)e^{-\frac{t}{400}} = \frac{9}{200}$$

Stačilo derivácií, čas nechať ich oddýchnuť:

$$k'(t) = \frac{9}{200e^{-\frac{t}{400}}}$$

$$k'(t) = \frac{9}{200}e^{\frac{t}{400}}$$

$$\int k'(t) = \int \frac{9}{200}e^{\frac{t}{400}}dt$$

$$k(t) = \frac{9}{200}\int e^{\frac{t}{400}} + K$$

$$k(t) = \frac{9}{200} \times \frac{1}{\frac{1}{400}}e^{\frac{t}{400}} + K$$

$$k(t) = 18e^{\frac{t}{400}} + K$$

Dosaďme $k(t)$ do očakávanej rovnice:

$$u_c(t) = (18e^{\frac{t}{400}} + K) \times e^{-\frac{t}{400}}$$

$$u_c(t) = 18 + Ke^{-\frac{t}{400}}$$

Konečne už len K a potom kontrola. Vypočítame ho podľa zadanej podmienky; $u_c(t) = 3V$ kde $t = 0s$:

$$u_c(0) = 18 + Ke^0$$

$$3 = 18 + K$$

$$K = -15$$

Kontrola:

1. krokom bude vyjadrenie si u_c':

$$u_c' + U_c \frac{1}{400} = \frac{9}{200}$$

$$\underline{\underline{u_c(t) = 18 - 15e^{-\frac{t}{400}}}}$$

$$u_c' + \frac{18}{400} - \frac{15e^{-\frac{t}{400}}}{400} = \frac{9}{200}$$

$$u_c' = \frac{15e^{-\frac{t}{400}}}{400}$$

Dosadíme do diferenciálnej rovnice. Prinesieme si so sebou naše novonadobudnuté $K = -15$ a $t = 0$:

$$u_c' + U_c \times \frac{1}{400} = \frac{9}{200}$$

$$\frac{15e^{-\frac{t}{400}}}{400} + \frac{18-15e^{-\frac{t}{400}}}{400} = \frac{9}{200}$$

$$\frac{15e^0}{500} + \frac{18-15e^0}{400} = \frac{9}{200}$$

$$\frac{15}{400} + \frac{18-15}{400} = \frac{9}{200}$$

$$\frac{18}{400} = \frac{9}{200}$$

$$0 = 0$$

Shrnutí výsledků

Příklad	**Skupina**	**Výsledky**	
1	E	$U_{R2} = 50.8628V$	$I_{R2} = 0.0771A$
2	H	$U_{R6} = 59.0252V$	$I_{R6} = 0.2361A$
3	A	$U_{R4} = 59.8478V$	$I_{R4} = 1.5346A$
4	E	$\lvert U_{L_2} \rvert = 6.0334V$	$\varphi_{L_2} = 16.8957°$
5	H	$u_C = 18 - 15e^{-\frac{t}{400}}$	

LaTeX