

Symbol	Definition
$G(C, R)$	Ontology graph
$C = \{c_1, c_2, \dots, c_n\}$	Set of concepts constituting the vertices of the ontology graph
$R = \{R_{ij} \mid i = 1, \dots, n, j = 1, \dots, n, j > i\}$	Set of relations constituting the edges of the ontology graph
$R_{ij} = \{r_{ij}^1, r_{ij}^2, \dots, r_{ij}^m, m < n\}$	Set of relations between concepts c_i and c_j in $G(C, R)$
$Q = \{(k_i, c_i)\}$	Query as a collection of pairs (keyword, concept)
$G_Q(C_Q, R_Q)$	Query sub-graph for query Q
$C_Q = \{c_i \mid (k_i, c_i) \in Q\}$	Set of concepts constituting the query sub-graph
$R_Q = \{\bar{R}_{ij} \mid 1 \leq i \leq n, 1 \leq j \leq n, j > i\}$	Set of relations constituting the query sub-graph
$\bar{R}_{ij} = \{\bar{r}_{ij} \mid c_i, c_j \in C_Q, \bar{R}_{ij} \geq 1\}$	Set of relations between c_i and c_j in the query sub-graph
$\eta_{ij} = \bar{R}_{ij} $	Number of relations between c_i and c_j in $G_Q(C_Q, R_Q)$
$A = (AC, AR)$	Graph-based annotation (AC and AR are the sets of concepts and relations)
$AR_{ij} = \{r_{ij}^d \mid r_{ij}^d \in AR, 1 \leq d \leq m\}$	Set of relations between c_i and c_j in AR
$G_{Q,p}(C_{Q,p}, R_{Q,p})$	Page sub-graph for page p given the query Q
$C_{Q,p} = \{c_i \mid c_i \in C_Q \cap AC\}$	Set of concept of page sub-graph $G_{Q,p}(C_{Q,p}, R_{Q,p})$
$R_{Q,p} = \{\bar{r}_{ij} \mid c_i, c_j \in G_{Q,p}\}$	Set of relations of page sub-graph $G_{Q,p}(C_{Q,p}, R_{Q,p})$
$\delta_{ij} = AR_{ij} $	Number of relations c_i and c_j in $G_Q(C_Q, R_Q)$
$\tau_{ij} = P(\bar{r}_{ij}, p) = \delta_{ij} / \eta_{ij}$	Relation probability for \bar{r}_{ij} in page p given the query Q
$SF_{Q,p}(l)$	Set of spanning forests (l edges) for page p and query Q
$SF_{Q,p}^f(l)$	f -th spanning forest (l edges) for page p and query Q
$\sigma_{Q,p}(l) = SF_{Q,p}(l) $	Number of spanning forests (l edges) for page p and query Q
$P(Q, p, l)$	Constrained relevance score for page p , query Q , relevance class l
$ps_{Q,p}$	Relevance score of page p for a given query Q

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Label the edges in  $G_{Q,r}$  with an index ranging from 1 to  $R_{Q,r}$ 
Define variables  $e$  and  $a$  to index graph edges
Set  $\eta_i = \eta_j$  //number of relations linking concepts
//  $i$  and  $j$  in the ontology graph (edge  $e \in R_{Q,r}$ )
Set  $\delta_i = \delta_j$  //number of relations linking concepts
//  $i$  and  $j$  in the page sub-graph (edge  $e \in R_{Q,r}$ )
Set  $r_i = \eta_i / \delta_i$  //relation probability for edge  $e$ 
Mark all the edges in  $G_{Q,r}$  as not visited
Allocate weight vector  $W$  of size  $|C_{Q,r}|-1$ 
//  $W[l]$  stores the accumulated constrained
//probabilities for page forests of length  $l$ 
Allocate vector  $\Sigma$  of size  $|C_{Q,r}|-1$ 
//  $\Sigma[l]$  stores the number of page forests
//for a given length  $l$ 
Initialize  $W$  and  $\Sigma$  to zero

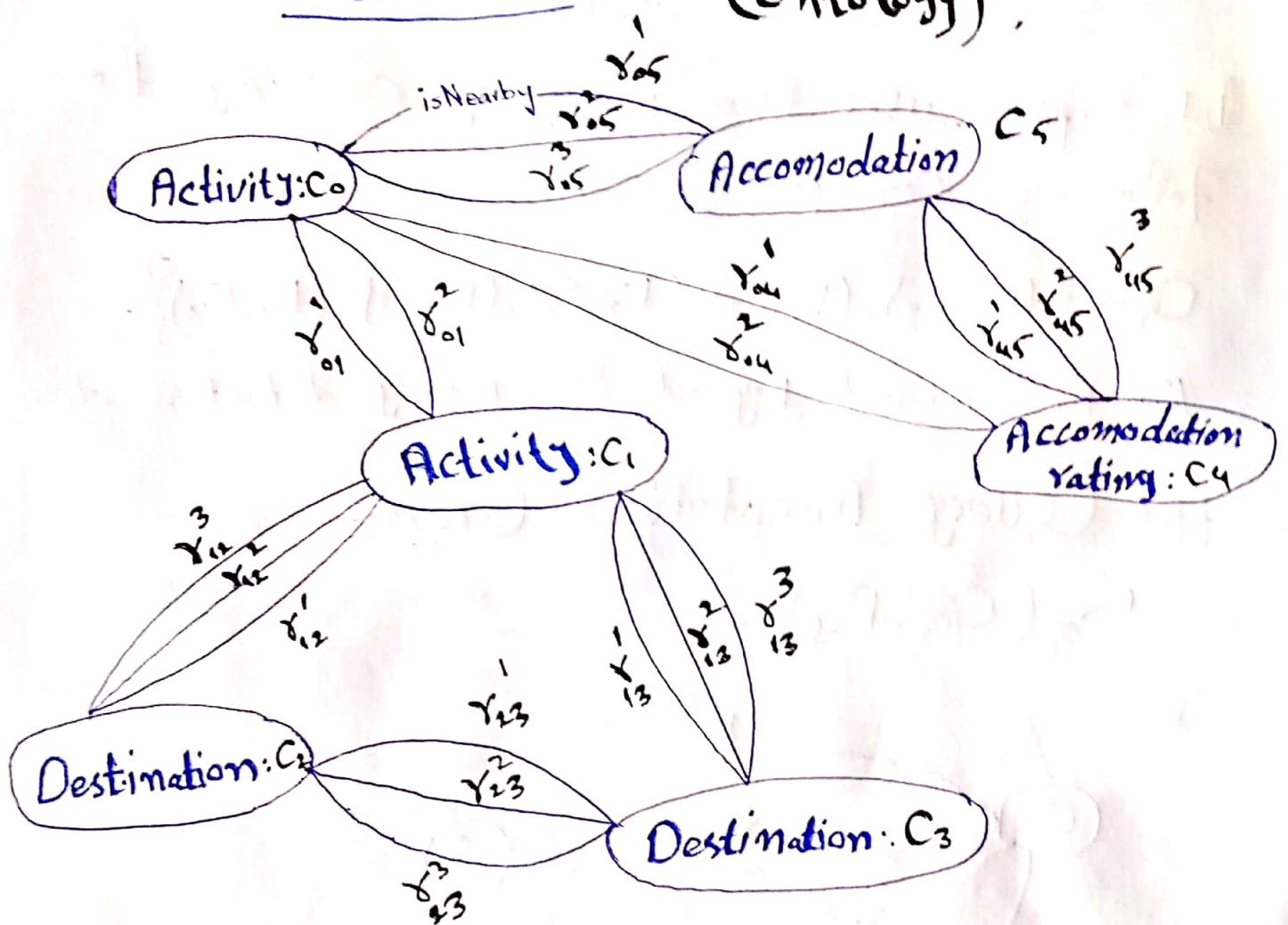
for  $e=1, e \leq |R_{Q,r}|, e=e+1$ 
    mark edge  $e$  as visited
    visit( $e, e, 1, r_i$ )
     $W[1] = W[1] + r_i$ 
     $\Sigma[1] = \Sigma[1] + 1$ 

function visit( $o, e, l, s$ )
     $a = e + 1$ 
    while  $a \leq |R_{Q,r}|$  and  $l \leq |C_{Q,r}|-1$ 
        if  $a$  is not visited and  $a$  is safe
            //(does not introduce cycles, checked through DFS)
            mark edge  $a$  as visited
            visit( $o, a, l+1, s \times r$ )
             $W[l+1] = W[l+1] + s$ 
             $\Sigma[l+1] = \Sigma[l+1] + 1$ 
            set edge  $a$  as not visited
        else
             $a = a + 1$ 

```

Dry Run

Knowledge Base (Ontology).



$$C = \{C_0, C_1, C_2, C_3, C_4, C_5\}$$

$$R = \{R_{01}, R_{04}, R_{05}, R_{12}, R_{13}, R_{23}, R_{45}\}$$

Here Every each element of set R represent different set.
Such as:

$$R_{01} = \{R_{01}^1, R_{01}^2\}$$

Step#01

Uses Query

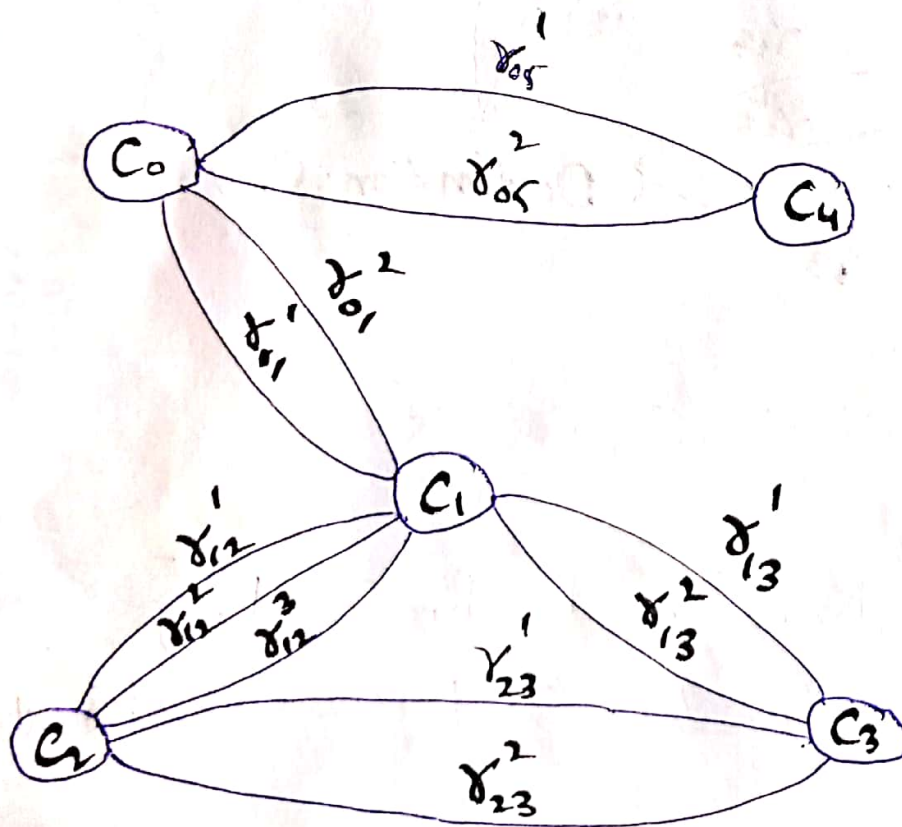
Lets Suppose user types the query consisting the following concepts:-

$$Q = \{ (K_0, C_0), (K_1, C_1), (K_2, C_2), (K_3, C_3), (K_4, C_4) \}.$$

Here K_i represent keywords like Activity, destination etc.

~~At~~ Query Annotation Graph:-

$G_q(C_q, R_q)$:-

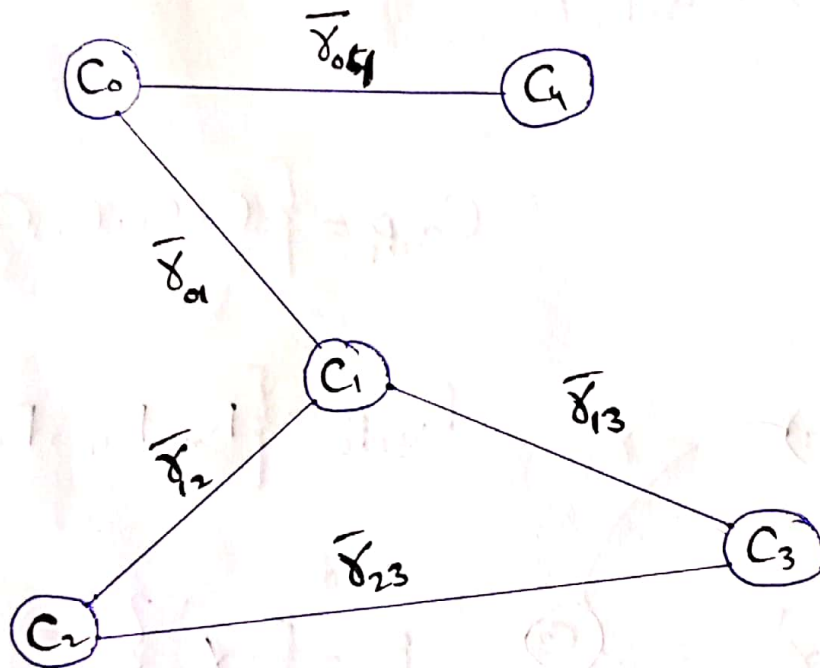


$$C_q = \{ C_0, C_1, C_2, C_3, C_4 \}.$$

$$R_q = \{ R_{01}, R_{04}, R_{12}, R_{13}, R_{23} \}.$$

and $R_{01} = \{ \delta^1_{01}, \delta^2_{01} \}.$

Query Sub-Graph:-



Number of relation b/w i and j Concepts:-

δ_{ij}	η_{ij}	r_{ij}
$\bar{\delta}_{01}$	η_{01}	2
$\bar{\delta}_{12}$	η_{12}	3
$\bar{\delta}_{04}$	η_{04}	2
$\bar{\delta}_{13}$	η_{13}	2
$\bar{\delta}_{23}$	η_{23}	2

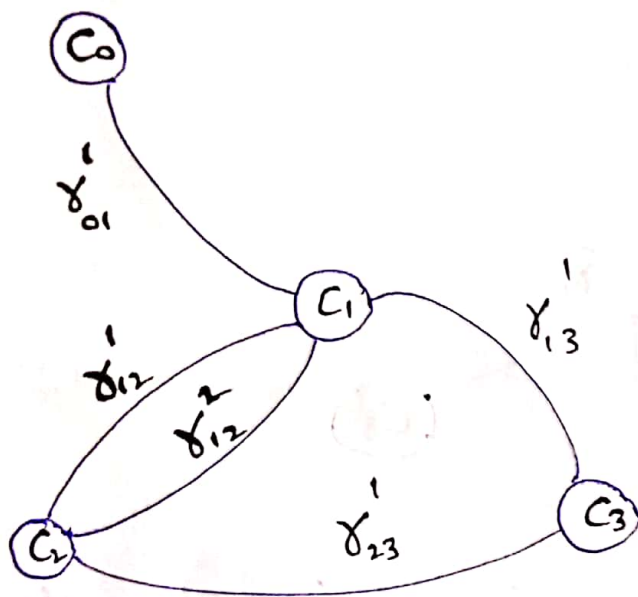
(Table:1)

Step#02

When user presses the Search button then suppose there are two pages where the concepts mentioned in the query are found.
So,

Page one Annotation Graph:-

$G_{\mathcal{A}, P_1} (C_{\mathcal{A}, P_1}, R_{\mathcal{A}, P_1}) :-$



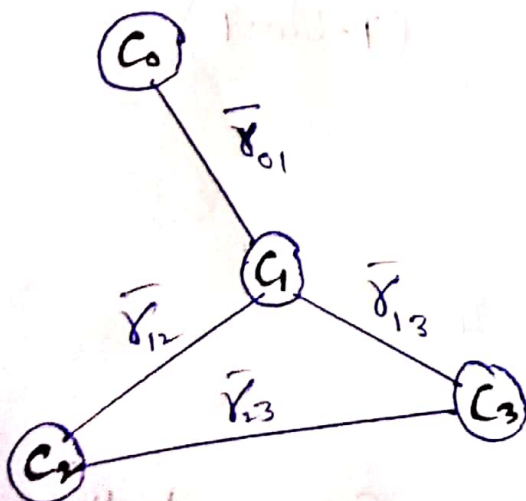
$$C_{\mathcal{A}, P_1} = \{C_0, C_1, C_2, C_3\}$$

$$R_{\mathcal{A}, P_1} = \{R_{01}, R_{12}, R_{13}, R_{23}\}$$

and

$$R_{01} = \{\delta_{01}^1\}$$

Page One Sub-Graph:-



$\bar{\delta}_{ij}$	δ_{ij}	fre
$\bar{\delta}_{01}$	δ_{01}	1
$\bar{\delta}_{12}$	δ_{12}	2
$\bar{\delta}_{13}$	δ_{13}	1
$\bar{\delta}_{23}$	δ_{23}	1

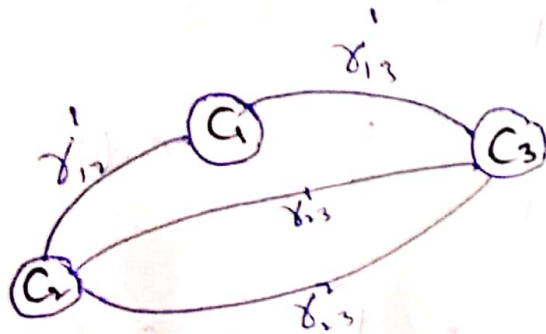
Table (i)

Page Two Annotation Graph:-

$G_{a,p_2}(C_{a,p_2}, R_{a,p_2})$:-

(C₀)

∴ $\begin{cases} \delta_{ij} = \text{Number of relations} \\ \text{b/w } i \text{ and } j \text{ concepts.} \end{cases}$

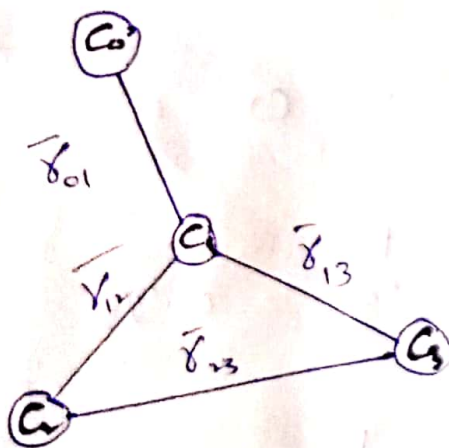


$$C_{a,p_2} = \{C_0, C_1, C_2, C_3\}$$

$$R_{a,p_2} = \{R_{01}, R_{12}, R_{13}, R_{23}\}$$

$$R_{01} = \{ \}$$

Page Two Sub-Graph:-



$\bar{\delta}_{ij}$	δ_{ij}	$\sum \delta_{ij}$
$\bar{\delta}_{01}$	δ_{01}	0
$\bar{\delta}_{12}$	δ_{12}	1
$\bar{\delta}_{13}$	δ_{13}	1
$\bar{\delta}_{23}$	δ_{23}	2

Table (3).

Step#03.

Relational probability (\mathcal{T}_{ij}):-

Page one:

$$P(\bar{x}_{01}, P_1) = \mathcal{T}_{01} = \frac{\delta_{01}}{\eta_{01}} = \frac{1}{2} = 0.5$$

$$P(\bar{x}_{12}, P_1) = \mathcal{T}_{12} = \frac{\delta_{12}}{\eta_{12}} = \frac{2}{3} = 0.66$$

$$P(\bar{x}_{13}, P_1) = \mathcal{T}_{13} = \frac{\delta_{13}}{\eta_{13}} = \frac{1}{2} = 0.5$$

$$P(\bar{x}_{23}, P_1) = \mathcal{T}_{23} = \frac{\delta_{23}}{\eta_{23}} = \frac{1}{2} = 0.5$$

Page Two:-

$$P(\bar{x}_{01}, P_2) = \mathcal{T}_{01} = \frac{\delta_{01}}{\eta_{01}} = \frac{0}{2} = 0$$

$$P(\bar{x}_{12}, P_2) = \mathcal{T}_{12} = \frac{\delta_{12}}{\eta_{12}} = \frac{1}{3} = 0.33$$

$$P(\bar{x}_{13}, P_2) = \mathcal{T}_{13} = \frac{\delta_{13}}{\eta_{13}} = \frac{1}{2} = 0.5$$

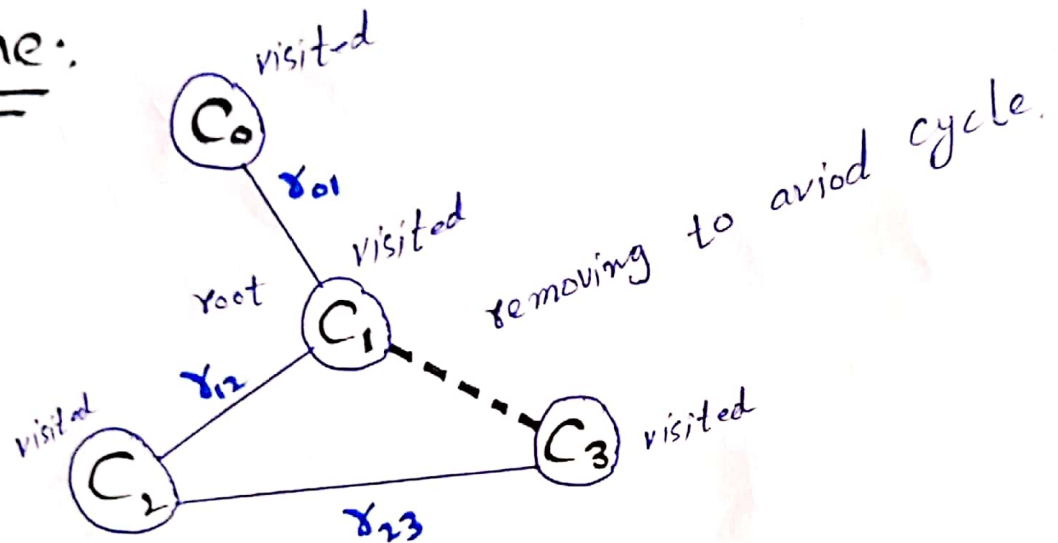
$$P(\bar{x}_{23}, P_2) = \mathcal{T}_{23} = \frac{\delta_{23}}{\eta_{23}} = \frac{2}{2} = 1$$

Step # 04

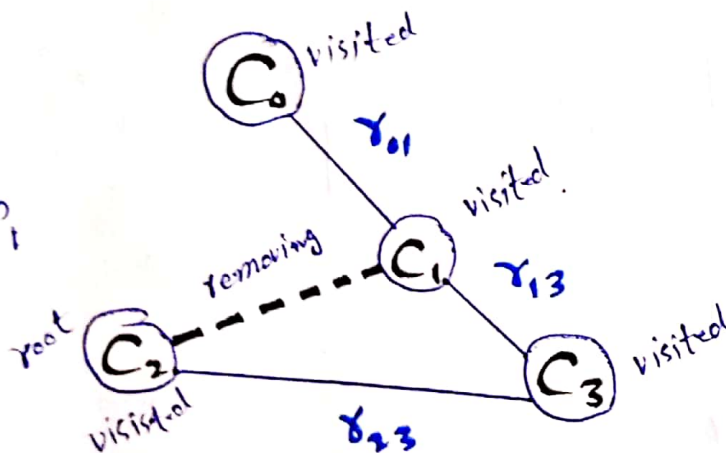
Generating page spanning forests:-

Page one:

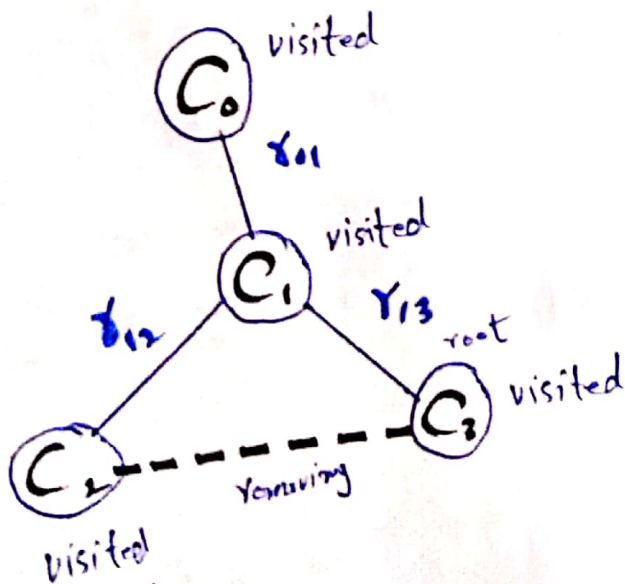
\Rightarrow
 SF_{G, P_1}^1



\Rightarrow
 SF_{G, P_1}^2



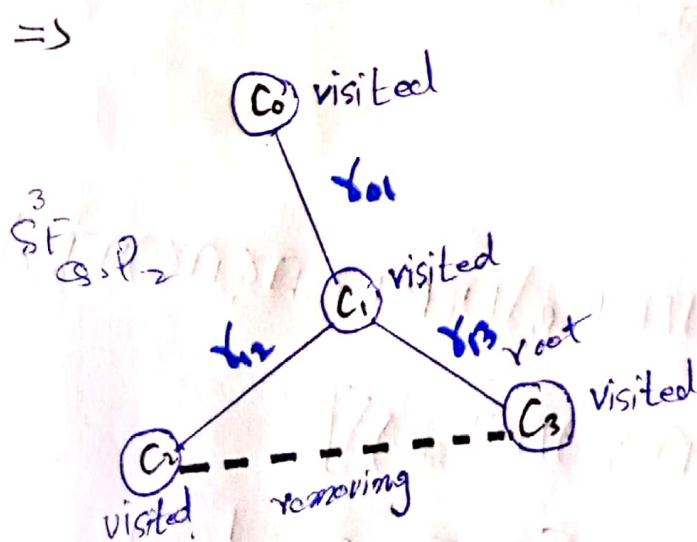
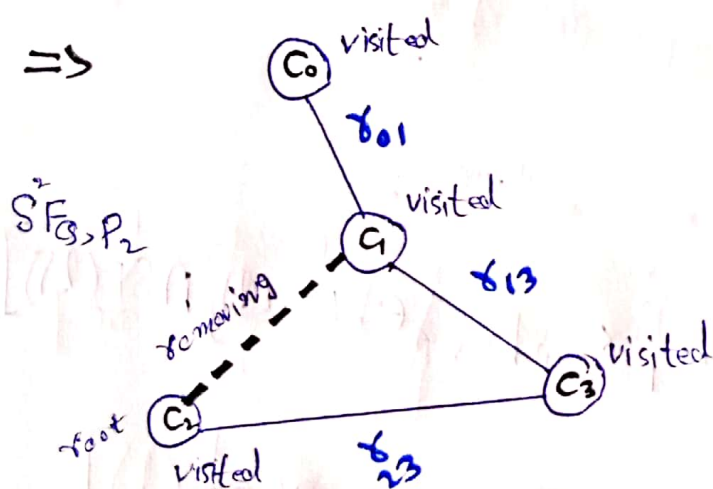
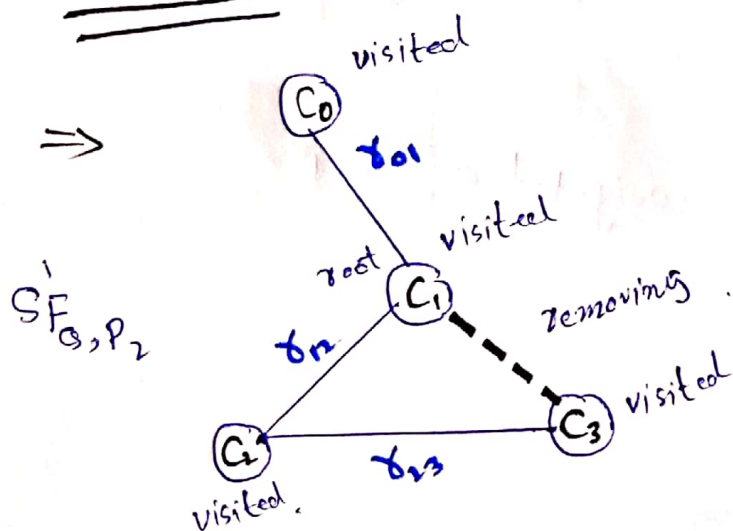
SF_{G, P_1}^3



\therefore Number of spanning forest = $\sigma_{G, P}$

$$\Rightarrow SF_{G, P_1}^1 = SF_{G, P_1}^2 = SF_{G, P_1}^3 = \frac{1}{\sigma_{G, P_1}} = \frac{1}{3}$$

Page Two



$\sigma_{G,P}$ = Number of spanning forest

$$\Rightarrow SF_{G,P_2}^1 = SF_{G,P_2}^2 = SF_{G,P_2}^3 = \frac{1}{\sigma_{G,P_2}} = \frac{1}{3}$$

Step#05:
Minimum Length of spanning forest.

Page one:

$l = \text{No. of Non-zero weighted edges} - 1$

$$l = 4 - 1 = 3$$

Page Two:

$$l = 3 - 1 = 2$$

Step#06:-

Page Relevance Score:-

$$P(\mathcal{G}, P, l) = P \left[\bigcup_{T=1}^{|\mathcal{SF}_{\mathcal{G}, P}(l)|} \left(\bigcap \left\{ x_{ij, p} \mid x_{ij, p} \in \mathcal{SF}_{\mathcal{G}, P}^T(l) \right\} \cap \mathcal{SF}_{\mathcal{G}, P}^T(l) \right) \right]$$

probability for node i to be in $\mathcal{SF}_{\mathcal{G}, P}(l)$ is:

$$\sum_{T=1}^{|\mathcal{SF}_{\mathcal{G}, P}(l)|} \prod_{x_{ij, p} \in \mathcal{SF}_{\mathcal{G}, P}^T(l)} P(\bar{x}_{ij, p}) \cdot P(\mathcal{SF}_{\mathcal{G}, P}^T(l))$$

Page One:

$$\begin{aligned} P(\mathcal{G}, P, 3) &= \left[P((x_{01} \wedge x_{12} \wedge x_{23}) \cap \mathcal{SF}_{\mathcal{G}, P}^1) \cup (P((x_{01} \wedge x_{13} \wedge x_{23}) \cap \mathcal{SF}_{\mathcal{G}, P}^2) \right. \\ &\quad \left. \cup (P((x_{01} \wedge x_{12} \wedge x_{13}) \cap \mathcal{SF}_{\mathcal{G}, P}^3)) \right] \\ &= \left[T_{01} \cdot T_{12} \cdot T_{23} \cdot \frac{1}{\sigma_{\mathcal{G}, P}} \right] + \left[T_{01} \cdot T_{13} \cdot T_{23} \cdot \frac{1}{\sigma_{\mathcal{G}, P}} \right] + \\ &\quad \left[T_{01} \cdot T_{13} \cdot T_{12} \cdot \frac{1}{\sigma_{\mathcal{G}, P}} \right] \end{aligned}$$

$$= \left[(\tau_{01} \cdot \tau_{12} \cdot \tau_{23}) + (\tau_{01} \cdot \tau_{13} \cdot \tau_{23}) + (\tau_{01} \cdot \tau_{13} \cdot \tau_{12}) \right] / \sigma_{\mathcal{G}, P_1}$$

$$= \left[\frac{1}{2} \times \frac{2}{3} \times \frac{1}{2} + \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} + \frac{1}{2} \times \frac{1}{2} \times \frac{2}{3} \right] / 3$$

$$= \left(\frac{1}{6} + \frac{1}{8} + \frac{1}{6} \right) / 3$$

$$P(\mathcal{G}, P_3) = 0.1527$$

Page Two:-

$$P(\mathcal{G}, P_2, 2) = \left[P((x_{12} \cap x_{23}) \cap \sigma_{\mathcal{G}, P_2}^1) \cup (P((x_{12} \cap x_{13}) \cap \sigma_{\mathcal{G}, P_2}^2) \cup (P((x_{23} \cap x_{13}) \cap \sigma_{\mathcal{G}, P_2}^3)) \right]$$

$$= \left[\tau_{12} \cdot \tau_{23} \cdot \frac{1}{\sigma_{\mathcal{G}, P_2}} \right] + \left[\tau_{12} \cdot \tau_{13} \cdot \frac{1}{\sigma_{\mathcal{G}, P_2}} \right] + \left[\tau_{23} \cdot \tau_{13} \cdot \frac{1}{\sigma_{\mathcal{G}, P_2}} \right]$$

$$= (\tau_{12} \cdot \tau_{23} + \tau_{12} \cdot \tau_{13} + \tau_{23} \cdot \tau_{13}) / \sigma_{\mathcal{G}, P_2}$$

$$= \left(\frac{1}{3} \times 1 + \frac{1}{3} \times \frac{1}{2} + 1 \times \frac{1}{2} \right) / 3$$

$$= \left(\frac{1}{3} + \frac{1}{6} + \frac{1}{2} \right) / 3$$

$$= 0.3333$$

Step#07

Final page Relevance Scores-

$$PS_{q,p} = P(q,p, \text{Max}(l)) + \text{max}(l) \mid P(q,p,l) \neq 0$$

Page One :-

$$PS_{q,p_1} = 0.1527 + 3$$

$$= 3.1527$$

$$, \in [l, l+1]$$

→ satisfied.

Page Two:-

$$PS_{q,p_2} = 0.3333 + 2$$

$$= 2.3333$$

$$, \in [l, l+1]$$

→ satisfied

Step#08:-

Ranking:-

Arranging page relevance score in decreasing order

$$\Rightarrow \begin{array}{l} p_1 \quad 3.1527 \\ p_2 \quad 2.3333 \end{array} \downarrow$$

Hence page one will be shown before the page two.