Tides II

How is tidal deformation linked to the interior structure?

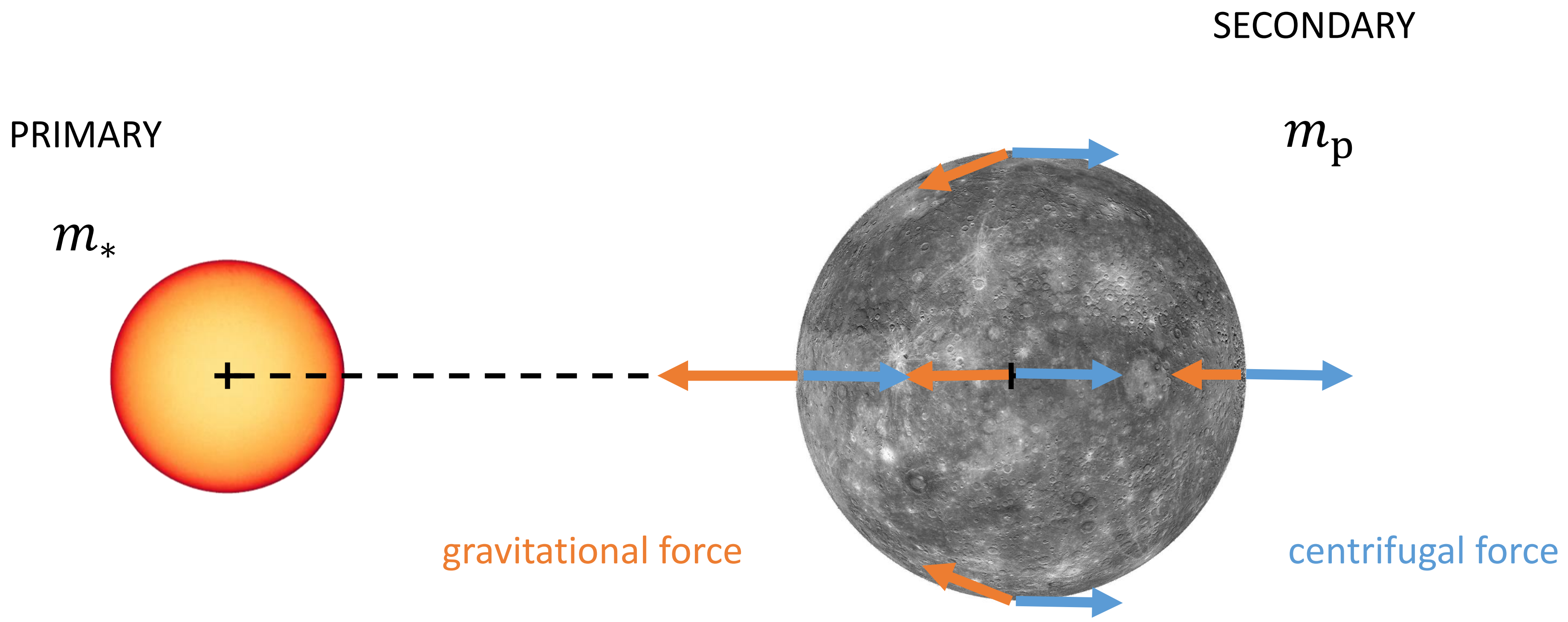
What are tidal Love numbers?

How do tides affect planetary rotation and orbital evolution?

And what about tidal heating?

What are tides?

Consequence of a differential gravitational field acting on an extended body

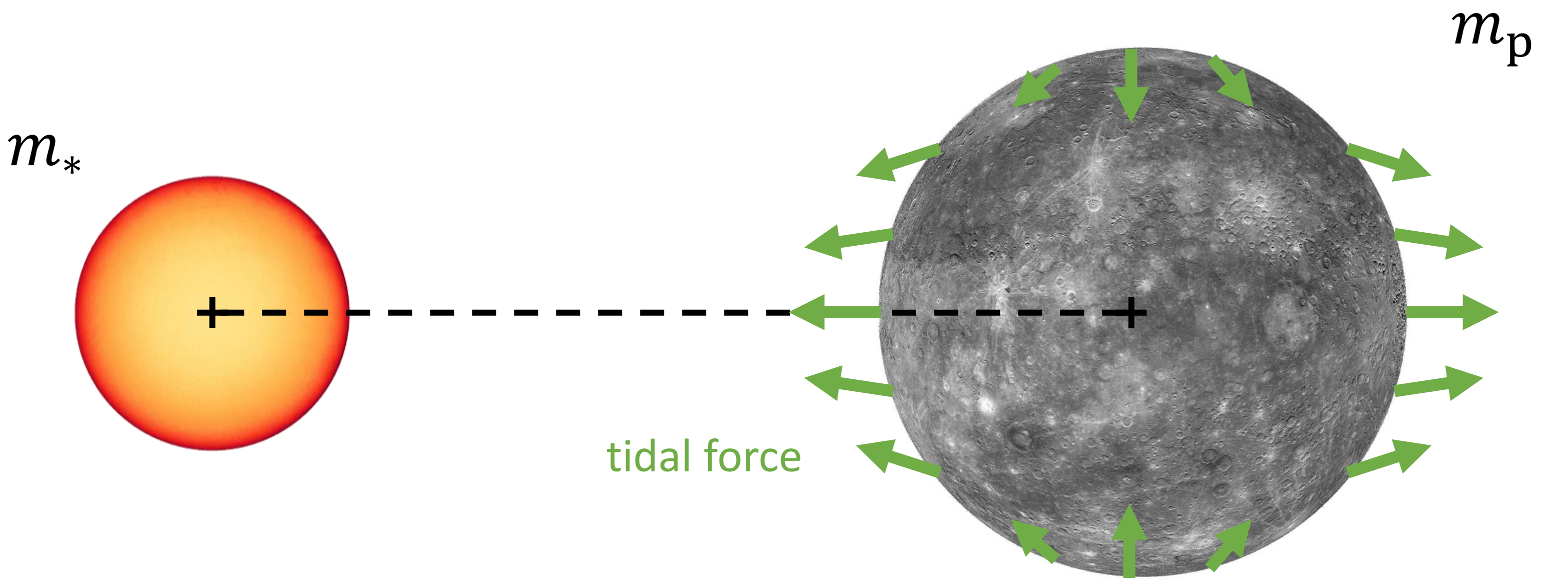


Reference frame coorbiting with SECONDARY around the PRIMARY

Image credits: Solar Orbiter/PHI Team/ESA & NASA

What are tides?

Consequence of a differential gravitational field acting on an extended body



Introduction of the tidal potential: $F_t = \nabla \mathcal{U}$

Image credits: Solar Orbiter/PHI Team/ESA & NASA

What are tides?

Consequence of a differential gravitational field acting on an extended body

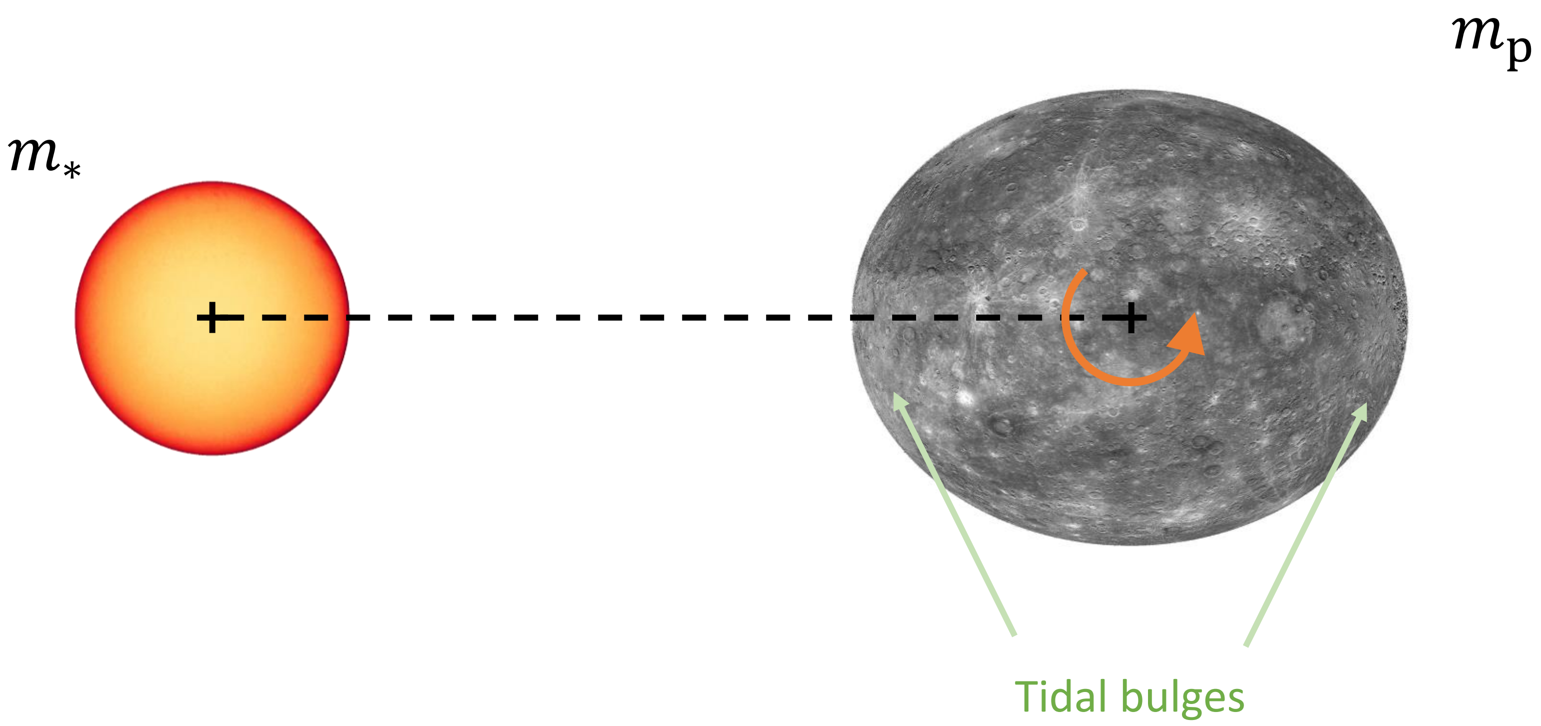
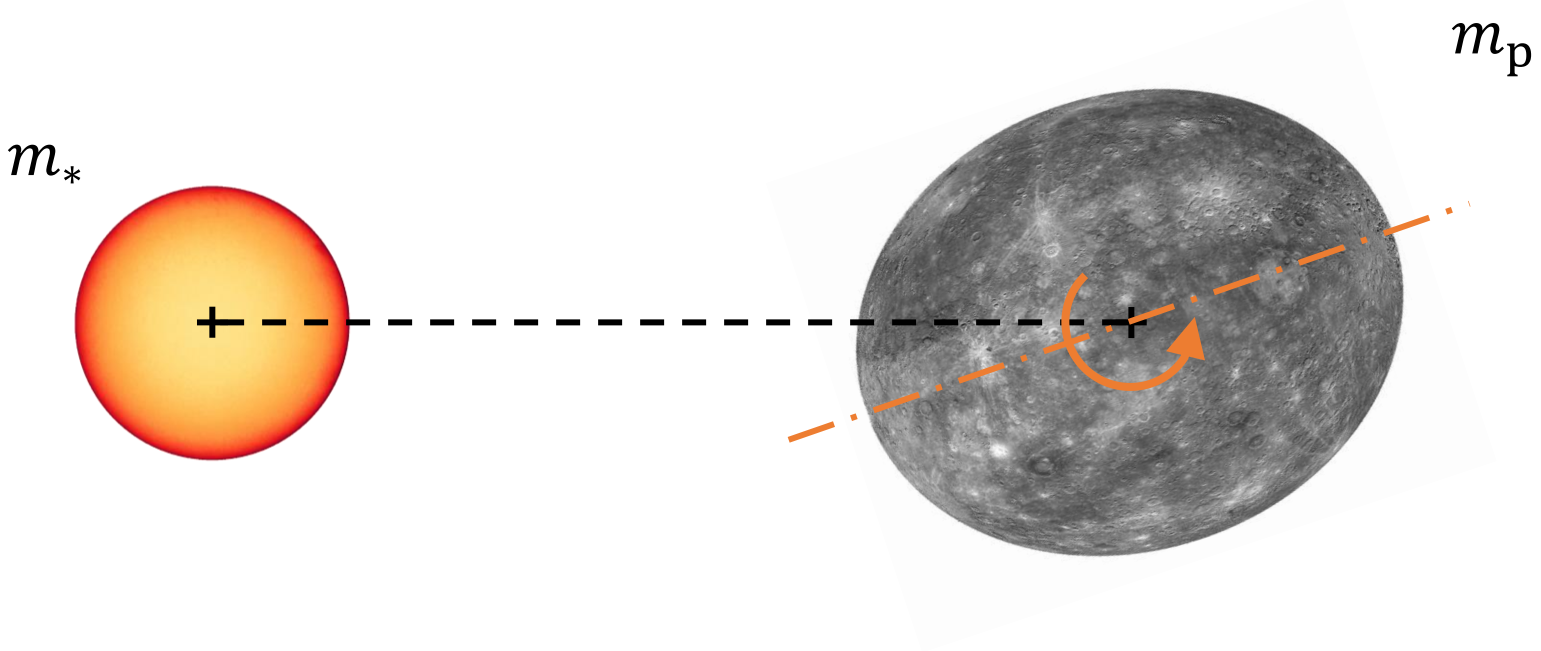


Image credits: Solar Orbiter/PHI Team/ESA & NASA

Tidal lagging

Celestial bodies need time to attain their equilibrium shape – the symmetry axis of the deformed body does NOT generally point toward the perturber.

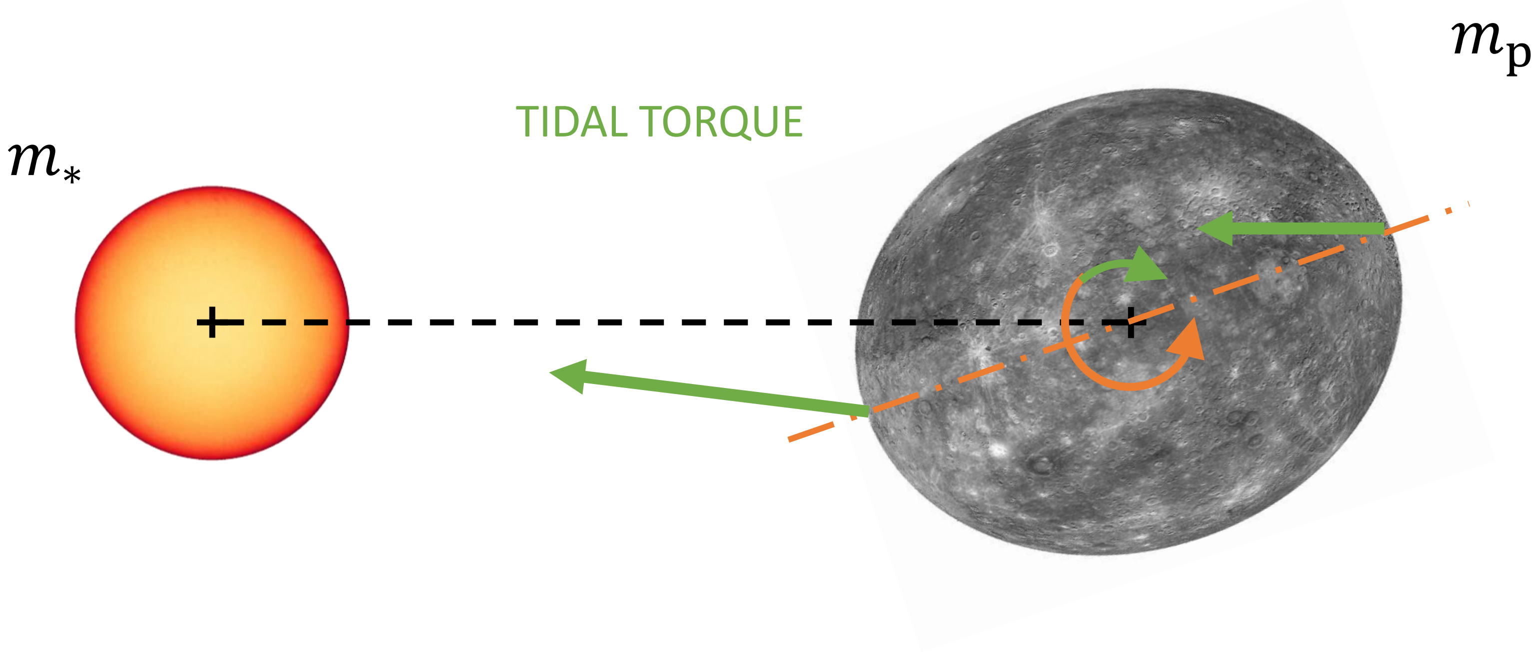


internal friction (heating) → deformation is not instantaneous → tidal lagging

Image credits: Solar Orbiter/PHI Team/ESA & NASA

Tidal torque

Primary (=star) acting on the tidal bulge of the deformed secondary (=planet) → force couple → torque



tidal torque is responsible for rotational and orbital evolution

Image credits: Solar Orbiter/PHI Team/ESA & NASA

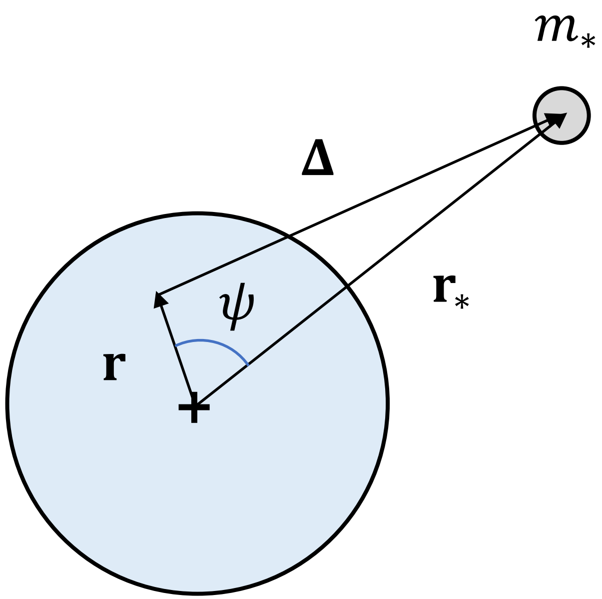
Tidal potential

Coordinates of the observer: r, ϑ, φ

Coordinates of the perturber: $r_*, \vartheta_*, \varphi_*$

Legendre polynomials

$$\mathcal{U}(r,\vartheta,\varphi) = \frac{\mathcal{G}m_*}{\Delta} = \frac{\mathcal{G}m_*}{r_*}\sum_{l=2}^{\infty}\left(\frac{r}{r_*}\right)^l \mathcal{P}_l(\cos\psi)$$



Addition theorem for Legendre polynomials:

associated Legendre polynomials

$$\mathcal{P}_l(\cos\psi) = \sum_{m=0}^{l}\frac{(l-m)!}{(l+m)!}(2-\delta_{0m})\,\mathcal{P}_{lm}(\cos\vartheta)\,\mathcal{P}_{lm}(\cos\vartheta_*)\,\cos m(\varphi-\varphi_*)$$

Rewrite the immediate position of the perturber in terms of Keplerian elements and time (Darwin-Kaula expansion)

Darwin-Kaula expansion

George H. Darwin (1880), William M. Kaula (1961, 1964)

$$\mathcal{U}(r,\vartheta,\varphi) = \frac{\mathcal{G}m_*}{\Delta} = \frac{\mathcal{G}m_*}{r_*}\sum_{l=2}^{\infty}\left(\frac{r}{r_*}\right)^l \sum_{m=0}^{l}\frac{(l-m)!}{(l+m)!}(2-\delta_{0m})\,\mathcal{P}_{lm}(\cos\vartheta)\,\mathcal{P}_{lm}(\cos\vartheta_*)\;\cos m(\varphi-\varphi_*)$$

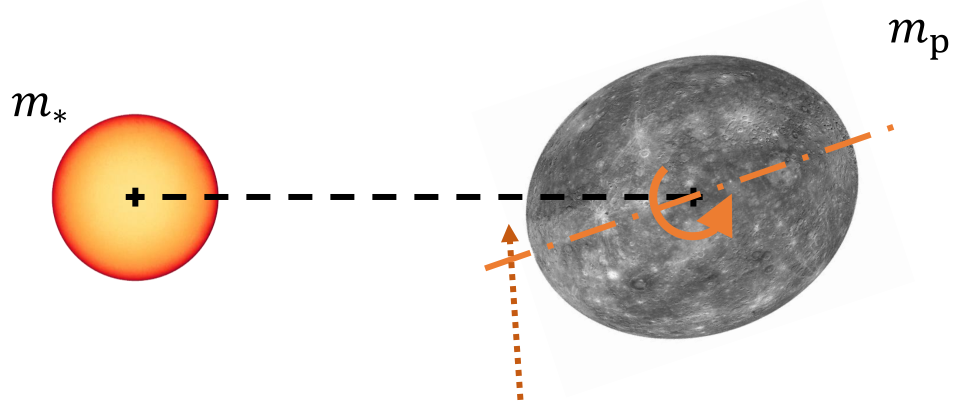
inclination functions and eccentricity functions

$$\mathcal{U}(r,\vartheta,\varphi) = \frac{\mathcal{G}m_*}{a}\sum_{l=2}^{\infty}\left(\frac{r}{a}\right)^l \sum_{m=0}^{l}\frac{(l-m)!}{(l+m)!}(2-\delta_{0m})\,\mathcal{P}_{lm}(\cos\vartheta)\sum_{p=0}^{l}\sum_{q=-\infty}^{\infty}\mathcal{F}_{lmp}(i)\,\mathcal{G}_{lpq}(e)$$

$$\times\left[\cos m\varphi\begin{Bmatrix}\cos\\ \sin\end{Bmatrix}_{l-m\text{ odd}}^{l-m\text{ even}}\{(l-2p)\omega+(l-2p+q)\mathcal{M}+m(\Omega-\theta)\}\right.$$

$$\left.+\;\sin m\varphi\begin{Bmatrix}\sin\\ -\cos\end{Bmatrix}_{l-m\text{ odd}}^{l-m\text{ even}}\{(l-2p)\omega+(l-2p+q)\mathcal{M}+m(\Omega-\theta)\}\right]$$

Darwin-Kaula expansion



Tidal potential:

$$\mathcal{U}(r,\vartheta,\varphi) = \sum_{l,m,p,q} \mathcal{U}_{lmpq}\,(r,\vartheta,\varphi)$$

{*lmpq*} = tidal modes = individual tidal waves

Additional potential due to tidal deformation:

planet radius

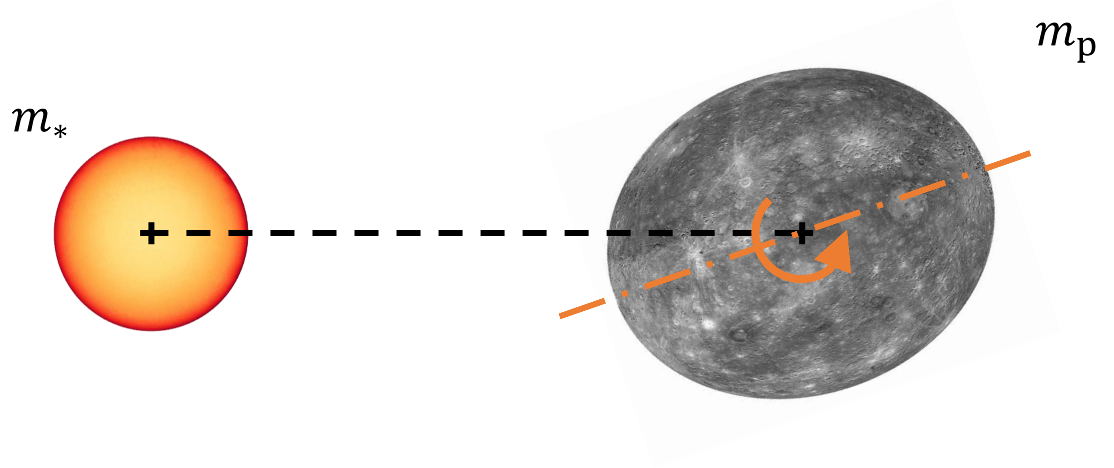
$$\delta\mathcal{U}(r,\vartheta,\varphi) = \sum_{l,m,p,q} \delta\mathcal{U}_{lmpq}\,(r,\vartheta,\varphi) \qquad \text{where} \qquad \delta\mathcal{U}_{lmpq} = \left(\frac{R}{r}\right)^{l+1} k_l \left[\mathcal{U}_{lmpq}\right]_{\mathrm{lag}}$$

Linear tidal theory (small deformations, laterally homogeneous medium)

= each mode of tidal potential gives rise to a corresponding mode of the additional potential

Shida-Love numbers

Link between the interior structure and tidal deformation



Radial deformations: $\frac{h_l U_l}{g}$

Lateral deformations: $\frac{l_l U_l}{g}$

Additional potential: $k_l U_l$

How much does a planet deform under tidal loading?

Tidal deformation can be measured: variable term in the gravity field $\rightarrow k_l$, variable shape $\rightarrow h_l, l_l$

Shida-Love numbers of a homogeneous sphere

From now on, we will only consider degree-2 deformation:

deformed planet = triaxial spheroid

$$k_2 = \frac{3}{2} \frac{1}{1 + \frac{19}{2} \frac{\mu}{\rho g R}}$$

POTENTIAL

$$h_2 = \frac{5}{2} \frac{1}{1 + \frac{19}{2} \frac{\mu}{\rho g R}}$$

RADIAL

$$l_2 = \frac{3}{4} \frac{1}{1 + \frac{19}{2} \frac{\mu}{\rho g R}}$$

LATERAL

ρ … mean density
g … surface gravity

R … radius
μ … rigidity

For planets with a nonhomogeneous interior structure, tidal Love numbers cannot be written analytically → numerical calculation

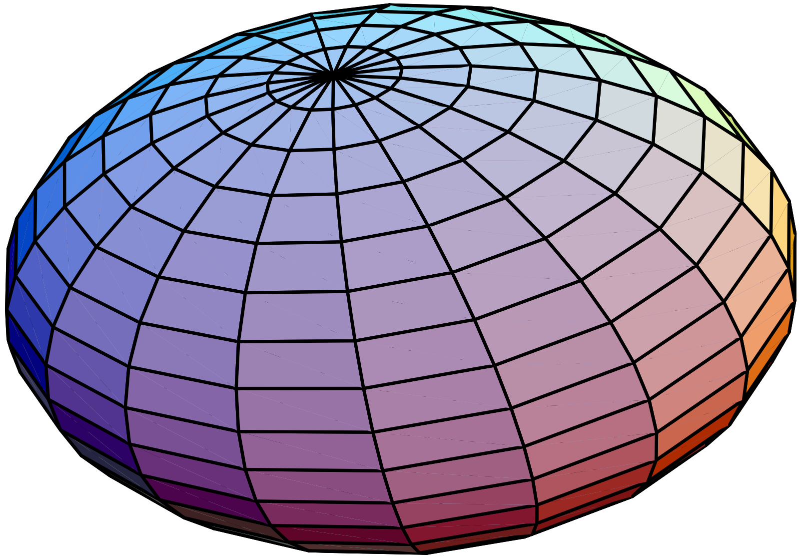


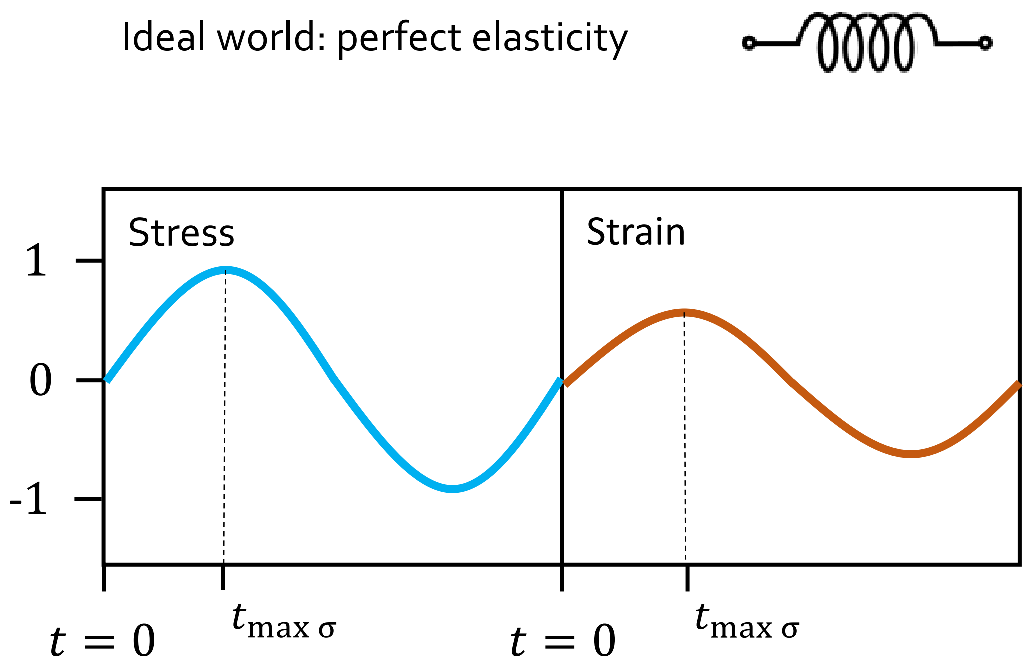
Image credits: Wolfram MathWorld

Tidal lagging

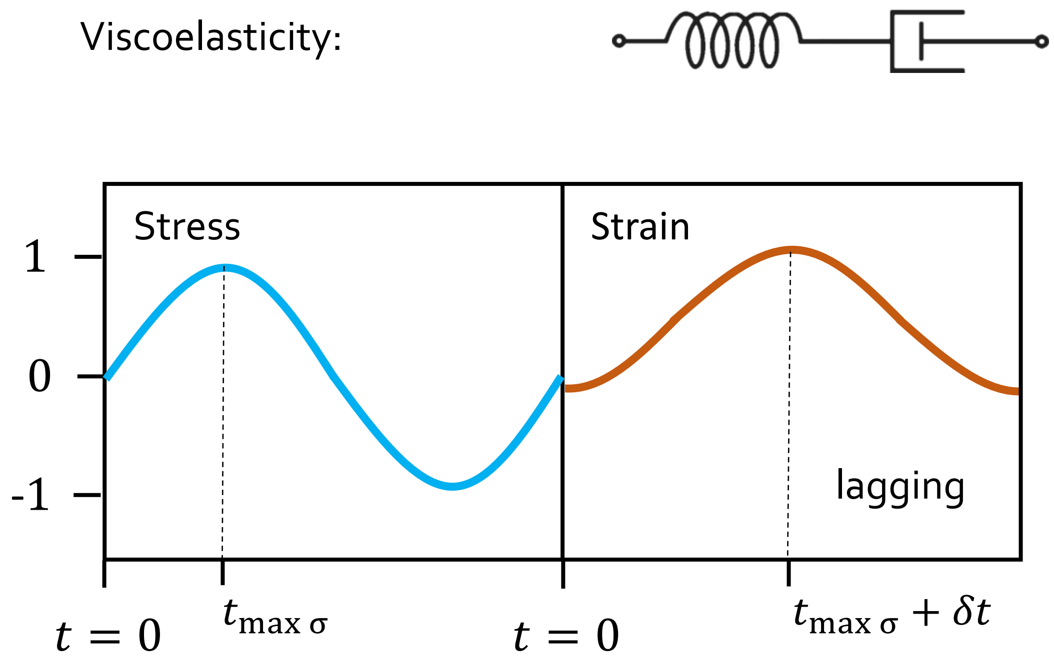
In elastic medium, the application of an external force is followed by an immediate deformation

In viscoelastic medium (planetary materials), deformation is lagging behind the external force

Ideal world: perfect elasticity



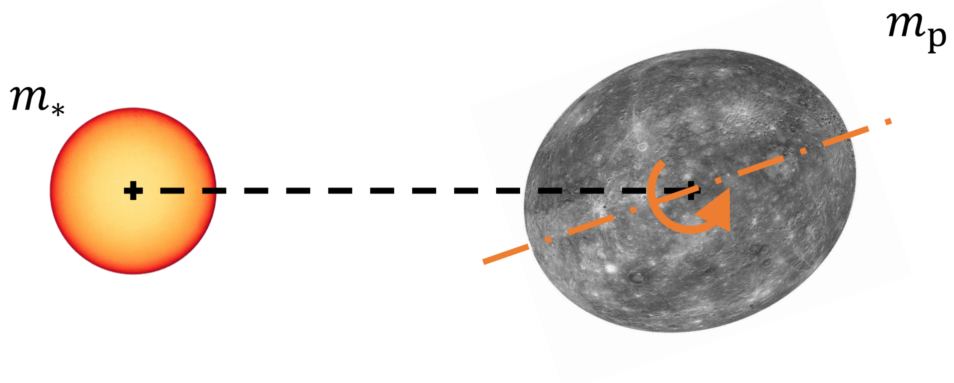
Viscoelasticity:



Tidal lagging

In elastic medium, the application of an external force is followed by an immediate deformation
In viscoelastic medium (planetary materials), deformation is lagging behind the external force

lagging = energy loss



In addition to the Love number k_2, let us define tidal phase lag ε_2

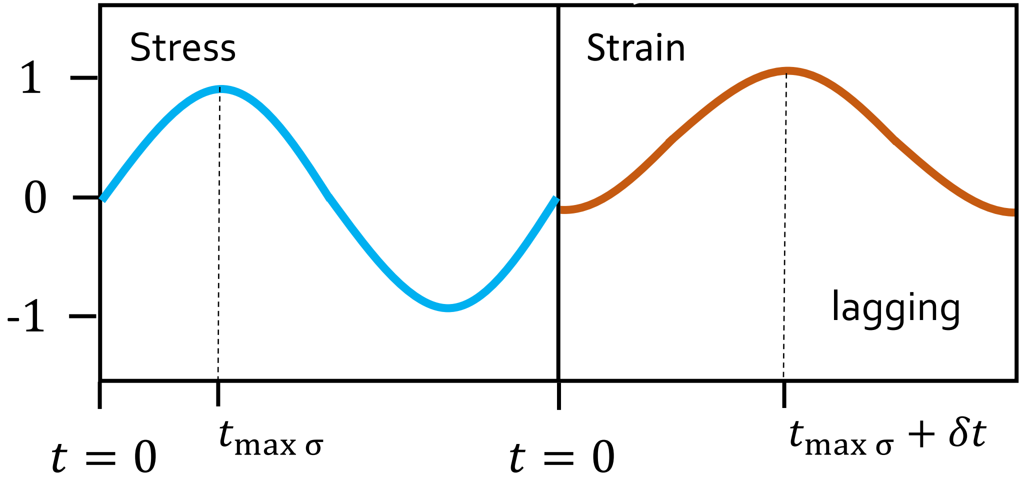
Energy loss is also parameterised by tidal quality factor Q

energy dissipated during a loading cycle

$$Q^{-1}(\chi) = \sin|\varepsilon_2| = \frac{\Delta E(\chi)}{2\pi E_{\mathrm{peak}}(\chi)}$$

peak energy stored in the body

during a loading cycle



Potential Love numbers (k_2) of celestial bodies

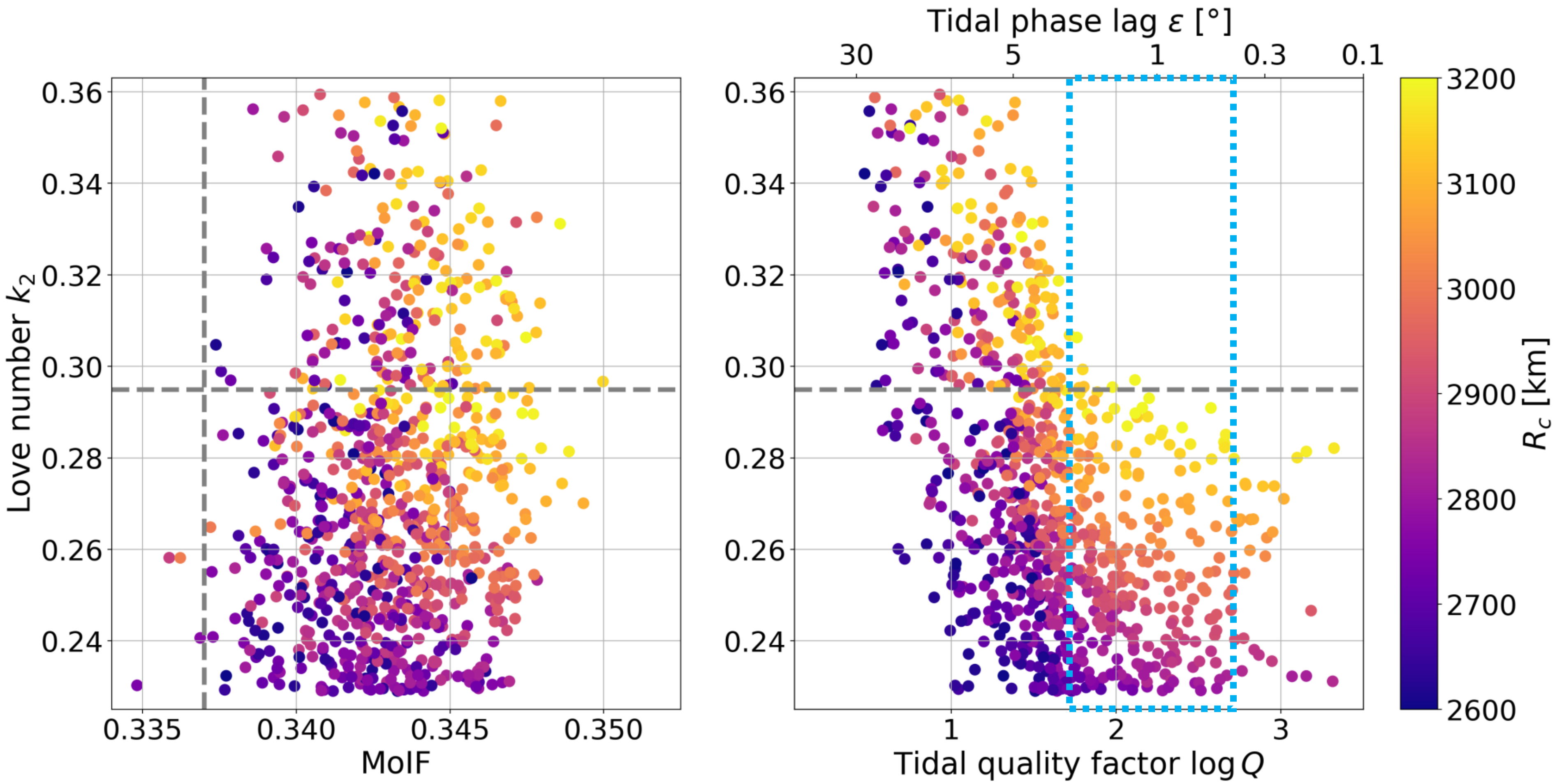
Object	Love number k2	Quality factor Q
Solid Earth	~ 0.353	~ 280
Venus	~ 0.295	unknown
Mars	~ 0.174	~ 100
Mercury	~ 0.569	unknown
Moon	~ 0.024	~ 40
Jupiter	~ 0.565	$10^5 - 10^6$
stars	< 0.1	$10^5 - 10^6$

Love numbers probe the elastic structure of planets – sensitive to core size and state

Quality factors reated to the amount of energy dissipation – sensitive to interior temperature

$$k_2 = \frac{3}{2} \frac{1}{1 + \frac{19}{2} \frac{\rho g R}{\mu}}$$

Tidal Love number and the core size (of Venus)



Typically: bigger core → larger k_2

Tidal effects: probes to the deep interior

Potential tidal Love number k_2 can be determined by measuring the perturbations to a satellite's orbit

Additional potential due to tidal deformation = disturbing function in **Lagrange's planetary equations**!

$$\frac{\mathrm{d}a}{\mathrm{d}t} = \frac{2}{na}\frac{\partial \mathcal{R}}{\partial \epsilon} ,$$

$$\frac{\mathrm{d}e}{\mathrm{d}t} = -\frac{\sqrt{1-e^2}}{na^2 e}(1-\sqrt{1-e^2})\frac{\partial \mathcal{R}}{\partial \epsilon} - \frac{\sqrt{1-e^2}}{na^2 e}\frac{\partial \mathcal{R}}{\partial \varpi} ,$$

$$\frac{\mathrm{d}i}{\mathrm{d}t} = -\frac{\tan\frac{i}{2}}{na^2\sqrt{1-e^2}}\left(\frac{\partial \mathcal{R}}{\partial \epsilon} + \frac{\partial \mathcal{R}}{\partial \varpi}\right) - \frac{1}{na^2\sqrt{1-e^2}\sin i}\frac{\partial \mathcal{R}}{\partial \Omega} ,$$

$$\frac{\mathrm{d}\Omega}{\mathrm{d}t} = \frac{1}{na^2\sqrt{1-e^2}\sin i}\frac{\partial \mathcal{R}}{\partial i} ,$$

$$\frac{\mathrm{d}\varpi}{\mathrm{d}t} = \frac{\sqrt{1-e^2}}{na^2 e}\frac{\partial \mathcal{R}}{\partial e} + \frac{\tan\frac{i}{2}}{na^2\sqrt{1-e^2}}\frac{\partial \mathcal{R}}{\partial i} ,$$

$$\frac{\mathrm{d}\epsilon}{\mathrm{d}t} = -\frac{2}{na}\frac{\partial \mathcal{R}}{\partial a} - \frac{1-e^2}{na^2 e}(1-\sqrt{1-e^2})\frac{\partial \mathcal{R}}{\partial e} + \frac{\tan\frac{i}{2}}{na^2\sqrt{1-e^2}}\frac{\partial \mathcal{R}}{\partial i} .$$

Tidal disturbing function (for dissipation in a single body):

$$\boxed{\mathcal{R} = \sum_{lmpq} \delta\mathcal{U}_{lmpq}\,(a, e, i, \Omega, \varpi, \epsilon)}$$

Evolution equations = sum of multiple modes

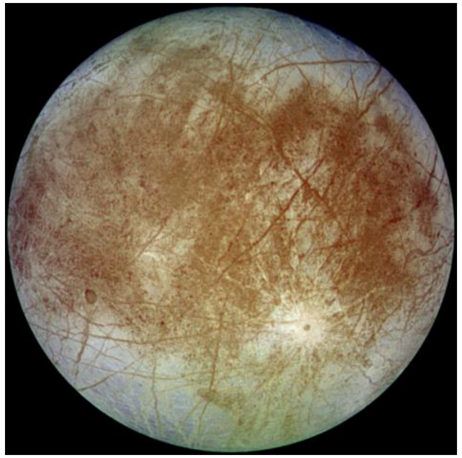
Tidal effects: probes to the deep interior

Tidal interaction contributes to orbital precession, evolution of semi-major axis, eccentricity, and inclination

Angular momentum is conserved: orbital evolution goes hand in hand with spin evolution of both bodies

Energy in not conserved: tidal loading results in tidal dissipation/heating

Changing stress field in a moon or planet – triggering of earthquakes (Moon), timing of geysers (Enceladus)…



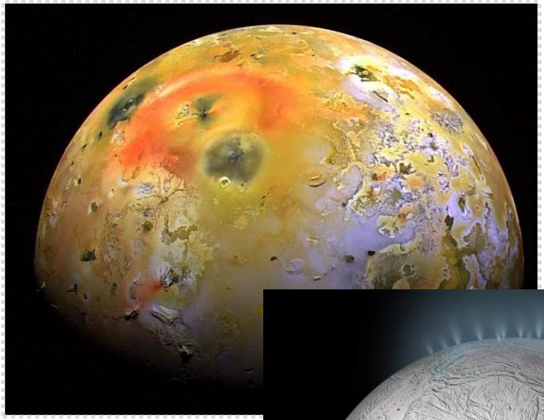
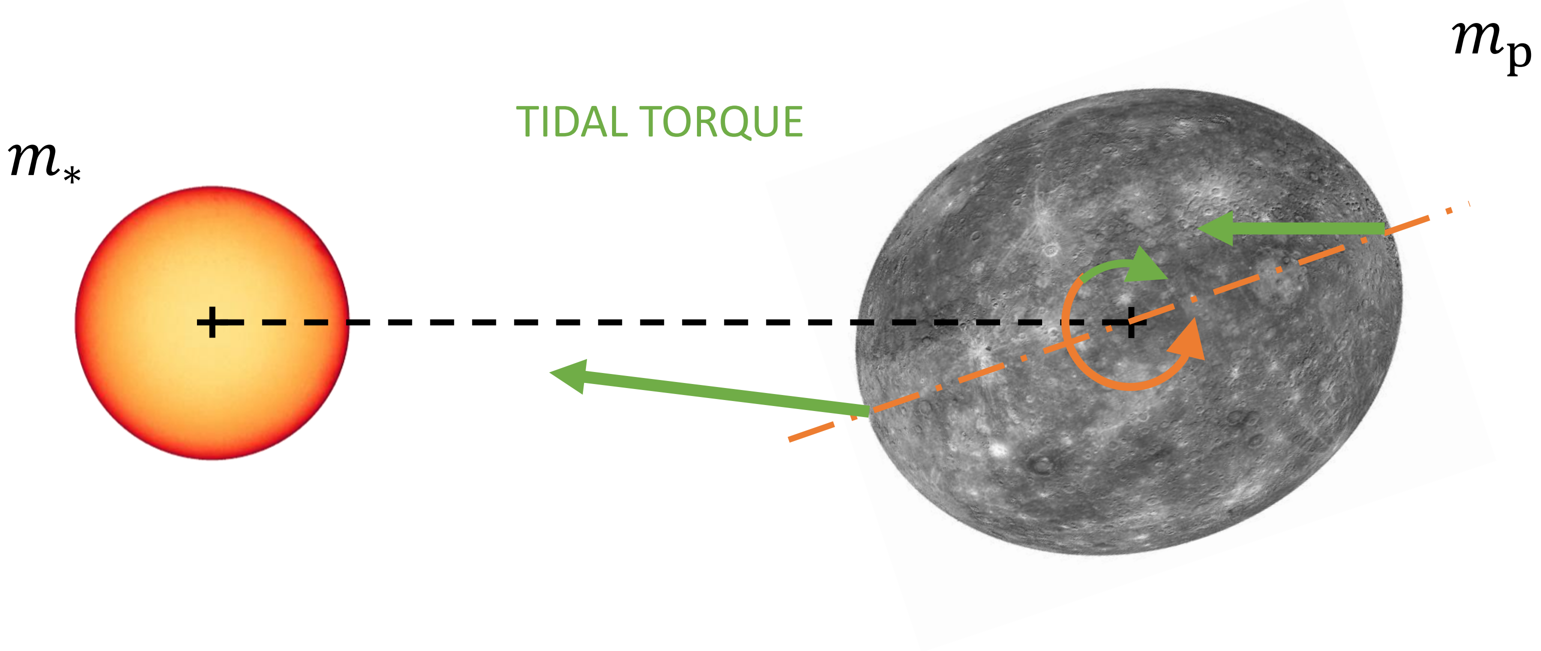


Image credits: NASA/JPL-Caltech/University of Arizona/ASU/DLR

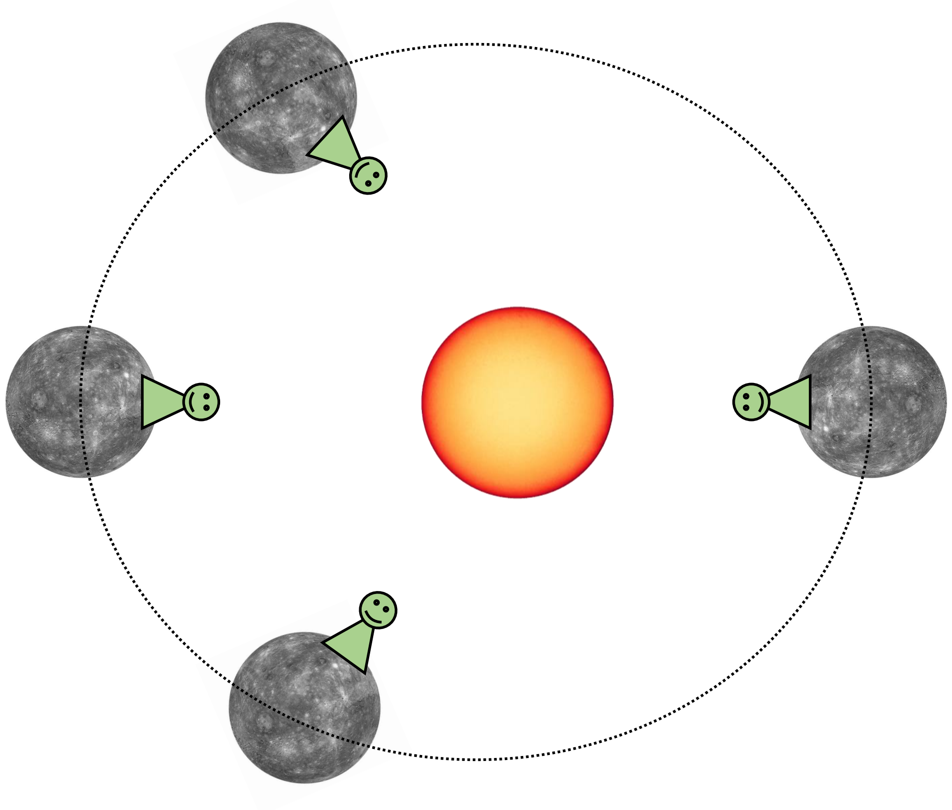
Tidal torque

Primary (=star) acting on the tidal bulge of the deformed secondary (=planet) → force couple → torque



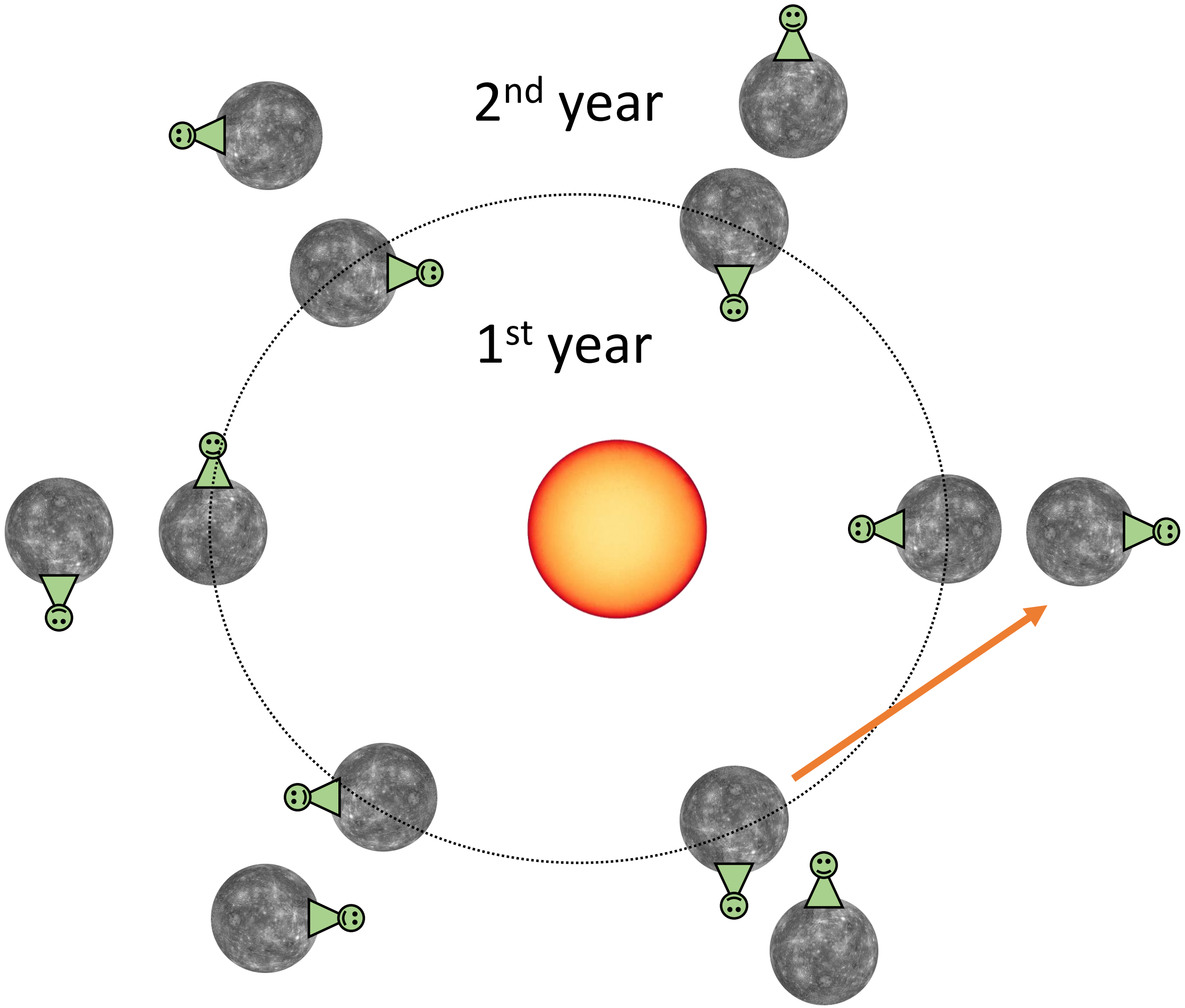
tidal torque is responsible for rotational (and orbital) evolution → spin-orbit coupling

Spin-orbit resonances



Synchronous rotation = **1:1 spin-orbit resonance** – rotation frequency equals mean orbital frequency (mean motion)

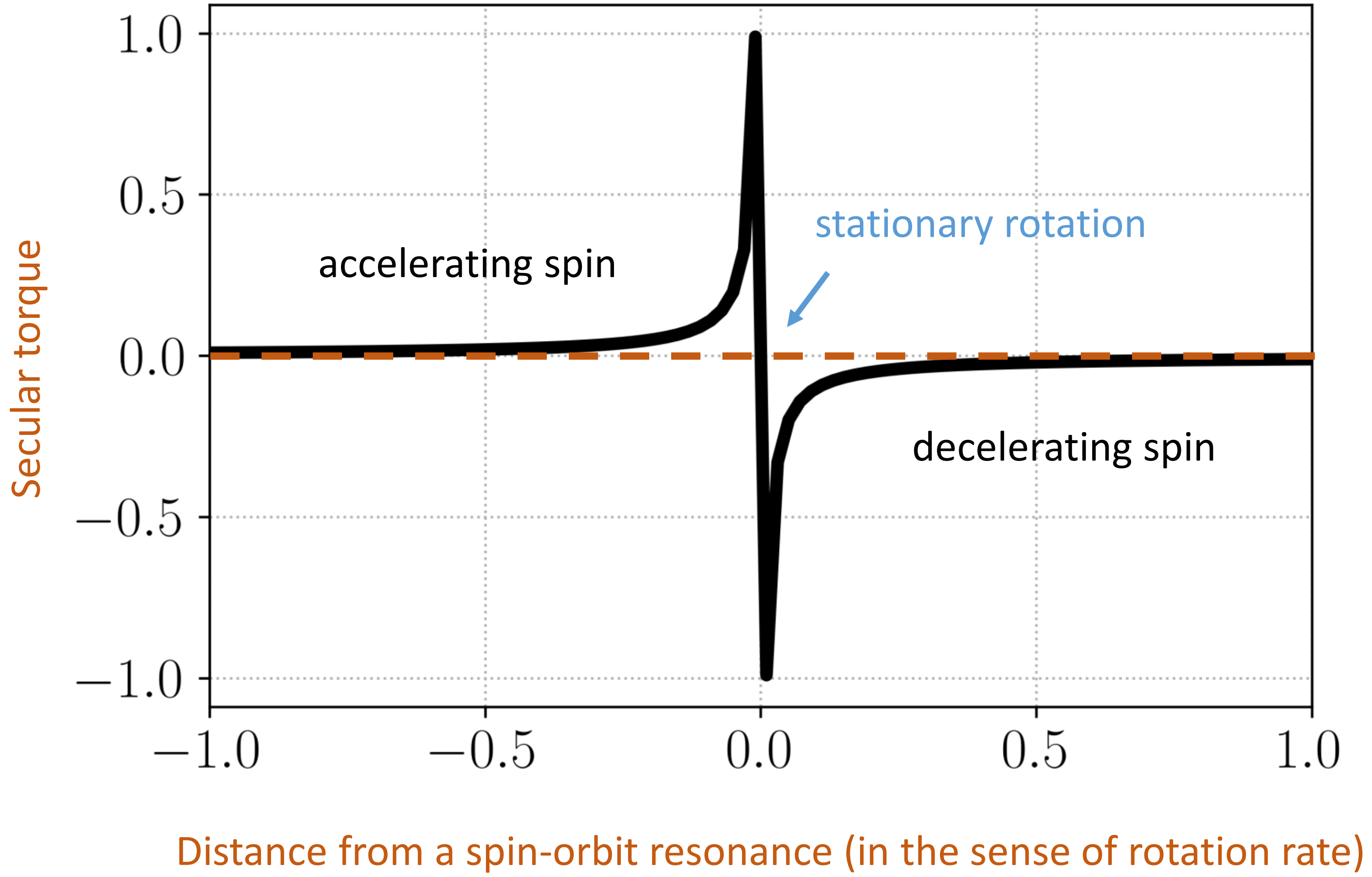
Spin-orbit resonances



3:2 spin-orbit resonance – rotation frequency equals 3/2-times the mean orbital frequency (mean motion)

Secular tidal torque

= tidal torque averaged over one orbital period



Secular tidal torque and stable spin states

a) Circular orbit, zero obliquity

→ only synchronous rotation stable

b) Eccentric orbit, zero obliquity

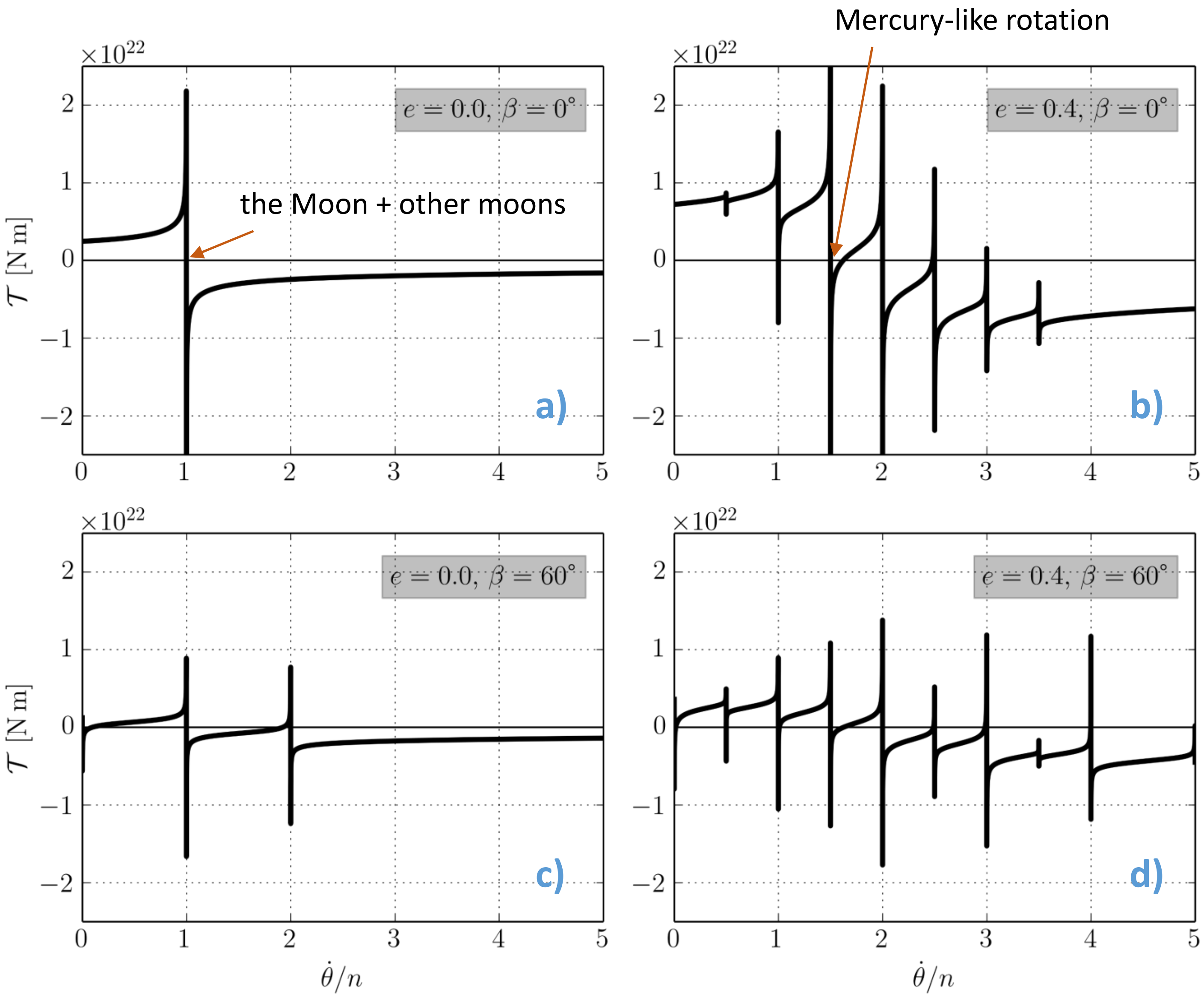
→ multiple spin-orbit resonances

c) Circular orbit, nonzero obliquity

→ only integer resonances stable

d) Eccentric orbit, nonzero obliquity

→ many resonances of varying stability

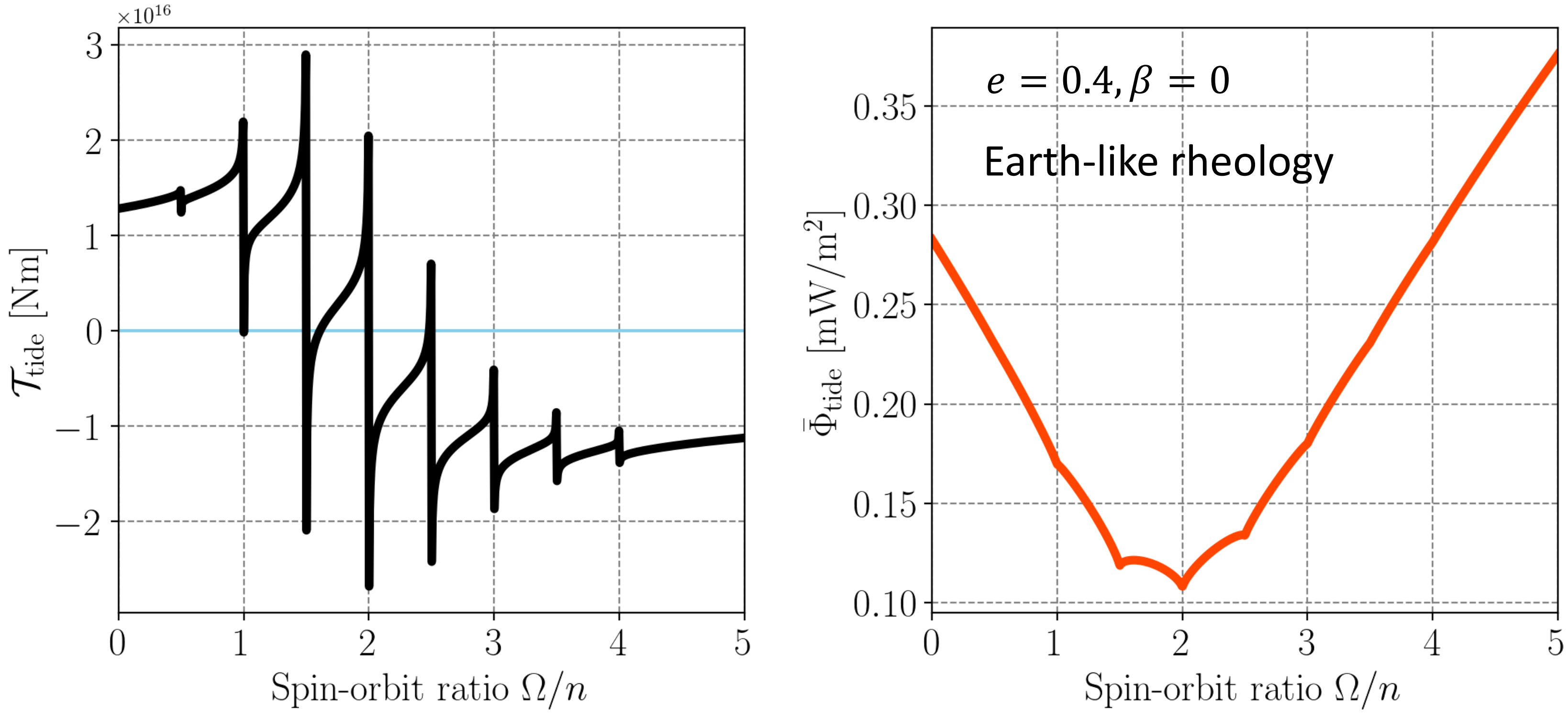


Tidal locking + tidal heating

Surface tidal heat flux

$$\Phi_{\text{tide}} = \frac{\bar{P}^{\text{tide}}}{4\pi R^2}$$

The most stable equilibrium rotation rate also minimizes tidal heating!



Tidal heating

Average tidal heating: *(Efroimsky and Makarov, 2014)*

eccentricity and obliquity

interior structure

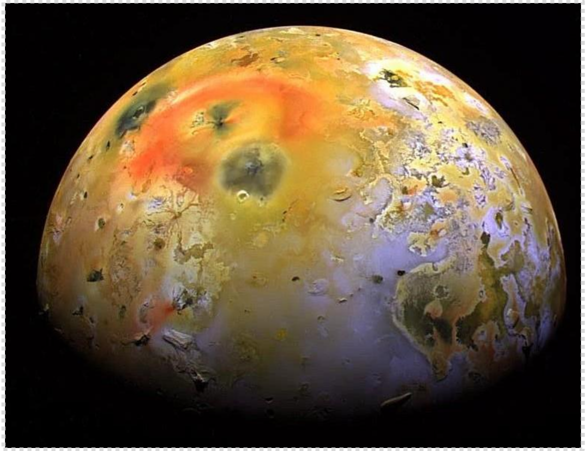
$$\bar{P}^{\text{tide}} = -\frac{\mathcal{G}m_*^2}{a}\sum_{lmpq}\left(\frac{R}{a}\right)^{2l+1}(2-\delta_{m0})\,\frac{(l-m)!}{(l+m)!}\left[\mathcal{G}_{lpq}(e)\right]^2\left[\mathcal{F}_{lmp}(\beta)\right]^2\omega_{lmpq}\,\mathrm{Im}\{\bar{k}_l(\omega_{lmpq})\}$$

distance from the star + size of planet (and star)

$$\mathrm{Im}\{\bar{k}_l(\omega_{lmpq})\} \equiv -\frac{k_l}{Q}$$

Special case: synchronous rotation + slightly eccentric orbit (e.g., Galilean satellites)

$$\boxed{\bar{P}^{\text{tide}} = -\frac{21}{2}\,\mathcal{G}m_*^2\,\frac{R^5}{a^6}e^2 n\;\mathrm{Im}\{k_2(n)\}}$$



The curious case of Io

Surface tidal heat flux $$\Phi_{\text{tide}} = \frac{\bar{P}^{\text{tide}}}{4\pi R^2}$$

$$\bar{P}^{\text{tide}} = -\frac{21}{2}\,\mathcal{G}m_*^2\,\frac{R^5}{a^6}\,e^2 n\;\operatorname{Im}\{k_2(n)\}$$

$a = 421\,700$ km

$e = 0.0041$

$n = 4.11 \times 10^{-5}$ rad/s

$R = 1822$ km

$m_* = 1.8982 \times 10^{27}$ kg

$\bar{P}^{\text{tide}} = -6.23 \times 10^{15}\,\operatorname{Im}\{k_2(n)\}$ W

Let us also assume that $\operatorname{Im}\{k_2(n)\} = -k_2/Q$ is Earth-like:

$k_2 \approx 0.3 \qquad Q \approx 100$

$\bar{P}^{\text{tide}} = 18.6$ TW => $\Phi_{\text{tide}} = 0.45\ \text{W/m}^2$

The total surface heat flux on the Earth (due to all sources) is $0.089\ \text{W/m}^2$

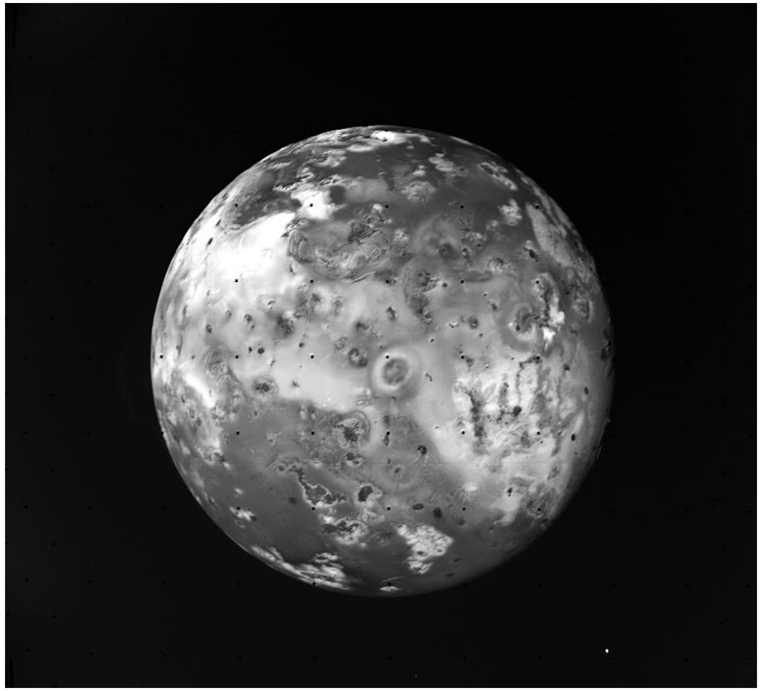
The curious case of Io

Peale et al. (Science, 1979), published on 2 March 1979:

Melting of Io by Tidal Dissipation

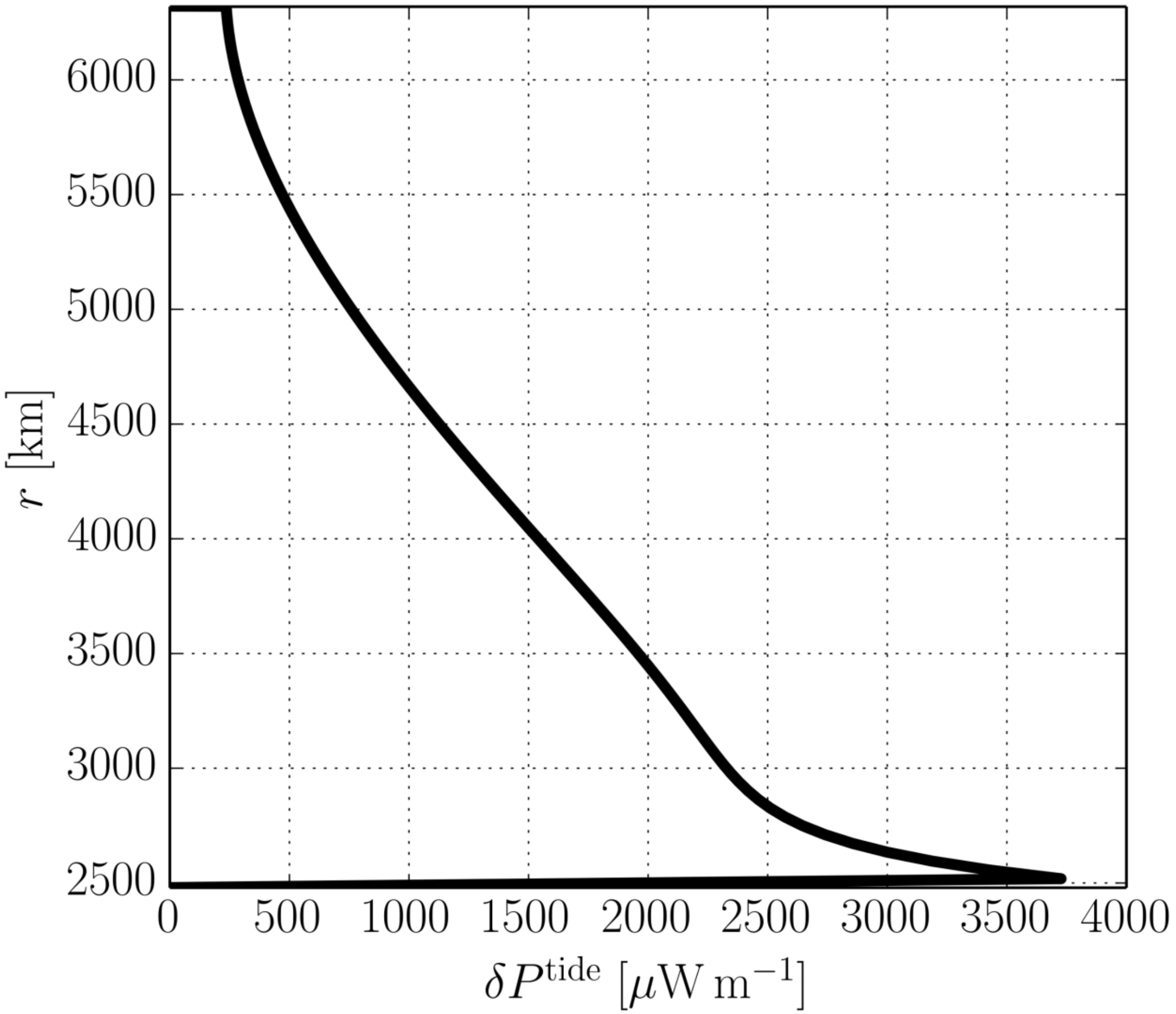
Abstract. *The dissipation of tidal energy in Jupiter's satellite Io is likely to have melted a major fraction of the mass. Consequences of a largely molten interior may be evident in pictures of Io's surface returned by Voyager I.*

Pictures from Voyager I, available on 4 March 1979:

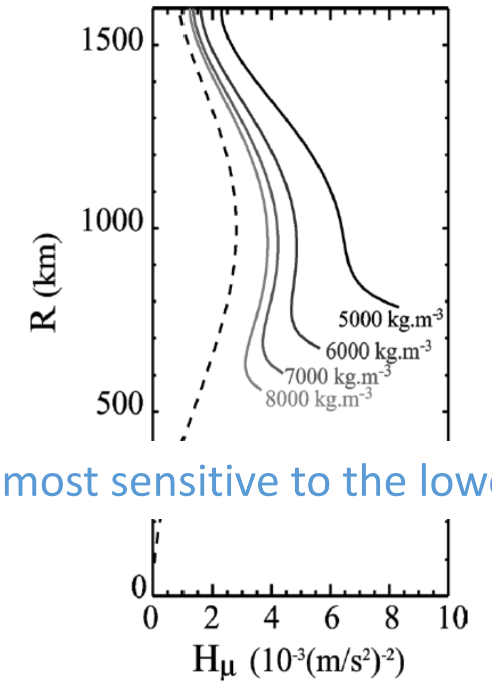


Io is a volcanic moon!

Depth-dependence of tidal heating



Most energy is released just above the liquid core (if there is any)

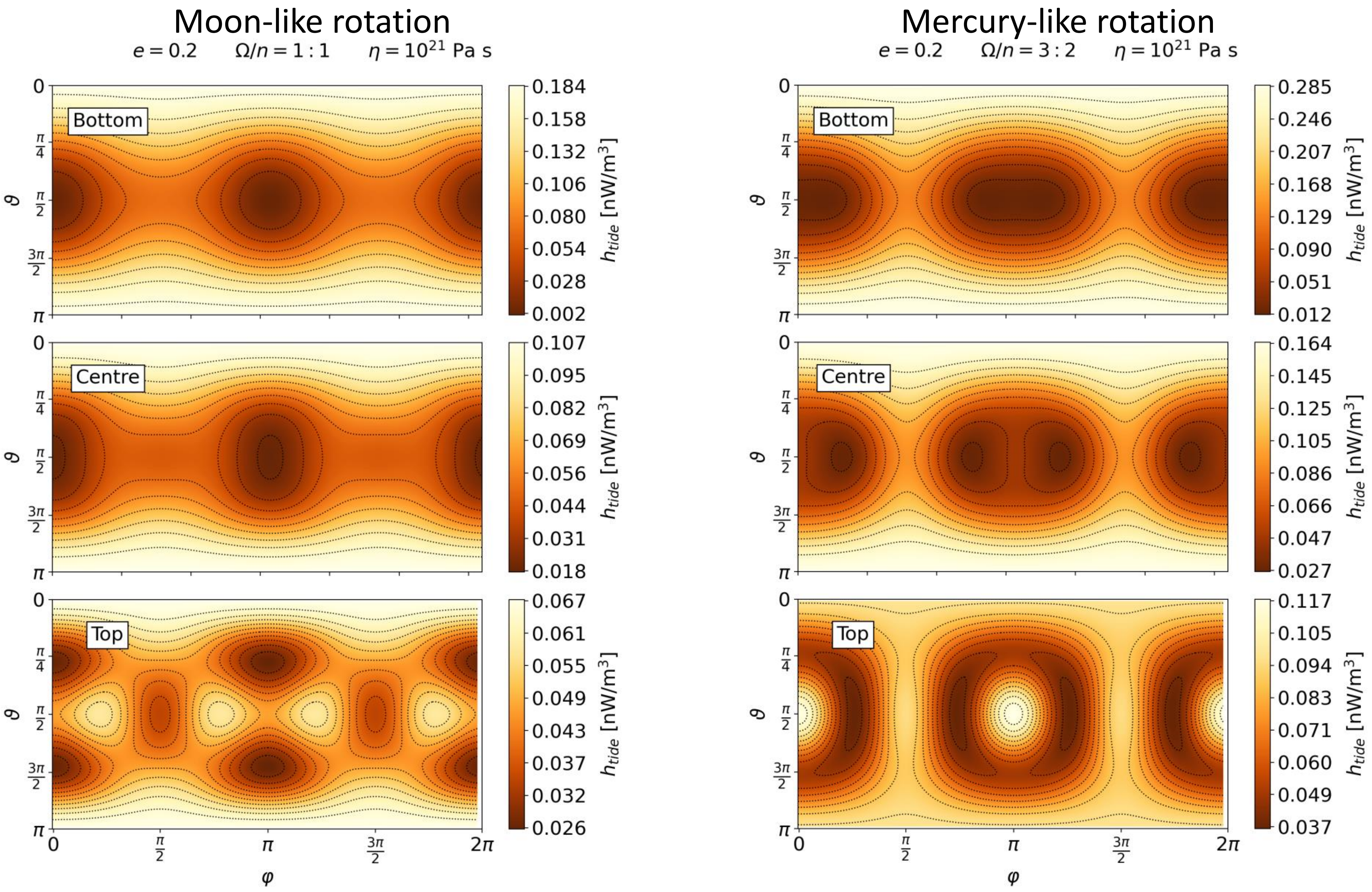


Tidal dissipation is most sensitive to the lower mantle!

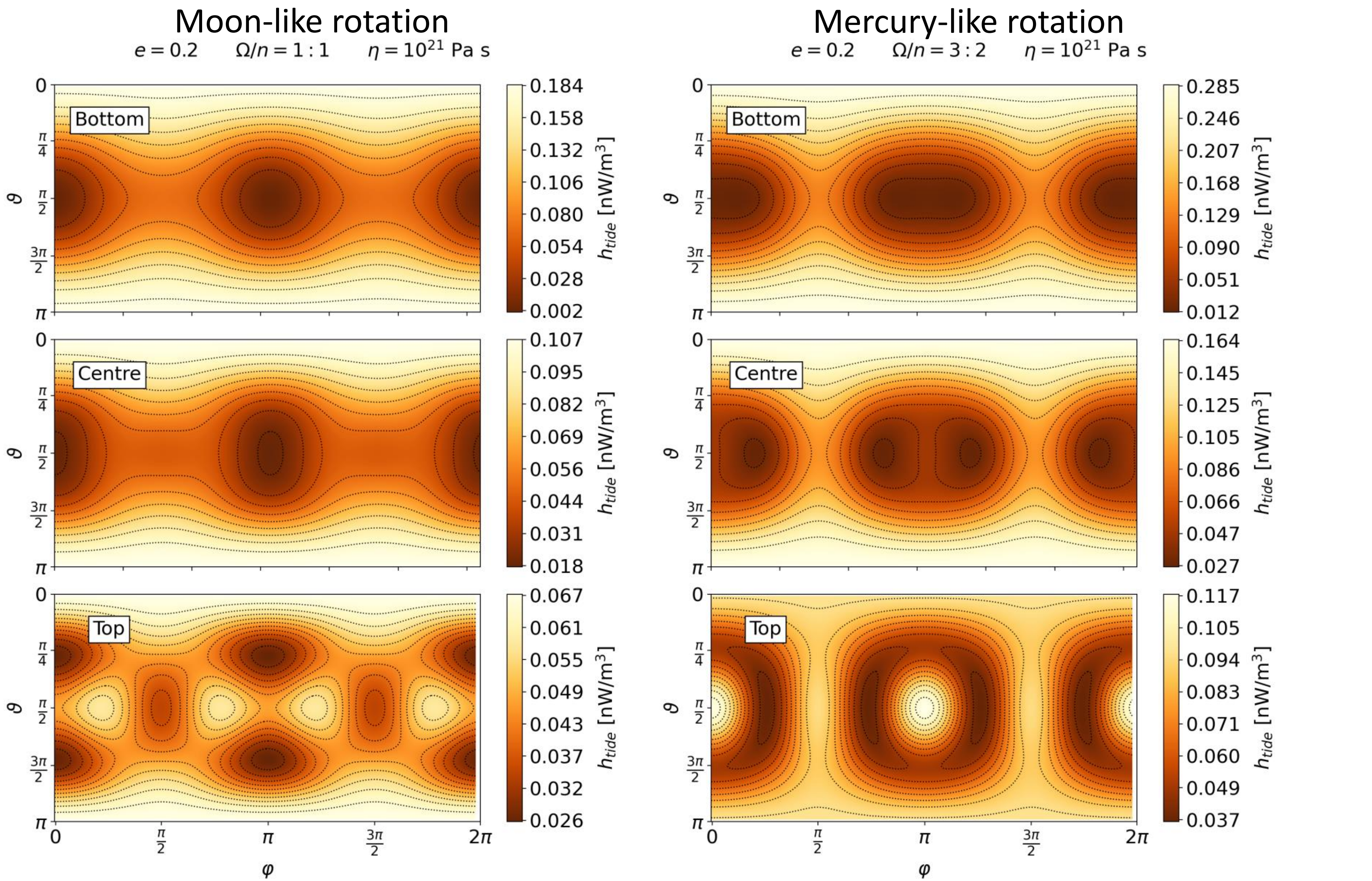
Fig. 3. Sensitivity parameter H_μ within the silicate mantle for different values of the core density (solid curves) and within an homogeneous undifferentiated interior (dashed curve).

Tobie et al. (Icarus, 2005)

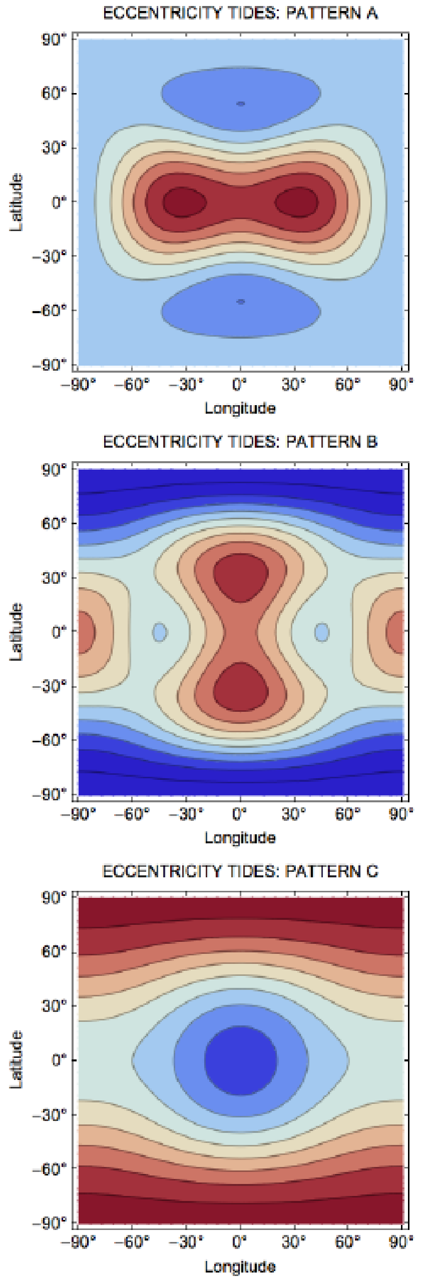
3d pattern of tidal heating



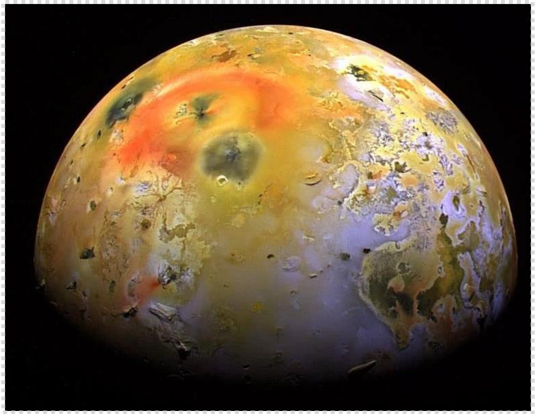
3d pattern of tidal heating



Beuthe (Icarus, 2013):

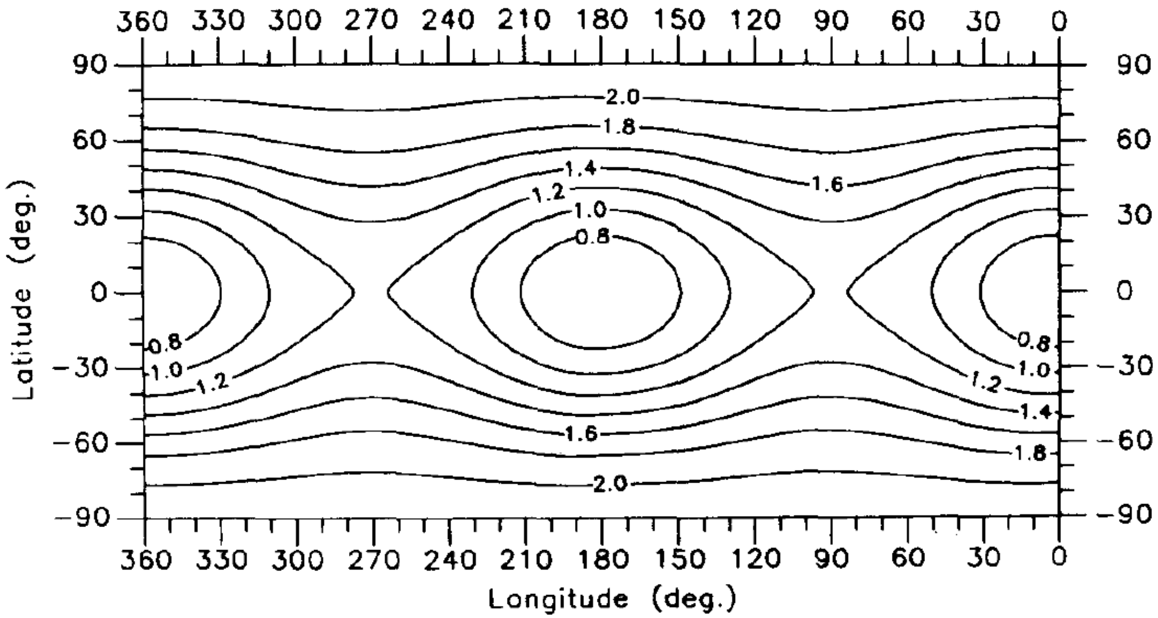


Tidal heat flux at the surface

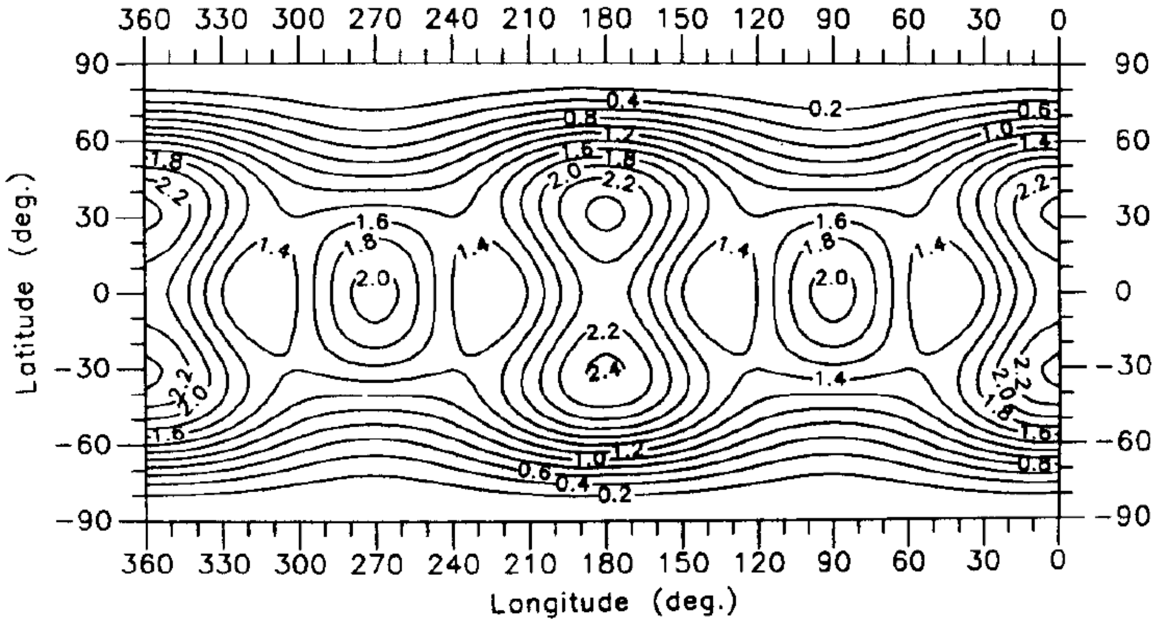


Homogeneous body: maximum heating on poles

Subsurface ocean/asthenosphere: maximum heating on equator



homogeneous Io

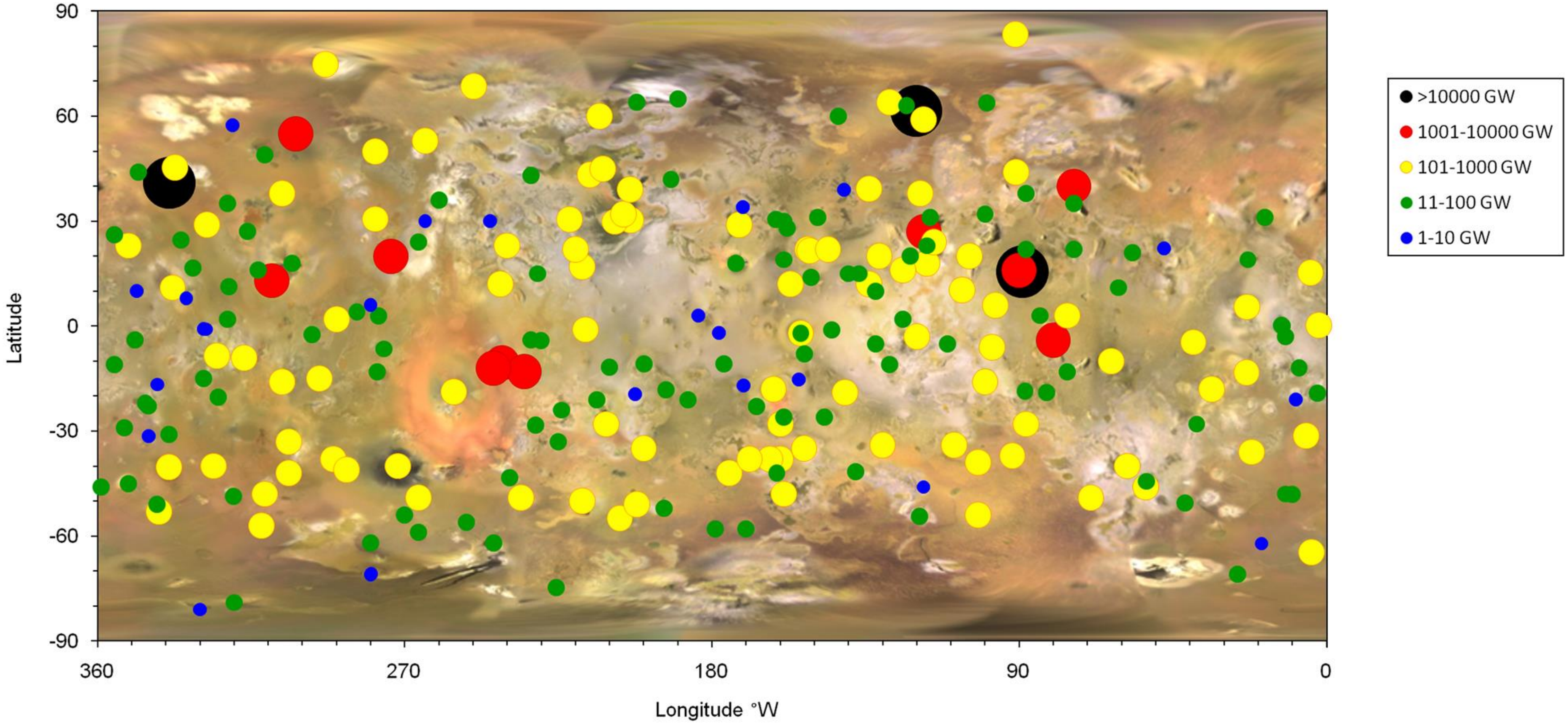


Io with a partially molten subsurface

Segatz et al. (Icarus, 1988)

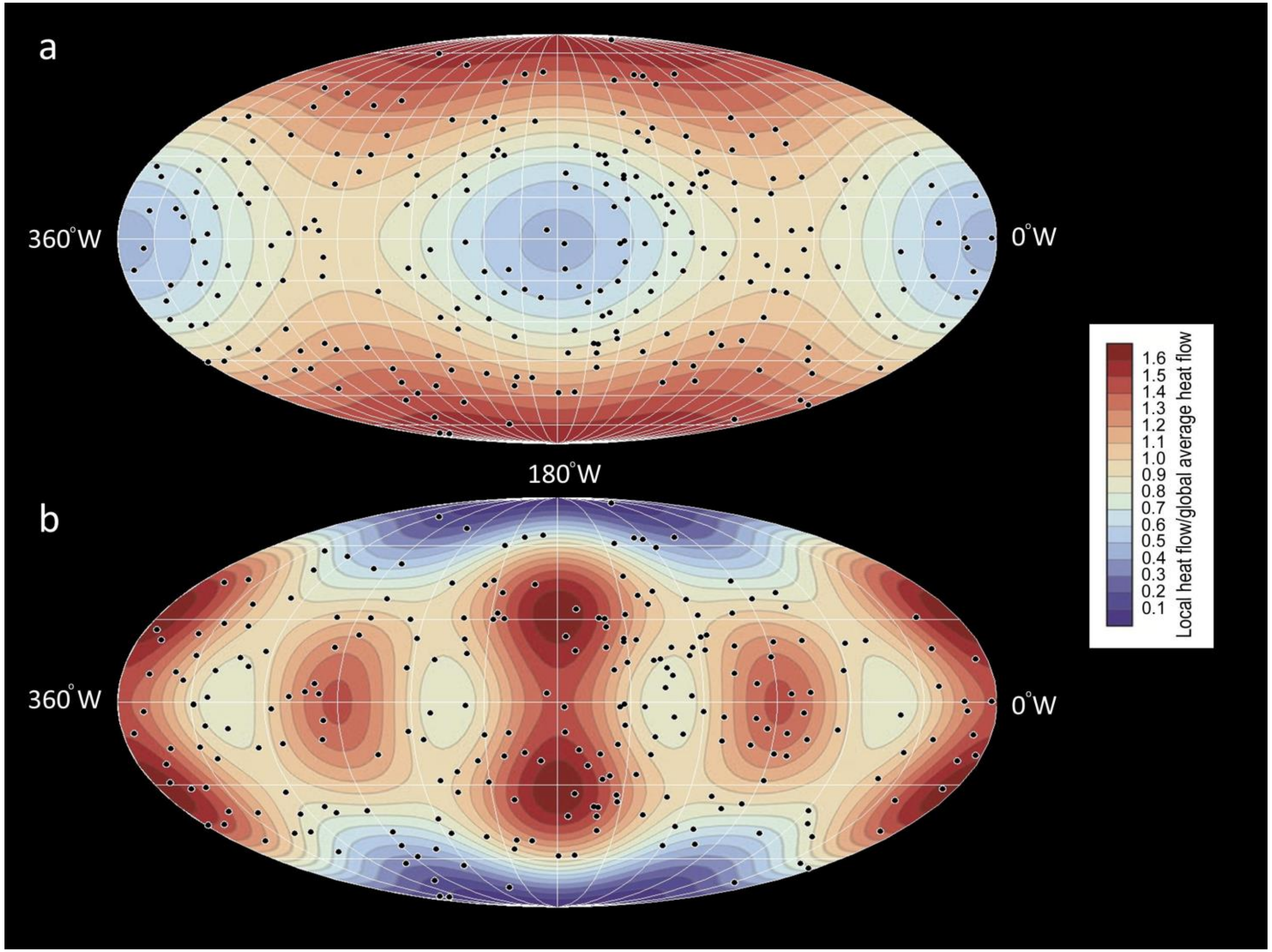
Volcanoes on Io

Is the heat flow pattern (~positions of volcanoes) consistent with any of the models of surface heat flow pattern?



Davies et al. (Icarus, 2015)

Volcanoes on Io



Davies et al. (Icarus, 2015)

The observed positions of volcanoes are **not** correlated with any of the models of tidal heat flow pattern

→ more detailed models of Io's interior needed

Tidal heating pattern is still a valuable constraint on the interior structure!

Tidally-induced orbital evolution

$\mathrm{Im}\{\bar{k}_l(\omega_{lmpq})\} \equiv -\frac{k_l}{Q}$

Special case: synchronous rotation + eccentric orbit (e.g., Galilean satellites)

SATELLITE (subscript #)

HOST PLANET (subscript *)

$$\dot{a} = 57\frac{\mathcal{G}m_* R_\#^5}{na^7} e^2\,\mathrm{Im}\{\bar{k}_{2,\#}(n)\} + \frac{9}{4}\frac{\mathcal{G}m_\# R_*^5}{na^7} e^2\,\mathrm{Im}\{\bar{k}_{2,*}(n)\} + \text{[tidal waves on a quickly rotating primary]}$$

$$\dot{e} = \frac{21}{2}\frac{\mathcal{G}m_* R_\#^5}{na^8} e\,\mathrm{Im}\{\bar{k}_{2,\#}(n)\} + \frac{9}{8}\frac{\mathcal{G}m_\# R_*^5}{na^8} e\,\mathrm{Im}\{\bar{k}_{2,*}(n)\} + \text{[tidal waves on a quickly rotating primary]}$$

< 0

< 0 for $n > \dot{\theta}$

> 0 for $n < \dot{\theta}$

Relatively large satellites orbiting on tight orbits around massive planets are subjected to fast tidal evolution

Tidally-induced orbital evolution

SATELLITE

HOST PLANET

$$\dot{a} = 57\frac{\mathcal{G}m_* R_\#^5}{na^7} e^2 \,\mathrm{Im}\{\bar{k}_{2,\#}(n)\} + \frac{9}{4}\frac{\mathcal{G}m_\# R_*^5}{na^7} e^2 \,\mathrm{Im}\{\bar{k}_{2,*}(n)\} + \text{[tidal waves on a quickly rotating primary]}$$

$$\dot{e} = \frac{21}{2}\frac{\mathcal{G}m_* R_\#^5}{na^8} e \,\mathrm{Im}\{\bar{k}_{2,\#}(n)\} + \frac{9}{8}\frac{\mathcal{G}m_\# R_*^5}{na^8} e \,\mathrm{Im}\{\bar{k}_{2,*}(n)\} + \text{[tidal waves on a quickly rotating primary]}$$

< 0

< 0 for $n > \dot{\theta}$

> 0 for $n < \dot{\theta}$

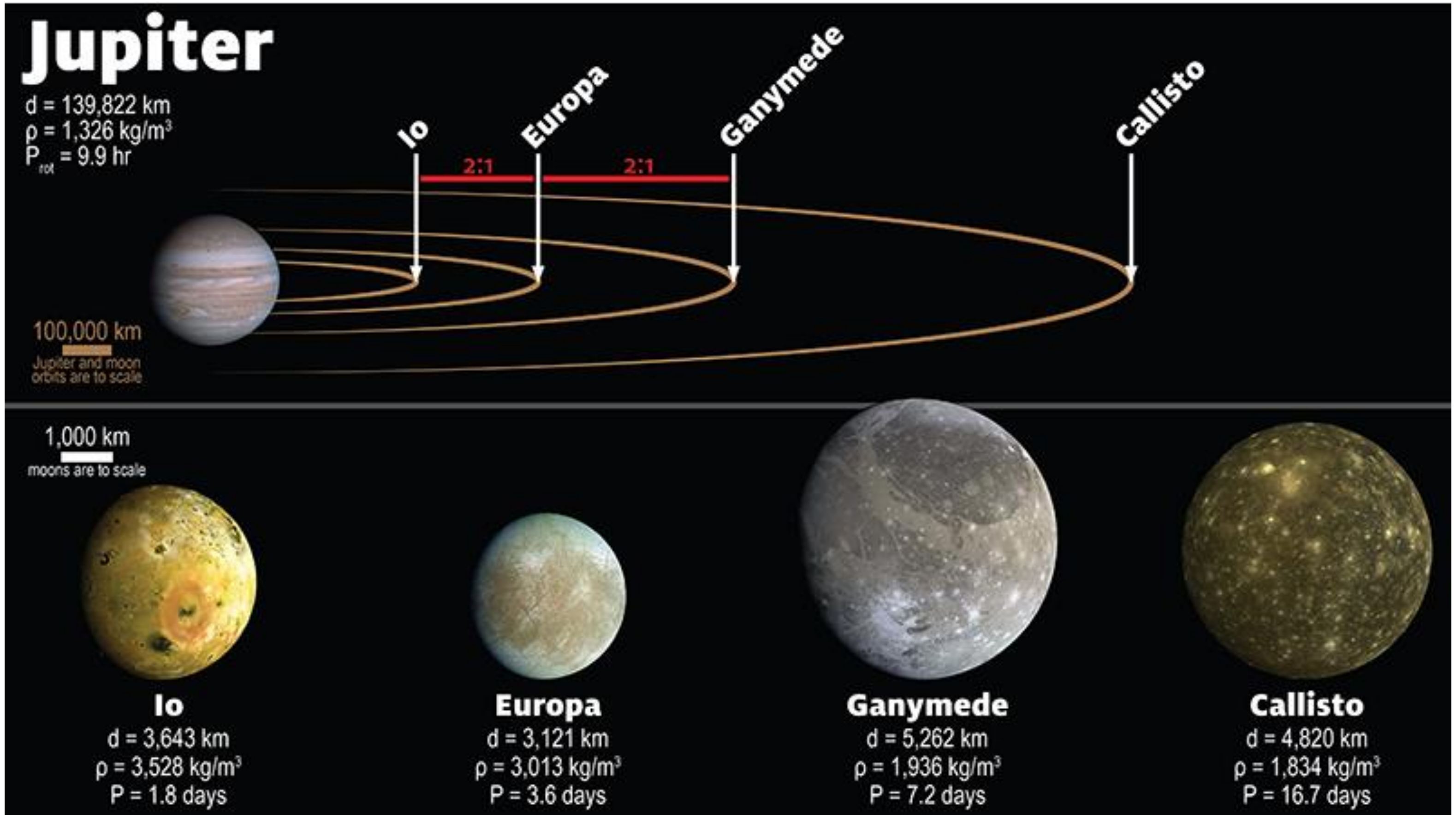
A tidally locked satellite tries to reduce its orbital eccentricity and semi-major axis

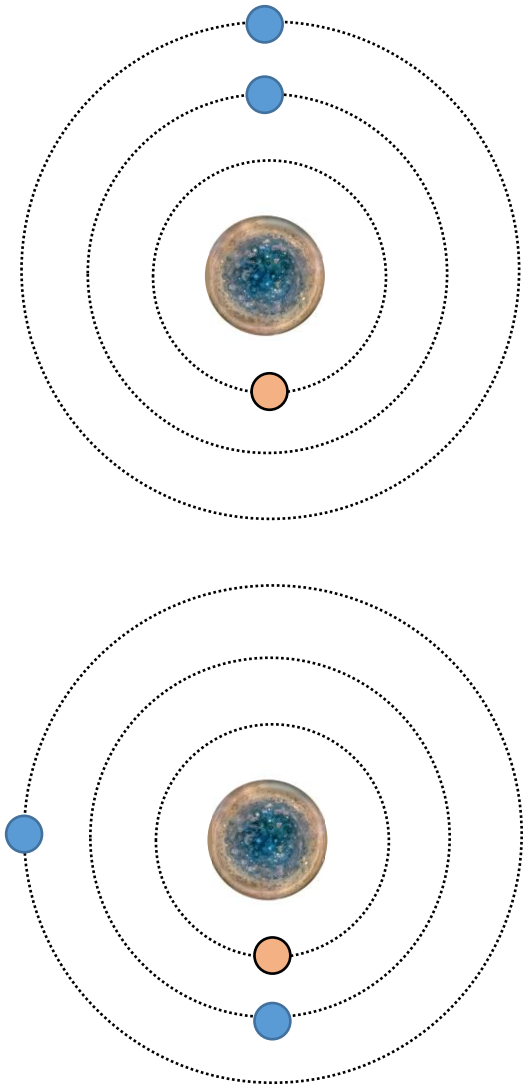
A quickly rotating host planet tends to increase orbital eccentricity and semi-major axis of the satellite

Who wins? Depends on the amount of dissipation in the two bodies…

Expected orbital dynamics of Galilean satellites

Image credits: James Tuttle Keane/Keck Institute for Space Studies





Laplace resonance: the ratio of orbital periods of Ganymede:Europa:Io is 4:2:1

Mean-motion resonances help to keep nonzero orbital eccentricities of the moons

Expected orbital dynamics of Galilean satellites

Image credits: NASA/JPL/DLR

high eccentricity → large amount of heating → partially molten, highly dissipative interior → orbital evolution controlled by the satellite → low eccentricity → small amount of heating → frozen interior → orbital evolution controlled by the planet (quickly rotating Jupiter) and other satellites → high eccentricity…

Take-home messages

All tidal effects (tidal deformation, surface pattern of tidal heating, orbital evolution, stability of spin-orbit resonances, tectonic features) **probe the interior of planets and satellites**

All celestial bodies are subjected to tidal forces, however…

…the timescales of tidal evolution depend strongly on the **mutual distance**

…bigger moons affected more than smaller ones

…mass of the perturber plays an important role, too

Tidal theory can also be applied to other stars and extrasolar planets: tidal Love numbers and rate of orbital evolution of binary stars and giant exoplanets are already being measured

When are tides important?

Below: Darwin-Kaula series

Tidal dissipation:

distance from the star

eccentricity, obliquity, spin rate

interior structure

$$\bar{P}^{\text{tide}} = -\mathcal{G}m_*^2 \frac{R^5}{a^6} \sum_{m=0}^{2} (2-\delta_{m0}) \frac{(2-m)!}{(2+m)!} \sum_{p=0}^{2} \left[\mathcal{F}_{2mp}(\beta)\right]^2 \sum_{q=-\infty}^{+\infty} \left[\mathcal{G}_{2pq}(e)\right]^2 \omega_{lmpq} \operatorname{Im}\{\bar{k}_l(\omega_{lmpq})\}$$

mass of the host star + planet size

Rotational evolution:

$$\mathcal{T}_{\text{tide}} = \mathcal{G}m_*^2 \frac{R^5}{a^6} \sum_{m=0}^{2} (2-\delta_{m0}) \frac{(2-m)!}{(2+m)!}\, m \sum_{p=0}^{2} \left[\mathcal{F}_{2mp}(\beta)\right]^2 \sum_{q=-\infty}^{+\infty} \left[\mathcal{G}_{2pq}(e)\right]^2 \operatorname{Im}\{\bar{k}_l(\omega_{lmpq})\}$$

Orbital evolution:

$$\dot{a}_2 = \frac{2\mathcal{G}m_*}{na} \frac{R^5}{a^6} \sum_{m=0}^{2} (2-\delta_{0m}) \frac{(2-m)!}{(2+m)!} \sum_{p=0}^{2} \left[\mathcal{F}_{2mp}(\beta)\right]^2 \sum_{q=-\infty}^{+\infty} \left[\mathcal{G}_{2pq}(e)\right]^2 (l-2p+q) \operatorname{Im}\{\bar{k}_l(\omega_{lmpq})\}$$