GW Open Data Workshop
April 18-20, 2024

Continuous Gravitational Waves Answer

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GW Open Data Workshop #7, April 18-20, 2024 @ Taichung, Taiwan

2. F-statistic

Quiz: How does the contour plot of the posterior look like? What is the origin of this shape?

Answer: It seems elongated meaning the parameters $f^{(0)}$ and $f^{(1)}$ are degenerated. It can be understood as follows.

Let's consider the phase evolution of the signals having slightly different $f^{(0)}$ and $f^{(1)}$ values. We assume the initial phases are same.

The phase at the end can be written by $\phi = 2\pi \left(f^{(0)} + \frac{1}{2} f^{(1)} T_{\mathrm{obs}} \right) T_{\mathrm{obs}}$

If $f^{(0)}$ and $f^{(1)}$ are slightly different, the phase at the end will be $\phi' = 2\pi \left(f^{(0)} + \Delta f^{(0)} + \frac{1}{2} (f^{(1)} + \Delta f^{(1)}) T_{\mathrm{obs}} \right) T_{\mathrm{obs}}$

The difference is $\Delta\phi = \phi' - \phi = 2\pi \left(\Delta f^{(0)} + \frac{1}{2} \Delta f^{(1)} T_{\mathrm{obs}} \right) T_{\mathrm{obs}}$

If $\Delta f^{(0)} + \frac{1}{2} \Delta f^{(1)} T_{\mathrm{obs}} = 0$ satisfies, $\Delta\phi$ becomes zero. This is the reason why the posterior have a degeneracy.

2. F-statistic

Q2. How large is the size of the contour? Why the contour have such a size?

A2. It is about $\Delta f^{(0)} \sim 10^{-7}$ Hz, $\Delta f^{(1)} \sim 10^{-14}$ Hz/sec. They relate to the frequency resolution, $(T_{\rm obs})^{-1}$.

Tobs ~ 10^7 sec is directly means $\Delta f^{(0)} \sim (T_{\rm obs})^{-1} \sim 10^{-7}$ Hz. For f(1), the frequency change will be

$\Delta f^{(1)}\, T_{\rm obs} \sim (T_{\rm obs})^{-1}$. Then, $\Delta f^{(1)} \sim (T_{\rm obs})^{-2} \sim 10^{-14}$ Hz/sec.

3. Frequency Hough

Q1: Why are there seemingly negative slopes of the lines in this map? Can you guess which parameter you need to change to obtain all positive slopes or a mix of positive and negative sloped lines?

A1: Because we set the reference time (the time at when we define $f^{(0)}$) is the start of the data.

Remind $f = f^{(0)} + f^{(1)}(t - t_{\mathrm{ref}}) \Rightarrow f^{(1)} = -\frac{1}{t - t_{\mathrm{ref}}} f^{(0)} + \frac{f}{t - t_{\mathrm{ref}}}$. The slope is $-(t - t_{\mathrm{ref}})^{-1}$.

If we set t_{ref} at the start time, the slope is always negative.

```
alltimes = times["H1"] ### the times of the analysis
df = 1/Tsft # the frequency bin size
ref_perc_time = 0.0 * 100 # reference time for the Hough at which f0 is determined, set any number
between 0 (beginning), 100 (end)
sig_fdot = inj['F1'] #spin-down of injected signal
dsd = 1/(Tsft * duration) # step in spin-down: dsd = df / Tobs
sdgrid = pyhough.hm.make_sd_grid(sig_fdot,dsd) ## grid of spin-downs to search over
```

3. Frequency Hough

Q2: If the true signal frequency and spin-down are F0 and F1, respectively, why are there multiple pixels in this map surrounding the true values?

A2: A frequency track of a neighboring pixel may overlap the true frequency track with some extent. This makes the number counts of the neighboring pixels can be high.

3. Frequency Hough

Q3: Do you see the difference between using the wrong and correct source positions to correct the peakmap? What can you conclude about the ability of this method to localize sources in the sky?

A3: When we use the correct position, the peakmap shows a straight line. When we use the wrong position, a line in the peakmap is curved.

Q4: What do you think will happen to a monochromatic noise line after the Doppler correction is applied? Is this good or not? Why?

A4: Doppler correction will modulate the line noise frequency. So, the peamap will show a curve line. It will help to discriminate a line noise and an astrophysical signal.

3. Frequency Hough

Q5: Compare the peak number count at the source frequency and spin-down in this Hough map and the one run on the properly corrected peakmap. Why is there a reduction in the number count?

A5: The peak number count is the number of peaks summed over the frequency track corresponding to the grid. If you use a wrong source position for the Doppler correction, the peaks will be off the track. It leads to the reduction in the number count.