1. Go to Wikipedia http://en.wikipedia.org/ and read the pages on Colour Spaces (http://en.wikipedia.org/wiki/Colour_spaces), Optical Illusions (http://en.wikipedia.org/wiki/Optical_illusion) and M. C. Escher (http://en.wikipedia.org/wiki/M._C._Escher)

2. Buy book (*Hearn, Baker and Carithers*) and read all of Chapters 1 and begin reading Chapter 2

Mathematics Refresher I

1. Time and time again during the course we will see the idea that we can express a point in terms of two others come up in several guises.

   Limiting ourselves to 1-D this can be expressed mathematically by saying that, given two x-values, any value in between them can be expressed as multiples of them.

   Suppose the end points are x_s and x_e then the distance between them is $x_e - x_s$. Call this Δ and now consider any fraction of this distance, α. Starting from x_s, walk a distance $\alpha\Delta$ to arrive at a new value $x_n = x_s + \alpha\Delta$. Some important points to know about this

   (a) If $\alpha > 0$ we're walking from x_s in the direction towards x_e

   (b) If $\alpha < 0$ we're walking from x_s in the direction away from x_e

   (c) If $0 \leq \alpha \leq 1$ then we're somewhere between x_s and x_e

   In all three cases

$$\begin{aligned} x_n &= x_s + \alpha\Delta \\ &= x_s + \alpha(x_e - x_s) \\ &= x_s + \alpha x_e - \alpha x_s \\ &= (1-\alpha)x_s + \alpha x_e \end{aligned}$$

   So x_n is a "blend" of the two end points or, as is more frequently said a *linear combination* of the two end points.

   The very important case (c), where $0 \leq \alpha \leq 1$ is known as a *convex combination* of the two end points. In this case note that since $0 \leq \alpha \leq 1$ it is also the case that $\alpha' = 1 - \alpha$ has the same limits, $0 \leq \alpha' \leq 1$ and so

$$x_n = \alpha' x_s + (1-\alpha')x_e, \quad \alpha' = (1-\alpha)$$

   In all three cases we can work backwards and, for given x_s, x_e and x_n, figure out the values of α and Δ.

   Q. 1: If $x_s = 90$, $x_e = 110$ and $x_n = 105$, find Δ, α.

Note that α is a relative measure: how far x_n is from x_s relative to the full distance $\Delta = x_e - x_s$.

Q. 2: If $x_s = 110$, $x_e = 90$ and $x_n = 105$, find Δ, α.

So $x_n = 105$ is either $3/4$ of the way from $x_s = 90$ to $x_e = 110$ or $1/4$ of the way from $x_s = 110$ to $x_e = 90$. What this second example also shows is that as long as we have the stomach to see through the negative numbers which end we start from doesn't matter: we can still express x_n with either point as the start point.

Note that the two Δs were the negative of each other – we're going in opposite directions in the two examples – and the αs added to 1 – three quarter's of the way from one side is one quarter to go to the other! This well and truly covers the case of $0 \leq \alpha \leq 1$, but what about cases (a) and (b) earlier?

Q. 3: If $x_s = 90$, $x_e = 110$ and $x_n = 290$, find Δ, α.

2. All of this scales up to two dimensions, and beyond. Instead of x_s we will write p_s, etc and we can say

$$p_n = (1 - \alpha)p_s + \alpha p_e$$

How we get to this is a little trickier in 2-D (or beyond) than in 1-D. We can best think about Δ in 2-D as being

$$\Delta = \frac{(y_e - y_s)}{(x_e - x_s)} = \frac{\Delta_y}{\Delta_x}$$

This, of course, is the slope, m, of the line going through p_s and p_e and is interpreted as the number of y-units to traverse relative to x-units in order to keep to the line. That is, for any two points p and q *on the line*, if the "x difference" between them is $\Delta_x = q_x - p_x$ then the "y difference" between them must be $\Delta_y = q_y - p_y = \Delta_x \Delta$.

Put another way, if $\Delta_x = q_x - p_x = \alpha(x_e - x_s)$ then to maintain the ratio Δ, it will have to hold that $\Delta_y = q_y - p_y$, the "y difference", will need to respect $\alpha(y_e - y_s)$.

So what we are saying, then, is that any point p_n on this line has x component that is some multiple, α, of the x part of the step, Δ, and y component that is *the same* multiple, α, of the y part of Δ away from the start point. The step can be in the positive direction or the negative direction.

In 2-D if we think of Δ as being steps in each dimension then we can say that for two points on the line

$$p' = p + \alpha\Delta$$

and this Δ maintains the aspect ratio.

In some respects, even though we are in 2-D there is only *one* degree of freedom: we may move in either the x direction or the y direction but once we move in one direction by an amount the change in the other is enforced by having to maintain the same ratio.

(Later we will see that Δ in either one or two dimensions is really a vector.)

3. In the 1-D case earlier we looked at case (c) $0 \leq \alpha \leq 1$, the convex combination of two points, as being those points between x_s and x_e on the line through them.

   We can do the same in 2-D with *three* points p_1, p_2 and p_3 and look at the convex combination of these. In this case it is not too difficult to show that the points in the convex combination of the three is somewhere in the triangle formed by the points. That is, p_n is in the triangle $p_1p_2p_3$ iff (if and only if)

$$p_n = \alpha_1 p_1 + \alpha_2 p_2 + \alpha_3 p_3, \quad \alpha_1 + \alpha_2 + \alpha_3 = 1, \quad \alpha_i \geq 0$$

   If we allow $\alpha_i < 0$ then, analogous to the 1-D case in item 1 above, *every* point in the 2-D plane can be expressed as

$$p_n = \alpha_1 p_1 + \alpha_2 p_2 + \alpha_3 p_3, \quad \alpha_1 + \alpha_2 + \alpha_3 = 1$$

4. Very frequently in graphics we want to *scale* a figure up or down in size. Suppose the rectangle that encloses the original figure has dimensions (w, h) and we want to enlarge it to (W, H).

   For a point (x, y) of the original image find what its new co-ordinates will be. Hint: think of the x-dimension and y-dimension separately.

   Note that this is different from Q. 2 where the aspect ratio was fixed. This exercise will be very useful later in the semester when we come to converting co-ordinates as viewed in a camera (camera view) to how they are presented on a computer screen (viewport).