CS4815 **Week08 Tutorial**

Review of Vectors

The cardinal rule of vector spaces is that they are pointless: vectors have no pre-ordained position. They just have magnitude and direction.

1. If a vector $\mathbf{v}$ in 3-D has components a, b and c in the three axes – that is, $\mathbf{v} = (a, b, c)$ – compute $|\mathbf{v}|$ the length of $\mathbf{v}$.
2. What is the vector $\mathbf{w}$ that has the same direction as $\mathbf{v}$ but is of unit-length; that is $|\mathbf{w}| = 1$. This new vector is called the *normalized* version of $\mathbf{v}$.
3. Use the *algebraic* definition of dot product to show that $\mathbf{v} \cdot \mathbf{v} = |\mathbf{v}|^2$ and so show that if $\mathbf{w}$ is unit-length then $\mathbf{w} \cdot \mathbf{w} = 1$
4. The *trigonometric* definition of dot product is

   $$\mathbf{v} \cdot \mathbf{w} = |\mathbf{v}||\mathbf{w}| \cos \theta$$

   where θ is the angle between the two vectors. Use this to show that if two vectors are perpendicular then their dot product is 0.
5. What is the vector way of expressing a plane in 3-D?
6. Use the dot product operator to determine what is the perpendicular projection of a point p onto a plane?
7. One important use of the dot product operator in computer graphics / games is to tell if a polygon (that is part of the wire frame that defines an object) is visible to the camera; the ones at the back of the object won't be!

   This idea relies on every polygon having a normal defined for it and that points in the direction of the outside world. If we are looking towards the object with a viewing direction given by a vector $\mathbf{v}$ what is the relationship that $\mathbf{v}$ must have with the polygon normal $\mathbf{n}$ for the face to be visible?