Computer Graphics

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Spring 2021–2022

Outline

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Subspaces

- A subspace is a set of points, all of which "share some property."
  The set $S_1 = \{(x, y)|y = 2x\}$ is a subspace; so too is $S_2 = \{(x, y)|y = 2x + 5\}$
- A linear subspace is one with the restrictions: 1) if $u \in S$ then $cu \in S$, for a multiplier (scalar) c; and, 2) if $u, v \in S$ then $u + v \in S$.
  For a subspace to be linear it must contain (0,0).
- An results from adding a constant vector to every element of a linear subspace.
  Lines, planes not going through (0,0) and arbitrary points are affine subspaces; they require a *translation* (shift) away from $(0, 0)$

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  The set $S_1 = \{(x, y)|y = 2x\}$ is a linear subspace; the set $S_2 = \{(x, y)|y = 2x + 5\}$ is **not**. For a subspace to be linear it must contain (0,0).
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  Affine subspace: the set S_2 above; $S_3 = \{(x, y) = (4, 5)\}$.
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Transformations

- A linear transformation on a set is a function that is *linear*. That is, if $f : S \to S'$ 1) $f(cu) = cf(u), u \in S$; and, 2) $f(u+v) = f(u) + f(v), u, v \in S$
  See "Examples of linear transformation matrices" section
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- Linear transformations are preserved under composition; that is, when we apply two LTs, one after the other.
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Affine Transformations

- The trouble with linear transformations is that they only work for lines and planes that go through the origin
- An affine transformation consists of a linear transformation followed by a (possibly 0) translation. Read this
- In d-dimensional space, $\vec{u}$ and $\vec{b}$ will be d-dim vectors and A will be a $d \times d$ matrix

$$\vec{u} = A\vec{u} + \vec{b}$$

$$\begin{pmatrix} u'_1 \\ u'_2 \\ \vdots \\ u'_d \end{pmatrix} = \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1d} \\ a_{21} & a_{22} & \cdots & a_{2d} \\ \vdots & & \cdots & \vdots \\ a_{d1} & a_{d2} & \cdots & a_{dd} \end{pmatrix} \begin{pmatrix} u_1 \\ u_2 \\ \vdots \\ u_d \end{pmatrix} + \begin{pmatrix} b_1 \\ b_2 \\ \vdots \\ b_d \end{pmatrix}$$

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Affine Transformations

- We can combine the $\boxed{}$ by a trick of appending an extra component of 1 to the vectors at posn d+1:

$$\begin{pmatrix} u'_1 \\ u'_2 \\ \vdots \\ u'_d \\ 1 \end{pmatrix} = \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1d} & b_1 \\ a_{21} & a_{22} & & a_{2d} & b_2 \\ \vdots & & \cdots & \vdots & \\ a_{d1} & a_{d2} & & a_{dd} & b_d \\ 0 & 0 & & 0 & 1 \end{pmatrix} \begin{pmatrix} u_1 \\ u_2 \\ \vdots \\ u_d \\ 1 \end{pmatrix}$$

- The transformation can now be written in a compact form as

$$\vec{u'} = \begin{bmatrix} A & b \\ 0 & 1 \end{bmatrix} \vec{u}$$

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Rabhadh: matrix on right is still $(d+1) \times (d+1)$: A is $d \times d$

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