CS4815 **Week04 Tutorial**

1. The rotation matrix R for rotating points around the origin through a positive (anti-clockwise) angle θ is

$$R = \begin{pmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

   That is, the point $p = (x, y)$ gets rotated to $q = Rp$.

   What is the matrix for rotating the opposite (clockwise) way?

2. You would expect that if you rotated a point anti-clockwise through some angle, θ, first and then rotated the result by the same angle but in the opposite (clockwise) direction, $-\theta$ that you would get back to where you started.

   Show that this is true by taking a point, say, $p = (1, 1)$, rotating it through θ and then rotate the result through $-\theta$. That is, compute

$$p_1 = R_\theta p$$

   and then

$$p_2 = R_{-\theta} p_1.$$

   How does this compare to multiplying the two *matrices* together first of all and then multiplying the point by the result as in

$$q = (R_{-\theta} R_\theta) p$$

   Why might the second approach be better in general cases where one rotation doesn't cancel the other out?

3. By multiplying together a translation matrix, a rotation matrix and an un-translation matrix, build a matrix for rotating the point $(2, 1)$ through an angle of $\theta = 30\,\text{deg}$ around the point $(1, 1)$.

4. The *determinant* of a matrix gives valuable information about how that matrix transforms a polygon. We will not justify this but it is a fact that the determinant of a matrix tells how the area of the polygon changes when transformed by applying the matrix transformation on every point that comprises the polygon. That is, the determinant of a matrix is the ratio of after-to-before areas.

   Clearly, rotating a square or a triangle or any polygon will leave its area intact.

   (a) By taking the determinant of a rotation matrix verify the above property.

(b) What is the matrix that achieves the *flip* transformation? You can either flip about the x-axis or the y axis.

(c) How should the area be affected by a flip transformation?

(d) Verify this by taking the determinant of your matrix. Anything mildly unsettling about your answer?