

# Computer Graphics

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Spring 2021–2022



# Outline

- 1 Lighting Models: §10 (contd.)
  - Illumination Models: §10-3


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  - Illumination Models: §10-3

# Reflections

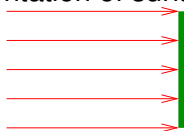
- From the types of light sources seen, what is the effect of the three different types of light reflection (**ambient**, **diffuse**, **specular**) off a surface?
- As surfaces vary so do the amounts of light that they reflect
- For a given surface we denote the fraction of light that is reflected **diffusely** by  $k_d$  and **specularly** by  $k_s$
- Although some models permit a separate  light reflectivity component  $k_a$  **usually**, not coming from a particular source,  light gets reflected diffusely and the intensity of the reflected light is

$$k_d I_a$$

- Since  light is “all around” the intensity of light due to ambient light,  $I_a$ , is constant at all points in space

# Diffuse Reflection

- When a surface is illuminated by a light source the **intensity** of the light falling on it,  $I_l$ , is the **amount** of light that falls per **unit area of the surface**; this depends on orientation of surface



- The amount depends on the **cosine** of the angle between the incident path and the normal to the surface,  $N$

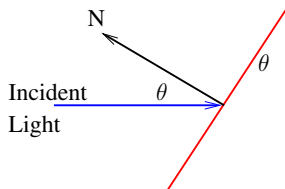
$$I_{l,\text{incident}} = I_l \cos \theta$$

- So that the diffuse reflection that results is

$$I_{l,\text{diff}} = k_d I_{l,\text{incident}} = k_d I_l \cos \theta$$

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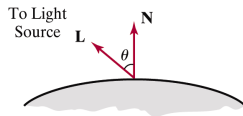
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# Diffuse Reflection (contd.)

- In general



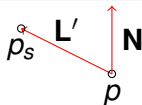
and, by using **normalised** vectors for  $\mathbf{L}$  and  $\mathbf{N}$ ,  $\cos \theta$  is easily computed as the  so that

$$I_{l,\text{diff}} = \begin{cases} k_d I_l (\mathbf{N} \cdot \mathbf{L}) & \text{if } \mathbf{N} \cdot \mathbf{L} > 0 \\ 0.0 & \text{if } \mathbf{N} \cdot \mathbf{L} \leq 0 \end{cases}$$

- We can calculate the vector  $\mathbf{L}$ , as the (normalised) difference of the light source  $p_s$  and the point on the surface  $p$ ; see picture over



# Diffuse Reflection (contd.)



- Since  $p + L' = p_s$ ,  $L' = p_s - p$  and

$$L = \frac{L'}{|L'|} = \frac{p_s - p}{|p_s - p|}$$

- If we allow for a different reflectivity coefficient for ambient light,  $k_a$ , then the intensity of diffuse reflection due to ambient light and diffuse reflection from a light source is given by

$$I_{\text{diff}} = \begin{cases} k_a I_a + k_d I_l (\mathbf{N} \cdot \mathbf{L}) & \text{if } \mathbf{N} \cdot \mathbf{L} > 0 \\ k_a I_a & \text{if } \mathbf{N} \cdot \mathbf{L} \leq 0 \end{cases}$$

# Diffuse Reflection (contd.)



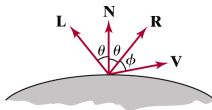
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# Specular Reflection

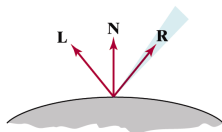
- Specular reflection is the result of total, or near total, reflection of the light in a region around the **specular-reflection angle**



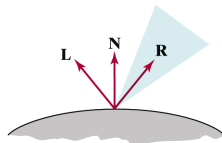
- Light from the direction  $L$  falling on a surface will be specularly reflected in the direction  $R$
- The “outgoing” angle  $\theta$  will be the same as the “incoming” angle  $\theta$
- For an ideal reflector (a perfect mirror) incident light is reflected in the **specular direction** only ( $\phi = 0$  above)

# Specular Reflection (contd.)

- With the viewer situated at angle  $\phi$  to **R** some light may be visible depending on the surface



Shiny Surface  
(Large  $n_s$ )



Dull Surface  
(Small  $n_s$ )

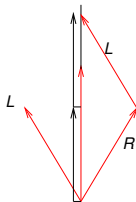
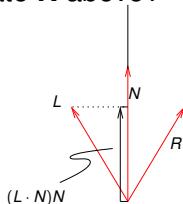
- According to the **Phong specular-reflection model** the intensity of light at angle  $\phi$  is given by  $\cos^{n_s} \phi$  where  $n_s$  is particular to a material (wood, metal, glass, etc.)
- Shiny surfaces have larger  $n_s$  so, for a given  $\phi$ , intensity will be *smaller*

# Specular Reflection (contd.)

- The intensity of the specular reflection due to a point light source of intensity  $I_l$  at a surface point is

$$I_{l,\text{spec}} = \begin{cases} k_s I_l (\mathbf{V} \cdot \mathbf{R})^{n_s} & \text{if } \mathbf{V} \cdot \mathbf{R} > 0 \text{ and } \mathbf{N} \cdot \mathbf{L} > 0 \\ 0.0 & \text{if } \mathbf{V} \cdot \mathbf{R} < 0 \text{ and } \mathbf{N} \cdot \mathbf{L} \leq 0 \end{cases}$$

- How to calculate  $\mathbf{R}$  above?



# Specular Reflection (contd.)

- From the scalar product operator definition  $\mathbf{L} \cdot \mathbf{N} = \mathbf{R} \cdot \mathbf{N} =$  projection of  $\mathbf{L}$  onto  $\mathbf{N}$
- But from adding vectors  $\mathbf{L}$  and  $\mathbf{R}$ ,  $\mathbf{L} + \mathbf{R}$  is twice projection of  $\mathbf{L}$  onto  $\mathbf{N}$  and in the direction  $\mathbf{N}$
- So

$$\mathbf{R} + \mathbf{L} = (2\mathbf{L} \cdot \mathbf{N})\mathbf{N}$$

$$\mathbf{R} = (2\mathbf{L} \cdot \mathbf{N})\mathbf{N} - \mathbf{L}$$

$\mathbf{L}$ ,  $\mathbf{N}$ ,  $\mathbf{V}$  are all unit-length so  $\mathbf{R}$  will be automatically. (Why?)

# Specular Reflection (contd.)

- Because it is different at every point on the surface calculating **R** can be expensive; although **N**, **V** and **L** are also unique to each point on surface **N** will be known in advance and, **if the light and viewer are far enough away**, **V** and **L** can be taken to be the same everywhere
- An alternative strategy that gives almost as good results and that avoids the additional computational burden is the **halfway vector H**
- **H** is the bisector (average) of the angle between **L** and **V** and can be computed by

$$\mathbf{H} = \frac{\mathbf{L} + \mathbf{V}}{|\mathbf{L} + \mathbf{V}|}$$

# Specular Reflection (contd.)

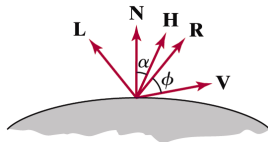
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# Specular Reflection (contd.)

- **Fun fact:** if all of this is taking place in a 2-D world (or if all vectors are co-planar) then  $\alpha = \phi/2$
- In effect, we replace the empirically-based parameter,  $\cos \phi = \mathbf{V} \cdot \mathbf{R}$ , by another empirically-based value,  $\cos \alpha = \mathbf{N} \cdot \mathbf{H}$ , that is cheaper to compute
- When computing lighting in a scene it is important to remember that, generally, the light source(s) and the viewer position are fixed; if they're far enough away we can say that at every point in the scene the vectors  $\mathbf{V}$  and  $\mathbf{L}$  are more-or-less constant. **Rabhadh: this is a simplification and may lead to poor results.**
- We may then (independently) choose to work with the halfway vector

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# Combined Diffuse and Specular Reflections

- We can model the combined ambient, diffuse and specular reflections from a point on a surface illuminated by a single point light as

$$= k_a I_a + k_d I_l (\mathbf{N} \cdot \mathbf{L}) + k_s I_l (\mathbf{V} \cdot \mathbf{R})^{n_s}$$

- The surface is illuminated only with ambient light if the light source is behind the surface; there are no specular effects if  $\mathbf{V}$  and  $\mathbf{L}$  are on the same side of  $\mathbf{N}$
- For multiple light sources ( $n$  of them)

$$\begin{aligned} I &= I_{\text{ambdiff}} + \sum_{l=1}^n (I_{l,\text{diff}} + I_{l,\text{spec}}) \\ &= k_a I_a + \sum_{l=1}^n I_l (k_d (\mathbf{N} \cdot \mathbf{L}) + k_s (\mathbf{V} \cdot \mathbf{R})^{n_s}) \end{aligned}$$