

# Understanding Percentiles and Quartiles

## Statistics Course Notes

### Introduction to Percentiles and Quartiles

Percentiles and quartiles are fundamental measures of position in a dataset:

- **Percentile:** Value below which a specific percentage of observations fall
- **Quartile:** Special percentiles that divide data into four equal parts
- Applications: Exam rankings (CAT, GATE), income distribution analysis, test scoring

### Percentile Calculation

#### Key Formula

For a value  $x$  in a dataset with  $n$  observations:

$$P_x = \left( \frac{\text{Number of values below } x}{n} \right) \times 100$$

#### Example Calculation

Dataset:  $\{2, 2, 3, 4, 5, 5, 6, 7, 8, 8, 8, 9, 9, 10\}$  ( $n = 14$ )

Find percentile for  $x = 9$ :

Values below 9 = 11

$$P_9 = \left( \frac{11}{14} \right) \times 100 = 78.57\%$$

**Interpretation:** 78.57% of values in the dataset are less than 9.

### Finding Values for Specific Percentiles

#### Formula for Position

$$\text{Position} = \left( \frac{P}{100} \right) \times (n + 1)$$

Where  $P$  is the desired percentile.

## 25th Percentile Example

$$\text{Position} = \left( \frac{25}{100} \right) \times (14 + 1) = 0.25 \times 15 = 3.75$$

Since position is fractional:

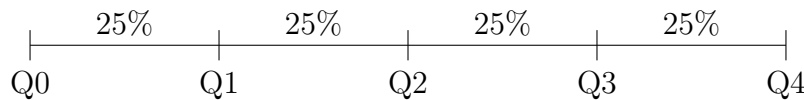
$$\text{Value} = \frac{\text{3rd value} + \text{4th value}}{2} = \frac{3 + 4}{2} = 3.5$$

Position	Value	Note
1	2	
2	2	
3	3	3rd value
4	4	4th value

**Interpretation:** 25% of values are below 3.5.

## Quartiles: Special Percentiles

Quartiles divide data into four equal parts:



- $Q_1$  (First Quartile) = 25th percentile
- $Q_2$  (Second Quartile) = 50th percentile (Median)
- $Q_3$  (Third Quartile) = 75th percentile

## Quartile Calculation for Example Dataset

$$Q_1 = 25^{\text{th}} \text{ percentile} = 3.5$$

$$Q_2 = 50^{\text{th}} \text{ percentile} :$$

$$\text{Position} = 0.5 \times 15 = 7.5$$

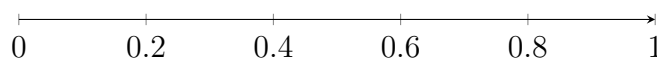
$$\text{Value} = \frac{6^{\text{th}} \text{ value} + 7^{\text{th}} \text{ value}}{2} = \frac{5 + 6}{2} = 5.5$$

$$Q_3 = 75^{\text{th}} \text{ percentile} :$$

$$\text{Position} = 0.75 \times 15 = 11.25$$

$$\text{Value} = \frac{11^{\text{th}} \text{ value} + 12^{\text{th}} \text{ value}}{2} = \frac{8 + 9}{2} = 8.5$$

## Box Plot: Visualizing Quartiles



## Key Concepts

- **Percentile Formula:**  $P_x = \frac{\text{count below } x}{n} \times 100$
- **Position Formula:**  $\text{Position} = \frac{P}{100} \times (n + 1)$
- **Fractional Positions:** Average adjacent values
- **Quartile Relationships:**
  - $Q_1 = 25\text{th percentile}$
  - $Q_2 = 50\text{th percentile (Median)}$
  - $Q_3 = 75\text{th percentile}$
- **Interquartile Range (IQR):**  $Q_3 - Q_1$  (Measure of spread)

## Practical Applications

1. **Exam Rankings:** "99th percentile" means 99% of test-takers scored lower
2. **Data Analysis:** Identify distribution characteristics and outliers
3. **Salary Comparisons:** Position individual income within population

## Common Mistakes to Avoid

- Forgetting to sort data before calculations
- Confusing percentage with percentile
- Incorrectly handling fractional positions
- Using  $n$  instead of  $n + 1$  in position formula