Introduction to Probability

1 Introduction to Probability

Probability quantifies the likelihood of an event occurring. For any event A:

$$P(A) = \frac{\text{Number of favorable outcomes}}{\text{Total possible outcomes}}$$

Examples

- **Coin Toss:** Sample space = {Head, Tail}

$$P(\text{Head}) = \frac{1}{2}, \quad P(\text{Tail}) = \frac{1}{2}$$

- **Dice Roll:** Sample space = {1, 2, 3, 4, 5, 6}

$$P(X = 1) = \frac{1}{6}, \quad P(X = 5) = \frac{1}{6}$$

2 Mutually Exclusive Events

Two events are mutually exclusive if they **cannot occur simultaneously**.

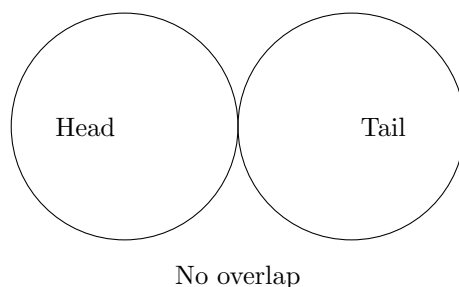### Properties

- No overlap in outcomes
- $P(A \cap B) = 0$

Examples

- Coin: Getting Head *and* Tail in one toss
- Dice: Rolling 1 *and* 5 simultaneously

Addition Rule

For mutually exclusive events A and B:

$$P(A \cup B) = P(A) + P(B)$$

Venn Diagram and Calculation

No overlap

$$P(\text{Head} \cup \text{Tail}) = P(\text{Head}) + P(\text{Tail}) = \frac{1}{2} + \frac{1}{2} = 1$$

3 Non-Mutually Exclusive Events

Two events are non-mutually exclusive if they **can occur simultaneously**.

Properties

- Possible outcome overlap
- $P(A \cap B) > 0$

Example: Card Deck

- 52 cards: 4 suits (13 cards each)
- Events:

$$A : \text{Drawing a King} \quad (P(A) = \frac{4}{52})$$
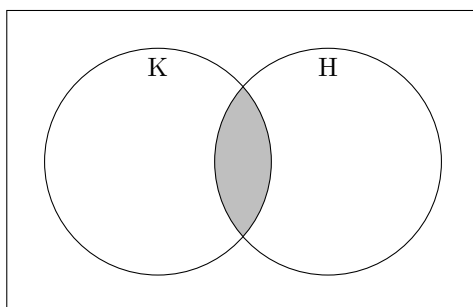$$B : \text{Drawing a Heart} \quad (P(B) = \frac{13}{52})$$
$$A \cap B : \text{King of Hearts} \quad (P(A \cap B) = \frac{1}{52})$$

Addition Rule

For non-mutually exclusive events A and B:

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

Venn Diagram and Calculation

$$P(\text{King} \cup \text{Heart}) = \frac{4}{52} + \frac{13}{52} - \frac{1}{52} = \frac{16}{52} = \frac{4}{13}$$

Summary

Property	Mutually Exclusive	Non-Mutually Exclusive
Overlap	None	Possible
Formula	$P(A) + P(B)$	$P(A) + P(B) - P(A \cap B)$
Example	Coin toss	Card deck