Probability: Multiplication Rules and Conditional Probability

1 Independent vs. Dependent Events

- **Independent Events:** Occurrence of one event *does not affect* the probability of the other

$$P(A \cap B) = P(A) \times P(B)$$

- **Dependent Events:** Occurrence of one event *affects* the probability of the other

$$P(A \cap B) = P(A) \times P(B|A)$$

2 Independent Events: Examples

Coin Toss

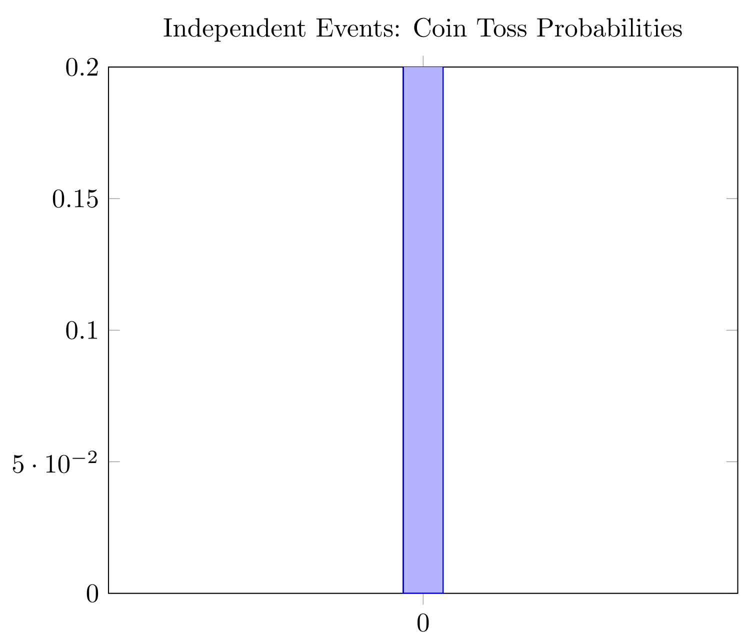
Probability of Head *then* Tail:

$$P(\mathrm{H} \cap \mathrm{T}) = P(\mathrm{H}) \times P(\mathrm{T}) = \frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$$

Dice Roll

Probability of 1 *then* 2:

$$P(X = 1 \cap X = 2) = \frac{1}{6} \times \frac{1}{6} = \frac{1}{36}$$



3 Dependent Events: Card Example

Scenario

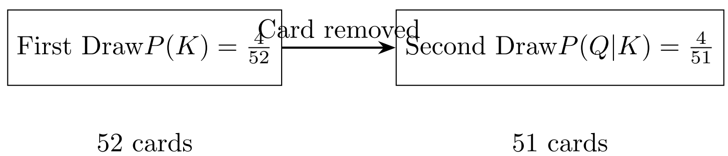
- Draw King *then* Queen from 52-card deck
- *Without replacement*

Probability Calculation

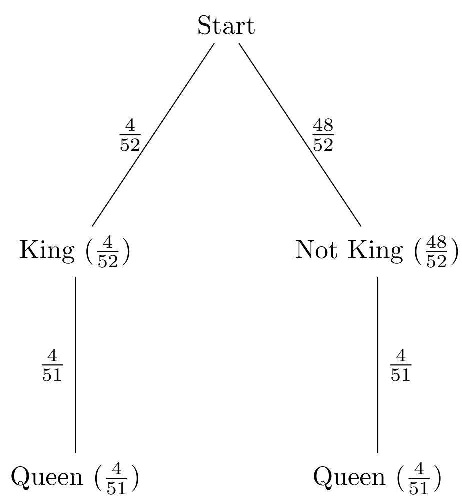
$$P(\mathrm{K} \cap \mathrm{Q}) = P(\mathrm{K}) \times P(\mathrm{Q}|\mathrm{K}) = \frac{4}{52} \times \frac{4}{51} = \frac{16}{2652} = \frac{4}{663}$$

Conditional Probability

$$P(B|A) = \frac{P(A \cap B)}{P(A)} \quad \text{(Probability of B given A has occurred)}$$



4 Probability Tree: Dependent Events



5 Connection to Machine Learning

- Conditional probability forms the foundation for **Bayes' Theorem**

$$P(A|B) = \frac{P(B|A) \cdot P(A)}{P(B)}$$

- Used in **Naive Bayes classifiers** for:
  - Spam detection
  - Text classification
  - Recommendation systems
- Key assumption: Features are conditionally independent

Summary Table

Property	Independent Events	Dependent Events
Affect on probability	None	Significant
Formula	$P(A) \times P(B)$	$P(A) \times P(B\|A)$
Example	Coin tosses	Card draws without replacement
Machine learning use	Feature independence	Naive Bayes classifiers