09. Random Variables in Statistics

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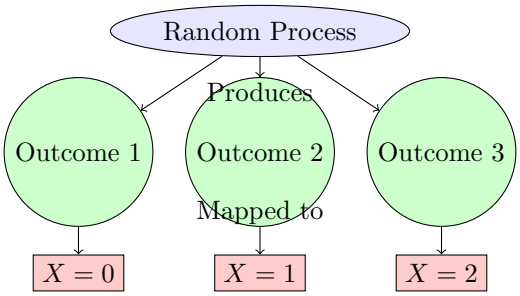
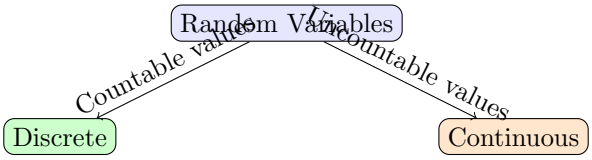
Definition of Random Variable

- A **random variable** (denoted X, Y, etc.) is a *function* that derives values from random processes or experiments
- Maps outcomes of random processes to numerical values
- **Key characteristic**: Values are determined by chance



Types of Random Variables



1. Discrete Random Variables

- Take **countable** number of distinct values
- Usually whole numbers (but not necessarily)
- Values derived from countable outcomes

Examples:

- *Coin Toss:* $X = \begin{cases} 0 & \text{heads} \\ 1 & \text{tails} \end{cases}$
- *Die Roll:* $X \in \{1, 2, 3, 4, 5, 6\}$
- *Number of customers:* $X = 0, 1, 2, 3, ...$

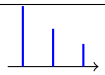
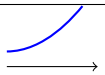
2. Continuous Random Variables

- Take **any value** in a continuous range
- Can be fractions or decimals
- Infinite possible values

Examples:

- *Rainfall:* $X = 1.1$", 5.5", 10.75" (any non-negative real number)
- *Height:* $X = 150.0$ cm, 160.1 cm, 175.5 cm
- *Temperature:* $X = 98.6°F$, $101.2°F$

Key Differences

Feature	Discrete	Continuous
Values	Distinct, countable	Any value in interval
Notation	$X \in \{x_1, x_2, ...\}$	$X \in [a, b]$
Examples	Coin toss (0,1) Die roll (1-6)	Rainfall Height
Probability	Probability mass function	Probability density function
Graph		

Why Random Variables Matter

- Foundation for probability distributions
- Essential for statistical modeling
- Critical in machine learning algorithms
- Enable quantification of uncertain outcomes