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Section I (Induction)

QNo1

A Sequence $d_1, d_2, d_3 \ldots$ is defined by lattering $d_1 = 2$ and $d_k = d_{k-1}$ for all integers $k \geq 2$ Show that^k

$d_n = \frac{n}{2!}$ for all integer $n \geq 1$ using mathematical induction

Sol:

$$d_1 = 2$$

and

$$d_k = \frac{d_{k-1}}{k} \qquad k \geq 2$$

for $n = 1$ $\qquad d_n = \frac{n}{2!}$

$$d_1 = 2 = \frac{2}{1!} = 2$$

$2 = 2$ first true.

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$$d_k = \frac{2}{K!} \quad \textbf{for} \quad n=K$$

for $n = K+1$

$$d_{k+1} = \frac{d(K+1)-1}{K+1}$$

$$= \frac{d_k}{(K+1)}$$

$$d_{k+1} = d_k \cdot \frac{1}{K+1}$$

$$d_{(k+1)} = \frac{2}{K!} \cdot \frac{1}{K+1}$$

$$d_{(k+1)} = \frac{2}{(k+1)(K!)} = \frac{2}{(K+1)!}$$

$$d_{(k+1)} = \frac{2}{(K+1)!}$$

So d_n is true for $n = K+1$ Hence by Principle of mathematical induction, d_n is true for all values of $n \in N$

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QNo2:

$$\frac{1}{1.2}+\frac{1}{2.3}+\cdots\cdots\frac{1}{n(n+1)}=\frac{n}{n+1} \quad \text{for all } n\geq 1$$

Sol:

for $n=1$

$$\frac{n}{n+1}=\frac{1}{1+1}=\frac{1}{2}$$

Hence it is true for $n=1$

Now we check it for $n=K$

$$\boxed{\frac{1}{1.2}+\frac{1}{2.3}+\cdots+\frac{1}{K(K+1)}=\frac{K}{K+1}} \quad -\text{eq } 1$$

Now we Prove it for $n=K+1$

$$\frac{1}{1.2}+\frac{1}{2.3}+\quad+\frac{1}{K(K+1)}+\frac{1}{(K+1)(K+2)}=\frac{K}{K+1}+\frac{1}{(K+1)(K+2)}$$

$$=\frac{K}{K+1}+\frac{1}{(K+1)(K+2)}=\frac{K(K+2)+1}{(K+1)(K+2)}=\frac{K^2+2K+1}{(K+1)(K+2)}$$

$$=\frac{(K+1)^2}{\cancel{(K+1)}(K+2)}=\frac{K+1}{K+2}=\frac{K+1}{(K+1+1)}$$

$$=\frac{K+1}{(K+1)+1}$$

Proved.

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Section II Modular Arithmetic

(3) Encrypt and decrypt following words Using Affine Cipher

19 14 15 8 2

① TOPIC ② FORCE

(a) K(19,4) (b) K(9,5)

→ Encryption

$y = (xa+b) \bmod 26$

K(19,4)

$y_1 = (19.19+4) \bmod 26$

$y_1 = 365 \bmod 26$

$y_1 = 1 = B$

$y_2 = (14.19+4) \bmod 26$

$y_2 = 270 \bmod 26$

$y_2 = 10 = K$

$y_3 = (15.19+4) \bmod 26$

$y_3 = 289 \bmod 26$

$y_3 = 3 = D$

$y_4 = (6.19+4) \bmod 26$

$y_4 = 156 \bmod 26$

$y_4 = 0 = A$

$y_5 = (2.19+4) \bmod 26$

$y_5 = 42 \bmod 26$

$y_5 = 16 = Q$

BKDAQ

K(9,5)

$y_1 = (19.9+5) \bmod 26$

$y_1 = 176 \bmod 26$

$y_1 = 20 = U$

$y_2 = (14.9+5) \bmod 26$

$y_2 = 131 \bmod 26$

$y_2 = 1 = B$

$y_3 = (15.9+5) \bmod 26$

$y_3 = 140 \bmod 26$

$y_3 = 10 = K$

$y_4 = (8.9+5) \bmod 26$

$y_4 = 77 \bmod 26$

$y_4 = 25 = Z$

$y_5 = (2.9+5) \bmod 26$

$y_5 = 23 \bmod 26$

$y_5 = 23 = X$

UBKZX

Decryption: $x = a^{-1}(y-b) \bmod Mod$

BKDAQ

$\downarrow$

1 10 3 0 16

- $K(19,4)$

$a^{-1} \bmod 26$

$19^{-1} \bmod 26$

$11 \bmod 26$

$x_1 = 11(1-4) \bmod 26$

$x_1 = -33 \bmod 26$

$x_1 = 19 \bmod 26$

$x_1 = 19 = T$

$x_2 = 11(10-4) \bmod 26$

$x_2 = 66 \bmod 26$

$x_2 = 14 = O$

$x_3 = 11(3-4) \bmod 26$

$x_3 = -11 \bmod 26$

$x_3 = 15 = P$

$x_4 = 11(0-4) \bmod 26$

$x_4 = -44 \bmod 26$

$x_4 = 8 = I$

$x_5 = 11(16-4) \bmod$

$x_5 = 132 \bmod 26$

$x_5 = 132 \bmod 26 = 2 = C$

TOPIC

UBKZX

$\downarrow$

20 1 10 25 23

- $K(9,5)$

$a^{-1} \bmod 26$

$9^{-1} \bmod 26$

$4 \bmod 26 \cong 3 \bmod 26$

$x_1 = 3(20-5) \bmod 26$

$x_1 = 45 \bmod 26$

$x_2 = 19 \bmod 26$

$x_2 = 19 = T$

$x_2 = 3(1-5) \bmod 26$

$x_2 = -12 \bmod 26$

$x_2 = 14 = O$

$x_3 = 3(10-5) \bmod 26$

$x_3 = 15 \bmod 26$

$x_3 = 15 = P$

$x_4 = 3(25-5) \bmod 26$

$x_4 = 60 \bmod 26$

$x_4 = 8 \bmod I = I$

$x_5 = 3(23-5) \bmod 26$

$x_5 = 54 \bmod 26$

$x_5 = 2 = C$ Ans.

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5 14 17 2 4

FORCE

Encryption.

K (19,4)

$y_1 = (19.5+4) \bmod 26$

$y_1 = 99 \bmod 26$

$y_1 = 21 = V$

$y_2 = (19.14+4) \bmod 26$

$y_2 = 270 \bmod 26$

$y_2 = 10 = K$

$y_3 = (19.17+4) \bmod 26$

$y_3 = 327 \bmod 26$

$y_3 = 15 = P$

$y_4 = (19.2+4) \bmod 26$

$y_4 = 42 \bmod 26$

$y_4 = 16 = Q$

$y_5 = (19.4+9) \bmod 26$

$y_5 = 80 \bmod 26$

$y_5 = 2 = B$

K (9,5)

$y_1 = (9.5+5) \bmod 26$

$y_1 = (49+1) \bmod 26$

$y_1 = 23+1 = 24 = Y$

$y_2 = (9.14+5) \bmod 26$

$y_2 = (130+1) \bmod 26$

$y_2 = 1 \bmod 26 = B$

$y_3 = ((9.17)+5) \bmod 26$

$y_3 = 158 \bmod 26$

$y_3 = 2 = C$

$y_4 = (9.2+5) \bmod 26$

$y_4 = 23 \bmod 26$

$y_4 = 23 = X$

$y_5 = (9.4+5) \bmod 26$

$y_5 = 41 \bmod 26$

$y_5 = 15 = P$

So, encrypt value are

V K P Q B

Y B C X P

Decryption.

$x = a^{-1}(y-b) \bmod 26$

V	K	P	Q	B
↓	↓	↓	↓	↓
21	10	15	16	2

Y	B	C	X	P
↓	↓	↓	↓	↓
24	1	2	23	15

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- K(19,4)

$a^{-1} \bmod 26$

$19^{-1} \bmod 26$

$11 \bmod 26$

$x_1 = 11(21)-4 \bmod 26$

$x_1 = 187 \bmod 26$

$x_1 = 5 = F$

$x_2 = 11(10-4) \bmod 26$

$x_2 = 66 \bmod 26$

$x_2 = 14 \bmod 26 = O$

$x_3 = 11(15-4) \bmod 26$

$x_3 = 121 \bmod 26$

$x_3 = 17 = R$

$x_4 = 11(25-4) \bmod 26$

$x_4 = 132 \bmod 26$

$x_4 = 2 = C$

$x_5 = 11(2-4) \bmod 26$

$x_5 = -22 \bmod 26$

$x_5 = 4 = E$

FORCE

Proved

- K(9,5)

$a^{-1} \bmod 26$

$9^{-1} \bmod 26$

$= 3 \bmod 26$

$x_1 = 3(24-5) \bmod 26$

$x_1 = 57 \bmod 26$

$x_1 = 5 \bmod 26$

$x_1 = 5 = F$

$x_2 = 3(1-5) \bmod 26$

$x_2 = -12 \bmod 26$

$x_2 = 14 = O$

$x_3 = 3(2-5) \bmod 26$

$x_3 = -9 \bmod 26$

$x_4 = 17 \bmod 26 = R$

$x_4 = 3(23-5) \bmod 26$

$x_4 = 54 \bmod 26$

$x_4 = 2 = C$

$x_5 = 3(15-5) \bmod 26$

$x_5 = 30 \bmod 26$

$x_5 = 4 = E$

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Section Two

(Combination)

Q: How many three digit numbers Can be formed from the Six digits 2, 3, 4, 5, 7, and 9 with replacement.

Sol:

TOTAL Digit = 6

Order Set = 3

$n^6 = 6^3 = 216$

Thus we Can form 216 three digit Numbers

Q2:

How many three digits Numbers Can be formed from the Six digits 2, 3, 4, 5, 7 and 9 without replacement.

total Numbers = n = 6

order = r = 3

Replacement Not allowed.

$$\frac{n!}{(n-6)!} = \frac{6!}{6-3!} = \frac{6!}{3!}$$

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$$= \frac{6!}{3!} = \frac{6 \times 5 \times 4 \times \cancel{3!}}{\cancel{3!}}$$

$$= 120$$

Thus, we can form 120 three digit numbers without replacement.

Q3: A box contain 10 different colored light bulbs. Find the number of ordered sample of size 3 with replacement.

Sol:

Total bulbs $= n = 10$

Order $= r = 3$

As Replication is not allowed

$$n^r = 10^3 = 1000$$

Thus, There will be 1000 different combinations of order 3 with replacement.

~End~