Probability:-

Quantitative way of counting

Permutation:-

→ BYTES ordered

r- Permutations

$$^nP_r = P(n,r) = \frac{n!}{(n-r)!}$$

way to choose 1 object $= n$

" " " 2nd object $= n-1$

" " " 3rd object $= n-2$

" " " 4th object $= n-3$

" " " rth object $= n-(r-1)$

$= n-(r-1)$

$= n-r+1$

$$^nP_r = n \times (n-1) \times (n-2) \times (n-3) \times \cdots (n-r+1)$$

$$^nP_r = n \times (n-1) \times (n-2) \times (n-3) \times \text{---} \times (n-r+1) \times \frac{(n-r)!}{(n-r)!}$$

$$^nP_r = \frac{n!}{(n-r)!}$$

Multiply and divide by $(n-r)!$

$$^8P_3 = 8 \times 7 \times 6 \times 5 \times 4 \times \frac{3!}{3!} = \frac{8!}{3!} \qquad \frac{(8-5)!}{(8-5)!}$$

$$^5P_2 = \frac{5!}{(5-2)!} = \frac{5 \times 4 \times 3!}{3!} = 20$$

How many permutation when $n=7$ and $r=4$

$$^7P_4 = \frac{7!}{(7-4)!} = \frac{7 \times 6 \times 5 \times 4 \times \cancel{3!}}{\cancel{3!}} = 840$$

How many arrangements in row of no more than three letters for word NETWORK.

$^7P_3 + {^7P_2} + {^7P_1} = 259$

$210 \quad 42 \quad 7$

Pigeon hole Principle:-

If 'n' pigeons fly into 'n' pigeon holes and $n > m$ than at least one pigeon hole must contain two or more pigeons.

let $A = \{1, 2, 3, 4, 5, 6, 7, 8\}$

(a) How many integers must be selected from A if we want a pair of integers having sum = 9.

$(1, 8), (2, 7), (3, 6), (4, 5)$

→ Ordered Selection (Permutation)

→ Unordered Selection (Combinations)

Combinations are always less the permutations.

$AB = BA \rightarrow$ Combinations

$AB \neq BA \rightarrow$ Permutations

$$^nC_r = \binom{n}{r}$$

$$^nC_r = \frac{n!}{(n-r)!\,r!}$$

How many unordered relations of two (or) elements from set $\{0, 1, 2, 3\}$

$$\binom{4}{2} = \frac{4!}{(4-2)!\,2!} = \frac{4 \times 3 \times \cancel{2!}}{(4-2)!\,\cancel{2!}} = \frac{12}{2!} = 6$$

How many ordered Selections of two from set $\{0,1,2,3\}$

$${}^nP_r = \frac{n!}{(n-r)!} = \frac{4!}{2!} = \frac{4\times3\times2!}{2!} = 12$$

Ordered Selection:

$S = \{0,1,2,3\}$

$\mathcal{E} = \left\{\begin{matrix}01,02,03,10,12,13\\20,21,23,30,31,32\end{matrix}\right\} \rightarrow$ Permutations

Unordered Selection

$\mathcal{E}_2 = \{01,02,03,12,13,23\} \rightarrow$ Combinations

$S = \{A,B,C,D\}$

Orderd Selections-

$n=4,\ r=2$

r- Permutation

$${}^4P_2 = \frac{4!}{(4-2)!} = \frac{4\times3\times\cancel{2!}}{\cancel{2!}} = 12$$

$\{AB, AC, AD, BA, BC, BD, CA, CB, \cancel{CC}, CD, DA, DB, DC, \cancel{DD}\}$

Un Ordered Selection

r- Combination

$${}^4C_2 = \frac{4!}{(4-2)!\,2!} = \frac{4\times3\times\cancel{2!}}{2!\,\cancel{2!}} = \frac{4\times3}{2\times1} = \frac{\cancel{12}\,6}{\cancel{2}} = 6$$

$n=12,\ r=5$

$$\frac{\binom{10}{3}+\binom{10}{5}}{\binom{12}{5}}$$

$$= \frac{\text{occuring event}}{\text{Total event}}$$

Suppose two members of a 12 person group insist on working together. Select a team containing either both or ~~none~~. How many 5-person teams are possible.

Pascal's Formula:-

$$\binom{n+1}{r} = \binom{n}{r-1} + \binom{n}{r}$$

$\binom{6}{2} = 15$ $\quad \binom{7}{3} \binom{7}{4} \binom{7}{5}$

$\binom{6}{3} = 20$ $\quad \binom{n+1}{r} = \binom{n}{r-1} + \binom{n}{r}$

$\binom{6}{4} = 15$ $\quad \binom{7}{3} = \binom{6}{2} + \binom{6}{3}$

$\binom{6}{5} = 6$ $\quad = 15 + 20 = 35$

$$\binom{7}{5} = \binom{6}{4} + \binom{6}{5}$$

$$= 15 + 6 = 21$$

Binomial Theorem:-

$(a+b)^2 = a^2 + 2ab + b^2$

$(a+b)^n = a^n + \binom{n}{1}$

$$(a+b)^n = a^n + \binom{n}{1} a^{n-1} b + \binom{n}{2} a^{n-2} b^2 + \text{------}$$

$$\text{------} + \binom{n}{n-1} a \cdot b^{n-1} + b^n$$

$$(a+b)^3 = a^3 + \binom{3}{1} a^{3-1} b + \binom{3}{2} a^{3-2} b^2 + \text{------}$$

$$\text{------} + \binom{3}{3-1} a \cdot b^{3-1} + b^3$$

$$= a^3 + 3a^2 b + 3ab^2 + b^3$$

$$\binom{n}{r} = \binom{n}{n-r}$$

$$\binom{10}{5} = \binom{10}{10-5}$$

$$\binom{10}{5} = \binom{10}{5}$$

find the coefficient of term using binomial expression x^6y^3 in $(x+y)^9$

$\binom{9}{3} x^{9-3} y^3$ $\binom{n}{3} x^{n-3} y^3$

$x^8 . y$ $(x+y)^9$

$\binom{9}{1} x^{9-1} y$