# Sequence & Series

## Sequences:

A particular order in which related things follow each other. or a set of related things.

- ⇒ Geometric Progression
- ⇒ Arithmetic sequence

} Types

① Find the Sequences Given by Explicit formula.

1- $a_k = \dfrac{k}{k+1} \quad k \geq 1$

$$a_1 = \frac{1}{1+1} \Rightarrow \frac{1}{2}$$

$$a_2 = \frac{2}{2+1} \Rightarrow \frac{2}{3}$$

$$a_3 = \frac{3}{3+1} \Rightarrow \frac{3}{4}$$

$$a_4 = \frac{4}{4+1} \Rightarrow \frac{4}{5}$$

$$\frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \frac{4}{5}$$

(2) $b_i = \frac{i-1}{i}$   $i \geq 2$

$b_2 = \frac{2-1}{2} \Rightarrow \frac{1}{2}$

$b_3 = \frac{3-1}{3} \Rightarrow \frac{2}{3}$

$b_4 = \frac{4-1}{4} \Rightarrow \frac{3}{4}$

$\frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \ldots$

Alternative method.

Compute first five terms of Sequence

$C_j = (-1)^j$   $j \geq 0$

$C_0 = (-1)^0 \Rightarrow 1$

$C_1 = (-1)^1 \Rightarrow -1$

$C_2 = (-1)^2 \Rightarrow 1$

$C_3 = (-1)^3 \Rightarrow -1$

$C_4 = (-1)^4 \Rightarrow 1$

$1, -1, 1, -1, 1$

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find the explicit formula to fit the given initial terms

$1, \frac{-1}{4}, \frac{1}{9}, -\frac{1}{16}, \frac{1}{25} + \ldots$

as we want to find explicit formula.

| index | 1 | 2 | 3 | 4 | 5 |
| --- | --- | --- | --- | --- | --- |
| Value | 1 | $\frac{-1}{4}$ | $\frac{1}{9}$ | $\frac{-1}{16}$ | $\frac{1}{25}$ |

$$a_k = \frac{(-1)^{k+1}}{k^2}$$

2)

2, 6, 12, 20, 30, 42

| index | 1 | 2 | 3 | 4 | 5 | 6 |
| --- | --- | --- | --- | --- | --- | --- |
| Value | 2 | 6 | 12 | 20 | 30 | 42 |
| | 1x2 | 2x3 | 3x4 | 4x5 | 5x6 | 6x7 |

$$a_k = k(k+1) \qquad k \geq 1$$

: Types :

i) Arithmetic Sequences

ii) Geomtric Sequences.

# Arithmetic Sequences.

A sequence in which each term is achieved by adding a constant term in previous one.

Example:

1, 2, 3, 4, 5 ....

Formula:

$$\underbrace{a_n}_{\text{find term}} = \underbrace{a}_{\text{first term}} + (\overset{\text{index}}{n}-1)\underset{\text{Differance}}{d}$$

Example:

find 20th term of Sequence.

$S = 3, 9, 15, 21, \ldots$

$a = 3$

$d = 9 - 3 \Rightarrow 6$

$n = 20 \qquad a_{20} = ?$

$a_{20} = a + (n-1)d$

$3 + (20-1)6$

$3 + 19 \times 6 \Rightarrow 117$

(2)

find the 36th term of A.P whose 3rd term is 7 & 8th term is 17

$a_{36} = ?$

$a_3 = 7$

$a_8 = 17$

$a_3 = a + (3-1)d$

$7 = a + 2d$ — i)

$a_8 = a + (8-1)d$

$17 = a + 7d$ — ii)

Subtract (i) & (ii)

$$\begin{aligned} 7 &= \cancel{a} + 2d \\ \pm 17 &= \pm \cancel{a} \pm 7d \\ \hline -10 &= -5d \end{aligned}$$

$\boxed{d = 2}$

put $d = 2$ in (i)

$7 = a + 2(2)$

$7 = a + 4$

$7 - 4 = a$

$\boxed{a = 3}$

$a_{36} = a + (36-1)d$

$3 + 35 \times 2$

$a_{36} = 73$

## Geometic Sequence

A sequence in which each term except first term achieved by multipling previous term with a constant.

Formula:

$$a_n = ar^{n-1} \quad \text{for all integers } n \geq 1$$

Example:

Find 8th term of Sequence by G.S

4, 12, 36, 108

$a_n = ar^{n-1}$

$a_8 = ?$

$a = 4 \Rightarrow$ initial term

$r = \frac{12}{4} = 3 \Rightarrow$ Comon ratio

$n =$ term

$a_8 = 4(3)^{8-1}$

$4(3)^7$

$4(2187) \Rightarrow 8748$

(Example No 2)

which term of A.P is 1/8 if its 1st term is 4 and $r = \frac{1}{2}$.

$n =$ find $= ?$

$a_n = \frac{1}{8}$

$a_n = ar^{n-1}$

$\frac{1}{8} = 4\left(\frac{1}{2}\right)^{n-1} \Rightarrow \frac{1}{8 \times 4} = \left(\frac{1}{2}\right)^{n-1}$

$\frac{1}{32} = \left(\frac{1}{2}\right)^{n-1}$

$\left(\frac{1}{2}\right)^5 = \left(\frac{1}{2}\right)^{n-1} \Rightarrow 5 = n-1$

$\boxed{n = 6}$

Example No 3

find the G.P with terms whose 2nd term is 9 & 4th term is one.

$a_2 = 9$

$a_4 = 1$

$a_2 = ar^{2-1}$

$a_2 = ar$ — (i)

$9 = ar$

$r = \frac{9}{a}$

$a_4 = ar^{4-1}$

$= ar^3$

put value of r

$1 = a\left[\frac{9}{a}\right]^3$

put value of a

$r = \frac{9}{27}$

$r = \frac{1}{3}$

$1 = \cancel{a}\,\frac{(9)^3}{a^{\cancel{3}2}}$

$a^2 = 9^3$

$a^2 = 729$

$a = 27$

$a = ar^{n-1}$

$27\left(\frac{1}{3}\right)$

## Series:-

Sum of terms of Sequence is called Series.

⇒ we just use (+) sign.

Representation:

use summation notation / Sigma.

$$\sum_{i=0}^{3} (2i-1) = [2(1)-1]+[2(2)-1]+[2(3)-1].$$
$$= 1+3+5 \Rightarrow 9$$

### :Propertise:

i) $\sum_{k=m}^{n} c \cdot a_k = c \sum_{k=m}^{n} a_k$

ii- $\sum_{k=1}^{n} c = n \cdot c$

iii) $\sum_{k=n}^{n} (a_k + b_k) = \sum_{k=n}^{n} a_k + \sum_{k=m}^{n} b_k$

### :Types:

* Arithmetic Series
* Geometric Series

## Arithmetic Series

Sum of arithmetic sequences is arithmetic series.

formula.

$$= \frac{n}{2}\left[2a + (n-1)d\right]$$

or

$$\frac{n}{2}(\underset{\text{initial term}}{\underline{a}} + \underset{\text{final term}}{\underline{a_n}})$$

Examples:

$S = 1+2+3+4+\cdots$

$n = 50$

$\frac{n}{2}\left[2a + (n-1)d\right]$

$a = 1 \quad d = 2-1 = 1$

$n = 50$

$\frac{50}{2}\left(2(1) + (50-1)1\right)$

$25(2+49)$

$25(51) \Rightarrow 1275$

Example : 2

$S_{10} = ?$

$n = 10$

$S = 1+2+3+4\ldots$

$\frac{n}{2}(2a+(n-1)d)$

$\frac{10}{2}(2(1)+(9)(1))$

$5\ (2+9)$

$5(11) \Rightarrow 55$

Example:

$S = 3+9+15+21$

$S_{10} = ?$

$a = 3 \quad d = 6$ ∴ not correct

$\frac{10}{2}(2(3)+(10-1)6)$

$5(6+(9)(6))$

$5(6+54)$

$5 \times 60 \Rightarrow 300$

⇒ by using other formula:

$= \frac{10}{2}(3+21)$ Correct

$= 5(24) \Rightarrow 120$

Example No 3:

$1+4+7+10+13$

$a = 1 \qquad S_8 = ?$

$d = 3$

$= \frac{n}{2}(2a+(n-1)d)$

$\frac{8}{2}(2(1)+(8-1)(3))$

$4(2+7(3))$

$4(2+21) \Rightarrow 23 \times 4$

92 Ans

## Geometric Series:

Sum of geometric sequence is

$ar^0 + ar^1 + ar^2 + \cdots\cdots + ar^{n-1}$

$\sum_{i=0}^{n-1} ar^i =$ Sigma notation

formula:

$$S = \frac{a(r^n - 1)}{r-1}$$

Example No 1:

$2 + 2^2 + 2^3 + 2^4 + \cdots\cdots$

$a = 2 \quad n = 10$

$r = 2$

$S_{10} = ?$

$$S_{10} = \frac{2(2^{10}-1)}{2-1}$$

$$\frac{2(1023)}{1} \Rightarrow 2046$$

Example No 2:

$4+12+36+108+\cdots$

$S_6=?$

$a=4$ , $r=\frac{12}{4}=3$ , $n=6$

$$S_6=\frac{4(3^6-1)}{3-1}$$

$$\frac{4(728)}{2}\Rightarrow 1456$$

Example No 3:

$\frac{9}{4}+\frac{3}{2}+1+\frac{2}{3}$

$a=\frac{9}{4}$ $r=\frac{\cancel{3}/\cancel{2}}{\cancel{9}3/\cancel{4}12}\Rightarrow\frac{2}{3}$ , $n=6$

$S_6=?$

$$S_6=\frac{\frac{9}{4}\left[\left(\frac{2}{3}\right)^6-1\right]}{\frac{2}{3}-1}\Rightarrow\frac{665}{108}\text{ Ans}$$