Eigen Values & Vectors

$$A = \begin{bmatrix} 2 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 2 \end{bmatrix}$$ (Question solved in the class)

$$|A - \lambda I| = 0$$

$$\left| \begin{bmatrix} 2 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 2 \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \right| = 0$$

$$\left| \begin{bmatrix} 2 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 2 \end{bmatrix} - \begin{bmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{bmatrix} \right| = 0$$

$$\left| \begin{bmatrix} 2-\lambda & 2 & 1 \\ 1 & 3-\lambda & 1 \\ 1 & 2 & 2-\lambda \end{bmatrix} \right| = 0$$

$$(2-\lambda) \begin{vmatrix} 3-\lambda & 1 \\ 2 & 2-\lambda \end{vmatrix} - 2 \begin{vmatrix} 1 & 1 \\ 1 & 2-\lambda \end{vmatrix} + 1 \begin{vmatrix} 1 & 3-\lambda \\ 1 & 2 \end{vmatrix} = 0$$

$$(2-\lambda)\left[(3-\lambda)(2-\lambda) - 2\right] - 2(2-\lambda-1) + 1(2-3+\lambda) = 0$$

$$(2-\lambda)(6-5\lambda+\lambda^2-2) - 2(1-\lambda) + 1(-1+\lambda) = 0$$

$$(2-\lambda)(\lambda^2-5\lambda+4) - 2(1-\lambda) + 1(\lambda-1) = 0$$

$$2\lambda^2 - 10\lambda + 8 - \lambda^3 + 5\lambda^2 - 4\lambda - 2 + 2\lambda + \lambda - 1 = 0$$

$$-\lambda^3 + 7\lambda^2 - 11\lambda + 5 = 0$$

$\boxed{\lambda = 1, 1, 5}$

↳ Eigen values

	-1	7	-11	5
1	↓	-1	6	-5
	-1	6	-5	0

$-\lambda^2 + 6\lambda - 5 = 0$

$\lambda^2 - 6\lambda + 5 = 0$

~~$\lambda^2 - 3\lambda - 2\lambda + 5 = 0$~~

$\lambda^2 - 5\lambda - \lambda + 5 = 0$

$\lambda(\lambda - 5) - 1(\lambda - 5) = 0$

$(\lambda - 5)(\lambda - 1) = 0$

$\lambda = 1, 5$

~~Put $\lambda = 5$~~

$$\begin{bmatrix} 2-\lambda & 2 & 1 \\ 1 & 3-\lambda & 1 \\ 1 & 2 & 2-\lambda \end{bmatrix} = 0 \longrightarrow (A)$$

Put $\lambda = 5$

$$\begin{bmatrix} -3 & 2 & 1 \\ 1 & -2 & 1 \\ 1 & 2 & -3 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = 0$$

$-3v_1 + 2v_2 + v_3 = 0$

$v_1 - 2v_2 + v_3 = 0$

$v_1 + 2v_2 - 3v_3 = 0$

Using Gauss's Elimination Method

$$[A|B] = \left[\begin{array}{ccc|c} -3 & 2 & 1 & 0 \\ 1 & -2 & 1 & 0 \\ 1 & 2 & -3 & 0 \end{array}\right]$$

- $R_2 + \frac{1}{3}R_1$
- $R_3 + \frac{1}{3}R_1$

$$[A|B] = \left[\begin{array}{ccc|c} -3 & 2 & 1 & 0 \\ 0 & -4/3 & 4/3 & 0 \\ 0 & 8/3 & -8/3 & 0 \end{array}\right]$$

- $R_3 + 2R_2$

$$[A|B] = \left[\begin{array}{ccc|c} -3 & 2 & 1 & 0 \\ 0 & -4/3 & 4/3 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$$-3v_1 + 2v_2 + v_3 = 0 \rightarrow (I)$$

$$-\frac{4}{3}v_2 + \frac{4}{3}v_3 = 0 \rightarrow (II)$$

From (II)

$$-\frac{4}{3}v_2 = -\frac{4}{3}v_3$$

$$v_2 = v_3$$

$$v_2 = v_3 = a \qquad \text{let } v_2 = a$$

eq (I) becomes

$$-3v_1 + 2a + a = 0$$

$$-3v_1 = -3a$$

$$v_1 = a$$

$$v = \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = \begin{bmatrix} a \\ a \\ a \end{bmatrix} = a\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

For $\lambda = 5$

$$\begin{bmatrix} 1 & 1 & 1 \end{bmatrix}^t$$

Scanned with CamScanner

For $\lambda = 1$

put $\lambda = 1$ in eq. (A)

$$\begin{bmatrix} 1 & 2 & 1 \\ 1 & 2 & 1 \\ 1 & 2 & 1 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = 0$$

$v_1 + 2v_2 + v_3 = 0$

$v_1 + 2v_2 + v_3 = 0$

$v_1 + 2v_2 + v_3 = 0$

Using Gauss Elimination Method

$$[A|B] = \left[\begin{array}{ccc|c} 1 & 2 & 1 & 0 \\ 1 & 2 & 1 & 0 \\ 1 & 2 & 1 & 0 \end{array}\right]$$

- $R_2 - R_1$
- $R_3 - R_1$

$$[A|B] = \left[\begin{array}{ccc|c} 1 & 2 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$v_1 + 2v_2 + v_3 = 0$

$\therefore v_3 = a$

$v_2 = b$

$v_1 + 2b + a = 0$

$v_1 = -a - 2b$

$v_1 = -(a + 2b)$

Eigen values = 1, 1, 5

- For $\lambda = 1$

$$\begin{bmatrix} -(a+2b) & b & a \end{bmatrix}^t$$

- For $\lambda = 1$

$$\begin{bmatrix} -(a+2b) & b & a \end{bmatrix}^t$$

- For $\lambda = 5$

$$\begin{bmatrix} 1 & 1 & 1 \end{bmatrix}^t$$

$$V = \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = \begin{bmatrix} -(a+2b) \\ b \\ a \end{bmatrix}$$

For

$\lambda = 1$

$$\begin{bmatrix} -(a+2b) & b & a \end{bmatrix}^t$$

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① $\begin{bmatrix} 3 & 1 & 1 \\ 2 & 4 & 2 \\ 1 & 1 & 3 \end{bmatrix}$

$$A = \begin{bmatrix} 3 & 1 & 1 \\ 2 & 4 & 2 \\ 1 & 1 & 3 \end{bmatrix}$$

$$|A - \lambda I| = 0$$

$$\left| \begin{bmatrix} 3 & 1 & 1 \\ 2 & 4 & 2 \\ 1 & 1 & 3 \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \right| = 0$$

$$\left| \begin{bmatrix} 3 & 1 & 1 \\ 2 & 4 & 2 \\ 1 & 1 & 3 \end{bmatrix} - \begin{bmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{bmatrix} \right| = 0$$

$-8+40 \cdot 56 \cdot 24$

$$\left|\begin{bmatrix} 3-\lambda & 1 & 1 \\ 2 & 4-\lambda & 2 \\ 1 & 1 & 3-\lambda \end{bmatrix}\right| = 0$$

$$(3-\lambda)\begin{vmatrix} 4-\lambda & 2 \\ 1 & 3-\lambda \end{vmatrix} - 1\begin{vmatrix} 2 & 2 \\ 1 & 3-\lambda \end{vmatrix} + 1\begin{vmatrix} 2 & 4-\lambda \\ 1 & 1 \end{vmatrix} = 0$$

$(3-\lambda)[(4-\lambda)(3-\lambda)-2] - 1(6-2\lambda-2) + 1(2-4+\lambda) = 0$

$(3-\lambda)(12-7\lambda+\lambda^2-2) - (4-2\lambda) + (-2+\lambda) = 0$

$(3-\lambda)(\lambda^2-7\lambda+10) - (4-2\lambda) + (\lambda-2) = 0$

$3\lambda^2 - 21\lambda + 30 - \lambda^3 + 7\lambda^2 - 10\lambda - 4 + 2\lambda + \lambda - 2 = 0$

$-\lambda^3 + 10\lambda^2 - 28\lambda + 24 = 0$

	-1	10	-28	24
2	↓	-2	16	-24
	-1	8	-12	0

$\boxed{\lambda = 2, 2, 6}$

↳ Eigen Values

$-\lambda^2 + 8\lambda - 12 = 0$

$\lambda^2 - 8\lambda + 12 = 0$

$\lambda^2 - 6\lambda - 2\lambda + 12 = 0$

$\lambda(\lambda-6) - 2(\lambda-6) = 0$

$\lambda = 2, 6$

For $\lambda = 6$

$$\begin{bmatrix} 3-\lambda & 1 & 1 \\ 2 & 4-\lambda & 2 \\ 1 & 1 & 3-\lambda \end{bmatrix} = 0$$

put $\lambda = 6$

$$\begin{bmatrix} -3 & 1 & 1 \\ 2 & -2 & 2 \\ 1 & 1 & -3 \end{bmatrix}\begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = 0$$

Scanned with CamScanner

$$-3v_1 + v_2 + v_3 = 0 \rightarrow (\mathrm{I})$$
$$2v_1 - 2v_2 + 2v_3 = 0 \rightarrow (\mathrm{II})$$
$$v_1 + v_2 - 3v_3 = 0 \rightarrow (\mathrm{III})$$

Using Gauss Elimination

$$[A|B] = \left[\begin{array}{ccc|c} -3 & 1 & 1 & 0 \\ 2 & -2 & 2 & 0 \\ 1 & 1 & -3 & 0 \end{array}\right]$$

- $\underline{R_2 + \frac{2}{3} R_1}$
- $\underline{R_3 + \frac{1}{3} R_1}$

$$[A|B] = \left[\begin{array}{ccc|c} -3 & 1 & 1 & 0 \\ 0 & -4/3 & 8/3 & 0 \\ 0 & 4/3 & -8/3 & 0 \end{array}\right]$$

- $\underline{R_3 + R_2}$

$$[A|B] = \left[\begin{array}{ccc|c} -3 & 1 & 1 & 0 \\ 0 & -4/3 & 8/3 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$$-3v_1 + v_2 + v_3 = 0 \rightarrow (\mathrm{I})$$
$$-\frac{4}{3} v_2 + \frac{8}{3} v_3 = 0 \rightarrow (\mathrm{II})$$
$$-\frac{4}{3}(v_2 - 2v_3) = 0$$
$$v_2 - 2v_3 = 0$$

let
$\therefore V_3 = a$

$$v_2 = 2v_3$$
$$\boxed{v_2 = 2a}$$

eq. (I)

$$-3v_1 + 2a + a = 0$$
$$\boxed{v_1 = a}$$

$$V = \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = \begin{bmatrix} a \\ 2a \\ a \end{bmatrix} = a \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}$$

For $\lambda = 6$

$$[1 \quad 2 \quad 1]^t$$

Now, put $\lambda = 2$

$$\begin{bmatrix} 1 & 1 & 1 \\ 2 & 2 & 2 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = 0$$

$v_1 + v_2 + v_3 = 0$

$2v_1 + 2v_2 + 2v_3 = 0$

$v_1 + v_2 + v_3 = 0$

Using Gauss Elimination

$$[A|B] = \left[\begin{array}{ccc|c} 2 & 2 & 2 & 0 \\ 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 0 \end{array}\right]$$

- $R_2 - \frac{1}{2} R_1$
- $R_3 - \frac{1}{2} R_1$

$$[A|B] = \left[\begin{array}{ccc|c} 2 & 2 & 2 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$$2v_1 + 2v_2 + 2v_3 = 0$$

$$v_1 + v_2 + v_3 = 0$$

$$\therefore \quad v_3 = a, \quad v_2 = b$$

$$v_1 + b + a = 0$$

$$v_1 = -(a+b)$$

$$V = \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = \begin{bmatrix} -(a+b) \\ b \\ a \end{bmatrix}$$

For $\lambda = 1$

$$[-(a+b) \quad b \quad a]^t$$

Eigen Values $= 1, 1, 6$

- For $\lambda = 1$

  $$[-(a+b) \quad b \quad a]^t$$

- For $\lambda = 1$

  $$[-(a+b) \quad b \quad a]^t$$

- For $\lambda = 6$

  $$[1 \quad 2 \quad 1]^t$$

(2) $\begin{bmatrix} 1 & 2 & 2 \\ 1 & 2 & -1 \\ -1 & 1 & 4 \end{bmatrix}$

$$A = \begin{bmatrix} 1 & 2 & 2 \\ 1 & 2 & -1 \\ -1 & 1 & 4 \end{bmatrix}$$

$$|A - \lambda I| = 0$$

$$\left| \begin{bmatrix} 1 & 2 & 2 \\ 1 & 2 & -1 \\ -1 & 1 & 4 \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \right| = 0$$

$$\left| \begin{bmatrix} 1 & 2 & 2 \\ 1 & 2 & -1 \\ -1 & 1 & 4 \end{bmatrix} - \begin{bmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{bmatrix} \right| = 0$$

$$\left| \begin{bmatrix} 1-\lambda & 2 & 2 \\ 1 & 2-\lambda & -1 \\ -1 & 1 & 4-\lambda \end{bmatrix} \right| = 0$$

$$(1-\lambda)\begin{vmatrix} 2-\lambda & -1 \\ 1 & 4-\lambda \end{vmatrix} - 2\begin{vmatrix} 1 & -1 \\ -1 & 4-\lambda \end{vmatrix} + 2\begin{vmatrix} 1 & 2-\lambda \\ -1 & 1 \end{vmatrix} = 0$$

$$(1-\lambda)[(2-\lambda)(4-\lambda)+1] - 2(4-\lambda-1) + 2(1+2-\lambda) = 0$$

$$(1-\lambda)(8-6\lambda+\lambda^2+1) \cancel{-2(3-\lambda)} + \cancel{2(3-\lambda)} = 0$$

$$(1-\lambda)(\lambda^2-6\lambda+9) = 0$$

$$\lambda^2 - 6\lambda + 9 - \lambda^3 + 6\lambda^2 - 9\lambda = 0$$

$$-\lambda^3 + 7\lambda^2 - 15\lambda + 9 = 0$$

$\boxed{\lambda = 1, 3, 3}$

$\hookrightarrow$ Eigen values

$$\begin{array}{c|cccc} & -1 & 7 & -15 & 9 \\ 1 & \downarrow & -1 & 6 & -9 \\ \hline & -1 & 6 & -9 & 0 \end{array}$$

$$-\lambda^2 + 6\lambda - 9 = 0$$
$$\lambda^2 - 6\lambda + 9 = 0$$
$$\lambda^2 - 3\lambda - 3\lambda + 9 = 0$$
$$\lambda(\lambda - 3) - 3(\lambda - 3) = 0$$
$$\lambda = 3, 3$$

$$\begin{bmatrix} 1-\lambda & 2 & 2 \\ 1 & 2-\lambda & -1 \\ -1 & 1 & 4-\lambda \end{bmatrix} = 0$$

put $\lambda = 3$

$$\begin{bmatrix} -2 & 2 & 2 \\ 1 & -1 & -1 \\ -1 & 1 & 1 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = 0$$

$$-2v_1 + 2v_2 + 2v_3 = 0 \rightarrow (\mathrm{I})$$
$$v_1 - v_2 - v_3 = 0 \rightarrow (\mathrm{II})$$
$$-v_1 + v_2 + v_3 = 0 \rightarrow (\mathrm{III})$$

Using Gauss Elimination

$$[A|B] = \left[\begin{array}{ccc|c} -2 & 2 & 2 & 0 \\ 1 & -1 & -1 & 0 \\ -1 & 1 & 1 & 0 \end{array}\right] \begin{array}{l} \cdot R_2 + \frac{1}{2}R_1 \\ \cdot R_3 - \frac{1}{2}R_1 \end{array}$$

Scanned with CamScanner

$$[A|B] = \left[\begin{array}{ccc|c} -2 & 2 & 2 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$$-2v_1 + 2v_2 + 2v_3 = 0$$

$$\Leftrightarrow v_1 - v_2 - v_3 = 0$$

$$\therefore v_3 = a, \quad v_2 = b$$

$$v_1 - b - a = 0$$

$$v_1 = a + b$$

$$v = \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = \begin{bmatrix} a+b \\ b \\ a \end{bmatrix}$$

For $\lambda = 3$

$$[a+b \quad b \quad a]^t$$

put $\lambda = 1$

$$\begin{bmatrix} 0 & 2 & 2 \\ 1 & 1 & -1 \\ -1 & 1 & 3 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = 0$$

$$2v_2 + 2v_3 = 0 \longrightarrow (\mathrm{I})$$

$$v_1 + v_2 - v_3 = 0 \longrightarrow (\mathrm{I})$$

$$-v_1 + v_2 + 3v_3 = 0 \longrightarrow (\mathrm{III})$$

Using Gauss Elimination

$$[A|B] = \left[\begin{array}{ccc|c} 1 & 1 & -1 & 0 \\ 0 & 2 & 2 & 0 \\ -1 & 1 & 3 & 0 \end{array}\right] \quad \cdot R_3 + R_1$$

$$[A|B] = \left[\begin{array}{ccc|c} 1 & 1 & -1 & 0 \\ 0 & 2 & 2 & 0 \\ 0 & 2 & 2 & 0 \end{array}\right] \quad \cdot R_3 - R_2$$

$$[A|B] = \left[\begin{array}{ccc|c} 1 & 1 & -1 & 0 \\ 0 & 2 & 2 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$$v_1 + v_2 - v_3 = 0 \rightarrow (\mathrm{I})$$

$$2v_2 + 2v_3 = 0 \rightarrow (\mathrm{II})$$

$$v_2 + v_3 = 0$$

$$v_2 = -v_3 \qquad \therefore \text{let } v_3 = a$$

$$v_2 = -a$$

Eq (I)

$$v_1 - a - a = 0$$

$$v_1 = -2a$$

$$v = \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = \begin{bmatrix} -2a \\ -a \\ a \end{bmatrix} = a\begin{bmatrix} -2 \\ -1 \\ 1 \end{bmatrix}$$

For $\lambda = 1$

$$[-2 \quad -1 \quad 1]^t$$

Eigen values $= 1, 3, 3$

- For $\lambda = 1$

$$[-2 \quad -1 \quad 1]^t$$

- For $\lambda = 3$

$$[a+b \quad b \quad a]^t$$

- For $\lambda = 3$

$$[a+b \quad b \quad a]^t$$

(4) $\begin{bmatrix} -3 & 1 & -1 \\ -7 & 5 & -1 \\ -6 & 6 & -2 \end{bmatrix}$

$$A = \begin{bmatrix} -3 & 1 & -1 \\ -7 & 5 & -1 \\ -6 & 6 & -2 \end{bmatrix}$$

$$|A - \lambda I| = 0$$

$$\left| \begin{bmatrix} -3 & 1 & -1 \\ -7 & 5 & -1 \\ -6 & 6 & -2 \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \right| = 0$$

$$\left| \begin{bmatrix} -3 & 1 & -1 \\ -7 & 5 & -1 \\ -6 & 6 & -2 \end{bmatrix} - \begin{bmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{bmatrix} \right| = 0$$

$$\left| \begin{bmatrix} -3-\lambda & 1 & -1 \\ -7 & 5-\lambda & -1 \\ -6 & 6 & -2-\lambda \end{bmatrix} \right| = 0$$

$$-3-\lambda \begin{vmatrix} 5-\lambda & -1 \\ 6 & -2-\lambda \end{vmatrix} - 1 \begin{vmatrix} -7 & -1 \\ -6 & -2-\lambda \end{vmatrix} - 1 \begin{vmatrix} -7 & 5-\lambda \\ -6 & 6 \end{vmatrix} = 0$$

$$(-3-\lambda)\left[(5-\lambda)(-2-\lambda)+6\right] - 1(+14+7\lambda-6) - 1(-42+30-6\lambda) = 0$$

$$(-3-\lambda)(-10-5\lambda+2\lambda+\lambda^2+6) - (7\lambda+8) - (-12-6\lambda) = 0$$

$$(-3-\lambda)(\lambda^2-3\lambda-4) - (7\lambda+8) - (-12-6\lambda) = 0$$

$$-3\lambda^2+9\lambda+12-\lambda^3+3\lambda^2+4\lambda-7\lambda-8+12+6\lambda=0$$

$$-\lambda^3+12\lambda+16=0$$

	-1	0	12	16
-2		2	-4	-16
	-1	2	8	0

$$\boxed{\lambda = 4, -2, -2}$$

$$-\lambda^2+2\lambda+8=0$$

$$-\lambda^2+4\lambda-2\lambda+8=0$$

$$\lambda(-\lambda+4)+2(-\lambda+4)$$

$$(\lambda+2)(4-\lambda)=0$$

$$\lambda = 4, -2$$

$$\begin{bmatrix} -3-\lambda & 1 & -1 \\ -7 & 5-\lambda & -1 \\ -6 & 6 & -2-\lambda \end{bmatrix} = 0$$

Put $\lambda = 4$

$$\begin{bmatrix} -7 & 1 & -1 \\ -7 & 1 & -1 \\ -6 & 6 & -6 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} = 0$$

$-7v_1 + v_2 - v_3 = 0 \rightarrow (I)$

$-7v_1 + v_2 - v_3 = 0 \rightarrow (III)$

$-6v_1 + 6v_2 - 6v_3 = 0 \rightarrow (II)$

Using Gauss Elimination

$$[A|B] = \left[\begin{array}{ccc|c} -7 & 1 & -1 & 0 \\ -6 & 6 & -6 & 0 \\ -7 & 1 & -1 & 0 \end{array}\right]$$

- $R_2 - \frac{6}{7}R_1$
- $R_3 - R_1$

$$[A|B] = \left[\begin{array}{ccc|c} -7 & 1 & -1 & 0 \\ 0 & 36/7 & -36/7 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

Scanned with CamScanner

$-7v_1 + v_2 - v_3 = 0 \rightarrow (\mathrm{I})$

$\frac{36}{7} v_2 - \frac{36}{7} v_3 = 0 \rightarrow (\mathrm{II})$

$\therefore$ let $v_3 = a$

$v_2 = v_3$

$v_2 = a$

eq(I)

$-7v_1 + a - a = 0$

$v_1 = 0$

$$v = \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = \begin{bmatrix} 0 \\ a \\ a \end{bmatrix} = a \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$$

For $\lambda = 4$

$[0 \quad 1 \quad 1]^t$

put $\lambda = -2$

$$\begin{bmatrix} -1 & 1 & -1 \\ -7 & 7 & -1 \\ -6 & 6 & 0 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$-v_1 + v_2 - v_3 = 0 \rightarrow (\text{III})$$
$$-7v_1 + 7v_2 - v_3 = 0 \rightarrow (\text{I})$$
$$-6v_1 + 6v_2 \quad = 0 \rightarrow (\text{II})$$

Using Gauss Elimination

$$[A|B] = \left[\begin{array}{ccc|c} -7 & 7 & -1 & 0 \\ -6 & 6 & 0 & 0 \\ -1 & 1 & -1 & 0 \end{array}\right] \quad \bullet\ \underbrace{R_2 - \tfrac{6}{7}R_1} \quad \bullet\ \underbrace{R_3 - \tfrac{1}{7}R_1}$$

$$[A|B] = \left[\begin{array}{ccc|c} -7 & 7 & -1 & 0 \\ 0 & 0 & 6/7 & 0 \\ 0 & 0 & -6/7 & 0 \end{array}\right]$$

$$-7v_1 + 7v_2 - v_3 = 0 \rightarrow (\text{I})$$
$$\frac{6}{7}v_3 = 0 \rightarrow (\text{II})$$
$$v_3 = 0$$

$$\therefore v_2 = a$$

eq (I)

$$-7v_1 + 7v_2 - v_3 = 0$$
$$-7v_1 + 7a = 0$$
$$v_1 = a$$

$$V = \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = \begin{bmatrix} a \\ a \\ 0 \end{bmatrix} = a \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$$

For $\lambda = -2$

$$\begin{bmatrix} 1 & 1 & 0 \end{bmatrix}^t$$

$\lambda = 4, -2, -2$

- For $\lambda = 4$

$[0 \quad 1 \quad 1]^t$

- For $\lambda = -2$

$[1 \quad 1 \quad 0]^t$

- For $\lambda = -2$

$[1 \quad 1 \quad 0]^t$

(3) $\begin{bmatrix} 1 & -3 & 3 \\ 3 & -5 & 3 \\ 6 & -6 & 4 \end{bmatrix}$

$$A = \begin{bmatrix} 1 & -3 & 3 \\ 3 & -5 & 3 \\ 6 & -6 & 4 \end{bmatrix}$$

$$|A - \lambda I| = 0$$

$$\left| \begin{bmatrix} 1 & -3 & 3 \\ 3 & -5 & 3 \\ 6 & -6 & 4 \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \right| = 0$$

$$\left| \begin{bmatrix} 1 & -3 & 3 \\ 3 & -5 & 3 \\ 6 & -6 & 4 \end{bmatrix} - \begin{bmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{bmatrix} \right| = 0$$

$$\left|\begin{bmatrix}1-\lambda & -3 & 3\\ 3 & -5-\lambda & 3\\ 6 & -6 & 4-\lambda\end{bmatrix}\right| = 0$$

$$(1-\lambda)\begin{vmatrix}-5-\lambda & 3\\ -6 & 4-\lambda\end{vmatrix} + 3\begin{vmatrix}3 & 3\\ 6 & 4-\lambda\end{vmatrix} + 3\begin{vmatrix}3 & -5-\lambda\\ 6 & -6\end{vmatrix} = 0$$

$$(1-\lambda)[(-5-\lambda)(4-\lambda)+18] + 3(12-3\lambda-18) + 3(-18+30+6\lambda) = 0$$

$$(1-\lambda)(-20+5\lambda-4\lambda+\lambda^2+18)+3(-3\lambda-6)+3(12+6\lambda)=0$$

$$(1-\lambda)(\lambda^2+\lambda-2)+3(-3\lambda-6)+3(12+6\lambda)=0$$

$$\cancel{\lambda^2}+\lambda-2-\lambda^3-\cancel{\lambda^2}+2\lambda-9\lambda-18+36+18\lambda=0$$

$$-\lambda^3+12\lambda+16=0$$

	-1	0	12	16
-2	↓	2	-4	-16
	-1	2	8	0

$$\boxed{\lambda = 4, -2, -2}$$

↳ Eigen values

$$-\lambda^2+2\lambda+8=0$$

$$-\lambda^2+4\lambda-2\lambda+8=0$$

$$-\lambda(\lambda-4)-2(\lambda-4)=0$$

$$\lambda = 4, -2$$

$$\begin{bmatrix} 1-\lambda & -3 & 3 \\ 3 & -5-\lambda & 3 \\ 6 & -6 & 4-\lambda \end{bmatrix}$$

put $\lambda = 4$

$$\begin{bmatrix} -3 & -3 & 3 \\ 3 & -9 & 3 \\ 6 & -6 & 0 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$-3v_1 - 3v_2 + 3v_3 = 0 \rightarrow (III)$

$3v_1 - 9v_2 + 3v_3 = 0 \rightarrow (II)$

$6v_1 - 6v_2 = 0 \rightarrow (I)$

Using Gauss Elimination

$$[A|B] = \left[\begin{array}{ccc|c} 6 & -6 & 0 & 0 \\ 3 & -9 & 3 & 0 \\ -3 & -3 & 3 & 0 \end{array}\right]$$

- $R_2 - \frac{1}{2} R_1$
- $R_3 + \frac{1}{2} R_1$

$$[A|B] = \left[\begin{array}{ccc|c} 6 & -6 & 0 & 0 \\ 0 & -6 & 3 & 0 \\ 0 & -6 & 3 & 0 \end{array}\right]$$

- $R_3 - R_2$

$$[A|B] = \left[\begin{array}{ccc|c} 6 & -6 & 0 & 0 \\ 0 & -6 & 3 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$6v_1 - 6v_2 = 0 \rightarrow (I)$

$-6v_2 + 3v_3 = 0 \rightarrow (II)$

$-6v_2 = -3v_3$

$v_2 = \frac{1}{2} v_3$ $\quad\quad\quad$ $\therefore$ let $v_3 = a$

$v_2 = \frac{1}{2} a$

eq (I)

$6v_1 - 6\left(\frac{1}{2}\right) a = 0$

$6v_1 - 3a = 0$

$6v_1 = 3a$

$v_1 = \frac{1}{2} a$

$$v = \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = \begin{bmatrix} \frac{1}{2} a \\ \frac{1}{2} a \\ a \end{bmatrix} = a \begin{bmatrix} \frac{1}{2} \\ \frac{1}{2} \\ 1 \end{bmatrix}$$

For $\lambda = 4$

$$\begin{bmatrix} \frac{1}{2} & \frac{1}{2} & 1 \end{bmatrix}^t$$

put $\lambda = -2$

$$\begin{bmatrix} 3 & -3 & 3 \\ 3 & -3 & 3 \\ 6 & -6 & 6 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$3v_1 - 3v_2 + 3v_3 = 0 \to (\text{III})$

$3v_1 - 3v_2 + 3v_3 = 0 \to (\text{II})$

$6v_1 - 6v_2 + 6v_3 = 0 \to (\text{I})$

Gauss Elimination

$$[A|B] = \left[\begin{array}{ccc|c} 6 & -6 & 6 & 0 \\ 3 & -3 & 3 & 0 \\ 3 & -3 & 3 & 0 \end{array}\right] \quad \begin{array}{l} \cdot R_2 - \frac{1}{2}R_1 \\ \cdot R_3 - \frac{1}{2}R_1 \end{array}$$

$$[A|B] = \left[\begin{array}{ccc|c} 6 & -6 & 6 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$6v_1 - 6v_2 + 6v_3 = 0 \to (\text{I})$

$v_1 - v_2 + v_3 = 0$ $\quad \therefore v_3 = a$

$v_1 - b + a = 0$ $\quad v_2 = b$

$v_1 = b - a$

$$v = \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = \begin{bmatrix} -a+b \\ b \\ a \end{bmatrix}$$

$v = Q$[illegible]

For $\lambda = -2$

$$\begin{bmatrix} -a+b & b & a \end{bmatrix}^t$$

Eigen values = $4, -2, -2$

- For $\lambda = 4$

$$\begin{bmatrix} \tfrac{1}{2} & \tfrac{1}{2} & 1 \end{bmatrix}^t$$

- For $\lambda = -2$

$$\begin{bmatrix} -a+b & b & a \end{bmatrix}^t$$

- For $\lambda = -2$

$$\begin{bmatrix} -a+b & b & a \end{bmatrix}^t$$