Monday

Gauss Elimination Method

Q:1

$3x+y-z=3 \rightarrow (I)$ highest mode value of x

$2x-8y+z=-5 \rightarrow (II)$ highest mode value of y

$x-2y+9z=8 \rightarrow (III)$

$$[A|B]=\left[\begin{array}{ccc|c} 3 & 1 & -1 & 3 \\ 2 & -8 & 1 & -5 \\ 1 & -2 & 9 & 8 \end{array}\right]$$

- $R_2 - \frac{2}{3}R_1$
- $R_3 - \frac{1}{3}R_1$

$\Rightarrow R_2 - \frac{2}{3}R_1$

- $2-\frac{2}{3}(3)=0$
- $-8-\frac{2}{3}(1)=\frac{-26}{3}$
- $1-\frac{2}{3}(-1)=\frac{5}{3}$
- $-5-\frac{2}{3}(3)=-7$

$\Rightarrow R_3 - \frac{1}{3}R_1$

- $1-\frac{1}{3}(3)=0$
- $-2-\frac{1}{3}(1)=\frac{-7}{3}$
- $9-\frac{1}{3}(-1)=\frac{28}{3}$
- $8-\frac{1}{3}(3)=7$

$$[A|B]=\left[\begin{array}{ccc|c} 3 & 1 & -1 & 3 \\ 0 & \boxed{-26/3} & 5/3 & -7 \\ 0 & \boxed{-7/3} & 28/3 & 7 \end{array}\right]$$

- $R_3 - \frac{\left(\frac{-7}{3}\right)}{\left(\frac{-26}{3}\right)}(R_2)$
- $R_3 - \frac{7}{26}(R_2)$

If $\left|\frac{-7}{3}\right| > \left|\frac{-26}{3}\right|$ shift R_2 & R_3

- Highest mode value will be 2nd eq
- Lowest mode value will be 3rd eq

$\Rightarrow R_3 - \frac{7}{26}R_2$

- $0-\frac{7}{26}(0)=0$
- $-\frac{7}{3}-\left(\frac{7}{26}\right)\left(\frac{-26}{3}\right) = -\frac{7}{3}+\frac{7}{3}=0$
- $\frac{28}{3}-\frac{7}{26}\left(\frac{5}{3}\right) = \frac{28}{3}-\frac{35}{78}=\frac{231}{26}$
- $7-\frac{7}{26}(-7) = 7+\frac{49}{26} = \frac{231}{26}$

$$[A|B]=\left[\begin{array}{ccc|c} 3 & 1 & -1 & 3 \\ 0 & -26/3 & 5/3 & -7 \\ 0 & 0 & \frac{231}{26} & \frac{231}{26} \end{array}\right]$$

$3x+y-z=3 \rightarrow (I)$

$-\frac{26}{3}y+\frac{5}{3}z=-7 \rightarrow (II)$

$\frac{231}{26}z=\frac{231}{26} \rightarrow (III)$

From (III)

$\boxed{z=1}$

put $z=1$ in (II)

$-\frac{26}{3}y+\frac{5}{3}=-7$

$-\frac{26}{3}y=-7-\frac{5}{3}$

$-\frac{26}{3}y=-\frac{26}{3}$

$\boxed{y=1}$

put $y=1$ & $z=1$ in (I)

$3x+1-1=3$

$3x=3$

$\boxed{x=1}$

$\boxed{x=1}, \boxed{y=1}, \boxed{z=1}$

Q: $4x + 11y + z = 33 \rightarrow (II)$

$8x + 3y + 2z = 20 \rightarrow (I)$

$2x + y + 4z = 12 \rightarrow (III)$

If 2 x values are equal, then any eq. can be (I)

$$[A|B] = \left[\begin{array}{ccc|c} 8 & 3 & 2 & 20 \\ 4 & 11 & 1 & 33 \\ 2 & 1 & 4 & 12 \end{array}\right]$$

- $R_2 - \frac{4}{8} R_1$
- $R_2 - \frac{1}{2} R_1$

$\Rightarrow R_2 - \frac{1}{2} R_1$

- $4 - \frac{1}{\cancel{2}}(\overset{4}{8}) = 0$
- $11 - \frac{1}{2}(3) = \frac{19}{2}$
- $1 - \frac{1}{2}(2) = 0$
- $33 - \frac{1}{2}(20) = \frac{46}{2} = 23$

- $R_3 - \frac{2}{8} R_1$
- $R_3 - \frac{1}{4} R_1$

$\Rightarrow R_3 - \frac{1}{4} R_1$

- $2 - \frac{1}{4}(8) = 0$
- $1 - \frac{1}{4}(3) = \frac{1}{4}$
- $4 - \frac{1}{4}(2) = 4 - \frac{1}{2} = \frac{7}{2}$
- $12 - \frac{1}{4}(20) = 7$

$$[A|B] = \left[\begin{array}{ccc|c} 8 & 3 & 2 & 20 \\ 0 & 19/2 & 0 & 23 \\ 0 & 1/4 & 7/2 & 7 \end{array}\right]$$

- $R_3 - \frac{1/4}{19/2} R_2$
- $R_3 - \frac{1}{4}\left(\frac{2}{19}\right) R_2$
- $R_3 - \frac{1}{38} R_2$

$\Rightarrow R_3 - \frac{1}{38} R_2$

- $0 - \frac{1}{38}(0) = 0$
- $\frac{1}{4} - \frac{1}{38}\left(\frac{19}{2}\right) = \frac{1}{4} - \frac{1}{4} = 0$
- $\frac{7}{2} - \frac{1}{38}(0) = \frac{7}{2}$
- $7 - \frac{1}{38}(23) = \frac{243}{38}$

$$[A|B] = \left[\begin{array}{ccc|c} 8 & 3 & 2 & 20 \\ 0 & 19/2 & 0 & 23 \\ 0 & 0 & 7/2 & 243/38 \end{array}\right]$$

$8x + 3y + 2z = 20 \rightarrow (I)$

$\frac{19}{2} y = 23 \rightarrow (II)$

$y = 23 \times \frac{2}{19} = \frac{46}{19}$

$\boxed{y = \frac{46}{19}}$

$\frac{7}{2} z = \frac{243}{38} \rightarrow (III)$

$z = \frac{243}{38} \times \frac{2}{7}$

$\boxed{z = \frac{243}{133}}$

put $y = \frac{46}{19}$ & $z = \frac{243}{133}$ in (I)

$8x + 3\left(\frac{46}{19}\right) + 2\left(\frac{243}{133}\right) = 20$

$8x = 20 - \frac{138}{19} - \frac{486}{133} \Rightarrow 8x = \frac{1208}{133} \Rightarrow x = \frac{1208}{133} \times \frac{1}{8}$

$\boxed{x = 151/133}$

Monday

Gauss Jordan Method

Q:1

$x_1 - 2x_2 + 10x_3 = 9 \rightarrow (\text{III})$

$x_1 + 10x_2 - x_3 = 10 \rightarrow (\text{II})$

$10x_1 + x_2 + x_3 = 12 \rightarrow (\text{I})$

$$[A|B] = \left[\begin{array}{ccc|c} 10 & 1 & 1 & 12 \\ 1 & 10 & -1 & 10 \\ 1 & -2 & 10 & 9 \end{array}\right]$$

- $R_2 - \frac{1}{10}R_1$
- $R_3 - \frac{1}{10}R_1$

$\Rightarrow R_2 - \frac{1}{10}R_1$

- $1 - \frac{1}{10}(10) = 0$
- $10 - \frac{1}{10}(1) = \frac{99}{10}$
- $-1 - \frac{1}{10}(1) = \frac{-11}{10}$
- $10 - \frac{1}{10}(12) = \frac{88}{10} = \frac{44}{5}$

$\Rightarrow R_3 - \frac{1}{10}R_1$

- $1 - \frac{1}{10}(10) = 0$
- $-2 - \frac{1}{10}(1) = \frac{-21}{10}$
- $10 - \frac{1}{10}(1) = \frac{99}{10}$
- $9 - \frac{1}{10}(12) = \frac{39}{5}$

$$[A|B] = \left[\begin{array}{ccc|c} 10 & 1 & 1 & 12 \\ 0 & 99/10 & -11/10 & 44/5 \\ 0 & -21/10 & 99/10 & 39/5 \end{array}\right]$$

- $R_3 - \frac{\left(\frac{-21}{10}\right)}{\left(\frac{99}{10}\right)}R_2$
- $R_3 + \frac{21}{99}R_2$

$\Rightarrow R_3 + \frac{21}{99}R_2$

- $0 + \frac{21}{99}(0) = 0$
- $\frac{-21}{10} + \left(\frac{21}{99}\right)\left(\frac{99}{10}\right) = \frac{-21}{10} + \frac{21}{10} = 0$
- $\frac{99}{10} + \frac{21}{99}\left(\frac{-11}{10}\right) = \frac{99}{10} - \frac{231}{990} = \frac{29}{3}$
- $\frac{39}{5} + \frac{21}{99}\left(\frac{44}{5}\right) = \frac{39}{5} + \frac{924}{495} = \frac{29}{3}$

$$[A|B] = \left[\begin{array}{ccc|c} 10 & 1 & 1 & 12 \\ 0 & 99/10 & -11/10 & 44/5 \\ 0 & 0 & 29/3 & 29/3 \end{array}\right]$$

- $R_2 - \frac{\left(\frac{-11}{10}\right)}{\left(\frac{29}{3}\right)}R_3$
- $R_2 + \frac{11}{10}\left(\frac{3}{29}\right)R_3$
- $R_2 + \frac{33}{290}R_3$

$\Rightarrow R_2 + \frac{33}{290}R_3$

- $0 + \frac{33}{290}(0) = 0$
- $\frac{99}{10} + \frac{33}{290}(0) = \frac{99}{10}$
- $\frac{-11}{10} + \frac{33}{290}\left(\frac{29}{3}\right) = 0$
- $\frac{44}{5} + \frac{33}{290}\left(\frac{29}{3}\right) = \frac{99}{10}$

$\Rightarrow R_1 - \frac{3}{29}R_3$

- $10 - \frac{3}{29}(0) = 10$
- $1 - \frac{3}{29}(0) = 1$
- $1 - \frac{3}{29}\left(\frac{29}{3}\right) = 0$
- $12 - \frac{3}{29}\left(\frac{29}{3}\right) = 11$

- $R_1 - \frac{1}{\left(\frac{29}{3}\right)}R_3$

$R_1 - \frac{3}{29}R_3$

$$[A|B] = \left[\begin{array}{ccc|c} 10 & 1 & 0 & 11 \\ 0 & 99/10 & 0 & 99/10 \\ 0 & 0 & 29/3 & 29/3 \end{array}\right]$$

- $R_1 - \frac{1}{\left(\frac{99}{10}\right)}R_2$
- $R_1 - \frac{10}{99}R_2$

$\Rightarrow R_1 - \frac{10}{99}R_2$

- $10 - \frac{10}{99}(0) = 10$
- $1 - \frac{10}{99}\left(\frac{99}{10}\right) = 0$
- $0 - \frac{10}{99}(0) = 0$
- $11 - \frac{10}{99}\left(\frac{99}{10}\right) = 10$

$$[A|B] = \left[\begin{array}{ccc|c} 10 & 0 & 0 & 10 \\ 0 & 99/10 & 0 & 99/10 \\ 0 & 0 & 29/3 & 29/3 \end{array}\right]$$

$10x = 10$ $\quad \frac{99}{10}y = \frac{99}{10}$ $\quad \frac{29}{3}z = \frac{29}{3}$

$\boxed{x = 1}$ $\quad \boxed{y = 1}$ $\quad \boxed{z = 1}$

Q:2

$3x + y + 2z = 16 \rightarrow$ III

$2x - 6y + 8z = 24 \rightarrow$ II

$5x + 4y - 3z = 2 \rightarrow$ I

$$[A|B] = \left[\begin{array}{ccc|c} 5 & 4 & -3 & 2 \\ 2 & -6 & 8 & 24 \\ 3 & 1 & 2 & 16 \end{array}\right]$$

- $R_2 - \frac{2}{5}R_1$
- $R_3 - \frac{3}{5}R_1$

$\Rightarrow R_2 - \frac{2}{5}R_1$

- $2 - \frac{2}{5}(5) = 0$
- $-6 - \frac{2}{5}(4) = -\frac{38}{5}$
- $8 - \frac{2}{5}(-3) = \frac{46}{5}$
- $24 - \frac{2}{5}(2) = \frac{116}{5}$

$\Rightarrow R_3 - \frac{3}{5}R_1$

- $3 - \frac{3}{5}(5) = 0$
- $1 - \frac{3}{5}(4) = -\frac{7}{5}$
- $2 - \frac{3}{5}(-3) = \frac{19}{5}$
- $16 - \frac{3}{5}(2) = \frac{74}{5}$

$$[A|B] = \left[\begin{array}{ccc|c} 5 & 4 & -3 & 2 \\ 0 & -38/5 & 46/5 & 116/5 \\ 0 & -7/5 & 19/5 & 74/5 \end{array}\right]$$

- $R_3 - \frac{(-7/5)}{(-38/5)}R_2$
- $R_3 - \frac{7}{38}R_2$

$\Rightarrow R_3 - \frac{7}{38}R_2$

- $0 - \frac{7}{38}(0) = 0$
- $-\frac{7}{5} - \left(\frac{7}{38}\right)\left(-\frac{38}{5}\right) = 0$
- $\frac{19}{5} - \left(\frac{7}{38}\right)\left(\frac{46}{5}\right) = \frac{19}{5} - \frac{322}{190} = \frac{40}{19}$
- $\frac{74}{5} - \left(\frac{7}{38}\right)\left(\frac{116}{5}\right) = \frac{74}{5} - \frac{812}{190} = \frac{200}{19}$

$$[A|B] = \left[\begin{array}{ccc|c} 5 & 4 & -3 & 2 \\ 0 & -38/5 & 46/5 & 116/5 \\ 0 & 0 & 40/19 & 200/19 \end{array}\right]$$

- $R_2 - \frac{(46/5)}{(40/19)}R_3$
- $R_2 - \left(\frac{46}{5}\right)\left(\frac{19}{40}\right)R_3$
- $R_2 - \frac{437}{100}R_3$
- $R_1 - \frac{(-3)}{(40/19)}R_3$
- $R_1 + (3)\left(\frac{19}{40}\right)R_3$
- $R_1 + \frac{57}{40}R_3$

$\Rightarrow R_2 - \frac{207}{100}R_3$

- $0 - \frac{437}{100}(0) = 0$
- $-\frac{38}{5} - \frac{437}{100}(0) = -\frac{38}{5}$
- $\frac{46}{5} - \frac{437}{100}\left(\frac{40}{19}\right)$

$= \frac{46}{5} - \frac{46}{5} = 0$

- $\frac{116}{5} - \frac{437}{100}\left(\frac{200}{19}\right)$

$= \frac{116}{5} - 46 = -\frac{114}{5}$

$\Rightarrow R_1 + \frac{57}{40}R_3$

- $5 + \frac{57}{40}(0) = 5$
- $4 + \frac{57}{40}(0) = 4$
- $-3 + \frac{57}{40}\left(\frac{40}{19}\right) = 0$
- $2 + \frac{57}{40}\left(\frac{200}{19}\right)$

$= 2 + 15 = 17$

$$[A|B] = \left[\begin{array}{ccc|c} 5 & 4 & 0 & 17 \\ 0 & -38/5 & \cancel{\quad} 0 & -114/5 \\ 0 & 0 & 40/19 & 200/19 \end{array}\right] \quad \begin{array}{l} \cdot\ R_1 - \frac{4}{\left(-\frac{38}{5}\right)} R_2 \\ \cdot\ R_1 + \frac{20}{38} R_2 \\ \cdot\ R_1 + \frac{10}{19} R_2 \end{array}$$

$\Rightarrow R_1 + \frac{10}{19} R_2$

- $5 + \frac{10}{19}(10) = 5$
- $4 + \frac{10}{19}\left(-\frac{38}{5}\right) = 0$
- $0 - \frac{10}{19}(0) = 0$

- $17 + \frac{10}{19}\left(-\frac{114}{5}\right)$

$= 17 - 12 = 5$

$$[A|B] = \left[\begin{array}{ccc|c} 5 & 0 & 0 & 5 \\ 0 & -38/5 & 0 & -114/5 \\ 0 & 0 & 40/19 & 200/19 \end{array}\right]$$

$5x = 5 \rightarrow$ (I)

$\boxed{x = 1}$

$-\frac{38}{5} y = -\frac{114}{5}$

$y = \frac{114}{38}$

$\boxed{y = 3}$

$\frac{40}{19} z = \frac{200}{19}$

$z = \frac{200}{40}$

$\boxed{z = 5}$

Thursday

LU-Decomposition
Method of Factorization
Triangularisation Method

Q.1

$$x + 3y + 8z = 4$$
$$x + 4y + 3z = -2$$
$$x + 3y + 4z = 1$$

$$L = \begin{bmatrix} 1 & 0 & 0 \\ l_{21} & 1 & 0 \\ l_{31} & l_{32} & 1 \end{bmatrix}$$

$AX = B$

$$\begin{bmatrix} 1 & 3 & 8 \\ 1 & 4 & 3 \\ 1 & 3 & 4 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 4 \\ -2 \\ 1 \end{bmatrix}$$

$$U = \begin{bmatrix} U_{11} & U_{12} & U_{13} \\ 0 & U_{22} & U_{23} \\ 0 & 0 & U_{33} \end{bmatrix}$$

Use -ve sign, if value is -ve, don't use mode

$LU = A$

↳ A ko decompose krna hai, upper Δ matrix or lower Δ matrix mai

$$LU = A$$

$$\begin{bmatrix} 1 & 0 & 0 \\ l_{21} & 1 & 0 \\ l_{31} & l_{32} & 1 \end{bmatrix}\begin{bmatrix} U_{11} & U_{12} & U_{13} \\ 0 & U_{22} & U_{23} \\ 0 & 0 & U_{33} \end{bmatrix} = \begin{bmatrix} 1 & 3 & 8 \\ 1 & 4 & 3 \\ 1 & 3 & 4 \end{bmatrix}$$

$$\begin{bmatrix} U_{11} & U_{12} & U_{13} \\ l_{21}U_{11} & l_{21}U_{12}+U_{22} & l_{21}U_{13}+U_{23} \\ l_{31}U_{11} & l_{31}U_{12}+l_{32}U_{22} & l_{31}U_{13}+l_{32}U_{23}+U_{33} \end{bmatrix} = \begin{bmatrix} 1 & 3 & 8 \\ 1 & 4 & 3 \\ 1 & 3 & 4 \end{bmatrix}$$

$\boxed{U_{11}=1}$ $\boxed{U_{12}=3}$ $\boxed{U_{13}=8}$

$l_{21}U_{11}=1$
$l_{21}(1)=1$
$\boxed{l_{21}=1}$

$l_{21}U_{12}+U_{22}=4$
$1(3)+U_{22}=4$
$U_{22}=4-3$
$\boxed{U_{22}=1}$

$l_{21}U_{13}+U_{23}=3$
$(1)(8)+U_{23}=3$
$U_{23}=3-8$
$\boxed{U_{23}=-5}$

$l_{31}U_{11}=1$
$l_{31}(1)=1$
$\boxed{l_{31}=1}$

$l_{31}U_{12}+l_{32}U_{22}=3$
$(1)(3)+l_{32}(1)=3$
$\boxed{l_{32}=0}$

$l_{31}U_{13}+l_{32}U_{23}+U_{33}=4$
$(1)(8)+(0)(-5)+U_{33}=4$
$8+U_{33}=4$
$\boxed{U_{33}=-4}$

$$L = \begin{bmatrix} 1 & 0 & 0 \\ l_{21} & 1 & 0 \\ l_{31} & l_{32} & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$$

$$U = \begin{bmatrix} U_{11} & U_{12} & U_{13} \\ 0 & U_{22} & U_{23} \\ 0 & 0 & U_{33} \end{bmatrix} = \begin{bmatrix} 1 & 3 & 8 \\ 0 & 1 & -5 \\ 0 & 0 & -4 \end{bmatrix}$$

$AX = B$
$(LU)X = B$
$L(UX) = B$
let $Y = UX$
$L(Y) = B$

$$\begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}\begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} 4 \\ -2 \\ 1 \end{bmatrix}$$

$y_1 = 4$
$y_1 + y_2 = -2$
$y_1 + y_3 = 1$

$4 + y_2 = -2$
$\boxed{y_2 = -6}$

$y_3 = 1 - y_1$
$y_3 = 1 - 4 = -3$
$\boxed{y_3 = -3}$

$UX = Y$

$$\begin{bmatrix} 1 & 3 & 8 \\ 0 & 1 & -5 \\ 0 & 0 & -4 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 4 \\ -6 \\ -3 \end{bmatrix}$$

$x + 3y + 8z = 4 \rightarrow (I)$

$y - 5z = -6 \rightarrow (II)$

$-4z = -3 \rightarrow (III)$

$z = \frac{3}{4}$

put $z = \frac{3}{4}$ in (II)

$y - 5\left(\frac{3}{4}\right) = -6$

$y - \frac{15}{4} = -6$

$y = -6 + \frac{15}{4}$

$y = \frac{-24+15}{4} = \frac{-9}{4}$

put $y = \frac{-9}{4}$ & $z = \frac{3}{4}$ in eq.(I)

$x + 3\left(\frac{-9}{4}\right) + 8\left(\frac{3}{4}\right) = 4$

$x = 4 - 6 + \frac{27}{4}$

$\boxed{x = \frac{19}{4}}$

Q:2 $2x + y + 4z = 12$

$8x + 3y + 2z = 20$

$4x + 11y + z = 33$

$AX = B$

$$\begin{bmatrix} 2 & 1 & 4 \\ 8 & 3 & 2 \\ 4 & 11 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 12 \\ 20 \\ 33 \end{bmatrix}$$

$AX = B$

$(LU)X = B$

$L(UX) = B$

let:

$UX = Y$

$L(Y) = B$

$LU = A$

$$\begin{bmatrix} 1 & 0 & 0 \\ l_{21} & 1 & 0 \\ l_{31} & l_{32} & 1 \end{bmatrix}\begin{bmatrix} U_{11} & U_{12} & U_{13} \\ 0 & U_{22} & U_{23} \\ 0 & 0 & U_{33} \end{bmatrix} = \begin{bmatrix} 2 & 1 & 4 \\ 8 & 3 & 2 \\ 4 & 11 & 1 \end{bmatrix}$$

$$\begin{bmatrix} U_{11} & U_{12} & U_{13} \\ l_{21}U_{11} & l_{21}U_{12}+U_{22} & l_{21}U_{13}+U_{23} \\ l_{31}U_{11} & l_{31}U_{12}+l_{32}U_{22} & l_{31}U_{13}+l_{32}U_{23}+U_{33} \end{bmatrix} = \begin{bmatrix} 2 & 1 & 4 \\ 8 & 3 & 2 \\ 4 & 11 & 1 \end{bmatrix}$$

$\boxed{U_{11} = 2}$ $\boxed{U_{12} = 1}$ $\boxed{U_{13} = 4}$

- $l_{21}U_{11} = 8$
  $l_{21}(2) = 8$
  $\boxed{l_{21} = 4}$
- $l_{21}U_{12} + U_{22} = 3$
  $(4)(1) + U_{22} = 3$
  $U_{22} = 3 - 4$
  $\boxed{U_{22} = -1}$
- $l_{21}U_{13} + U_{23} = 2$
  $(4)(4) + U_{23} = 2$
  $U_{23} = 2 - 16$
  $\boxed{U_{23} = -14}$

$l_{31}U_{11} = 4$

$l_{31}(2) = 4$

$\boxed{l_{31} = 2}$

$l_{31}U_{12} + l_{32}U_{22} = 11$

$(2)(1) + l_{32}(-1) = 11$

$-l_{32} = 11 - 2$

$-l_{32} = 9$

$\boxed{l_{32} = -9}$

$l_{31}U_{13} + l_{32}U_{23} + U_{33} = 1$

$(2)(4) + (-9)(-14) + U_{33} = 1$

$8 + 126 + U_{33} = 1$

$U_{33} = 1 - 134$

$\boxed{U_{33} = -133}$

$$L = \begin{bmatrix} 1 & 0 & 0 \\ l_{21} & 1 & 0 \\ l_{31} & l_{32} & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 4 & 1 & 0 \\ 2 & -9 & 1 \end{bmatrix}$$

$$U = \begin{bmatrix} U_{11} & U_{12} & U_{13} \\ 0 & U_{22} & U_{23} \\ 0 & 0 & U_{33} \end{bmatrix} = \begin{bmatrix} 2 & 1 & 4 \\ 0 & -1 & -14 \\ 0 & 0 & -133 \end{bmatrix}$$

$LY = B$

$$\begin{bmatrix} 1 & 0 & 0 \\ 4 & 1 & 0 \\ 2 & -9 & 1 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} 12 \\ 20 \\ 33 \end{bmatrix}$$

$y_1 = 12$

$4y_1 + y_2 = 20$

$2y_1 - 9y_2 + y_3 = 33$

$4(12) + y_2 = 20$

$y_2 = 20 - 48$

$\boxed{y_2 = -28}$

$y_3 = 33 - 2(12) + 9(-28)$

$y_3 = 33 - 24 - 252$

$\boxed{y_3 = -243}$

$UX = Y$

$$\begin{bmatrix} 2 & 1 & 4 \\ 0 & -1 & -14 \\ 0 & 0 & -133 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 12 \\ -28 \\ -243 \end{bmatrix}$$

$2x + y + 4z = 12$

$-y - 14z = -28$

$-133z = -243$

$$\boxed{z = \frac{243}{133}}$$

$y = -14z + 28$

$y = -14\left(\frac{243}{133}\right) + 28$

$$\boxed{y = \frac{46}{19}}$$

$2x = 12 - y - 4z$

$2x = 12 - \frac{46}{19} - 4\left(\frac{243}{133}\right)$

$2x = \frac{302}{133}$

$$\boxed{x = \frac{151}{133}}$$

Monday

Crout's Method

$$2x + y + 4z = 19$$
$$8x + 3y + 2z = 20$$
$$4x + 11y + z = 33$$

$AX = B$

$$\begin{bmatrix} 2 & 1 & 4 \\ 8 & 3 & 2 \\ 4 & 11 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 12 \\ 20 \\ 33 \end{bmatrix}$$

$$L = \begin{bmatrix} l_{11} & 0 & 0 \\ l_{21} & l_{22} & 0 \\ l_{31} & l_{32} & l_{33} \end{bmatrix}$$

$$U = \begin{bmatrix} 1 & U_{12} & U_{13} \\ 0 & 1 & U_{22} \\ 0 & 0 & 1 \end{bmatrix}$$

$A = LU$

$$\begin{bmatrix} l_{11} & 0 & 0 \\ l_{21} & l_{22} & 0 \\ l_{31} & l_{32} & l_{33} \end{bmatrix}\begin{bmatrix} 1 & U_{12} & U_{13} \\ 0 & 1 & U_{23} \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 2 & 1 & 4 \\ 8 & 3 & 2 \\ 4 & 11 & 1 \end{bmatrix}$$

$$\begin{bmatrix} l_{11} & l_{11}U_{12} & l_{11}U_{13} \\ l_{21} & l_{21}U_{12} + l_{22} & l_{21}U_{13} + l_{22}U_{23} \\ l_{31} & l_{31}U_{12} + l_{32} & l_{31}U_{13} + l_{32}U_{23} + l_{33} \end{bmatrix} = \begin{bmatrix} 2 & 1 & 4 \\ 8 & 3 & 2 \\ 4 & 11 & 1 \end{bmatrix}$$

$\boxed{l_{11} = 2}$

$l_{11}U_{12} = 1$

$\boxed{U_{12} = 1/2}$

$l_{11}U_{13} = 4$

$\boxed{U_{13} = 2}$

$\boxed{l_{21} = 8}$

$l_{21}U_{12} + l_{22} = 3$

$(8)\left(\frac{1}{2}\right) + l_{22} = 3$

$l_{22} = 3 - 4$

$\boxed{l_{22} = -1}$

$l_{21}U_{13} + l_{22}U_{23} = 2$

$(8)(2) + (-1)U_{23} = 2$

$16 - 2 = U_{23}$

$\boxed{U_{23} = 14}$

$\boxed{l_{31} = 4}$

$l_{31}U_{12} + l_{32} = 11$

$(4)\left(\frac{1}{2}\right) + l_{32} = 11$

$\boxed{l_{32} = 9}$

$l_{31}U_{13} + l_{32}U_{23} + l_{33} = 1$

$(4)(2) + (9)(14) + l_{33} = 1$

$l_{33} = 1 - 8 - 126$

$\boxed{l_{33} = -133}$

$AX = B$

$(LU)X = B$

$L(\underline{UX}) = B$

$L(Y) = B$

Let : $UX = Y$

$LY = B$

$$\begin{bmatrix} 2 & 0 & 0 \\ 8 & -1 & 0 \\ 4 & 9 & -133 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} 12 \\ 20 \\ 33 \end{bmatrix}$$

$2y_1 = 12$

$\boxed{y_1 = 6}$

$8y_1 - y_2 = 20$

$8(6) - y_2 = 20$

$48 - 20 = y_2$

$\boxed{y_2 = 28}$

$4y_1 + 9y_2 - 133y_3 = 33$

$4(6) + 9(28) - 133y_3 = 33$

$24 + 252 - 33 = 133y_3$

$133y_3 = 243$

$\boxed{y_3 = \frac{243}{133}}$

$UX = Y$

$$\begin{bmatrix} 1 & \frac{1}{2} & 2 \\ 0 & 1 & 14 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 6 \\ 28 \\ 243/133 \end{bmatrix}$$

$x + \frac{1}{2}y + 2z = 6$

$y + 14z = 28$

$\boxed{z = \frac{243}{133}}$

$y = 28 - 14\left(\frac{243}{133}\right) = \frac{46}{19}$

$\boxed{y = \frac{46}{19}}$

$x = 6 - \frac{1}{2}\left(\frac{46}{19}\right) - 2\left(\frac{243}{133}\right)$

$x = 6 - \frac{23}{19} - \frac{486}{133}$

$\boxed{x = \frac{151}{133}}$

(2)

$\frac{19}{4}, \frac{-9}{4}$

$\frac{3}{4}$

$x + 3y + 8z = 4$

$x + 4y + 3z = -2$

$x + 3y + 4z = 1$

$AX = B$

$$\begin{bmatrix} 1 & 3 & 8 \\ 1 & 4 & 3 \\ 1 & 3 & 4 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 4 \\ -2 \\ 1 \end{bmatrix}$$

$A = LU$

$$\begin{bmatrix} l_{11} & 0 & 0 \\ l_{21} & l_{22} & 0 \\ l_{31} & l_{32} & l_{33} \end{bmatrix} \begin{bmatrix} 1 & u_{12} & u_{13} \\ 0 & 1 & u_{23} \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 3 & 8 \\ 1 & 4 & 3 \\ 1 & 3 & 4 \end{bmatrix}$$

$$\begin{bmatrix} l_{11} & l_{11}u_{12} & l_{11}u_{13} \\ l_{21} & l_{21}u_{12} + l_{22} & l_{21}u_{13} + l_{22}u_{23} \\ l_{31} & l_{31}u_{12} + l_{32} & l_{31}u_{13} + l_{32}u_{23} + l_{33} \end{bmatrix} = \begin{bmatrix} 1 & 3 & 8 \\ 1 & 4 & 3 \\ 1 & 3 & 4 \end{bmatrix}$$

$\boxed{l_{11} = 1}$

$\boxed{l_{21} = 1}$

$\boxed{l_{31} = 1}$

$l_{11}u_{12} = 3$

$\boxed{u_{12} = 3}$

$l_{11}u_{13} = 8$

$\boxed{u_{13} = 8}$

$l_{21}u_{12} + l_{22} = 4$

$(1)(3) + l_{22} = 4$

$\boxed{l_{22} = 1}$

$l_{21}u_{13} + l_{22}u_{23} = 3$

$(1)(8) + (1)u_{23} = 3$

$\boxed{u_{23} = -5}$

$l_{31}U_{12} + l_{32} = 3$

$(1)(3) + l_{32} = 3$

$\boxed{l_{32} = 0}$

$l_{31}U_{13} + l_{32}U_{23} + l_{33} = 4$

$(1)(8) + (0)(-5) + l_{33} = 4$

$l_{33} = 4 - 8$

$\boxed{l_{33} = -4}$

$AX = B$

$(LU)X = B$

$L(UX) = B$

let: $UX = Y$

$L(Y) = B$

$$\begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 0 & -4 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} 4 \\ -2 \\ 1 \end{bmatrix}$$

$\boxed{y_1 = 4}$

$y_1 + y_2 = -2$

$\boxed{y_2 = -6}$

$y_1 - 4y_3 = 1$

$4 - 1 = 4y_3$

$\boxed{\frac{3}{4} = y_3}$

$UX = Y$

$$\begin{bmatrix} 1 & 3 & 8 \\ 0 & 1 & -5 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 4 \\ -6 \\ 3/4 \end{bmatrix}$$

$x + 3y + 8z = 4$

$y - 5z = -6$

$\boxed{z = \frac{3}{4}}$

$y = -6 + 5\left(\frac{3}{4}\right)$

$y = \frac{-24 + 15}{4} = -\frac{9}{4}$

$\boxed{y = -\frac{9}{4}}$

$x = 4 - 3\left(-\frac{9}{4}\right) - 8\left(\frac{3}{4}\right)$

$x = 4 + \frac{27}{4} - \frac{24}{4}$

$x = \frac{16 + 27 - 24}{4}$

$\boxed{x = \frac{19}{4}}$

Rank of a Matrix

The rank of a Matrix A is equal to ~~its~~ the number of non-zero rows in its **echelon form**

$$\begin{bmatrix} 1 & - & - \\ 0 & 1 & - \\ 0 & 0 & 1 \end{bmatrix}$$

A [

(1) $A = \begin{bmatrix} 5 & 9 & 3 \\ -3 & 5 & 6 \\ -1 & -5 & -3 \end{bmatrix}$

$R_1 \times \frac{1}{5}$

$A = \begin{bmatrix} 1 & 9/5 & 3/5 \\ -3 & 5 & 6 \\ -1 & -5 & -3 \end{bmatrix}$

$R_2 + 3 R_1$
$R_3 + R_1$

$A = \begin{bmatrix} 1 & 9/5 & 3/5 \\ 0 & 52/5 & 39/5 \\ 0 & -16/5 & -12/5 \end{bmatrix}$

$R_2 \times \frac{5}{52}$

$A = \begin{bmatrix} 1 & 9/5 & 3/5 \\ 0 & 1 & 39/52 \\ 0 & -16/5 & -12/5 \end{bmatrix}$

$R_3 + \frac{16}{5} R_2$

$A = \begin{bmatrix} 1 & 9/5 & 3/5 \\ 0 & 1 & 39/52 \\ 0 & 0 & \end{bmatrix}$

$\boxed{\text{Rank} \longrightarrow 3}$ $\boxed{R(A) = 3}$

$-\frac{12}{5} + \frac{16}{5}\left(\frac{39}{52}\right)$

- unique solution when diagonal has 1, 1, 1 & Rank (A) = 3

2

$$\begin{bmatrix} 1 & 3 & 1 & -2 & -3 \\ 1 & 4 & 3 & -1 & -4 \\ 2 & 3 & -4 & -7 & -3 \\ 3 & 8 & 1 & -7 & -8 \end{bmatrix}$$

- $R_2 - R_1$
- $R_3 - 2R_1$
- $R_4 - 3R_1$

$$A = \begin{bmatrix} 1 & 3 & 1 & -2 & -3 \\ 0 & 1 & 2 & 1 & -1 \\ 0 & -3 & -6 & -3 & 3 \\ 0 & -1 & -2 & -1 & 1 \end{bmatrix}$$

$R_3 + 3R_2$
$R_4 + R_2$

$$A = \begin{bmatrix} 1 & 3 & 1 & -2 & -3 \\ 0 & 1 & 2 & 1 & -1 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$R(A) = 2$

- This matrix has infinitely many solutions
- No unique solution

3

$$\begin{bmatrix} 1 & 0 & -2 & 0 \\ 0 & -1 & -3 & 3 \\ -2 & -1 & 1 & 3 \\ 0 & 1 & 3 & -4 \end{bmatrix}$$

- $R_3 + 2R_1$

$$\begin{bmatrix} 1 & 0 & -2 & 0 \\ 0 & -1 & -3 & 3 \\ 0 & -1 & -3 & 3 \\ 0 & 1 & 3 & -4 \end{bmatrix}$$

$R_2 \longleftrightarrow R_4$

$$\begin{bmatrix} 1 & 0 & -2 & 0 \\ 0 & 1 & 3 & -4 \\ 0 & -1 & -3 & 3 \\ 0 & -1 & -3 & 3 \end{bmatrix}$$

$R_3 + R_2$
$R_4 + R_2$

$$\begin{bmatrix} 1 & 0 & -2 & 0 \\ 0 & 1 & 3 & -4 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & -1 \end{bmatrix}$$

$R_3 / -1$

$$\begin{bmatrix} 1 & 0 & -2 & 0 \\ 0 & 1 & 3 & -4 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & -1 \end{bmatrix}$$

R

$R(A) = 4$

4

$$\begin{bmatrix} 1 & 3 \\ 0 & -2 \\ 5 & -1 \\ -2 & 3 \end{bmatrix}$$

- $R_3 - 5R_1$
- $R_4 + 2R_1$

$$= \begin{bmatrix} 1 & 3 \\ 0 & -2 \\ 0 & -16 \\ 0 & 9 \end{bmatrix}$$

$R_2 / -2$

$$= \begin{bmatrix} 1 & 3 \\ 0 & -2/-2 \\ 0 & -16 \\ 0 & 9 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 3 \\ 0 & 1 \\ 0 & -16 \\ 0 & 9 \end{bmatrix}$$

$R_3 + 16R_2$
$R_4 - 9R_2$

$$= \begin{bmatrix} 1 & 3 \\ 0 & 1 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$R(A) = 2$

5

$$A = \begin{bmatrix} 1 & 3 & 1 & -2 & -3 \\ 1 & 4 & 3 & -1 & -4 \\ 2 & 3 & -4 & -7 & -3 \\ 3 & 8 & 1 & -7 & -8 \end{bmatrix}$$

- $R_2 - R_1$
- $R_3 - 2R_1$
- $R_4 - 3R_1$

$$A = \begin{bmatrix} 1 & 3 & 1 & -2 & -3 \\ 0 & 1 & 2 & 1 & -1 \\ 0 & 3 & -6 & -3 & 3 \\ 0 & -1 & -2 & -1 & 1 \end{bmatrix}$$

- $R_3 - 3R_2$
- $R_4 + R_2$

$$A = \begin{bmatrix} 1 & 3 & 1 & -2 & -3 \\ 0 & 1 & 2 & 1 & -1 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$R(A) = 0$

LADE

Thursday

Eigen values & Eigen Vector

If A is $n \times n$ matrix over R, than a scalar $\lambda \in R$ is called an eigen value of A, if there exists a non-zero column vector $v \in R^n$ such that $Av = \lambda v$. In this case, v is called an eigen vector of A corresponding to eigen value λ,

$$Av = \lambda v \Rightarrow Av = \lambda I v$$

$$(A - \lambda I)v = 0$$

$$v \neq 0$$

$|A - \lambda I| = 0$ — characteristic polynomial

- over R → all values Real Numbers

$Ax = B$

$\vec{v} = \hat{i} + \hat{j} + \hat{k}$

$$\vec{v} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

$v \rightarrow$ column matrix
$\lambda \rightarrow$ scalar matrix
$A \rightarrow$ square matrix

R^2

$A \rightarrow 2 \times 2$

$$\begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

R^3

x, y, z

$A = 3 \times 3$

$AI = A$

$$\begin{bmatrix} 2 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 2 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 1 \end{bmatrix}$$

$$A = \begin{bmatrix} 2 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 2 \end{bmatrix}$$

$$|A - \lambda I| = 0$$

$$\left| \begin{bmatrix} 2 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 2 \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \right| = 0$$

$$\left| \begin{bmatrix} 2 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 2 \end{bmatrix} - \begin{bmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{bmatrix} \right| = 0$$

$$\left| \begin{bmatrix} 2-\lambda & 2 & 1 \\ 1 & 3-\lambda & 1 \\ 1 & 2 & 2-\lambda \end{bmatrix} \right| = 0$$

$$(2-\lambda)\begin{vmatrix} 3-\lambda & 1 \\ 2 & 2-\lambda \end{vmatrix} - 2\begin{vmatrix} 1 & 1 \\ 1 & 2-\lambda \end{vmatrix} + 1\begin{vmatrix} 1 & 3-\lambda \\ 1 & 2 \end{vmatrix} = 0$$

$$(2-\lambda)\left[(3-\lambda)(2-\lambda) - 2\right] - 2(2-\lambda-1) + 1(2-3+\lambda) = 0$$

$$(2-\lambda)(6-3\lambda-2\lambda+\lambda^2-2) - 2(1-\lambda) + 1(-1+\lambda) = 0$$

$$(2-\lambda)(\lambda^2-5\lambda+4) - 2(1-\lambda) + 1(-1+\lambda) = 0$$

$$2\lambda^2 - 10\lambda + 8 - \lambda^3 + 5\lambda^2 - 4\lambda - 2 + 2\lambda - 1 + \lambda = 0$$

$$\cancel{\lambda^3 + 8\lambda^2}$$

$$-\lambda^3 + 7\lambda^2 - 11\lambda + 5 = 0$$

$-\lambda^3 + 7\lambda^2 - 11\lambda + 5 = 0$

$\boxed{\lambda = 1, 1, 5} \rightarrow$ values of λ Eigen values,

$$\begin{array}{c|cccc} & -1 & 7 & -11 & 5 \\ \boxed{1} & \downarrow & -1 & 6 & -5 \\ \hline & -1 & 6 & -5 & 0 \end{array}$$

$-x^2 + 6x - 5 = 0$

$x^2 - 6x + 5 = 0$

$x^2 - 5x - x + 5 = 0$

$x(x-5) - 1(x-5) = 0$

$\boxed{\lambda = 1, 5}$

Eigen factor $\rightarrow$?

$$\begin{bmatrix} 2-\lambda & 2 & 1 \\ 1 & 3-\lambda & 1 \\ 1 & 2 & 2-\lambda \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

~~$(2-\lambda)v_1 + 2v_2 + v_3 = 0$~~

~~$-3v_1 + 2v_2 + v_3 = 0$~~

~~$v_1 - 2v_2 + v_3 = 0$~~

~~$v_1 + 2v_2 - 3v_3 = 0$~~

$\boxed{\text{put } \lambda = 5}$

$$\begin{bmatrix} -3 & 2 & 1 \\ 1 & -2 & 1 \\ 1 & 2 & -3 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$-3v_1 + 2v_2 + v_3 = 0$

$v_1 - 2v_2 + v_3 = 0$

$v_1 + 2v_2 - 3v_3 = 0$

Using Gauss's Elimination Method

Gauss's Elimination Method

$$[A|B] = \left[\begin{array}{ccc|c} -3 & 2 & 1 & 0 \\ 1 & -2 & 1 & 0 \\ 1 & 2 & -3 & 0 \end{array}\right]$$

$\cdot R_2 + \frac{1}{3}R_1$

$1 + \frac{1}{3}$

$\cdot R_3 + \frac{1}{3}R_1$

$$[A|B] = \left[\begin{array}{ccc|c} -3 & 2 & 1 & 0 \\ 0 & -\frac{4}{3} & \frac{4}{3} & 0 \\ 0 & \frac{8}{3} & -\frac{8}{3} & 0 \end{array}\right] \cdot R_3 + 2R_2$$

$\Rightarrow 2 + \frac{1}{3}(2) = \frac{8}{3}$

$$[A|B] = \left[\begin{array}{ccc|c} -3 & 2 & 1 & 0 \\ 0 & -\frac{4}{3} & \frac{4}{3} & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$\hookrightarrow$ If complete row is zeros, means it has infinitely many solution

$\boxed{-\frac{4}{3}v_2 + \frac{4}{3}v_3 = 0} \rightarrow$ (I)

$v_2 - v_3 = 0$

$v_2 = v_3$

$\boxed{v_2 = v_3 = a}$

$\boxed{-3v_3 + 2v_2 + v_3 = 0} \rightarrow$ (II)

$-3v_3 + 2a + a = 0$

$-3v_3 = -3a$

$\boxed{v_3 = a}$

$$V = \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = \begin{bmatrix} a \\ a \\ a \end{bmatrix} = a\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

For $\lambda = 5$

$[1 \quad 1 \quad 1]^t$

$\boxed{\text{put } \lambda = 1}$

$$\begin{bmatrix} 1 & 2 & 1 \\ 1 & 2 & 1 \\ 1 & 2 & 1 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$v_1 + 2v_2 + v_3 = 0$

$v_1 + 2v_2 + v_3 = 0$

$v_1 + 2v_2 + v_3 = 0$

Gauss Elimi

$$[A|B] = \left[\begin{array}{ccc|c} 1 & 2 & 1 & 0 \\ 1 & 2 & 1 & 0 \\ 1 & 2 & 1 & 0 \end{array}\right] \quad \begin{array}{l} \cdot\ R_2 - R_1 \\ \cdot\ R_3 - R_1 \end{array}$$

$$[A|B] = \left[\begin{array}{ccc|c} 1 & 2 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$v_1 + 2v_2 + v_3 = 0 \rightarrow (1)$

put $v_3 = a$

put $v_2 = b$

$v_1 + 2b + a = 0$

$v_1 = -a - 2b$

$$V = \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = \begin{bmatrix} -a-2b \\ a \\ a \end{bmatrix}$$

$\lambda = 1$

$$\begin{bmatrix} -1-2\left(\frac{b}{a}\right) & 1 & 1 \end{bmatrix}$$

$$= a \begin{bmatrix} -1-2\left(\frac{b}{a}\right) \\ 1 \\ 1 \end{bmatrix}$$

write twice because

Do it yourself

① $A = \begin{bmatrix} 3 & 1 & 1 \\ 2 & 4 & 2 \\ 1 & 1 & 3 \end{bmatrix}$

$|A - \lambda I| = 0$

Do it yourself

2

$$A = \begin{bmatrix} 1 & 2 & 2 \\ 1 & 2 & -1 \\ -1 & 1 & 4 \end{bmatrix}$$

Do it yourself

3

$$A = \begin{bmatrix} 1 & -3 & 3 \\ 3 & -5 & 3 \\ 6 & -6 & 4 \end{bmatrix}$$

Do it yourself

(4) $\begin{bmatrix} -3 & 1 & -1 \\ -7 & 5 & -1 \\ -6 & 6 & -2 \end{bmatrix}$

$|A - \lambda I_3| = 0$

Ye notes dekhne ki waja se

AP pr **MUJHE SHAWARMA KHILANA WAJIB** hogya hai :)