(ordered)

permutation — arrangement

→ arrangement is important

→ $AB \neq BA$

→ $^nP_r = \frac{n!}{(n-r)!}$

Detail:

ways to choose 1st object = n

" 2nd " = n-1

" 3rd " = n-2

" 4th " = n-3

⋮

rth = n-r+1

$^nP_r = n(n-1)(n-2)(n-3)\cdots\cdots(n-r+1)$

Multipling & dividing by $(n-r)!$

$$= \frac{n(n-1)(n-2)(n-3)\cdots\cdots(n-r+1)(n-r)!}{(n-r)!}$$

$$^nP_r = \frac{n!}{(n-r)!}$$

Example:

Bytes — Combination of 2 words.

$^nP_r \Rightarrow {}^5P_2$

$$\frac{5!}{(5-2)!} \Rightarrow \frac{5!}{3!} \Rightarrow \frac{5\times4\times\cancel{3!}}{\cancel{3!}} \Rightarrow 20$$

Example:

How many permutation when $n=7$, $r=4$

$^7P_4 = ?$

$$\frac{n!}{(n-r)!}$$

$$\frac{7!}{(7-4)!}$$

$$\frac{7!}{3!} \Rightarrow \frac{7\times6\times5\times4\times3!}{3!} \Rightarrow 7\times6\times5\times4 = 840$$

Example:

How many arrangements in row of no more then three letters for word NETWORK.

NETWORK

$= {}^7P_3 + {}^7P_2 + {}^7P_1$

$= 259$

No more than 3 letterers but 2 and 1 can be used

Pigeonhole Principle:

if "n" pigeons flag

into "m" pigeonholes and $n > m$ then atleast one pigeonhole must Contain two or more pigeons.

Example:

How many integers must be selected from "A" if we want a pair of integers having Sum = 9.

$A = \{1, 2, 3, 4, 5, 6, 7, 8\}$

(1, 8) (2, 7) (3, 6) (4, 5)

Combination: — Selection

$\Rightarrow$ $AB = BA$

$\Rightarrow$ $^{n}C_{r} = \dfrac{n!}{r!(n-r)!}$

$\Rightarrow$ replacement allow (unorder Selection)

Example:

$AB = BA$

COMPUTER

8 8 8 8 8 8 8 8

$${}^nC_r = \begin{bmatrix} n \\ r \end{bmatrix} - \frac{n!}{(n-r)!\,r!}$$

Example:

How many unordered selection of two elements from $\{0, 1, 2, 3\}$

$${}^nC_r = {}^4C_2$$

$$\frac{4!}{(4-2)!\,2!} \Rightarrow \frac{4\times3\times2!}{2!\,2!} \Rightarrow \frac{4\times3}{2} \Rightarrow \frac{12}{2} = 6$$

Example:

How many ordered selection of two element from $\{0, 1, 2, 3\}$

$${}^nP_r = \frac{n!}{(n-r)!}$$

$$\frac{4!}{(4-2)!} \Rightarrow \frac{4!}{2!} \Rightarrow \frac{4\times3\times2!}{2!} \Rightarrow 12$$

Example:

$$S = \{A, B, C, D\}$$

$n = 4$, $r = 2$

${}^nP_r = ?$

${}^nC_r = ?$

$${}^nP_r = \frac{4!}{(4-2)!} \Rightarrow \frac{4!}{2!} \Rightarrow \frac{4\times3\times2!}{2!} \Rightarrow 12$$

$${}^nC_r = \frac{4!}{(4-2)!\,2!} \Rightarrow \frac{4\times3\times2!}{2!\,2!} \Rightarrow \frac{4\times3}{2} = 6$$

ordered (Permutation)

{AB, BC, CA, AD, BA, BD, CB, CD
DA, DB, DC, AC}

unordered (Combination)

{AB, AC, AD, BC, BD, CD}

S = {0, 1, 2, 3}

ordered (Permutation)

{01, 02, 03, 10, 12, 13, 20, 21, 23,
30, 31, 32}

unordered (Combination)

{01, 02, 03, 12, 13, 23}

Example:

Suppose 2 members out of 12 members (people group) insist on working together. Select a team

Mon Tue Wed Thu Fri Sat

Containing either both or none.
How many possibilities.

5-person team are possible.

$\begin{bmatrix} 10 \\ 3 \end{bmatrix}$: 2 members working together

$\begin{bmatrix} 10 \\ 5 \end{bmatrix}$: 2 members not having same group

$$\frac{{}^{10}C_3 + {}^{10}C_5}{{}^{12}C_5} \Rightarrow \frac{\begin{bmatrix} 10 \\ 3 \end{bmatrix} + \begin{bmatrix} 10 \\ 5 \end{bmatrix}}{\begin{bmatrix} 12 \\ 5 \end{bmatrix}} \Rightarrow \frac{120 + 252}{792}$$

$$\frac{372}{792} \Rightarrow \frac{31}{66} \approx 0.469$$

Pascal's formula.

$$\begin{bmatrix} n+1 \\ r \end{bmatrix} = \begin{bmatrix} n \\ r-1 \end{bmatrix} + \begin{bmatrix} n \\ r \end{bmatrix}$$

Example: ${}^7C_5$

$$\begin{bmatrix} 7 \\ 5 \end{bmatrix} = \begin{bmatrix} 6 \\ 4 \end{bmatrix} + \begin{bmatrix} 6 \\ 5 \end{bmatrix}$$

$$15 + 6 \Rightarrow 21$$

Bionomial Theorem

$$\begin{bmatrix} n \\ r \end{bmatrix} = \begin{bmatrix} n \\ r-1 \end{bmatrix}$$

Example:

Find the Coefficient of team using Bionomial expansion

$$x^6 y^3_{r} \text{ in } (x+y)^{9=n}$$

$$\begin{bmatrix} n \\ r \end{bmatrix} = \begin{bmatrix} n \\ r-1 \end{bmatrix}$$

$$\begin{bmatrix} 9 \\ 3 \end{bmatrix} x^{9-3} y \Rightarrow \begin{bmatrix} 9 \\ 1 \end{bmatrix} x^6 y^3$$

Example

$$x^8 y \quad (x+y)^9$$

$$\begin{bmatrix} 9 \\ 1 \end{bmatrix} x^{9-1} y$$

$$\begin{bmatrix} 9 \\ 1 \end{bmatrix} x^8 y$$