Application of Linear System

Network Analysis

Network

Network is a set of branches through which smething "flow".

Branches might be electric wire (electricity flo
Pipes through which liquid/fluid flow
Traffic lanes through which traffic flow.

Nod/ Junction

In most networks, the branches meet at the points called nodes or junctions.

Flow Conservation

We will restrict our attention to the networks in which there is "flow conservation" at each node by which means that the rate of flow into any node is equal to the rate of flow out of that node."

Note: A common ~~network~~ problem in network analysis is to use known flow rates in certain branches to find the flow rate ~~a~~ in all branches.

Note Assumptions: If the directions in network are not given then assign arbitrary directions to unknown flow rates, if some direction are incoxed will be signled by negative value.

H-Anton

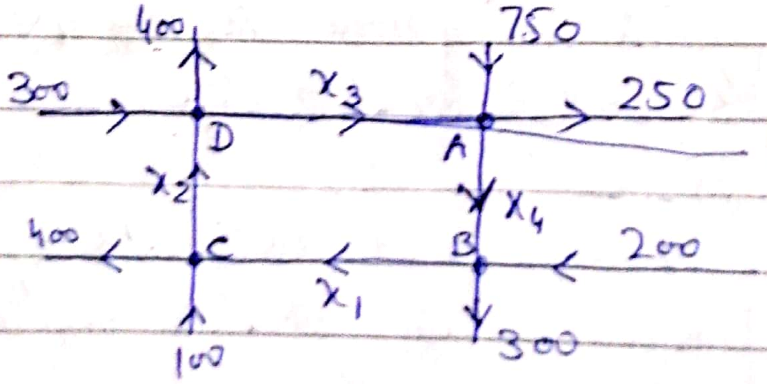
Ex 1.9

Q3 The accompanying fig shows a network of "one-way streets" with traffic flowing in the directions indicated. The flow rates along streets are "measured" as the average number of vehical per hour.

(a): Set up a linear system whose sol. provides the unknown flow rates.

(b): Solve the system for the unknown flow rates.

(c): If the flow along the road from A to B must be reduced for construction, what is the minimum flow that is required to keep traffic flowing on all roads?



At node	Flow In		Flow Out	
A	$x_3 + 750$	$=$	$x_4 + 250$	$\Rightarrow x_3 - x_4 = -500$
B	$x_4 + 200$	$=$	$x_1 + 300$	$\Rightarrow x_1 - x_4 = -100$
C	$x_1 + 100$	$=$	$x_2 + 400$	$\Rightarrow x_1 - x_2 = 300$
D	$x_2 + 300$	$=$	$x_3 + 400$	$\Rightarrow x_2 - x_3 = 100$

∴ linear system for given network is

(a)

$$\left.\begin{aligned} x_3 - x_4 &= -500 \\ x_1 \qquad - x_4 &= -100 \\ x_1 - x_2 \qquad &= 300 \\ x_2 - x_3 \qquad &= 100 \end{aligned}\right\}$$

$$A_b = \begin{bmatrix} 0 & 0 & 1 & -1 & -500 \\ 1 & 0 & 0 & -1 & -100 \\ 1 & -1 & 0 & 0 & 300 \\ 0 & 1 & -1 & 0 & 100 \end{bmatrix}$$

$$\underset{\sim}{R} \begin{bmatrix} 1 & -1 & 0 & 0 & 300 \\ 1 & 0 & 0 & -1 & -100 \\ 0 & 0 & 1 & -1 & -500 \\ 0 & 1 & -1 & 0 & 100 \end{bmatrix} R_{13}$$

$$\underset{\sim}{R} \begin{bmatrix} 1 & -1 & 0 & 0 & 300 \\ 0 & 1 & 0 & -1 & -400 \\ 0 & 0 & 1 & -1 & -500 \\ 0 & 1 & -1 & 0 & 100 \end{bmatrix} R_2 - R_1$$

$$\sim_R \begin{bmatrix} 1 & -1 & 0 & 0 & 300 \\ 0 & 1 & 0 & -1 & -400 \\ 0 & 0 & 1 & -1 & -500 \\ 0 & 1 & -1 & 0 & 100 \end{bmatrix}$$

$$\sim_R \begin{bmatrix} 1 & -1 & 0 & 0 & 300 \\ 0 & 1 & 0 & -1 & -400 \\ 0 & 0 & 1 & -1 & -500 \\ 0 & 0 & -1 & 1 & 500 \end{bmatrix} R_2 - R_4$$

$$\sim_R \begin{bmatrix} 1 & -1 & 0 & 0 & 300 \\ 0 & 1 & 0 & -1 & -400 \\ 0 & 0 & 1 & -1 & -500 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} R_4 + R_3$$

$\text{rank } A = 3 = \text{rank } A_b$, $n = 4$

$$\left.\begin{aligned} x_1 - x_2 &= 300 \\ x_2 - x_4 &= -400 \\ x_3 - x_4 &= -500 \end{aligned}\right\}$$

From last eq

$x_3 = -500 + x_4$ let $x_4 = t$

$x_3 = -500 + t$

$x_2 = -400 + t$

(b)

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(c) Si

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$$x_1 = 300 + x_2 = 300 - 400 + t = -100 + t$$

(b)

$$\therefore \left.\begin{aligned} x_1 &= -100 + t \\ x_2 &= -400 + t \\ x_3 &= -500 + t \\ x_4 &= t \end{aligned}\right\} \text{Infinite sol.} \rightarrow 1$$

From the physical constraints

(c) Since the average flow rate must be non-negative (since we assumed the streets to be one way and negative flow rate would indicate a flow in the wrong direction)

$$\therefore \; t \geq 500$$

To ~~flow~~ ~~from~~ keep the traffic flowing on all roads, the ~~rate~~ flow from 'A' to B must exceed 500 vehicles per hour.