16/03-22

~~DMA~~ ~~DMS~~

# DMS

→ In DMS, we focus on distinct objects
- logic building
- analytical

# Discrete Mathematics Structures

Discrete → discontinuity

⇒ Types of Numbers

① Discrete

② Continuous

↳ have precise pattern

i.e. movement of clock

* 4 bits → $2^4 = 16$ combinations

* ~~one~~ at least (1) must in each combinations
= 15 combinations

# Course Assessment

- Mid Terms → 30%
- Final Terms → 40%
- Quiz → 15%
- Assignment → 15%

(Quiz and Assignment) can be varied

# Topics covered in overall semester

→ logic
→ set
→ functions
→ counting
→ Induction/Deduction (proved everything by logic i.e. 2+3=4 →?)
→ Probability
→ Graph
→ Tree

## Book

K.H. Rosen 7th edition
→ Discrete Mathematics and its applications

ic everything by
i.e. 2+3=4 ...
dition
and

# Logic

- reason to present arguments
- to convince someone

→ i.e. Most of the students got D grade.

Logic (Reason)
- students not studied
- teacher don't give marks

## Propositional Logic:

⇒ statement either true or false

→ always true

→ always false

→ ~~true/false changes~~

⇒ **Fact** (similar to propositional logic)

- have Y/N (Yes/No)
- T/F (True/False)

i.e. religion

- vary geographically
- vary religionally
- vary on gender base

⇒ Proposition?

1. a declarative (descriptive) statement

→ Property either true or false

2. Statement that might be true or false

e.g: $\boxed{2+2=4}$

$\boxed{\begin{array}{c} 2+x=4 \\ x=2 \end{array}}$

propositions

$\boxed{2+x=4}$ → not a proposition

# Variables assign to Proposition

In DMS,

variables are $p, q, r, s, \ldots$

e.g:

This is discrete mathematics class. → proposition

⇒ Assigning variable:

$p$ = This is discrete mathematics class

$\neg$ → Negation

$\neg p$ = This is not discrete methamatics class

e.g: $\mathbb{N} = \{1, 2, 3, \ldots\}$

$10 \in \mathbb{N}$ → proposition

17/3-22 Thursday

# Propositional Logical Operator (connectives)

- we use logical operators to make new propositions from existing propositions.

## (i) Conjunction:

- denoted by ($\wedge$) $\Rightarrow$ AND Operator
- 2 propositions required

like AND && operator in PF
i.e. required at least 2 operands

e.g

p : Today is Thursday.
q : It is very hot today

$p \wedge q$ → connector

- outcome (Y/N) depends on individual conjunction

**Logically:**
- one false
- all false

## (ii) Disjunction: ($\vee$) $\Rightarrow$ OR Operator

e.g Students success

① p: Student got 50% marks in final
q: Student have 50% marks in sessional

② win Cricket
p: makes a century
q: takes 5 wickets

~~$p \wedge q$~~ $p \vee q$

# (Conditional Statement)

## (iii) Implication ($\longrightarrow$) $\Rightarrow$ implies

$\rightarrow p \rightarrow q$ (p implies q)

$\hookrightarrow$ ~~p q depends on p~~

q depends on p

- like if condition

p: Metric marks 950

q: Samsung mobile

$p \longrightarrow q$

- unidirectional

~~Conditional Statement~~

$p \rightarrow q$

- p (hypothesis, antecedent, premise)
- q (conclusion, consequence)

## (iv) Biimplication ($\longleftrightarrow$)

- Both propositions depend on each other
- bidirectional

same ← [ ① $p \longleftrightarrow q$
② $p \rightarrow q \wedge q \rightarrow q$ ]

- It is true when p & q have same truth values

---

Disjunction (OR (V) connective)

① Inclusive or

| p | q | $p \vee q$ |
|---|---|---|
| T | F | T |
| F | T | T |
| T | T | T |
| F | F | F |

② Exclusive or ($p \oplus q$)

- $p \oplus q$ is true when one proposition is true & other is false.

| p | q | $p \oplus q$ |
|---|---|---|
| T | F | T |
| F | T | T |

# Truth Table

| $p$ | $q$ | $p \wedge q$ | $p \vee q$ | $p \to q$ | $q \to p$ | $p \leftrightarrow q$ |
|---|---|---|---|---|---|---|
| T | T | T | T | T | T | T |
| T | F | F | T | F | T | F |
| F | T | F | T | T | F | F |
| F | F | F | F | T | T | T |

**Note (Ratta):**

Implication Operator $\to$ is only false when first proposition ($p$) is true & second proposition ($q$) is false

---

Q: $\sim p \longrightarrow \sim q = \sim(p \to q)$

Is L.H.S = R.H.S

| $p$ | $q$ | $\sim p$ | $\sim q$ | $\sim p \to \sim q$ (L.H.S) | $p \to q$ | $\sim(p \to q)$ (R.H.S) |
|---|---|---|---|---|---|---|
| T | T | F | F | T | T | F |
| T | F | F | T | T | F | T |
| F | T | T | F | F | T | F |
| F | F | T | T | T | T | F |

$\Rightarrow$ Not equal

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# Additional Operators/Conditions by using basic operators

$\boxed{p \rightarrow q}$ → original proposition

- ~~Converse~~ Inverse

  proposition $\neg p \rightarrow \neg q$

- ~~Inverse~~ Converse

  proposition $q \rightarrow p$

- **Contrapositive**

  proposition $\neg q \rightarrow \neg p$

p: Marks 950
q: Laptop
$p \rightarrow q$

⇒ All these operators (converse, inverse, contrapositive) produces same outcome.

# Equivalent Statement

When the truth table of two compound prepositions are equal.

- original preposition & contrapositive of original preposition are equivalent
- converse & inverse of a preposition are equal.

$\boxed{p \to q} \to$ original

| | | | | $p \to q$ | Co Inverse | Inverse | Contrapositive |
|---|---|---|---|---|---|---|---|
| $p$ | $q$ | $\neg p$ | $\neg q$ | $p \to q$ | $q \to p$ | $\neg p \to \neg q$ | $\neg q \to \neg p$ |
| T | T | F | F | T | T | T | T |
| T | F | F | T | F | T | T | F |
| F | T | T | F | T | F | F | T |
| F | F | T | T | T | T | T | T |

$\to$ Equal

equal

$(p \vee \neg q) \rightarrow (p \wedge q)$

24/3-22

# Operator Precedence in Proposition

① $\neg$

② $\wedge$

③ $\vee$

④ $\rightarrow$

⑤ $\leftrightarrow$

# Logic Equivalence

$\neg(p \wedge q) = \neg p \vee \neg q$

| $p$ | $q$ | $p \wedge q$ | $\neg(p \wedge q)$ | $\neg p$ | $\neg q$ | $\neg p \vee \neg q$ |
|---|---|---|---|---|---|---|
| T | T | T | F | F | F | F |
| T | F | F | T | F | T | T |
| F | T | F | T | T | F | T |
| F | F | F | T | T | T | T |

Equal

Tautology

# Predicate:

- having unknown value of a variable
- by knowing that ~~kno~~ unknown value, predicate becomes proposition

$x > 5$

$p(x): x > 5 \rightarrow$ predicate depends on proposition

$x = 3$

$p(3):$ False

$p(10):$ True

$y = x + 5$

$p(x, y)$ : $y = x + 5$

$P(2, 3)$

False

$p(x)$ : "X student envolved in DMS has passed PF."

↳ converted to propostion "T/F"

Domain {students enroll in DMS}

# Predicate Operators

(i.) $\forall$ ( for all )

(ii) $\exists$ ( this exist any)

↓

Quantifers

$\forall x\, p(x) \Rightarrow$ True

$p(x_1) \wedge p(x_2) \wedge \ldots\ldots \wedge p(x_n)$

- Evaluating every element of domain
- If true, predicate returns true

$Q(x)$; Student $x$ got $A^+$ in PF

$\exists x\, Q(x) \Rightarrow$ True

$Q(x_1) \vee Q(x_2) \vee \ldots\ldots \vee Q(x_n)$

- $\underbrace{\neg \forall x\, Q(x)}$ = $\underbrace{\exists x\, \neg Q(x)}$

↓ Not all students take $A^+$ grade in PF

↳ There exist any student x who don't take $A^+$ grade in PF

- $\underbrace{\neg \exists x\, Q(x)}$ = $\underbrace{\forall x\, \neg Q(x)}$

↓ Not any student take $A^+$ grade in PF

↳ All of the students not take $A^+$ grade in PF

# Disjunction

$P_1 \vee P_2 \vee \cdots\cdots \vee P_n \longrightarrow \bigvee_{j=1}^{n} P_j$

$\boxed{\text{In Demorgan's law:}}$

$$\neg\left(\bigvee_{j=1}^{n} P_j\right) = \bigwedge_{j=1}^{n} \neg P_j$$

# Conjunction

$P_1 \wedge P_2 \wedge \cdots\cdots \wedge P_n \longrightarrow \bigwedge_{j=1}^{n} P_j$

$\boxed{\text{In Demorgan's law}}$

$$\neg\left(\bigwedge_{j=1}^{n} P_j\right) = \bigvee_{j=1}^{n} \neg P_j$$

30/3/22

predicate:

" x is enrolled in DMS"

→ Domain specify

## Quantifiers:

$\forall$ → for all

$\exists$ → there exist any

- $\forall x(p(x) \wedge Q(x))$

Domain {SE 2021}

p(x) : x is enrolled in DMS

Q(x) : x is enrolled in OOP

→ All students enrolled in both DMS & OOP

- $\exists x(p(x) \vee Q(x))$

→ some students or may be a single student enrolled in both DMS & OOP.

"To get enroll in OOP & DMS, it is compulsory to pass the PF."

$p(x)$: x enroll in OOP and DMS

↓ Further break

$p(x)$: x enroll in OOP

$Q(x)$: x enroll in DMS

$R(x)$: x pass the PF

$$\boxed{R(x) \longrightarrow (p(x) \wedge Q(x))}$$

• $S(y,z)$: To ~~gend~~ get enroll in course y and course x

$R(x) \longrightarrow (p(x) \wedge Q(x))$

$R(x) \longrightarrow (S(z,y))$

$R(x) \rightarrow S(x, OOP, DMS)$

↳ for a particular student

Different ③ ways to define a prediate

• $(p(x) \wedge Q(x))$

↳ There are multiple predicates

• $S(x, OOP, DMS)$

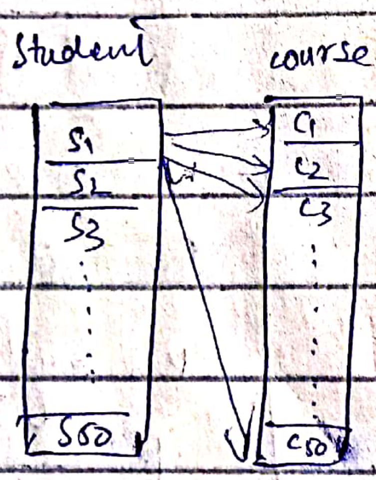
↳ multiple variables for one predicate

student {SE 2021}
course {spring}

$\begin{cases} \text{student : } (x) \\ \text{course : } (y) \end{cases}$

$P(x, y)$ : $x$ is enrolled in $y$ course

① $\forall x \forall y \, P(x, y)$




⇒ Every student is studying all courses

- returns false when all students not studying all courses

② $\exists x \forall y \, P(x, y)$

⇒ A particular student studying all courses

- returns false when no student is studying all courses

③ $\forall x \exists y\, P(x,y)$

⇒ Every student must be enrolled in a particular course

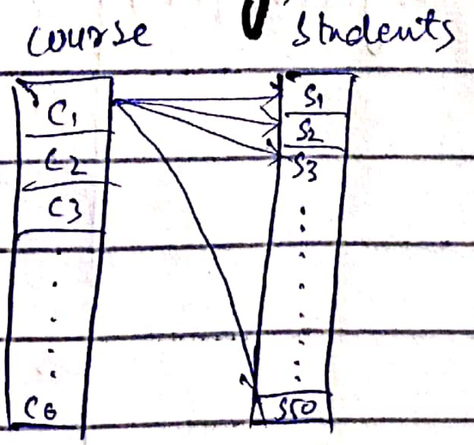
- returns false if any of the student is not studying any course

④ $\exists x \exists y\, P(x,y)$

⇒ Any student enrolled in any course

- returns false when none of the students are not studying any course

⑤ $\forall y \forall x\, P(x,y)$




⇒ Every course is studied by all students.

- returns false if any of the course is not studied by all students

# Set Theorey

Collection of unordered ~~unorder~~ distinct objects

$S = \{2, 4, 6, 8\}$

• elements in a set matter

$S = \{8, 4, 2, 6\}$

• order not matter

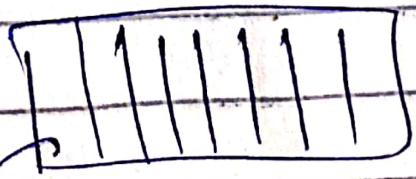
• Built-in Notation
• Set builder Notation
• Descriptive Notation

$S: \{x \,/\, 0 < x < 10 \text{ and } x \% 2 = 0\}$

Set of first 1000 Natural Numbers

$S = \{1, 2, 3, \ldots, 1000\}$

• 8 bits = 1 byte
• ~~2 bytes~~ $2^8 - 1 = 255$



• 2 bit store max value 3

| |
|---|
| 0 0 |
| 0 1 |
| 1 0 |
| 0 0 |

signed

0 + next no.

1 - next no.

$2^7 - 1 = 127$

# Subset:

$N = \{1, 2, 3, \ldots\ldots, 10\}$
$S = \{2, 4, 6, 8\}$

$\boxed{S \subseteq N} \rightarrow$ True

$p(x) : \{x \in S \rightarrow x \in N\}$

If:

$S = \{2, 4, 6, 11\}$

Then $\boxed{S \in N}$

$\hookrightarrow$ false

# Equal sets

$\hookrightarrow$ same elements

$S = \{1, 2, 3, 4, 5\}$
$N = \{1, 2, 5, 4, 3\}$

$p(x) : \{x \in S \longleftrightarrow x \in N\}$

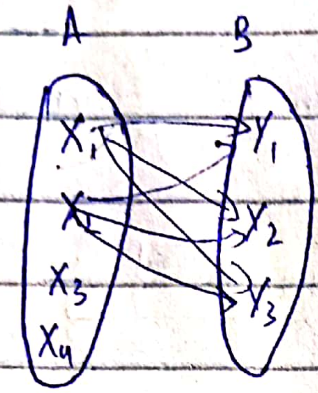
$\{(x \in S \rightarrow x \in N) \wedge (x \in N \rightarrow x \in S)\}$

3

## Singleton Set:

Set having only one element.

$A = \{x\}$



Total pairs = $4 \times 3 = 12$

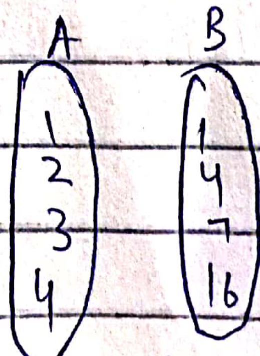
$(x_1, y_1), (x_1, y_2), (x_1, y_3) \ldots\ldots$

## Functions

$f : A \to B$

f is function defined on A & B

gives input
returns output



Mapping from A to B, we put some input A, which gives some output B.

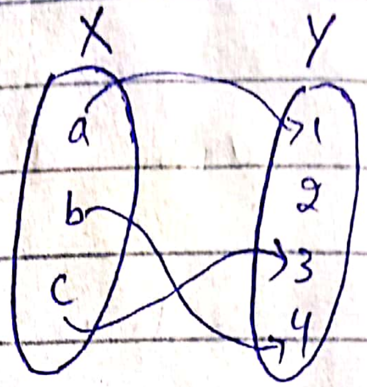
$f : \boxed{A} \to \boxed{B}$

domain (all possible inputs)

codomain (all possible outputs)

Range

actual outcome that appears



codomain

$Y = \{1, 2, 3, 4\}$

Range = $\{1, 3, 5\}$

## Image / Pre-image

$\boxed{f(a) = 1}$

- 1 is image of a
- a is pre-image of 1.

# Types of Functions

↳ output defined against each element of domain

(i) One to One (Injective)

(ii) Onto (Surjective)

(iii) One to One Corespondance (Bijective)

⇒ **Partial Functions**

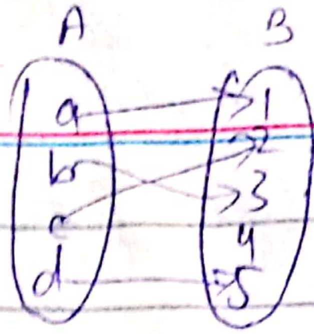
functions that not operate in some circumstance

i.e. Number % by zero

## (i) One to One (Injective)

• Every pre-image has a unique image.

Each element of domain must has a unique output




$f: A \rightarrow B$

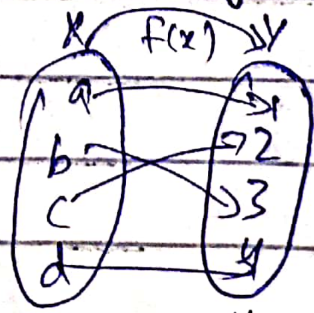
$|A| \leq |B|$

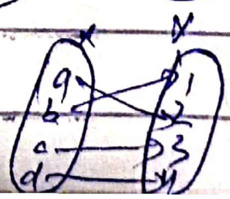
• size of codmain (B) must be greater than or equal to domain (A)

If $|A| = 4$, $|B| = 3$ — function but not one-to-one

## (ii) Onto (Surjective)

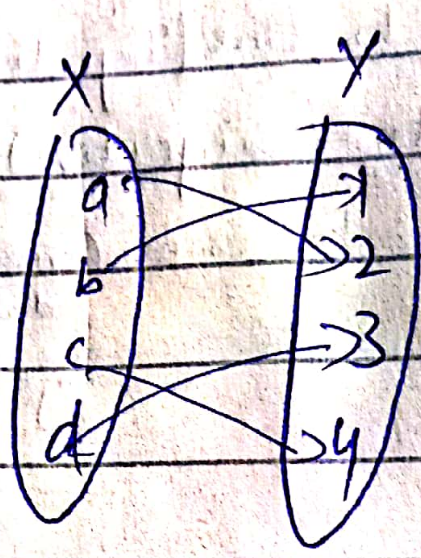
• codmain & range equal



# One - to - One ~~Correspondence~~ Correspondance
# (Bijective)

having property of one to one
& onto



# $\boxed{1-4}$

## Sequence

- finite sequence
- infinite sequence

$a_m + a_{m+1} + a_{m+2} + \dots + a_n$

$a$ → term; $m$ → index / subscript; $a_n$ → final term

$\boxed{a^k = 2^k}$ → Explicit form

## Finding term from Explicit formula

We can write whole sequence from explicit formula

$a_k = \frac{k}{k+1}$ 1st 6 terms

$\frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \frac{4}{5}, \frac{5}{6}, \frac{6}{7}$

$a_i = \frac{i-1}{i}$ for $i \geq 2$

first 6 terms

$= \frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \frac{4}{5}, \frac{5}{6}, \frac{6}{7}$

## write formula when intial term are given:

$1, -\frac{1}{4}, \frac{1}{9}, -\frac{1}{16}, \frac{1}{25}, -\frac{1}{36}$

| index | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| | $1$ | $-\frac{1}{4}$ | $\frac{1}{9}$ | $-\frac{1}{16}$ | $\frac{1}{25}$ | $-\frac{1}{36}$ |

$\frac{1}{1^2}, \frac{-1}{2^2}, \frac{1}{3^2}, \frac{-1}{4^2}, \frac{1}{5^2}, \frac{-1}{6^2}$

$\therefore k \geq 1$

$\boxed{a_k = \frac{(-1)^{k+1}}{k^2}}$

**2, 6, 12, 20, 30, 42, 56**

| index | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|---|
| | 2 | 6 | 12 | 20 | 30 | 42 | 56 |
| | 1×2 | 2×3 | 3×4 | 4×5 | 5×6 | 6×7 | 7×8 |

$\boxed{a_k = k(k+1)}$ for $k \geq 1$

# Arithmetic Sequence

also known as Arithmetic Progression (AP)

⇒ A sequence in which each term is achieved by adding a constant term in previous term.

1, 2, 3, 4, 5 .......

e.g.:

$\boxed{5}, 9, 13, 17, 21, \boxed{25}$

a

$d = 9-5 = 4$

$d = 13-9 = 4$

$a_n = 25$

• Same distance (4), so it is arithmetic sequence.

$$\boxed{a_n = a + (n-1)d}$$

↳ formula to find any term

Example:

Find the $20^{th}$ term

$S = 3, 9, 15, 21, \dots$

$n = 20$

$a = 3 \quad d = 9-3 = 6$

$a_{20} = 3 + (20-1)(6)$
$= 3 + (19 \times 6)$
$= 117$

4, 1, -2, .....

which term of this is -77

$a = 4$

$d = 1-4 = -3$

$\therefore a_n = a + (n-1)d$

$-77 = 4 + (n-1)(-3)$

$-77 = 4 - 3n + 3$

$-77 - 7 = -3n$

$-84 = -3n$

$n = \frac{84}{3} = 28$

$\boxed{n = 28}$

Find the $36^{th}$ term of AP whose $3^{rd}$ term is 7 & $8^{th}$ term is 17

$a_{36} = ?$

$a_n = a + (n-1)d$

$a_3 = a + (3-1)d \Rightarrow 7 = a + 2d$

$a_8 = a + (8-1)d \Rightarrow 17 = a + 7d$

$7 = \cancel{a} + 2d$

$\pm 17 = \cancel{\pm a} \pm 7d$

$-10 = -5d$

$\boxed{d = 2}$

put $d = 2$ in (i)

$7 = a + 2(2)$

$7 - 4 = a$

$\boxed{a = 3}$

$\therefore a_{36} = a + (36-1)d$

$a_{36} = 3 + 35(2)$

$\boxed{a_{36} = 73}$

# Geometric Sequence

A sequence in which each term except 1st term is achieved by multiplying proceeding term with a constant.

S: 1, 2, 4, 8, 16, ......

S: 0.1, 0.01, 0.001, 0.0001, ......

$\boxed{a_n = ar^{n-1}}$ → for all integers, $n \geq 1$

① Find the 8th term of sequence

4, 12, 36, 108, ......

$a = 4$

$\boxed{r = \frac{a_n}{a_{n-1}} = \frac{12}{4} = 3}$

$\therefore a_n = ar^{n-1}$

$a_8 = 4(3)^7$

$= 4(2187)$

$= 8748$

② Which term of G.P is $\frac{1}{8}$ if 1st term is 4 and $r = \frac{1}{2}$

$a = 4$

$r = \frac{1}{2}$

$a_n = \frac{1}{8}$

$a_n = ar^{n-1}$

$\frac{1}{8} = 4\left(\frac{1}{2}\right)^{n-1}$

$\frac{1}{32} = \left(\frac{1}{2}\right)^{n-1}$

$\left(\frac{1}{2}\right)^5 = \left(\frac{1}{2}\right)^{n-1}$

$5 + 1 = n$

$\boxed{n = 6}$

Write the G.P with the terms whose 2nd term is 9 & 4th term is 1

$a_4 = 1$

$a_2 = 9$

$\therefore a_n = ar^{n-1}$

$a_4 = ar^{4-1}$ ~~$\Rightarrow a_4 = ar^3$~~

$a_2 = ar^{2-1}$ ~~$\Rightarrow a_2 = ar$~~

$1 = ar^3$

$9 = ar$

$$\frac{1}{9} = \frac{\cancel{a}r^{\cancel{3}}}{\cancel{a}\cancel{r}}$$

$$\frac{1}{9} = r^2$$

$$\left(\frac{1}{3}\right)^2 = r^2$$

$$\boxed{\frac{1}{3} = r}$$

$$9 = a\left(\frac{1}{3}\right)$$

$$\boxed{a = 27}$$

Sequence

$27, 9, 3, 1, \frac{1}{3}, \frac{1}{9}, \ldots$

$a_n = ar^{n-1}$

$a_1 = 27\left(\frac{1}{3}\right)^0$

$a_1 = 27$

$a_2 = (27)\left(\frac{1}{3}\right)^{2-1}$

$a_2 = \overset{9}{\cancel{27}}\left(\frac{1}{\cancel{3}}\right)$

$a_2 = 9$

# Series

↳ all terms add

In series, we have summation (sum)

$2+4+8+16+32+\ldots\ldots$

$$\boxed{\sum_{i=1}^{5} 2^i} = 2^1+2^2+2^3+2^4+2^5$$

↳ Summation Notation / Sigma

$$\sum_{i=1}^{4}(2i-1) = [2(1)-1]+[2(2)-1]+[2(3)-1]+[2(4)-1]$$

$$\sum_{i=1}^{4}(2i-1) = 1+3+5+7$$

$$\sum_{i=0}^{5}\frac{(-1)^i}{i+1}$$

$$+1-\frac{1}{2}+\frac{1}{3}-\frac{1}{4}+\frac{1}{5}-\frac{1}{6}$$

# Properties of Summation

① $\displaystyle\sum_{k=m}^{n} c\cdot a_k = c\sum_{k=m}^{n} a_k$

② $\displaystyle\sum_{k=1}^{n} c = n\cdot c$

2 کو پانچ دفعہ add کرنے سے بہتر ہے 2x5 کریں

③ $\displaystyle\sum_{k=m}^{n}(a_k+b_k) = \sum_{k=m}^{n} a_k + \sum_{k=m}^{n} b_k$

# Arithmetic Series

$\Rightarrow$ Sum of terms of Arithmetic Progression is arithmetic Series.

$$S_n = \frac{n}{2}\left[2a + (n-1)d\right]$$

$$S = \underline{a} + \underline{a+d} + \underline{a+2d} + \cdots + \underline{a+(n-1)d}$$

$$S = \underline{a+(n-1)d} + \underline{a+(n-2)d} + \underline{a+(n-3)d} + \cdots + \underline{a}$$

$$a+a_n \quad a+a_n \quad a+a_n \quad \quad a+a_n$$

$$2S = n(a+a_n)$$

$$\boxed{a+d+a+2n-2d = a+(n-1)d}$$

$$S_n = \frac{n}{2}(a+a_n)$$

$$S_n = \frac{n}{2}\left[a + a + (n-1)d\right]$$

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