DMS lec # 1 16-03-2022

Discrete Mathematics Structure

Book: K. H. Rosen 7th

Discrete Mathematics & its app

→ Logic → Reasoning

Propositional logic → A statement whether true / False

e.g $2+3=4$ but when $\boxed{x+2=4}$ not a propositional logic.

$x=2$, $x+2=4$ ⇒ Propositional Logic.

p, q, r, s ... these variables are used

p ⇒ This is discrete Mathematics class. (T/F)

¬p This is not discrete Mathematics class.

↳ not operation.

① $(4y-x^2)dx+dy=0$

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Propositional logic Operation/Operator

Major Operators

i. **Conjunction** (AND) operator (.) (∧) (&&)

→ Binary operator (Require two operands)

→ (∧) symbolic rep.

p: Today is Thursday

q: It is very hot ~~to~~day.

$p \wedge q$

p	q	$p \wedge q$
T	T	T
F	F	F
T	F	F
F	T	F

ii) **Disjunction** (V) (OR) operator (||)(V)

p: Today is Thursday. / Students got 50% m in final.

q: It is very hot day. / Students have 50% m in sessional.

$p \vee q$

p	q	$p \vee q$
T	T	T
F	F	F
T	F	T
F	T	T

(iii) **Implication** ($\longrightarrow$) (implies that)

- If p is T then q is applied.
- q is dependent on the outcome of p.

p: Matric marks 90.

q: Samsung Mobile

p	q	$p \to q$
T	T	T
T	F	F
F	T	T
F	F	T

p is true but q is False.

base fact true ho jaye ~~(apny (end))~~ apni full koshish krna k true ho jaye but effect generate hona tha wo false ho jaye.

~~(اگر base فیکٹ فالز ہو اور دوسرا ٹرو ہو تو بھی فائنل آنسر آئے گا)~~

(اگر base فیکٹ ٹرو اور effect والا فالز ہو تو result false ho ga.)

(iv) **Biimplification** ($\longleftrightarrow$)

~~DA~~ $(p \to q) \wedge (q \to p)$

$q \to p$	~~$q \to p$~~	$p \leftrightarrow q$
T	~~T~~	T
T	~~F~~	F
F	~~T~~	F
T	~~T~~	T

$p \leftrightarrow q$
T
F
F
T

Additional Operator / Conditions

$p \rightarrow q$

Converse

$\cancel{\neg p \rightarrow \neg q}$ $\quad q \rightarrow p$

Inverse

$\cancel{q \rightarrow p}$ $\quad \sim p \rightarrow \sim q$

Contrapdosition

$\neg q \rightarrow \neg p$

p	q	$\sim p$	$\sim q$	$p \rightarrow q$	$\sim(p \rightarrow q)$	$\sim p \rightarrow \sim q$
T	T	F	F	T	F	T
T	F	F	T	T	F	T
F	T	T	F	F	T	F
F	F	T	T	T	F	T

Tautology

Always return true.

$P \vee \neg P$

P	$\neg P$	$P \vee \neg P$	$P \wedge \neg P$
T	F	T	F
T	F	T	F
F	T	T	F
F	T	T	F

Always return false

$P \wedge \neg P$

Predicate

Such statements about which we are not sure that it is T/F.

⇒ unknown variable

$x > 5$ (x is unknown)

Representation of exp:

$P(x) : x > 5$

------- Convert into proposition ------

$x = 3$

$P(3) \Rightarrow$ False

$x = 10$

$P(10) \Rightarrow$ True

Priority of Operator

$\neg p \vee \neg q$

1) Negation $\neg$
2) And $\wedge$
3) OR $\vee$
4) implies $\longrightarrow$
5) Biimplies $\longleftrightarrow$

--, ++, = R→L

Logical equivalence

$\neg(p \wedge q) = \neg p \vee \neg q$

p	q	$\neg p$	$\neg q$	$p \wedge q$	$\neg(p \wedge q)$	$\neg p \vee \neg q$
T	T	F	F	T	F	F
T	F	F	T	F	T	T
F	T	T	F	F	T	T
F	F	T	T	F	T	T

Predicate example

Students enroll in DMS. (No Specify student)

Eg Students enroll in DMS passed PF.

$P(X) = X$ (whole line).

Domain only passed students.

let X = Ahmed, then we can say T/F.

Operator (Quantifiers)

Quantity (countable)

i) $\forall$ (for all) Universal

ii) $\exists$ (This exist only)

$\forall x\, P(x)$ (Domain should be define)

Domain = { student enroll in DMS }

$P(x_1) \wedge P(x_2)$ ------

جو ہم Argument / claim کر رہے ہیں اگر وہ غلط ہو جائے تو فالز ہو جائے گا۔

$Q(x)$: Student x got A^+ in PF.

$\exists x\, Q(x) \Rightarrow T$ (atleast value)

x particular value in domain that give true.

اگر ہم فالز prove کرنا چاہتے ہیں تو ہمیں atleast 1 individual کو فالز prove کرنا ہوگا۔

$$\underbrace{\neg \forall x\, Q(x)} = \exists x \neg Q(x)$$

atleast one student don't have A^+

$$\neg \exists x\, Q(x) = \forall x \neg Q(x)$$

DMS Lec #

Predicates: single, multiple & with ... Evaluate -

Set theory

Collection of (finite), well defined, unordered, distinct objects.

set of 10 even numbers: {2, 4, 6, 8}

{x | 0 < x < 10 and x % 2 = 0}

Set 1 Set of first 1000 N. S{1, 2, 3 ... 1000}

Storage sy range fit hoti hy architecture sy.

I: {+ ....}, F: {.....}

DT: {I, F, B, C, ...} Data type

Subset: S is subset of N. Every element of S should be present in N set.

N = {1, 2, 3, ... 10}, S{2, 4, 6, 8}

$S \subseteq N$

ISP

P(x): {x ∈ S → x ∈ N} Predicate

For equal set

Equal number, Same Numbers

Of Element of element

{x ∈ S ↔ x ∈ N}

Biimplies

{x ∈ S → x ∈ N} ∧ {x ∈ N → x ∈ S}

DMS lec

Set theory:

. Database is application
. Array
. Struct

Methods to represent

- Roaster method

list down every element in parenthesis {...}

$A = \{1, 2, 3 --- 10\}$

- Set builder notation

$A = \{x \mid x \in N, 1 \le x < 10\}$ or $0 < x \le 10$

$A = \{x \mid x$ is set of first 10 Natural Numbers$\}$

Subset:

$A \subseteq B$: $A = \{1, 3, 5\}$, $B = \{1, 2, 3, 4 --- 10\}$

Element of set A must belongs to elements of B.

$\forall x (x \in A \rightarrow x \in B)$

$A \subset B \Rightarrow$ Proper set

A should not be equal to B

There must be atleast one unique element in B. that is not in A.

$\{1, 3, 4, 5\} \subset \{1, 2, 3, 4, 5\}$

$\forall x (x \in A \rightarrow x \in B) \wedge \exists x (x \in B \rightarrow x \notin A)$

Equal Set :

$A = \{1, 1, 3, 4\}$ when content is matched
$B = \{1, 3, 4\}$ both sets are equal. (Basically no. of elements are automatically matched)
$A = \{1, 3, 4\}$

$\forall x (x \in A \leftrightarrow x \in B)$ or

$\forall x (x \in A \rightarrow x \in B) \wedge \forall x (x \in B \rightarrow x \in A)$

Empty set:

Set with no elements

ϕ, $\{\}$ but $\{\phi\}$ is not empty set

$\{\{\}\}$ → There exists one element whose value is null.

Set cardinality :

Array → length and size

union → length and size

It gives no. of elements in a set.

$A = \{\cancel{1}, \cancel{2}, 3, 5, 7, \ldots 19\}$ $\frac{18}{2}$ 10

cardinality = 10 (no. of elements)

$|A| = 10$

$|A|$ of infinite set is infinite

$|A| = 0$ when $A = \phi$

Set intervals

$[a, z] \Rightarrow$ set of alphabets

Open interval	Close interval
◎ ( ), Here elements in brackets are not included.	◎ [ ], it means that elements in [a, z] a, z are part of set.
◎ (a, z], a is not included.	◎ (a, z], Here z is part of interval.

Theorem:- (must be non-empty set)

$A = \{ \quad \}$

How many "atleast" (elements) subsets can be there in non empty set.

$A = \{a\}$

$\{\} \subseteq A$, $A \subseteq A$ at-least there must be two subsets of non empty set.

Power Set:

Set consist of all subsets.

$A = \{a, b\}$ $\qquad 2^N = 2^2 = 4$

$P(A) = \{ \{\}, \{a\}, \{b\}, \{a, b\} \}$

Singleton Set:

Only one ~~set~~ element x in a set.

SET OPERATIONS:

(1) Union
(2) Intersection
(3) Difference
(iv) complement
(5) Cartesian Product
(6) Disjoint set

(1) Union

Resultant set = R, elements of R belongs to A and B.

$A = \{1, 2, 3, 4\}$, $B \{4, 5, 6, 7\}$

$A \cup B = \{1, 2, 3, 4, 5, 6, 7\}$

$|A \cup B| = 6 \Rightarrow \left[\begin{matrix} A = N \\ B = M \end{matrix}\right.$ $\quad |A| + |B| - \{\cancel{|A| \cap |B|}\} \; |A \cap B|$

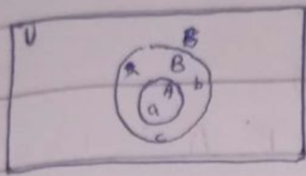
(2) Intersection

Common elements of A and B

$A = \{1, 3, 5, 7\}$, $B = \{2, 3, 5, 7\}$

$A \cap B = \{3, 5, 7\}$

$\boxed{A \subseteq B}$

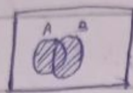


$A = \{a\}$

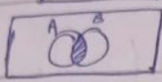
$B = \{b, a, c\}$

Universal set $\Rightarrow$ alphabets (A-

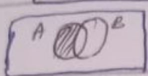
$\boxed{A \cup B}$



$\boxed{A \cap B}$



$\boxed{A \setminus B \text{ or } A - B}$



$A \{a, b, g, h\}$, $B = \{a, b, c, d\}$

$A - B = \{a, b\}$

(A'

$\forall x (x \in A \rightarrow x \notin B)$ ✗

$\exists x (x \in A \rightarrow x \notin B)$ ✓ atleast there exits one element that does not belong to B

Disjoint set

A disj B $\rightarrow$ must return { }.

$A \cap B = \phi$ tells A and B are disjoint set.

Complement

$\bar{A} = A' = U - A$

$U = \{1, 2, 3, 4, \ldots 10\}$

$A = \{1, 2, \text{---} 8\}$

$A' = \{9, 10\}$

Cartesian Product

$A = \{a_1, a_2, a_3, a_4, \ldots a_n\}$

$B = \{b_1, b_2, b_3, b_4, \ldots b_n\}$

$A \times B = \{(a_i, b_i) \mid a_i \in A,\ b_i \in B \wedge i = 1, 2, 3, \ldots n\}$

$A \times B = 6$

$A = \{1, 2, 3\}, \quad B = \{a, b\} \qquad |A| = 3, \ |B| = 2$

$A \times B = \{(1,a), (1,b), (2,a), (2,b), (3,a), (3,b)\}$

$A \times B \neq B \times A$

as order of pair changes (a,b) from $A \times B$ is $\neq$ (b,a) from $B \times A$

Relations

In $A \times B$ se ایسے pairs extract کرنے ہیں جن میں b آتا ہو

$\{(1,b), (2,b), (3,b)\}$

Student

id	Name	Contact
1	A	03--
2	B	03.-

Course

cid	Title	Teacher
01	OOP	X
02	DMS	Y

$S \times C$

id	Name	Contact	cid	Title	Teacher
1	A	A---	01	OOP	X
~~1~~	A	0--	02	Dms	Y

ان تمام students کا Data extract کرتا ہے۔ جو DMS پڑھ رہے ہیں

1	A	—	02	DMS	Y

aik ye row aur mazeed

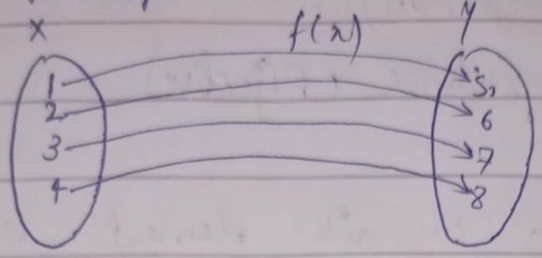
Outer join, Inner join

DMS Lec

Function

input → Black box → Output

provides ~~mechanism~~ from one set to other set.
mapping / transportation

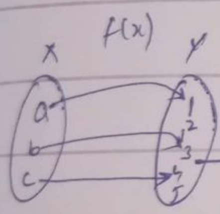


$f : A \to B$

domain — all possible inputs

codomain — all possible outputs / outcomes

outcomes that appears for specific function ⇒ range.



→ possible outcomes ⇒ codomain
(5) (1, 2, 3, 4, 5)

1, 3, 4 are assosicated with domain

Range {1, 3, 4}   Range is subset of codomain

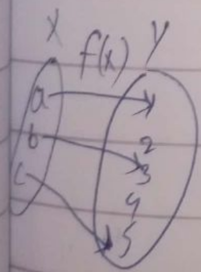


image → output

pre-image → input (ie b)

f(a) = 1   1 is image of a / a is preimage of 1

f(b) = 3

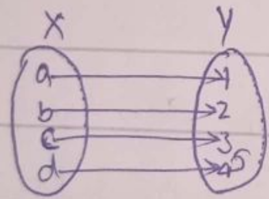
f(c) = 5

Types of functions

(i) One-One function (Injection)
(ii) Onto (Surjection)
(iii) One to One correspondence (Bijective)

Domain ka hr aik element k against koi na koi value ho. Kuch specific domain k against output nhi atti kuch functions ki un ko partial function kehty hain.

i)



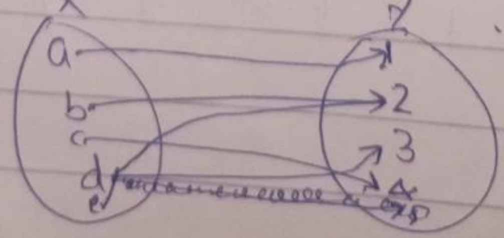
ہر pre img کا unique img ہو
ہر domain کے against unique output
یہ تو (1-1) فنکشن ہوگا

|X| should be less than or equal to |Y| ($f : x \to y$ $|x| \le |y|$)

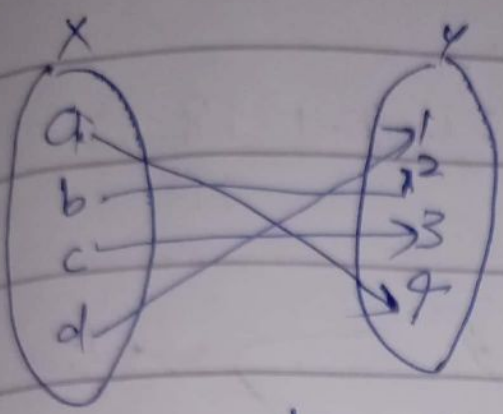
(ii) Onto (Surjection)

codomain = range

codomain 1234
range 1234



(iii)



Bijective

1-1 , codomain = range

DMS lec

Sequence and Series

To study Repeated process → (require) series

1	2	3	4	5
2	4	8	16	32 ....

index	1	2	3	4	5	6	7
value	2	4	8	16	32	64	128
	2^1	2^2	2^3	2^4	2^5	2^6	2^7

10 Generation ⇒ $2^k = a_k$

$a_{10} = 2^{10}$

$a_m, a_{m+1}, a_{m+2}, \dots a_n$

term index/subscript final term

جس میں فائنل ٹرم ہو وہ finite
اگر نہ ہو تو infinite seq ہوتی ہے

Explicit formula

وہ فارمولا جو ہم لکھ لیں بجائے ساری sequence لکھنے کے۔

$a^k = 2^k$

$$\boxed{a_k = \frac{k}{k+1}}$$ (1st . 6 terms as:)

$\frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \frac{4}{5}, \frac{5}{6} \dots$

$$\boxed{a_i = \frac{i-1}{i} \quad \text{for } i \geq 2}$$

$\frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \frac{4}{5}, \frac{5}{6} \dots$

Write formula when initial terms are given

$1, -\frac{1}{4}, \frac{1}{9}, -\frac{1}{16}, \frac{1}{25}, -\frac{1}{36}$ $\quad n \geq 1$

index	1	2	3	4	5	6
value	1	$-\frac{1}{4}$	$\frac{1}{9}$	$-\frac{1}{16}$	$\frac{1}{25}$	$-\frac{1}{36}$

$$a_k = \frac{(-1)^{k+1}}{k^2}$$

$\Rightarrow 2, 6, 12, 20, 30, 42, 56$

4 6 8 10 12 14

	1	2	3	4	5	6	7
index	2	6	12	20	30	42	56
value	1x2	2x3	3x4	4x5	5x6	6x7	7x8

pattern

$$a_k = k(k+1) \quad . \text{for } k \geq 1$$

$1(1+1)$

2

$2(2+1)$

2×3

$\Rightarrow \frac{1}{4}, \frac{2}{9}, \frac{3}{16}, \frac{4}{25}, \frac{5}{36}, \ldots$

$$a_k = \frac{k}{(k+1)^2}$$

Arithmetic Sequence / Progression

A sequence in which each term is achieved by adding a constant term in previous one.

$1, 2, 3, 4; 5, \ldots$

e.g: $5, 9, 13, 17, \ldots 21, 25$

a $\quad d = 9-5 \Rightarrow 4$ always constant in A.P

$a_n = 25$

$a_n = a + (n-1)d$

⇒ example: Find the 20^{th} term of sequence

$S = 3, 9, 15, 21, \ldots$

$d = 9-3$

$= 6$

$\begin{array}{r} 19 \\ \times 6 \\ \hline 114 \end{array}$

$a_{20} = 3 + (20-1)6$

$= 3 + (19 \times 6)$

$= 114 + 3 \Rightarrow 117$

⇒ $4, 1, -2, \ldots$

$4-1$

-3

$a_n = a + (n-1)d$

$-77 = 4 + (n-1)(-3)$

$-77 - 4 = -3n + 3$

$-81 - 3 = -3n$

$\frac{-84}{-3} = n$

$28 = n$

Find the 36^{th} term of AP whose 3rd term is 7, 8^{th} term is 17

$a_{36} = ?$

$a_3 = 7$

$a_8 = 17$

$a_3 = a + (n-1)d$

$a_8 = a + (n-1)d$

$7 = a + (n-1)d$

$17 = a + (n-1)d$

$7 = a + nd - d$, $17 = a + nd - d$

$7 - a = (n-1)d$

$\frac{7-a}{n-1} = d$

$\frac{17-a}{n-1} = d$

$\frac{7-a}{n-1} = \frac{17-a}{n-1}$

$7 - a = 17 - a$

$a_{36} = a + (n-1)d$

$a_8 = a + (8-1)d$

$a_8 = a + 7d$

$a_3 = a + 2d$

$10 = 5d$

$2 = d$

$a_8 = a + 2(7)$

$17 - 14 = a$

$3 = a$

$a_{36} = 3 + 35(2)$

$= 3 + 70$

$\boxed{a_{36} = 73}$

Geometric Sequence

A sequence in which each term except first term achieved by multiplying previous term with a constant.

S: 1, 2, 4, 8, ..., 16

: 0.1, 0.01, 0.001, 0.0001, ...

General formula:

$a_n = ar^{n-1}$ for all integers $n \geq 1$

① find 8th term of sequence

4, 12, 36, 108 ..

$r = \frac{12}{4} = 3$

$a_8 = 4(3)^{8-1}$

$= 4(3)^7$

$= 8748$

② which term of G.P is 1/8 if 1st term is 4, r = 1/2

$a_n = ar^{n-1}$

$\frac{1}{8} = 4 \times \left(\frac{1}{2}\right)^{n-1} \rightarrow \frac{1}{32} = \left(\frac{1}{2}\right)^{n-1}$

$\cancel{\frac{1}{2} = \left(\frac{1}{2}\right)^{n-1}}$

$\left(\frac{1}{2}\right)^5 = \left(\frac{1}{2}\right)^{n-1}$

$5 = n-1$

$5+1 = n$

$6 = n$

Trick

Write the GP with terms whose 2nd term is 9 and 4th term is 1.

$a_2 = 9 \quad , \quad a_4 = 1$

$a_2 = ar^{2-1}$

$a_2 = ar$

$9 = ar$

$\frac{9}{a} = r$

$a_4 = ar^{4-1}$

$a_4 = ar^3$

$1 = ar^3$

$1 = a\left(\frac{9}{a}\right)^3$

$= a\frac{(9)^3}{a^3}$

$a^2 = 9^3 \quad 9^2 \cdot 9$

$a = 9 \times 3$

$a = 27$

$\frac{9}{27} = r$

$\frac{1}{3} = r$

$a = ar^{n-1}$

$27, 27$

$= 27, 9, 3, 1, \frac{1}{3}, \text{-----}$

Series

$2 + 4 + 8 + 16 + 32 + \cdots$

That's why series, not sequence

Summation notation / ~~also~~ Sigma

$$\sum_{i=1}^{5} 2^i$$

$$\Rightarrow \sum_{i=1}^{4} (2i-1) = (2(1)-1) + (2(2)-1) + (2(3)-1) + (2(4)-1)$$
$$= (2-1) + (4-1) + (6-1) + (8-1)$$

$$\sum_{i=1}^{4} (2i-1) = 1 + 3 + 5 + 7 = 16$$

$$\sum_{i=0}^{5} \left(\frac{(-1)^i}{i+1}\right) \quad \frac{(-1)^0}{0+1} + \frac{(-1)^1}{1+1} + \frac{(-1)^2}{2+1} + \frac{(-1)^3}{3+1} + \frac{(-1)^4}{4+1} + \frac{(-1)^5}{5+1}$$

$$1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \frac{1}{5} - \frac{1}{6}$$

① $$\sum_{k=m}^{n} c \cdot a_k = c \sum_{k=m}^{n} a_k$$

② $$\sum_{k=1}^{n} c = n \cdot c$$

③ $$\sum_{k=n}^{n} (a_k + b_k) = \sum_{k=n}^{n} a_k + \sum_{k=m}^{n} b_k$$

Arithmetic Series

Sum of arithmetic series / progression is arithmetic series.

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

$$S = \underline{a} + \underline{a+d} + \underline{a+2d} + \underline{a+(n-1)d}$$

$$S = a+(n-1)d + a+(n-2)d + a+(n-3)d + a$$

$$S = a+a_n + a+a_n + a+a_n + a+a_n$$

$$2S_n = n(a+a_n)$$

$$S_n = \frac{n}{2}(a+a+(n-1)d)$$

$$= \frac{n}{2}(2a+(n-1)d)$$

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Arithmetic Series

$$S = \frac{n}{2}(2a + (n-1)d)$$

$$= \frac{n}{2}(a + a_n)$$

initial final term

$S = 1+2+3+4+ \cdots$

$n = 50$

$\frac{50}{2}(2(1) + (50-1))$

$25(2+49) \Rightarrow (51)$

51×25

1275

$S_{10} = 3+9+15+21$

$= \frac{10}{2}(2(3) + (10-1)6)$

$= 5(6+(9 \times 6))$

$= 300$

$= \frac{10}{2}(3+21)$

$= 5 \cdot (24)$

$= 120$

$1+4+7+10+13$

$S_8 = ?$, $a = 1$, $d = 3$

$S_8 = \frac{8}{2}(2(1) + (8-1)3)$

$= 4(2 + 7 \times 3)$

$= 4(2+21)$

$= 23 \times 4$

$= 92$

Geometric Series

$$ar^0 + ar^1 + ar^2 + \cdots + ar^{n-1}$$

$$\sum_{i=0}^{n-1} ar^i \rightarrow \text{sigma notation}$$

Sum of geometric series is if $r<1$ then

$$S = \frac{a(r^n - 1)}{r-1} \qquad r \rightarrow \text{common ratio} \qquad \Bigg| \quad \frac{a(1-r^n)}{1-r}$$

$\Rightarrow$ $a = 2$, $r = 2$ of $2 + 2^2 + 2^3 + 2^4 + \cdots$

$n = 10$

$$S_{10} = \frac{2(2^{10} - 1)}{2 - 1} \Rightarrow \frac{2(1023)}{1}$$

$S_{10} = 2046$ $\qquad 2^{10}$ KB

$\Rightarrow$ $4 + 12 + 36 + 108 + \cdots$

$a = 4$, $r = 3$

$$S_6 = \frac{4(3^6 - 1)}{3-1}$$

$$= \frac{4(728)}{2}$$

$$= 1456$$

$\frac{9}{4} + \frac{3}{2} + 1 + \frac{2}{3}$

$a = 9/4$, $r = 2/3$

$n = 6$

$$= \frac{\frac{9}{4}\left(\left(\frac{2}{3}\right)^6 - 1\right)}{\frac{2}{3} - 1}$$

$$= \frac{665}{108}$$

1) Principle of Mathematical Induction

Works on Dominos effect

$$1+2+3+4+\dots+n = \frac{n(n+1)}{2} \quad \text{for } n \geq 1$$

1- **Basis step:**

prove for $n=1$

$$1 = \frac{1(1+1)}{2} = 1$$

کسی بھی arbitrary ہے

2- **Inductive Step:**

let it be true for $n = K \geq 1$

$$1+2+3+\dots+k = \frac{k(k+1)}{2}$$

Now we have to prove it is true for $n = k+1$

$$1+2+3+\dots+k+(k+1) = \frac{k(k+1)}{2} + k+1$$

R.H.S

$$= \frac{k(k+1)}{2} + (k+1)$$

$$= \frac{k(k+1)+2(k+1)}{2}$$

$$= \frac{(k+1)(k+2)}{2}$$

$$= \frac{(k+1)(k+1+1)}{2}$$

Hence it is proved

prove that

$$\sum_{i=0}^{n} i =$$

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20/9 DMS lecture

2 methods → ① Basis ② Inductive

Prove that $\sum_{r=0}^{n} r^i = \frac{r^{n+1}-1}{r-1}$ for $n \geq 0$

① Let P(n) be the property

Basis Step: Let it be true for $n=0$

$$r^0 = \frac{r^{0+1}-1}{r-1} \Rightarrow \frac{r-1}{r-1} = 1 \quad , 1 = 1 , \ L.H.S = R.H.S$$

It means base case is true, P(0) is true.

Inductive Step: let P(k) be true for some $n = K$, $n > 0$

$$\sum_{i=0}^{K} r^i = \frac{r^{K+1}-1}{r-1}$$

We have to prove that P(K+1) is true:

$$\sum_{i=0}^{K} r^i + r^{K+1} = \frac{r^{K+1}-1}{r-1} + r^{K+1}$$

$$\sum_{i=0}^{K+1} r^i = \frac{r^{K+1} - 1 + r^{K+1}(r-1)}{r-1}$$

$$= \frac{\cancel{r^{K+1}} - 1 + r.r^{(K+1)} - \cancel{r^{K+1}}}{r-1}$$

$$= \frac{r r^{K+1} - 1}{r-1}$$

$$= \frac{r^{K+2}-1}{r-1} \Rightarrow \frac{r^{K+1+1}-1}{r-1}$$

So it is ~~prove~~ that P(K+1) is also true which completes the proof.

$1+3+5+ \cdots +(2n-1) = n^2$ for $n \geq 1$

Basis step

Prove $P(n)$ for $n=1$

$n^2 = (1)^2 \Rightarrow 1$

L.H.S = R.H.S

Inductive step $n=k$

Prove $P(k+1)$ for $k \geq 1$

$1+3+5+ \cdots + (2k-1) + (k+1) = k^2+(k+1)^2$

$= k^2+(k+1)^2$

$= k^2+k+1$

$= k^2+k+1$

So it is true for $P(k+1)$

$1+2+3+ \cdots +2k-1 = k^2$

$1+3+ \cdots +2k-1+2(k+1)-1 =$

$= k^2+2(k+1)-1$

$= k^2+2k+2-1$

$= k^2+2k+1$

$= (k+1)^2$

$1+6+11+ \cdots + 5n-4 = \frac{n(5n-3)}{2}$ $n \geq 1$

Let $P(n)$ be the property to prove.

Basis step $n=1$

$\frac{n(5n-3)}{2} = \frac{1(5(1)-3)}{2} = \frac{5-3}{2} = \frac{\cancel{2}}{\cancel{2}} = 1$

As L.H.S = R.H.S be true for $n=1$

Inductive step $n=k$, $n \geq 1$

$1+6+11+ \cdots + 5k-4 = \frac{k(5(k)-3)}{2}$

Prove for $P(k+1)$

$1+6+11+ \cdots +5(k+1)-4 + 5k-4 = \frac{k(5k-3)}{2} + 5(k+1)-4$

$\underbrace{1+6+11+ \cdots +5k-4}$ $+5(k+1)-4$

$\frac{k(5k-3)}{2} + 5(k+1)-4 = \frac{5k^2-3k+10k+10-8}{2}$

$= \frac{5k^2+7k+2}{2}$

$= \frac{5k^2+5k+2k+2}{2}$

$$= \frac{5k(k+1) + 2(k+1)}{2}$$

$$= \frac{(5k+2)(k+1)}{2}$$

R.H.S (what is needed into final step)

$$= \frac{(k+1)(5(k+1)-3)}{2}$$

$$= \frac{k+1(5k+5-3)}{2}$$

$$= \frac{(k+1)(5k+2)}{2}$$

For all integers $n \geq 0$ prove that $2^{2n}-1$ is divisible by 3

Basis step

let $P(n)$ is true for $n=0$, show that $P(0)$ is true

$= 2^{2(0)} - 1 \Rightarrow 2^0 - 1 \Rightarrow 1-1=0$

$\Rightarrow 3.0 = 0$. so $P(0)$ is proved.

Inductive step

let $n = k$, so, $k+1$

$3r = 2^{2k} - 1$

$3r = 2^{2(k+1)} - 1$

$3r = 2^{2k} \cdot 2^2 - 1$

$3r = 2^{2k} .4 - 1$

$r = 2^{2k} . \frac{4}{3} - \frac{1}{3}$

$3r = 2^{2k}(3+1) - 1$

let $P(k) = 2^k - 1$ is divisible by 3

$= 3.2^{2k} + 2^{2k} - 1$

$= 3.2^{2k} + 3r$

$= 3(2^{2k} + r)$

Hence $P(k+1)$ is divisible by 3

Principle Statement (Well ordering Principle)

Property of integer

ہمارے پاس ایک set ہے non negative integers کا اور اس میں ایک ایسا
نمبر ضرور ہوتا ہے جو سب سے چھوٹا ہوتا ہے۔

Every non-empty set S of non-negative integers contains a least element.

$$S = \{2, 4, 6, 5, 8\}$$

Sequences (Some have explicit formula but finding of some sequences is very diffcult i.e: 0 1 1 2 3 5 8 13 .... **Fibonacci Sequence**).

0 1 1 2 3 5 8 13 ...

$F_n = F_{n-1} + F_{n-2}$

↳ Such representation is recusion / recussive sequences also relation

1) Recusive relation (In equation form)

2) Initial conditions, $F_0 = 0$, $F_1 = 1$

$C_k = C_{k-1} + kC_{k-2} + 1$ for $k \geq 2$

$c_1 = 1, \quad c_2 = 2$

c_2: c_3, c_4, c_5

$C_3 = C_{3-1} + 3C_{3-2} + 1$
$= C_2 + 3C_1 + 1$
$= 2 + 3 + 1 \Rightarrow 6$

$C_4 = C_{4-1} + 4C_{4-2} + 1$
$= C_3 + 4C_2 + 1$
$= 6 + 8 + 1 \Rightarrow 15$

$C_5 = C_4 + 4C_3 + 1$
$= 15 + 28 + 1$
$= 46$

$C_5 = C_{5-1} + 5C_{5-2} + 1$
$= 15 + 30 + 1$
$= 46$

$F_n = F_{n-1} + F_{n-2}$

$F_0 = 1 \,,\, F_1 = 1$

$F_3 = ?$

$= F_{3-1} + F_{3-2}$

$= F_2 + F_1$

$= 2+1$

$F_3 = 3$

$F_2 = F_1 + F_0$

$= 1+1$

$= 2$

$a_k = 2a_{k-1} + k$ for all $k \geq 2$

$a_1 = 1$

$a_2 = 2a_1 + 2 \Rightarrow 2+2 = 4$

$a_3 = 2a_2 + 3 \Rightarrow 8+3 = 11$

$a_4 = 2a_3 + 4 \Rightarrow 22+4 = 26$

$S_k = S_{k-1} + 2S_{k-2}$ for $k \geq 2$

$S_0 = 1 \,,\, S_1 = 1$

$S_2 = S_1 + 2S_0 \Rightarrow 1+2 \Rightarrow 3$

$S_3 = S_2 + 2S_1 \Rightarrow 3+2 \Rightarrow 5$

$S_4 = S_3 + 2S_2 \Rightarrow 5+6 \Rightarrow 11$

Does an explicit formula statisfies a Recussion Relation (RR)

Let : $a_n = 3n+1$ for $n \geq 0$

Show that it satisfies RR

$a_k = a_{k-1} + 3$ for $k \geq 1$

L.H.S = R.H.S

let $n = k$, so

$a_k = 3k+1$

L.H.S = R.H.S,

$a_k = a_{k-1} + 3$

and $k = k-1$ so

$a_{k-1} = 3(k-1)+1$

$= 3k-3+1$

$a_{k-1} = 3k-2$

R.H.S $= a_{k-1} + 3$

$= 3k-2+3$

$= 3k+1 =$ L.H.S

Let : $b_n = 4^n$ for $n \geq 0$

Show that it satisfies RR

① ← $b_k = 4b_{k-1}$ for $k \geq 1$

let $n = k$, so

$b_k = 4^k$ → ①

Replace k with $k-1$

$b_{k-1} = 4^{k-1}$

Put in eq ①

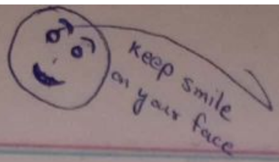
$b_k = 4(4^{k-1})$

$= 4.4^{k-1}$

$= 4^{k-1+1}$

$b_k = 4^k$

L.H.S = R.H.S



$t_n = 2+n$

$t_K = 2t_{K-1} - t_{K-2}$ for $K \geq 1$

$t_K = 2+K$

$t_{K-1} = 2+K-1$

$t_{K-1} = K+1$

$t_K = 2(K+1) - t_{K-2}$

$t_K = 2K+2-t_{K-2}$

$t_{K-2} = 2+K-2$

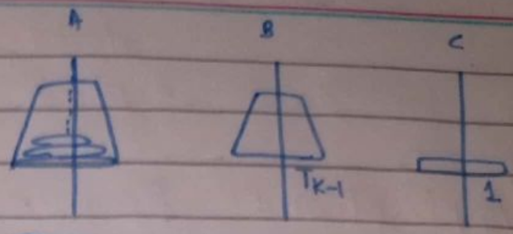
$= K$

$t_K = 2K+2-K$

$t_K = 3K+2$

$t_K = 2+3K$

L.H.S = R.H.S



$T_1 = 1,\ T_2 = 3,\ T_3 = 7$

$T_K = T_{K-1} + 1 + T_{K-1}$

$= 2T_{K-1} + 1$

$T_3 = 2T_{3-1} + 1$

$= 2T_2 + 1$

$= 2(3) + 1$

$= 6 + 1$

$= 7$

$T_6 = 2T_5 + 1$

$= 2(2T_4 + 1) + 1$

$= 2(2(2T_3 + 1)) + 1$

$= 2(2(2(7)+1)+1$

$= 2(2(15)+1$

$= 2(30) + 1 \Rightarrow 61$

Sequence and Series

Closed form

Can we drive explicit formula from given recursive sequence.

Application: Recursive Sequence used in Algorithms. i.e: To choose best algorithm for sorting 1000 numbers, recursive sequence will be used.

1/ **Iterative Method** (Analyze pattern and write formula)

$a_0, a_1, a_2, \ldots$

$a_k = a_{k-1} + 2$

$a_0 = 1$

Guess explicit formula $a_0 = 1 + (2 \times 0)$

$a_1 = 1+2 \Rightarrow 1 + (2 \times 1)$

$a_2 = 3+2 \ (1+2+2) \Rightarrow 1 + (2 \times 2)$

$a_3 = 5+2 \ (1+2+2+2) \Rightarrow 1 + (2 \times 3)$

$\boxed{a_n = 1 + 2n}$ Iterative method

2/ **let $a_k = r a_{k-1}$ for $k \geq 1$**

$a_0 = a$

$a_1 = ra \Rightarrow ra$

$a_2 = r[ra] \Rightarrow r^2 a$

$a_3 = r[rra] \Rightarrow r^3 a$

$a_4 = r[rrra] \Rightarrow r^4 a$

$\boxed{a_n = ar^n}$ General closed formula

3) $c_k = 3c_{k-1} + 1$ for $k \geq 2$, $c_1 = 1$

~~$c_1 = 1$~~
~~$c_2 = 3c_{2-1}+1 \to 3c_1 \to 3(1) + 1 \Rightarrow 3^{n-1}+1$ $\quad c_1 = 3^{1-1} = 3^0 = 1$~~
~~$c_3 = 3c_{3-1}+1 \to 3c_2 \to 3(3)(1) + 1$~~
~~$c_4 = 3c_{4-1}+1 \to 3c_3 \to 3.(3)(3)(1) + 1$~~
~~$c_5 = 3c_{5-1}+1 \to 3c_4 \to 3.(3)(3)(3)(1) + 1$~~

$c_1 = 1$, $c_2 = 3c.$

$c_2 = 3c_{2-1} + 1 \Rightarrow 3(1)+1 \qquad (3+1)$

$c_3 = 3c_{3-1} + 1 \Rightarrow 3(3+1) + 1 \qquad 3(3+1) + 1$

$c_4 = 3c_{4-1} + 1 \Rightarrow 3(3(3+1)+1 \qquad 3^2(3+1) + 1$

$c_5 = 3c_{5-1} + 1 \Rightarrow 3.(3(3(3+1)))+1 \qquad 3^3(3+1)+1$

$$c_n = 3^{n-1} + 3^{n-2} + ---- + 3 + 1 \to 3^0$$

$$= \sum_{i=0}^{n-1} 3^i$$

$$S_n = \frac{a(r^{n-1}-1)}{r-1} \quad \text{(Sum of Series General formula}$$

$$= \frac{1(3^{n-1}-1)}{3-1}$$

$$\boxed{c_n = \frac{3^{n-1}-1}{2}}$$

4) A worker is promised bonus if he can increase productivity by 2 units a day everyday for 30 days. If his productivity is 170 on day 0, what is the productivity on day 30.

$d_{30} = ?$ $d_0 = 170$.

$d_K = d_{K-1} + 2$

$d_1 = d_0 + 2 \to 170+2 \qquad \to 170 + 2\times 1$

$d_2 = d_1 + 2 \to (170+2)+2 \qquad \to 170 + 2\times 2$

$d_3 = d_2 + 2 \to 170 + 2+2+2 \to 170 + 2\times 3$

$d_4 = d_3 + 2 \to 170+2+2+2+2 \to 170 + 2\times 4$

~~$d_n = 170 + 2^n$~~ can't use power because $d_3 \to 6$ becomes 8

~~$d_{30} = 170 + 2^{30}$~~

~~$= 170 + 1073741824$~~

~~$= 1073741994$~~

$= 170 + 2n$

$= 170 + 2(30)$

$= 170 + 60$

$= 230$

5) A runner targets himself to improve a course 3 seconds a day. On day zero, he runs on track in 3 mins. how fast will he run on day 14.

$d_0 = 3\,min \Rightarrow 3\times 60 \Rightarrow 180$ sec $\qquad d_{14} = ?$

$d_1 = 180 - 3$

$d_K = d_{K-1} - 3 \qquad d_{14} = 180 - 3(14)$

$d_2 = d_1 - 3,\ 180-3-3 \qquad \boxed{= 138}$

$d_3 = 180-3-3-3$

$d_4 = 180-3-3-3-3$

$d_n = 180 - 3n$

* fast means time is reducing that's why minus krna hy.

Tower of Hanoi (Puzzle)

$T_1 = 1$

$T_2 = 3$ $\quad (2+1)$

$T_3 = 7$ $\quad 2(2+1)+1$

$T_k = 2T_{k-1}+1$

$2_4 = 2T_3+1$

$= 2(7)+1$ $\quad 2.(2(2+1)+1)+1$

$2_5 = 2.(2(7))+1$ $\quad 2(2(2(2+1)+1)+1)+1$ $\quad 2(2^2+2)+1$

$2_6 = 2(2^2(7)+1)$ $\quad 2(\cancel{2^3+2^2+})$

$2_7 = 2(2^3(7)+1)$

$= 2^4($

$T_1 = 1$

$T_2 = 3$

$T_3 = 7$

$T_k = 2T_{k-1}+1$

$T_1 = 1$

$T_2 = 2+1$

$T_3 = 2(2+1)+1 \Rightarrow 2^2+2+1$

$T_4 = 2(2^2+2+1)+1 \Rightarrow 2^3+2^2+2+1$

$T_n = \sum_{i=0}^{n-1} 2^i$

$= \frac{a(r^n-1)}{2-1}$

Modular Arithmetic

Cryptography is based on modular arithmetic
Its all about remainder.

$2 \overline{)19}$ quotient 9, 18, remainder 1 $\Rightarrow$ $19 \equiv 1 \text{ Mod } 2$

congruence

$\equiv$ congruent — way of doing math in the term of congruent

$2 | 19 - 1$ — divide

Mod 2 — Modulus of congruence

$25 = 4 \text{ Mod } 7$
$\Rightarrow 7 \mid 25 - 4$
$\Rightarrow 7 \mid 21$

Z integers $\Rightarrow (-\infty \leftrightarrow 0 \leftrightarrow +\infty)$ (Not fraction)

$-5 \equiv 2 \text{ Mod } 7$
$= 7 \mid -5 - 2$
$= 7 \mid -7$
$= -1$

$7 \overline{)-5}$ quotient 1, 7, 2

$-13 = 1 \text{ mod } 7$
$= 7 \mid -13 - 1$
$= 7 \mid -14$
$= -2$

$7 \overline{)-13}$ quotient 2, 7, -

$5 \mid 33 - 3$
$33 \equiv 3 \text{ Mod } 5$

$18 \equiv 16 \text{ Mod } 2$
All even are congruent to each other

Two integers 'a' and "b" Mod 'n' are congruent to each other (and) "mod n' if their remainder mod 'n' is same.

- $18 \equiv 0 \text{ Mod } 2$
- $16 \equiv 0 \text{ mod } 2$

Z Mod 2

0 remainder or 1 remainder

Z Mod 2

two classes

1 ; 0

odd ; even

- $15 \equiv 1 \bmod 2$
- $16 \equiv 0 \bmod 2$
- $17 \equiv 1 \bmod 2$
- $18 \equiv 0 \bmod 2$
- $19 \equiv 1 \bmod 2$

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DMS

$p(x)$: x is enrolled in DMS. {SE 2021}
belongs to domain, Δ should be spicified.

quantifiers : $\forall$, $\exists$

$\forall$: Domain ke hr element pr true honi chaiye

$\exists$: There must be atleast one value that hold the T value

$\forall x\,(P(x) \wedge Q(x))$

$P(x)$ x is enrolled in DMS

$Q(x)$ x is enrolled in OOP

This lec was missed in above images so i add it at last

- all students enrolled in DMS and OOP.

$\exists x\,(P(x) \wedge Q(x))$: There should be atleast one such student who is enrolled in DMS and PF. (at least true the result).

کسی بھی student کے لیے OOP اور DMS میں enroll
ہونے کے لیے PF پاس کرنا ضروری ہے۔

A $\exists$ $P(x)$, $D(x)$, $P(x)$

a $\exists\,(O(x) \wedge D(x))$ $\exists P(x)$

T | F | F

a $\exists$ (To enroll in OOP $\wedge$ DMS) $\wedge\, \exists P(x)$

✓ $R(x) \longrightarrow (P(x) \wedge Q(x))$

S(y, z): "Student enroll in course y and z"

$R(x) \to (S(y, z))$.

$R(x) \to (S(x, OOP, DMS))$

Nested Quantifiers

$P(x, y): x + y = 10$ Domain = {1, 2, 3, ... 10}

P(5, 2) will return False.

$\forall x \exists y \; P(x, y)$

x ki hr value aur y ki particular aysi value jo is equation ko true kr dy, value meet kr jaye.

$P(1, 9) \Rightarrow T$ $P(2, 8)$, $P(3, 7)$... P(10, no such value)

This scenario generates false for $(\forall x)$ that make quantifier $\forall x \exists y P(x, y)$ False for domain {1, 2, 3 ... 10}

$\Rightarrow$ $x + y = y + x$

$\forall x \forall y P(x, y)$

$5 + y = y + 5$

$\Rightarrow$ $x + y = 0$

$\forall x \exists y \; P(x, y) \Rightarrow$ True

for all x There is one y particular value

Date: / /

Day: M T W T F S

• Domain Student {SE 2021} Course {Spring 2022}

$\begin{cases} \text{student}: (x) \\ \text{course}: (y) \end{cases}$

$\forall_x \forall_y$

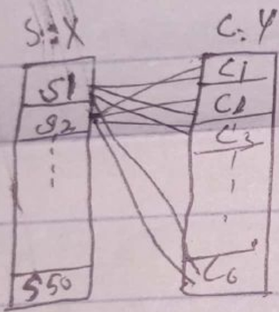
$\forall_x \exists_y$

$\exists_x \exists_y$

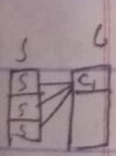
$\exists_x \forall_y$

$P(x,y)$: x student is enroll in y course.

$\forall_x \forall_y P(x,y)$ ہر کورس کو ہر student پڑھ رہا ہے



$\forall_x \exists_y P(x,y)$ میری کلاس کا ہر student کسی نہ کسی course میں (particular) میں لازمی enroll ہے۔



$\forall_y \forall_x P(x,y)$ تمام ~~particular~~ course میں ہر student enroll ہے

$\exists_x \exists_y (P(x,y))$

for(i=1 ; i<s.length ; i++)
{
for (j=1 ; j<c.length ; j++)
{
Registration(i,j)
}
};

- $\exists_x \forall_y$ agr aik x py value true ho jaye tu baqi phir check krny ki zaroort nhi.

$\exists_x \exists_y$ کوئی s کسی نہ کسی c میں enroll ہے۔

Is ka best case 1 hy.

Google search engine
By default every character me ~~and~~ OR operator lgata hy like UET new campus.

اسم: نام، چیز، جگہ (ii) اپنا معنی خود بتاتا ہے (iii) اس میں زمانہ نہیں ہو (No time)

فعل: کام کا کرنا یا ہونا (ii) ،، ،، (iii) اس میں زمانہ بتایا جاتا ہے (کام)

حرف: نہ اپنا معنی بتائے گا، نہ زمانہ، میں، فی، علی، پر (کنارہ)

اسم کی علامات: نام پر تنوین "ٌ" آخری حرف پر)، ۃ آخر میں، تثنیہ

ا۔ واحد، 2 ← تثنیہ، 3 ← جمع رجلٌ۔ ایک آدم، تثنیہ: رجلان، جمع: رجال

حروف جر: