Section ONE (Induction)

Q1: A Sequence $d1, d2, d3, \ldots\ldots$ is defined by letting $d_1 = 2$ and $d_K = \frac{d_{K-1}}{K}$ for all integers $K \geq 2$. Show that $d_n = \frac{2}{n!}$ for all integers $n \geq 1$ using Mathematical Induction.

Sol:

First Step: As $d_1 = 2$,

$d_K = \frac{d_{K-1}}{K}$, $K \geq 2$ and $d_n = \frac{2}{n!}$, $n \geq 1$

For $n = 1$

$d_1 = \frac{2}{1!} = 2$

So d_n is true for $\boxed{n = 1}$

Second Step:

Let d_n is true for $n = K$

$d_K = \frac{2}{K!}$

For $n = K+1$

$d_{K+1} = \frac{d_{K+1-1}}{K+1} = \frac{d_K}{K+1}$

$d_{K+1} = d_K \cdot \frac{1}{K+1}$

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$$d_{K+1} = \frac{2}{K!} \cdot \frac{1}{K+1}$$

$$d_{K+1} = \frac{2}{(K+1)(K!)} = \frac{2}{(K+1)!}$$

$$d_{(K+1)} = \frac{2}{(K+1)!}$$

So d_k is true for $n = K+1$
Hence by Principal of Mathematical Induction, d_n is true for all value of $n \in N$

Q#2

$$\frac{1}{1\cdot 2} + \frac{1}{2\cdot 3} + \cdots \frac{1}{n(n+1)} = \frac{n}{n+1}$$ for all integers $n \geq 1$

Sol: First Step:

For $n = 1$

$$\frac{n}{n+1} = \frac{1}{1+1} = \frac{1}{2}$$

Hence it is true for $n = 1$

Let it be true for $n = K$

$$\boxed{\frac{1}{1\cdot 2} + \frac{1}{2\cdot 3} + \cdots \frac{1}{K(K+1)} = \frac{K}{K+1}}$$ eq ①

Second Step:

Now let it be ~~true~~ Prove for $n = K+1$

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$$\frac{1}{1\cdot 2} + \frac{1}{2\cdot 3} + \cdots \frac{1}{K(K+1)} + \frac{1}{(K+1)(K+1+1)} = \frac{K}{K+1} + \frac{1}{(K+1)(K+}$$

$$= \frac{K}{(K+1)} + \frac{1}{(K+1)K+2)} = \frac{K(K+2)+1}{(K+1)(K+2)} = \frac{K^2+2K+1}{(K+1)(K+2)}$$

$$= \frac{(K+1)^2}{(K+1)(K+2)} = \frac{K+1}{K+2} = \frac{K+1}{(K+1+1)}$$

$$= \frac{(K+1)}{(K+1)+1}$$

Hence, Statement is true for $n = K+1$,
Thus Proved, it is True for all integers
$n \geq 1$

$$\frac{1}{1\cdot 2} + \frac{1}{2\cdot 3} + \cdots \frac{1}{n(n+1)} = \frac{n}{n+1}$$

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③

Section One (Modular Arithmetic)

3) Encrypt and Decrypt following words using Affine Cipher

① TOPIC ② Force

a) K (19,4)   b) K(9,5)

① TOPIC

T O P I C
19 14 15 8 2
x_1 x_2 x_3 x_4 x_5

K (19, 4)
a b

A F K P U Z
0 5 10 15 20 25

Encryption:

$y = (x \cdot a + b)$ Mod 26

⇒ K (19, 4)

$y_1 = (19 \cdot 19 + 4)$ Mod 26
$y_1 = 365$ Mod 26
$y_1 = 1 = B$

$y_2 = (14 \cdot 19 + 4)$ Mod 26
$y_2 = 270$ Mod 26
$y_2 = 10 = K$

$y_3 = (15 \cdot 19 + 4)$ Mod 26
$y_3 = 289$ Mod 26
$y_3 = 3 = D$

⇒ K (9, 5)

$y_1 = (19 \cdot 9 + 5)$ Mod 26
$y_1 = 176$ Mod 26
$y_1 = 20 = U$

$y_2 = (14 \cdot 9 + 5)$ Mod 26
$y_2 = 131$ Mod 26
$y_2 = 1 = B$

$y_3 = (15 \cdot 9 + 5)$ Mod 26
$y_3 = 140$ Mod 26
$y_3 = 10 = K$

$y_4 = (8\cdot19+4) \text{ Mod } 26$

$y_4 = 156 \text{ Mod } 26$

$y_4 = 0 = A$

$y_5 = (2\cdot19+4) \text{ Mod } 26$

$y_5 = 42 \text{ Mod } 26$

$y_5 = 16 = Q$

TOPIC
↓
K(19,4)
↓
BKDAQ

$y_4 = (8\cdot9+5) \text{ Mod } 26$

$y_4 = 77 \text{ Mod } 26$

$y_4 = 25 = Z$

$y_5 = (2\cdot9+5) \text{ Mod } 26$

$y_5 = 23 \text{ Mod } 26$

$y_5 = 23 = X$

TOPIC
↓
K(9,5)
↓
UBKZX

Decryption:

$$x = a^{-1}(y-b) \text{ Mod } 26$$

B	K	D	A	Q
↓	↓	↓	↓	↓
1	10	3	0	16
↓	↓	↓	↓	↓
y_1	y_2	y_3	y_4	y_5

• K(19,4)

$a^{-1} \text{ Mod } 26$

$19^{-1} \text{ Mod } 26$

$\Rightarrow 11 \text{ Mod } 26$

$19^{-1} \text{ Mod } 26 \cong 11 \text{ Mod } 26$

U	B	K	Z	X
↓	↓	↓	↓	↓
20	1	10	25	23
↓	↓	↓	↓	↓
y_1	y_2	y_3	y_4	y_5

• K(9,5)

$a^{-1} \text{ Mod } 26$

$9^{-1} \text{ Mod } 26$

$\Rightarrow 4 \text{ Mod } 26$

$9^{-1} \text{ Mod } 26 \cong 3 \text{ Mod } 26$

$x_1 = 11(1-4) \text{ Mod } 26$

$x_1 = -33 \text{ Mod } 26$

$x_1 = 19 \text{ Mod } 26$

$x_1 = 19 = T$

$x_2 = 11(10-4) \text{ Mod } 26$

$x_2 = 66 \text{ Mod } 26$

$x_2 = 14 = O$

$x_3 = 11(3-4) \text{ Mod } 26$

$x_3 = -11 \text{ Mod } 26$

$x_3 = 15 = P$

$x_4 = 11(0-4) \text{ Mod } 26$

$x_4 = -44 \text{ Mod } 26$

$x_4 = 8 = I$

$x_5 = 11(16-4) \text{ Mod } 26$

$x_5 = 132 \text{ Mod } 26$

$x_5 = 2 = C$

B K O A Q

Decrypt K(19,4)

T O P I C

$x_1 = 3(20-5) \text{ Mod } 26$

$x_1 = 45 \text{ Mod } 26$

$x_1 = 19 = T$

$x_2 = 3(1-5) \text{ Mod } 26$

$x_2 = 3(-4) \text{ Mod } 26$

$x_2 = -12 \text{ Mod } 26 = 14 \text{ Mod } 26$

$x_2 = 14 = O$

$x_3 = 3(10-5) \text{ Mod } 26$

$x_3 = 15 \text{ Mod } 26$

$x_3 = 15 = P$

$x_4 = 3(25-5) \text{ Mod } 26$

$x_4 = 60 \text{ Mod } 26$

$x_4 = 8 = I$

$x_5 = 3(23-5) \text{ Mod } 26$

$x_5 = 54 \text{ Mod } 26$

$x_5 = 2 = C$

U B K Z X

Decrypt K(4,5)

T O P I C

2) FORCE

$$\begin{array}{ccccc} \downarrow & \downarrow & \downarrow & \downarrow & \downarrow \\ 5 & 14 & 17 & 2 & 4 \\ x_1 & x_2 & x_3 & x_4 & x_5 \end{array}$$

Encryption: $y = (ax + b) \text{ Mod } 26.$

• K(19, 4)

$y_1 = (19 \cdot 5 + 4) \text{ Mod } 26$

$y_2 = 99 \text{ Mod } 26$

$y_1 = 21 = V$

$y_2 = (19 \cdot 14 + 4) \text{ Mod } 26$

$y_2 = 270 \text{ Mod } 26$

$y_2 = 10 = K$

$y_3 = (19 \cdot 17 + 4) \text{ Mod } 26$

$y_3 = 327 \text{ Mod } 26$

$y_3 = 15 = P$

$y_4 = (19 \cdot 2 + 4) \text{ Mod } 26$

$y_4 = 42 \text{ Mod } 26$

$y_4 = 16 = Q$

• K(9, 5)

$y_1 = (9 \cdot 5 + 5) \text{ Mod } 26$

$y_1 = 49 + 1 \text{ Mod } 26$

$y_1 = 23 + 1 = 24 = Y$

$y_2 = (9 \cdot 14 + 5) \text{ Mod } 26$

$y_2 = 130 + 1 \text{ Mod } 26$

$y_2 = 0 + 1 = 1 = B$

$y_3 = (9 \cdot 17 + 5) \text{ Mod } 26$

$y_3 = 158 \text{ Mod } 26$

$y_3 = 2 = C$

$y_4 = (9 \cdot 2 + 5) \text{ Mod } 26$

$y_4 = 23 \text{ Mod } 26$

$y_4 = 23 = X$

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$y_5 = (19 \cdot 4 + 9) \text{ Mod } 26$

$y_5 = 80 \text{ Mod } 26$

$y_5 = 2 = B$

$y_5 = (9 \cdot 4 + 5) \text{ Mod } 26$

$y_5 = 41 \text{ Mod } 26$

$y_5 = 15 = P$

So the Encrypted value are

V K P Q B

Y B C X P

Decryption:

$x = a^{-1}(y - b) \text{ Mod } 26$

V	K	P	Q	B
21	10	15	16	2
y_1	y_2	y_3	y_4	y_5

Y	B	C	X	P
24	1	2	23	15
y_1	y_2	y_3	y_4	y_5

- K(19, 4)

$a^{-1} \text{ Mod } 26$

$= 19^{-1} \text{ Mod } 26$

$= 11 \text{ Mod } 26$

- K(9, 5)

$a^{-1} \text{ Mod } 26$

$= 9^{-1} \text{ Mod } 26$

$= 3 \text{ Mod } 26$

$x_1 = 11(21 - 4) \text{ Mod } 26$

$x_1 = 187 \text{ Mod } 26$

$x_1 = 5 = F$

$x_1 = 3(24 - 5) \text{ Mod } 26$

$x_1 = 57 \text{ Mod } 26$

$x_1 = 5 = F$

$x_2 = 11(10 - 4) \text{ Mod } 26$

$x_2 = 66 \text{ Mod } 26$

$x_2 = 3(1 - 5) \text{ Mod } 26$

$x_2 = -12 \text{ Mod } 26$

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$x_2 = 14 = O$

$x_3 = 11(15-4) \text{ Mod } 26$

$x_3 = 121 \text{ Mod } 26$

$x_3 = 17 = R$

$x_4 = 11(16-4) \text{ Mod } 26$

$x_4 = 132 \text{ Mod } 26$

$x_4 = 2 = C$

$x_5 = 11(2-4) \text{ Mod } 26$

$x_5 = -22 \text{ Mod } 26$

$x = 4 = E$

$x_2 = 14 = O$

$x_3 = 3(2-5) \text{ Mod } 26$

$x_3 = -9 \text{ Mod } 26$

$x_3 = 17 = R$

$x_4 = 3(23-5) \text{ Mod } 26$

$x_4 = 54 \text{ Mod } 26$

$x_4 = 2 = C$

$x_5 = 3(15-5) \text{ Mod } 26$

$x_5 = 30 \text{ Mod } 26$

$x_5 = 4 = E$

So the Decrypted values are

$\boxed{\text{FORCE}}$ $\boxed{\text{FORCE}}$

Section TWO (Combinatorics)

Qs: How many three digits numbers can be formed from the six digits 2, 3, 4, 5, 7 and 9 with replacement

Sol:

Total Digits $= n = 6$

order set $= r = 3$

$$n^r = 6^3 = 216$$

Thus we can form 216 three digit numbers with repetition.

Q2: How many three digits numbers can be formed from the six digits 2, 3, 4, 5, 7 and 9 without replacement

total numbers $= n = 6$

order $= r = 3$

As repetition is not allowed thus

$$\frac{n!}{(n-r)!} = \frac{6!}{(6-3)!} = \frac{6!}{3!} = 6 \times 5 \times 4 = 120$$

Thus, we can form 120 three digit numbers without repetition.

Q3: A Box contain 10 different colored light bulbs. Find the of number of ordered sample of size 3 with replacement.

Sol: total bulbs $= n = 10$

order $= r = 3$

As Repitition is allowed thus

$$n^r = 10^3 = 1000$$

Thus, there will be 1000 different combinations of order 3 with replacement.