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Permeutation:

Ordered Selection ($AB \neq BA$)

$$^nP_r = \frac{n!}{(n-r)!}$$

Proof:

Ways to choose 1^{st} object $= n$

Ways to choose 2^{nd} object $= n-1$

ways to choose 3^{rd} object $= n-2$

ways to choose 4^{th} object $= n-3$

ways to choose r object $= n-(r-1)$

$= n-r+1$

$$^nP_r = n \times (n-1) \times (n-2) \times (n-3) \times \cdots (n-r+1)$$

Multiply and divided by $(n-r)!$

$$^nP_r = \frac{n \times (n-1) \times (n-2) \times (n-3) \times \cdots (n-r+1)(n-r)!}{(n-r)!}$$

$$^nP_r = \frac{n!}{(n-r)!}$$

Hence proved!

Examples:

* Find permeutation when $n = 7$ and $r = 4$.

$${}^nP_r = \frac{n!}{(n-r)!}$$

$${}^7P_4 = \frac{7!}{(7-4)!}$$

$${}^7P_4 = \frac{7!}{3!} = \frac{7.6.5.4.\cancel{3!}}{\cancel{3!}}$$

$${}^7P_4 = 840$$

* How many arrangements in a row of no more than three letters for word 'NETWORK' can be made?

Total arrangement for 1 letter word $= {}^7P_1 = 7$

Total arrangement for 2 letters word $= {}^7P_2 = 42$

Total arrangement for 3 letters word $= {}^7P_3 = 210$

Total arrangement for no more

than three letters word = 7 + 42 + 210
= 259

Pigeonhole principle:

If 'n' pigeons fly into 'm' pigeonholes and $n > m$ then at least one pigeonhole must contain two or more pigeons.

Example:

let $A = \{1, 2, 3, 4, 5, 6, 7, 8\}$

How many integers must be selected from A if we want a pair of integers having Sum = 9?

Possible pair of integers having sum = 9 are (1,8), (2,7), (3,6) and (4,5).

So 5 integers must be selected from A for a pair of integer having sum is equal to 9.

Combination:

Unordered Selection (AB = BA)

$$^nC_r = \frac{n!}{(n-r)!\,r!}$$

* How many unordered selection of two elements from set (0, 1, 2, 3) be made?

$n = 4$

$r = 2$

$$^nC_r = \frac{n!}{(n-r)!\,r!}$$

$$^4C_2 = \frac{4!}{(4-2)!\,2!}$$

$$= \frac{4!}{2!.2!} = \frac{4.3.\cancel{2!}}{2!.\cancel{2!}}$$

$$= \frac{12}{2} = 6$$

Ordered Selection:

$$^nP_r = \frac{n!}{(n-r)!}$$

$$^4P_2 = \frac{4!}{(4-2)!} = \frac{4.3.\cancel{2!}}{\cancel{2!}}$$

$^4P_2 = 12$

$S = \{0, 1, 2, 3\}$

ordered Selection:

$E_1 = \{01, 02, 03, 10, 12, 13, 20, 21, 23, 30, 31, 32\}$

unordered selection:

$E_2 = \{01, 02, 03, 12, 13, 23\}$

* How many unordered and ordered selection of two letter word can be made from $S = \{a, b, c, d\}$

Ordered Selection (Permutation)

$$^4P_2 = \frac{4!}{2!} = 4 \times 3 = 12$$

$E_1 = \{(ab), (ac), (ad), (ba), (bc), (b,d), (ca), (cb), (cd), (da), (db), (dc)\}$

unordered Selection (combination)

$$^4C_2 = \frac{4!}{(4-2)!\,2!} = \frac{4!}{2!.2!}$$

$$^4C_2 = \frac{^2\cancel{4}.3.\cancel{2!}}{\cancel{2}.\cancel{2!}} = 6$$

$$E_2 = \{(ab),(ac),(ad),(bc),(bd),(cd)\}$$

* Suppose two members of a 12 person group insist on working together. Select a team containing either both person or none. How many 5 person tearms are possible? Also find the probability of each.

When both are present in team:

Total arrangement of 5 members team when both are present $= {}^{10}C_3$

$N(E) = 120$

Total possible arrangements of 5 team member $= {}^{12}C_5$

$N(S) = 792$

$$\text{Probability} = \frac{N(E)}{N(S)}$$

$$= \frac{120}{792}$$

$$= \frac{5}{33}$$

When both are not present:

Total arrangement when both are not present in team $= {}^{10}C_5$

$N(E) = 252$

$$\text{Probability} = \frac{N(E)}{N(S)}$$

$$= \frac{252}{792}$$

$$= \frac{7}{22}$$

Pascal's Formula:

$$\binom{n+1}{r} = \binom{n}{r-1} + \binom{n}{r}$$

Let $\binom{6}{2} = 15$, $\binom{6}{3} = 20$, $\binom{6}{4} = 15$ and $\binom{6}{5} = 6$ then find $\binom{7}{3}$, $\binom{7}{4}$ and $\binom{7}{5}$ by using pascal's formula:

$$\binom{7}{3} = \binom{6}{2} + \binom{6}{3}$$

$$= 15 + 20$$

$$= 35$$

$$\binom{7}{4} = \binom{6}{3} + \binom{6}{4}$$

$$= 20 + 15$$

$$= 35$$

$$\binom{7}{5} = \binom{6}{4} + \binom{6}{5}$$

$$= 15 + 6$$

$$= 21$$

Binomial theorem

$$(a+b)^n = a^n + \binom{n}{1} a^{n-1} b + \binom{n}{2} a^{n-2} b^2 + \cdots \binom{n}{n-1} a b^{n-1} + b^n.$$

* Find $(a+b)^3$ by binomial theorem.

$$(a+b)^3 = a^3 + \binom{3}{1} a^{3-1} b + \binom{3}{2} a^{3-2} b^2 + \binom{3}{3} a^{3-3} b^3$$

$$= a^3 + 3a^2b + 3ab^2 + b^3$$

* Find the co-efficient of term $x^6 y^3$ in $(x+y)^9$ by using binomial expansion.

Let b be the co-efficient

$$(b)\, x^6 y^3 = \binom{9}{3} x^{9-3} y^3$$

$$b = \binom{9}{3}$$

$$b = 84$$

84 be the coefficient of $x^6 y$

In General

$$^{n}C_{r} = {}^{n}C_{n-r}$$