Date: Chp 6 Probability:-

Probability:-

Chances of doing something/Occurance

→ Random Experiment:-

Their term experiment means a planned activity or process who's results yield a set of data. A single performance of an experiment is called a $\boxed{\text{Trial}}$. The result obtain from an experiment or a Trial is called as an $\boxed{\text{outcome}}$

Example:-

n = 5 times

A coin is used to be tossed [It is planned]

Outcomes:- {H, T, H, H, T}

→ Sample Space:-

A set consisting of all possible outcomes that can result from random experiment is defined to be a sample space for an experiment. Denoted by letter S Each possible outcome is the member of sample space is called Sample point in that space:-

Example:-

* rolling a dice. *possible outcomes are S{1, 2, 3, 4, 5, 6}

S = {(1,1), (1,2), (1,3), (1,4), (1,5), (1,6), (2,1), (2,2), (2,3), (2,4), (2,5), (2,6), (3,1), (3,2), (3,3), (3,4), (3,5), (3,6), (4,1), (4,2), (4,3), (4,4), (4,5), (4,6), (5,1), (5,2), (5,3), (5,4), (5,5), (5,6), (6,1), (6,2), (6,3), (6,4), (6,5), (6,6)}.

$\boxed{n(S) = 36}$ Total number of sample space:-

→ Event:-

An event is an individual outcomes or any number of outcomes of a random experiment or a trial.

Date:

M T W T F S S

Rule of Permutation:- order matter

$^nP_r = \frac{n!}{(n-r)!}$

Rule of Combination:- order not matter

$^nC_r = \frac{n!}{r!(n-r)!}$

1. Example:-

A club consists of 4 members how many sample points are in sample space? When 3 officers, president, secretary & Treasurer. Are to be choosen.

Answer:-

As the order of selection matters so, we use rule of permutation:

$$\frac{n!}{(n-r)!} = \frac{4!}{1!} = 24.$$

2. A three persons commettee is to be formed from a list of 4 persons? How many sample points are associated with the experement?

Rule of combination is selected because there is no ranks on which we have to chose the three persons commettee.

$$\frac{n!}{r!(n-r)!} \quad \frac{4!}{3!} = 4. \qquad A, B, C, D.$$

$S = \{(A,B,C), (A,B,D), (B,C,D), (A,C,D)\}.$

Permutation

Date:

Let A be an event then propability of event A, denoted by P(A) is defined as.

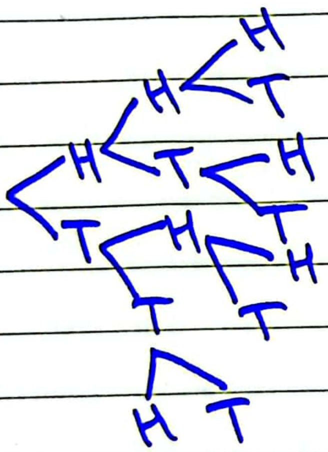
$$P(A) = \frac{\text{number of favourable outcomes}}{\text{Total no. of outcomes}} = \frac{m}{n} = \frac{n(E)}{n(S)}$$

Probability of an event is always greater or equal to 0

Example:

A faise coin is tossed three times what is the probability that at least one head appears?

{(H, H, H), (H, T, H), (H, H, T), (T, T, T), (H, T, T), (T, H, H) (T, T, H), (T, H, T)}



Sample space for this experiment is
$n(S) = 8$

Fair is when posibilities are same:-
biased coin is embosed having no same possibilities:-

* let A denote the event that at least one Head appers then.

A = {(H, H, H), (H, T, H), (H, H, T), (H, T, T), (T H H), (T, T, H), (T, H, T)}

$n(A) = 7$

$$\text{Probability of } A = P(A) = \frac{n(A)}{n(S)} = \frac{7}{8}$$

13 4 2 $13\sqrt{15}$

Date:__________ M T W T F S S

Example

If two fair dice are thrown what is the probability of getting

1* 6,6 2* A sum of 8 or more dots

① $P(A) = \frac{n(A)}{n(S)} = \frac{1}{36}$ ② B = {(2,6) (3,5) (3,6) (4,4) (4,5) (4,6) (5,3) (5,4) (5,5) (5,6) (6,2) (6,3) (6,4) (6,5) (6,6).

$P(B) = \frac{n(B)}{n(S)} = \frac{15}{36} = \frac{5}{13} = 0.42$.

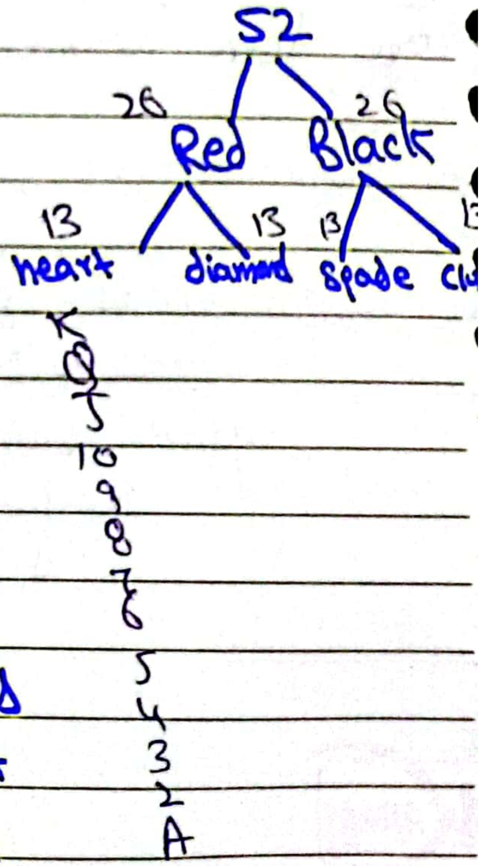
42%.

Example

If a card is drawn from an ordinary deck of 52 playing cards find the probability that:

1- The card is a red card 2- The card is a diamond

3- The card is a 10.



1- $P(1) = \frac{n(1)}{n(S)} = \frac{26}{52} = 50\%$. $\frac{13}{26} = \frac{1}{2}$

2- $P(2) = \frac{n(2)}{n(S)} = \frac{13}{52} = 0.25 = 25\% = \frac{1}{4}$

3- $P(3) = \frac{n(3)}{n(S)} = \frac{4}{52} = 7.69\% = \frac{1}{13}$

Example

6 white balls & 4 black balls which are indistinguisiable apart from the color & place in a bag. if 6 balls are taken from

Ex # 6.8, 6.9,

Date:

the bag find the probability of their being 3 white & 3 black

{(6 white), (5 w, 1 B), (4 w, 2 b) (3 w, 3 b) (2 w, 4 b)}

with replacement / without replacement:- We always assume that the sampling is done with replacement. We will do sampling with out replacement only when it is mention in the Q.

white / Black

$n(A) = 1 \Rightarrow P(A) = \frac{1}{5}$

$n(S) = 5$

$^nC_r = \frac{10!}{6!(10-6)!} = \frac{10 \cdot 9 \cdot 8 \cdot 7}{} = 210$

$n(A) = (^6C_3)(^4C_3)$

$= 80$

$P(A) = \frac{80}{210} = 0.38$

---

laws of Probability:-

→ **law of Complementation:-**

If you have any event A & add complement of A into event A which is equal to Sample space.

$A + \bar{A} = S$ where $S - A = \bar{A}$

$P(A) + P(\bar{A}) = P(S)$

$P(A) + P(\bar{A}) = 1$ $\because P(S) = n(S)/n(S) = 1$

$P(\bar{A}) = 1 - P(A)$

Example Date:

A coin is tossed 4 times in succession, what is the prob. that at least one head occurs.

Sol

$n(S) = 16$; $2^4 = 16$

let 'A' is atleast one head appears.

$\bar{A} \rightarrow$ let no head occurs. Tail occurs: 1, 2, 3, 4 } 4

$\bar{A} = \{(TTTT)\}$ $\boxed{P(\bar{A}) = 1/16}$

$P(A) = P(\bar{A}) + 1$

$P(A) = 1 - \frac{1}{16}$

$\boxed{P(A) = \frac{15}{16}}$

Example 6.11

A coin is biased so, that the prob. that it falls showing tails is 3/4 ① Find the probability of obtaining at least one head when the coin is tossed 5 times ② How many times must the coin be tossed so, that the prob. of obtaining at least one head is greater than 0.98?

Sol

(a) $n(S) = 2^5 = 32$

A → let at least one head appears

P(no head & tail appears) = 3/4

P(one head appears) = 1/4

$\bar{A} \rightarrow$ (no head appears) = 1/32

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$P(A) = P(\bar{A}) + 1$

$= -\frac{1}{32} + 1$

$= \frac{-1 + 32}{32}$

$= \frac{31}{32}$

$P(A) = 1 - P(\bar{A})$

$= 1 - (3/4)^5$

$= 0.763$

(b) $1 - \left(\frac{3}{4}\right)^n \geq 0.98$

Addition law.

Example

$P(A) + P(B) - P(A \cap B) = P(A \cup B)$

and / or

If one card is selected at random from a deck of 52 playing cards. What is the prob. that the card is a club or a face card or Both?

Sol

$n(S) = 52$

A → card is club

B → card is face

$A \cap B$ → card is club & card is face.

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$\frac{13}{52} + \frac{12}{52} - \frac{3}{52}$

$= \frac{22}{52} \Rightarrow$

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Example

A integer is chosen at random from 1st 200 +ve integer. Prob. that integer chosen is divisible by 6 or by 8.

$n(S) = 200$

A → integer divisible by 6.

B → integer divisible by 8.

$P(A\cup B) = P(A) + P(B) - P(A\cap B)$

$= \frac{33}{200} + \frac{25}{200} - \frac{8}{200}$

$= \frac{33+25-8}{200}$

$P(A\cup B) = \frac{50}{200} = \frac{1}{4}$

$n(A) = \left[\frac{200}{6}\right] = 33$

$n(B) = \left[\frac{200}{8}\right] = 25$

$A\cap B \rightarrow$ divisible by 6 or 8.

$n(A\cap B) = \left[\frac{200}{24}\right] = 8$

6.15

Three horses A, B & C are in a race. A is twice likely to win as B & B is twice as likely to win as C. Prob is of A or B is win?

$n(S) = 3$

A → twice likely to win as B.

B → twice likely to win as C.

$P(A\cup B) = ?$

$P(C) = p = 1/7$

$P(B) = 2p$ twice of C $\Rightarrow 2 \cdot 1/7 = 2/7$

$P(A) = 2(2p)$ twice of B $\Rightarrow$ 4/7

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we know. $p + 2p + 4p = 1$

$7p = 1$

$p = 1/7$

$P(A\cup B) = P(A) + P(B) - \cancel{P(A\cap B)}$

$\frac{4}{7} + \frac{2}{7} \Rightarrow \frac{6}{7}$

$\boxed{P(A\cup B) = 6/7}$

Addition law for three events:

$P(A\cup B\cup C) = P(A) + P(B) + P(C) - P(A\cap B) - P(A\cap C) - P(B\cap C) + P(A\cap B\cap C)$.

6.16 Do it yourself.

A card is drawn at random from a deck of ordinary plane cards. Prob. that it is a diamond or a face card or a king.

$n(S) = 52$

A → card is a diamond $n(A) = \frac{13}{52} \Rightarrow P(A)$

B → A face card $n(B) = \frac{12}{52}$

C → a king. $n(C) = \frac{4}{52}$

$P(A\cup B\cup C) = \frac{13}{52} + \frac{12}{52} + \frac{4}{52} = \frac{13+12+4}{52} \Rightarrow \frac{29}{52}$

$= \frac{13}{52} + \frac{12}{52} + \cancel{\frac{4}{52}} - \frac{3}{52} - \cancel{\frac{1}{52}} - \cancel{\frac{1}{52}} + \cancel{\frac{1}{52}}$

$= \frac{22}{52}$

Date:

$P(A\cap B) = \frac{3}{52}$ $P(A\cap C) = \frac{1}{52}$

$P(B\cap C) = \frac{4}{52}$ $P(A\cap B\cap C) = \frac{1}{52}$

Q#
A drawer contains 50 golds & 150 nuts. Half of the bolts & half of the nuts are rusted if one item is chosen at random. Prob. that it is rusted or a bolt.

Sol $P(A\cup B) = P(A) + P(B) - P(A\cap B)$

$n(S) = 200$
A → ~~bolt chosen is rusted~~
B → ~~It is a bolt.~~

~~$\frac{1}{150} + \frac{1}{50} \Rightarrow \frac{1+40}{150} \Rightarrow \frac{41}{150} \Rightarrow 0.27$~~

$n(S) = 200$
A → Rusted
B → bolt

~~$\frac{1}{150} + \frac{1}{50} \Rightarrow \frac{41}{150} = 0.27 \Rightarrow 27.$~~

$P(A) + P(B) - P(A\cap B)$

$\frac{100}{200} + \frac{50}{200} - \frac{25}{200} = 62.$

KARAM NOTES

Date:

A certain cartain of eggs have 3 bad eggs & 9 good eggs. & An omlette is made of 3 eggs randomly choosen from the cartain. Prob. that they're are

1- No bad eggs (0)
2- Atleast one bad egg (3)
3- Exactly 2 bad eggs in omelette. (2)

$P(A\cup B\cup C) = P(A)+P(B)+P(C)-P(A\cap B)-P(A\cap C)-P(B\cap C)+P(A\cap B\cap C)$

→ $n(S) = {}^{12}C_3 \Rightarrow 220$
A → No bad eggs $n(A) = 9$ $P(A) =$ ${}^9C_3$ ${}^nC_r$ $\frac{84}{220}$ ${}^3C_0$
B → Atleast one bad egg $n(B) = 1$ $P(B) =$
C → Exactly 2 bad eggs $n(C) = 2$ $P(C) =$

$B \to {}^nC_r = \left(\frac{1}{12}\right)\left(\frac{2}{12}\right)\left(\frac{3}{12}\right)$

KARAM NOTES

Date:

Conditional Probability:

$P(A/B) = \frac{P(A\cap B)}{P(B)}$ or $P(B/A) = \frac{P(A\cap B)}{P(A)}$, $P(A) \neq 0$

$P(B) \neq 0$.

Let A & B are two events in sample space S, if $P(B) \neq 0$ then the conditional Probability of event A is given that event B has occurred, written as $P(A|B)$ is defined by

$$P(A|B) = \frac{P(A\cap B)}{P(B)} \quad \because P(B) \neq 0$$

Answer of Probability is always greater than 0 & less then 1. $\cancel{0 \geq P(A/B) \leq 1}$ $0 \leq P(A/B) \geq 1$

It should be noted that $P(A/B)$ satisfies all the basic axioms of probability.

1 $\cancel{0 \geq P(A/B) \leq 1}$

1- $0 \leq P(A/B) \geq 1$

2- $P(S/B) = 1$ because $\frac{P(S\cap B)}{P(B)} = \frac{P(B)}{P(B)}$ S is a Sample space & greater than the set B when we take intersection b/w S & B A ~~sho~~ will be P(B). So that's why it always one.

Example 6.17

Two coins are tossed what is the conditional probability that two heads Result given that there is at least one head?

Date:

$S = \{(H,H)(H,T)(T,H)(T,T)\}$.

$n(S) = 4$.

A ⟶ Two heads Result $A = \{HH\}$ $n(A) = 1$

B ⟶ At least one head $B = \{HH, HT, TH\}$ $n(B) = 3$.

$P(A/B) = \frac{P(A\cap B)}{P(B)}$

$= \frac{1}{3}$.

$A\cap B = \{H,H\}$. $n(A\cap B) = 1$.

$P(A\cap B) = 1/4$. $P(B) = 3/4$

$P(A/B) = 1/3$, $P(A/B) = \frac{1/4}{3/4} = 1/3$.

Example

A man tosses two fair dice what is the conditional probability that the sum of the two die will be 7 given that

1- The sum is odd

2- The sum is greater than 6.

3- The two dice have the same outcome.

$n(S) = 36$.

S = {11, 12, 13, 14, 15, 16
21, 22, 23, 24, 25, 26
31, 32, 33, 34, 35, 36
41, 42, 43, 44, 45, 46
51, 52, 53, 54, 55, 56
61, 62, 63, 64, 65, 66}

A ⟶ sum of two die will be 7.

{(1,6)(2,5)(3,4)(4,3)(5,2)(6,1)} $n(A) = 6$ $\boxed{P(A) = 6/36 = 1/6}$

B ⟶ Sum is odd.

{(1,2)(1,4)(1,6)(2,5)(2,3)(2,1)(3,2)(3,4)(3,6)(4,5)
(4,3)(4,1)(5,2)(5,4)(5,6)(6,5)(6,3)(6,1)}

Date:

$n(B) = 18 \Rightarrow P(B) = 18/36 \Rightarrow \boxed{1/2 = P(B)}$

C ⟶ {(1,6) (2,5) (2,6) (3,4) (3,5) (3,6) (4,3) (4,4) (4,5) (6,6) (4,6) (5,2) (5,3) (5,4) (5,5) (5,6) (6,1) (6,2) (6,3) (6,4) (6,5)

$n(C) = 21$

$P(C) = 21/36 \Rightarrow \boxed{\frac{7}{12} = P(C)}$

D ⟶ {(1,1) (2,2) (3,3) (4,4) (5,5) (6,6)

$n(D) = 6$

$P(D) = \frac{6}{36} \Rightarrow \boxed{\frac{1}{36} = P(D)}$

① $P(A/B) = \frac{P(A\cap B)}{P(B)} = \frac{1/6}{1/2} = \frac{1}{6} \times \frac{2}{1} \Rightarrow \frac{1}{3}$

$\frac{n(A\cap B)}{n(S)} = \frac{6}{36} = \frac{1}{6}$

② $P(A/C) \Rightarrow \frac{P(A\cap C)}{P(C)} = \frac{1/6}{7/12} = \frac{1}{6} \times \frac{12}{7} \Rightarrow \frac{2}{7}$

③ $P(A/D) \Rightarrow \frac{P(A\cap D)}{P(D)} = \frac{0}{1/36} \Rightarrow \frac{0 \times 36}{1} = 0$

$\frac{1}{3} + \frac{1}{6} + \frac{2}{7} = \frac{}{35}$

$\frac{16+7+12}{42} \Rightarrow \frac{35}{42}$

n(S) = 5

3/5 2/5

2/5

n(S) = {

n(A) = 3

n(B) = 2

$n(A\cap B) = 2/52 = 1/26 = P(A\cap B)$

$n(S) = 52$

$\frac{2}{5} \times \frac{5}{2} = 1$ $P(B) = 2/5$ $\frac{1}{26} \times \frac{5}{2} \Rightarrow \frac{5}{52}$

Date:

$n(S) = {}^nC_r = {}^{52}C_5$

$\frac{52!}{5!(52-5)!} \Rightarrow \frac{52!}{5!(47)!}$

$\frac{52 \cdot 51 \cdot 50 \cdot 49 \cdot 48 \cdot 47}{5!}$ $\boxed{2598960}$

$n(A) = {}^nC_r$

Chp 8 Discrete Prob. distribution:-

Bionomial Prob. Distributions:-

Two types Discrete & Continous.

Chp 8 Chp 9.

If the probability of each outcome remains the same throughout the trials then such trials are called the bernoulli trials. & the experiment having n-bernoulli trials is called bionomial Experiment.

In other words an Experiment is called bionomial prob. Experiment if it posses the following 4 properties:

1- Outcome of each trial maybe classified into 1 of 2 categories conventionally called success & failure (F). it is to be noted that the outcome of interest is called a success & the other, is called a failure (F)

2- The probability of success denoted by P remains constant for all trials

3- The successive trials are all independent.

4- The experiment is repeated a fix number of times, say 'n'

Outcome of interest whose prob. we have to find.

Date:

When X denotes the number of successes in n-trials of a bionomial Probability experiment it is called a **bionomial Random Variable**. & its **p.d** (Probability distribution) is called **bionomial probability distribution**

$P(X=x) = ({}^nC_x)\, p^x q^{n-x}$; $x = 0, 1, 2, \dots n$

$X=x$ → assign a certain value

where
$p+q=1$
$\Rightarrow q = 1-p$ $\therefore p = 1-q$.

Bionomial p.d has 2 parameters n & p & is generally denoted by

$b(x; n, p)$

x → assigned value; n → number of experiment; b → bionomial distr.; p → Prob. of success

q is probability of failure
p is probability of success.

Example:
Toss a coin 5 times:- Explore whether it is a bionomial Prob. Experiment or not.

Example:- Rolling a die:-

③ Trials are independent coz no trial is dependent on other trial outcome.

② Remains constant that the out come is possible in all the way like toss a coin having head has prob. $\frac{1}{2}$ & tail has Prob. $\frac{1}{2}$ is same.

Date:

Example:-
A fair coin is tossed 5 times, find the probabilities of obtaining various number of heads

Probability to find in Qs is always success:-

Sol:
We have check all the four properties of bionomial prob distribution & it satisfies all our qs:-
The random variable x (no. of something) which denotes the no. of heads (successes) has a bionomial prob. dist. with $p=\frac{1}{2}$ & $n=5$ (no. of success). The possible values of x are

$q = 1-p$
$q = 1 - \frac{1}{2} \Rightarrow q = \frac{1}{2}$.

$P(X=0) = {}^5C_0\, p^0 q^5$

Hence:-

$P(\text{no head}) = P(X=0) = \frac{5!}{0!(5-0)!} \cdot \left(\frac{1}{2}\right)^0 \left(\frac{1}{2}\right)^5$

$= \frac{1}{32} = 0.03125$.

$P(X=1) = {}^5C_1\, p^1 q^4$

$= \frac{5!}{1!(4)!} \cdot \frac{1}{2} \cdot \left(\frac{1}{2}\right)^4 \Rightarrow 5 \cdot \frac{1}{2} \cdot \frac{1}{16} = \frac{5}{32} = 0.15625$.

$P(x=2) = {}^5C_2\, p^2 q^3$

$= \frac{5!}{2!\,3!} \cdot \left(\frac{1}{2}\right)^2 \cdot \left(\frac{1}{2}\right)^3 \Rightarrow 10 \cdot \frac{1}{4} \cdot \frac{1}{8} = \frac{10}{32} = 0.3125$

$P(x=3) = {}^5C_3\, p^3 q^2$

$= \frac{5!}{3!\,2!} \cdot \left(\frac{1}{2}\right)^3 \left(\frac{1}{2}\right)^2 \Rightarrow 10 \cdot \frac{1}{8} \cdot \frac{1}{4} = \frac{10}{32} = 0.3125$

$P(x=4) = {}^5C_4\, p^4 q$

$= \frac{5!}{4!} \cdot \left(\frac{1}{2}\right)^4 \left(\frac{1}{2}\right) = 5 \cdot \frac{1}{16} \cdot \frac{1}{2} = \frac{5}{32} = 0.15625$

$1-\frac{1}{3}$ $\frac{2}{3}$

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$P(X=5) = {}^5C_5\, p^5 q^0$

$= \frac{5!}{5!}\left(\frac{1}{2}\right)^5\left(\frac{1}{2}\right)^0 \Rightarrow \frac{1}{32} = 0.03125$

Example:-

let x have a bionomial distribution with $n=4$ & $p=\frac{1}{3}$ find

1) $P(X=1)$

${}^4C_1\left(\frac{1}{3}\right)\left(\frac{2}{3}\right)^3$

$= \frac{4!}{3!}\cdot\left(\frac{1}{3}\right)\left(\frac{2}{3}\right)^3 = 4\cdot\frac{1}{3}\cdot\frac{8}{27}\ \frac{32}{81}$

2) $P(X=3)$

${}^4C_3\left(\frac{1}{3}\right)^3\left(\frac{2}{3}\right)^1$

$\frac{4!}{3!(1!)} = \frac{1}{27}\cdot\frac{2}{3}$

$\frac{8}{81} =$

3) $P(X=6)$

${}^4C_6\left(\frac{1}{3}\right)^6\cdot\left(\frac{2}{3}\right)^{-2}$

$\frac{4!}{6!(4-6)!}\left(\frac{1}{729}\right)\left(\frac{9}{4}\right)$

This cannot happen.

0% percent chances:

4) $P(X\leq 2)$

${}^4C_0\cdot\left(\frac{1}{3}\right)\left(\frac{2}{3}\right)^4$

$\frac{4!}{4!} = \frac{1}{3}\cdot\frac{16}{81}$

$\frac{16}{243} + \frac{24}{81}$

$\frac{16+72}{243}$

$\frac{88}{243} + \frac{32}{81}$

$\frac{88+96}{243}$

${}^4C_2\left(\frac{1}{3}\right)^2\left(\frac{2}{3}\right)^2$

$\frac{4!}{2!(2!)}\left(\frac{1}{9}\right)\left(\frac{4}{9}\right)$

$\frac{4\cdot 3\cdot 2}{4}$

$\frac{24}{81}$

$\boxed{\frac{184}{243} = P(X\leq 2)}$

$1-\frac{2}{3}$ $\frac{2}{3}$

$\cancel{48}\cdot 7\cdot 6\cdot 5\cdot\cancel{4}$

$\cancel{4\times 3\times 2}$

Date:

$P(X=3/2) = 0$

Bcz a random variable x. with biono. prob. dist. takes only one of the integers $\{0,1,2,\ldots,n\}$ hence $X=3/2$ $=0$.

Example

A & B play a game in which A's prob. of winning is $\frac{2}{3}$ in a series of 8 games. What is the probability that A will win

(1) Exactly 4 games (2) at least 4 games (3) 6 or more games (4) from 3 to 6 games:-

$P(X=4) = {}^8C_4\left(\frac{2}{3}\right)^4\cdot\left(\frac{1}{3}\right)^4$:

$\frac{8!}{4!(4!)}\ \frac{16}{81}\cdot\frac{1}{81}$.

$70\cdot\frac{16}{81\cdot 81}$

$\frac{1120}{\cancel{6561}} \Rightarrow 0.1707$.

$\frac{40320}{24\cdot 24} \Rightarrow \frac{40320}{576} = 70$

$P(X\geq 4)$ $P(X=4)+P(X=5)+P(X=6)+P(X=7)$ $+P(X=8)$

$P(X=5)$

$\frac{8!}{5!(3!)}\cdot\left(\frac{2}{3}\right)^5\cdot\left(\frac{1}{3}\right)^3 \Rightarrow$

$56\cdot\frac{32}{243}\cdot\frac{1}{27} \Rightarrow \frac{1792}{6561} = 0.273$.

$P(X=6)$

$\frac{8!}{6!(2!)}\cdot\left(\frac{2}{3}\right)^6\left(\frac{1}{3}\right)^2$

$28\cdot\frac{64}{729}\cdot\frac{1}{9} = \frac{1792}{6561}$.

Date:

$P(X=7)$

$\frac{8!}{7!(1!)} \cdot \left(\frac{2}{3}\right)^7 \cdot \frac{1}{3}$

$8 \cdot \frac{128}{2187} \cdot \frac{1}{3}$

$\frac{1024}{6561}$

$P(X=8)$

$\frac{8!}{8!} \cdot \left(\frac{2}{3}\right)^8 \cdot \left(\frac{1}{3}\right)^0$

$\frac{256}{6561} \cdot 1$

$$\frac{1120}{6561} + \frac{1792}{6561} + \underbrace{\frac{1792}{6561} + \frac{1024}{6561} + \frac{256}{6561}} = \boxed{\frac{5984}{6561} = P(X \geq 4)}$$

$P(X \geq 6)$

$$\frac{1792}{6561} + \frac{1024}{6561} + \frac{256}{6561} = \frac{\quad}{6561}$$

Hypergeometric Prob. Dist.

① Same as Bionomial Either Success or failure:

② Probability changes of every success:-

③ The successive trials are dependent:

④ The Experiment is repeated a fix number of times:-

The number of Successes ~~x~~ X in a hypergeometric experiment is called hypergeometric ~~vari~~ random variable:- & its prob. dist. is given by the formula:-

$$P(X=x) = \frac{{}^{K}C_{x} \cdot {}^{N-K}C_{n-x}}{{}^{N}C_{n}} \quad \text{for } x = 0,1,2,\cdots m \text{ and } x = 0,1,2,\cdots,k$$

where number of units N = number of units in the set of population.

n = Number of units in a subset or sample. K = Number of successes in a set or population or population. a positive integer is less than or equal to N positive integer $\leq N$.

The HG P.D has three parameters N, n, K $\left(\text{or } N, n \text{ \& } p = \frac{K}{N}\right)$ $q = 1 - p$.

Example

A box contains 4 red balls & 6 black balls. A sample of 4 balls is selected from the box without replacement. let x be the number of Red balls contain in sample then find the probability distribution for x.

- if Sampling is without replacement then mostly the question will be hypergeometric.

let x be the number of red balls:

$N=10;\ n=4;\ k=4$

$$P(X=x)=\frac{({}^{K}C_x)({}^{N-K}C_{n-x})}{{}^{N}C_n}$$

$$P(0)=\frac{({}^{4}C_0)({}^{6}C_4)}{{}^{10}C_4}=\frac{1\cdot15}{210}=\frac{8}{70}=\frac{1}{14}=\frac{1}{14}$$

$$P(1)=\frac{({}^{4}C_1)({}^{6}C_3)}{{}^{10}C_4}=\frac{4\cdot20}{210}=\frac{8}{210}=\frac{4}{105}$$

$$P(2)=\frac{({}^{4}C_2)({}^{6}C_2)}{{}^{10}C_4}=\frac{6\cdot15}{210}=\frac{90}{210}=\frac{3}{7}$$

$$P(3)=\frac{({}^{4}C_3)({}^{6}C_1)}{({}^{10}C_4)}=\frac{4\cdot6}{210}=\frac{24}{210}=\frac{8}{70}$$

$$P(4)=\frac{({}^{4}C_4)({}^{6}C_0)}{{}^{10}C_4}=\frac{1\cdot1}{210}=\frac{1}{210}$$

Bionomical distribution means & variance.

Hypergeometric mean & variance.

Example

The names of 5 men & 5 women are written on slips of paper & place in a box. 4 names are drawn what is the prob. that 2 are men & 2 are women:-

$N = 10, n = 4, K = 5$

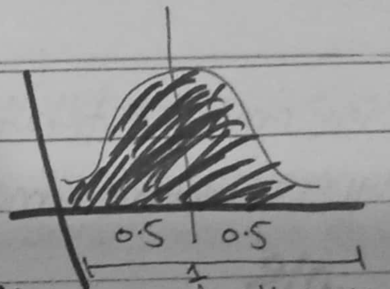
let x denote the number of women in population:-

$P(x = 2) = \frac{({}^{5}C_{2})({}^{5}C_{2})}{{}^{10}C_{4}} := \frac{10\cancel{0}}{21\cancel{0}} = \frac{10}{21}$

Chp 9:-

Continuous Prob. Distribution

→ Normal distribution:.



Total area under the curve is one 1

Example

let the random variable Z have the standard normal distribution find

i) $P(0 \le Z \le 1.20) = ?$ .3849. X = Normal variable / Z = Standard Normal Var

ii) $P(-1.65 \le Z \le 0) = ?$

X converted into Z.

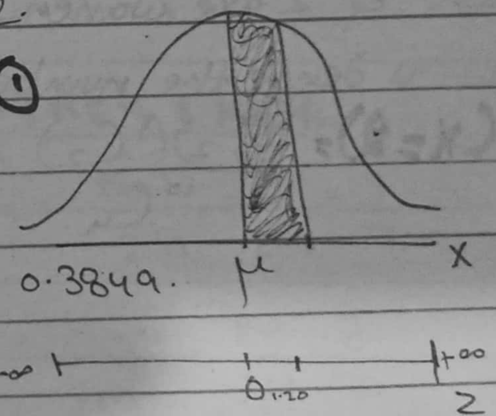
iii) $P(0.6 \le Z \le 1.67) = ?$ ~~.2486~~ .4525 − .2257 = .2268 ①

iv) $P(-1.30 \le Z \le 2.18) = ?$

v) $P(-1.96 \le Z \le -0.84) = ?$

vi) $P(Z \ge 1.96) = ?$

vii) $P(Z \le -2.15) = ?$

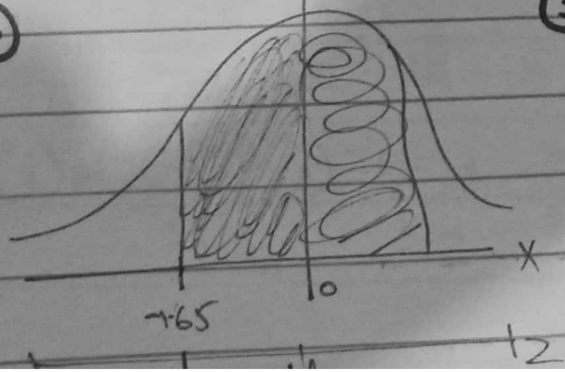


$$Z = \frac{X-\mu}{\sigma} = \frac{\bar{X}}{\sigma}$$

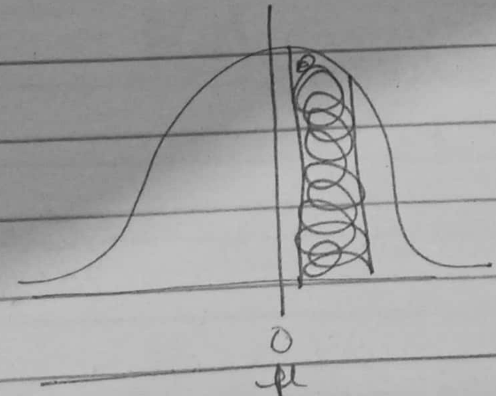
σ → variance

σ → standard deviation::

②



③



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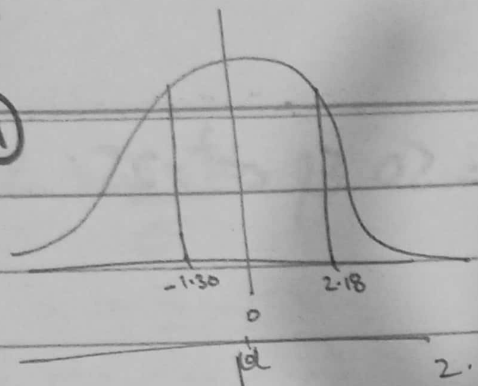
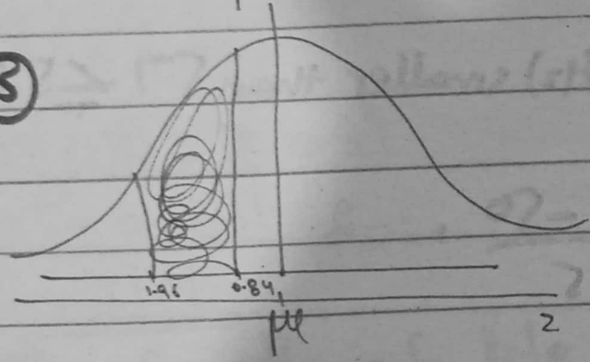


$-1.30 \le z \le 2.18$..

$(0 \le z \le 1.30) + (0 \le z \le 2.18)$.

$0.4032 + 0.4854 \Rightarrow 0.8886$

5



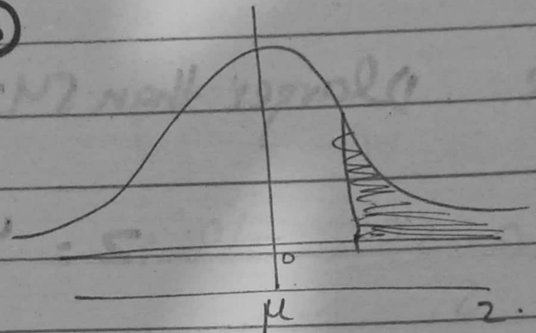
~~(0 − 0.84)~~

$(0 \to 1.96) - (0 - 0.84)$.

$0.4750 - 0.2995$

0.1755

6



$\boxed{z \ge 1.96}$

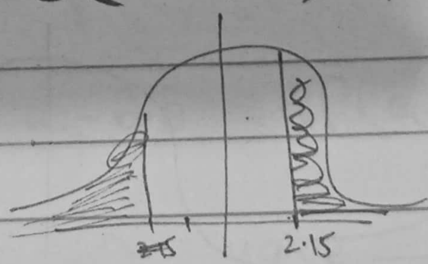
0.4750.

$0.5 - P(0 \le z \le 1.96)$

$0.5 - 0.4750$

0.025.

8 $(z \le -2.15)$



$0.5 - 0.4842$

~~0.9842~~

0.0158.

$0.5 - P(0 \le z \le 2.15)$.

$\mu = 40$ $\sigma = 8.$ $X \cdot 8$ $\frac{38-40}{8} = \frac{-.35}{8} = -0.675$

$X < 35)$

Example

A random var. X is normally dist. with $\mu = 50$ & $\sigma^2 = 25$.

find Prob. it will fall b/w:

1) 0 & 40

2) 55 & 100

that it will be 1) larger than 54 >54 2) smaller than 57. ≤57

$P(0 \leq Z \leq 40)$

$Z = \frac{X-\mu}{\sigma} \therefore Z = \frac{0-50}{5} = -10. \quad Z = \frac{40-50}{5} = -2.$

$P(55 \leq Z \leq 100)$

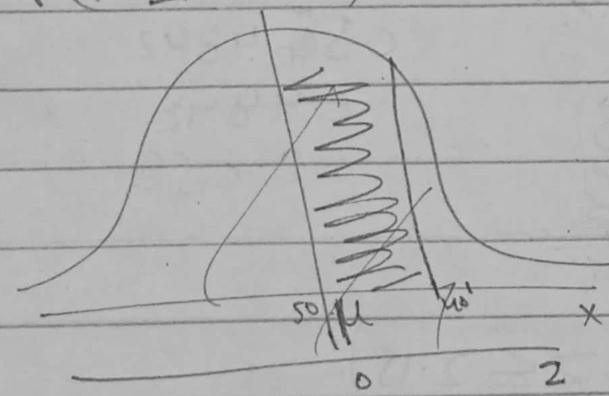
$Z = \frac{55-50}{5} = 1 \quad Z = \frac{100-50}{5} = 10.$

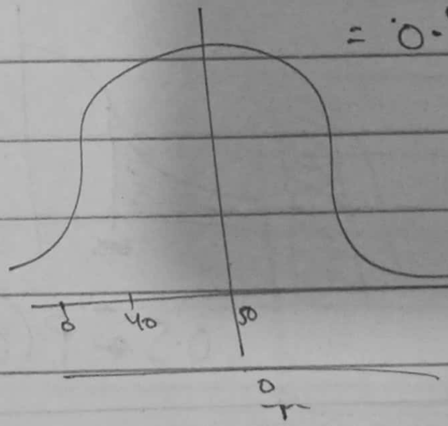
$P(0 \leq Z \leq 40).$

$P(-10 \leq Z \leq 0) - P(-2 \leq Z \leq 0) \Rightarrow P(0 \leq Z \leq 10) - P(0 \leq Z \leq 2).$

$= 0.5 - 0.4772.$

$= 0.0228$





$Z = \frac{x-\mu}{\sigma} \Rightarrow z = \frac{54-50}{5} = \frac{4}{5} = 0.80.$

$\frac{62}{1000} \times 100$ 0.62.

$0 \geq z \geq 54.$

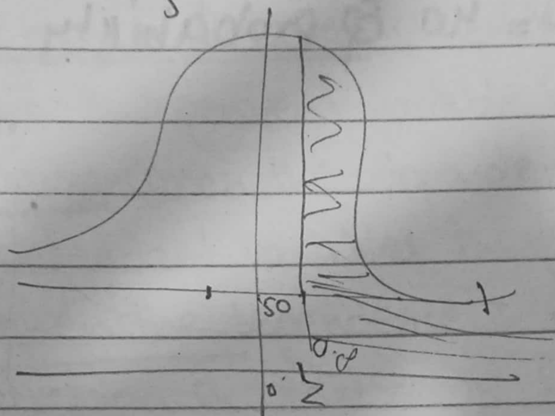
$P(+54 \leq z \leq 0)$

$P(z \geq 0.80)$

$P(0 \leq z \leq 0.8).$

$0.5 - 0.2881.$

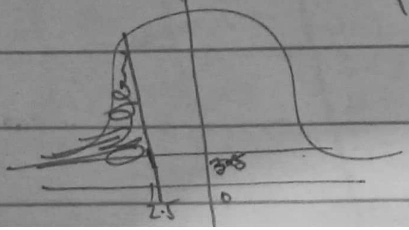
$0.2119.$



The length of life for an automated dish washer is ≈ normally dist. a mean of 3.5 years & a standard deviation of 1.0 years. if this type of dish washer is guaranted for 12 months what fraction of sales will require replacement.

$\sigma = 1.0, \mu = 3.5.$

$X = 12. = z = \frac{1.0-3.5}{1.0} \Rightarrow \frac{-2.5}{1.0} = -2.5$ $P(x \leq 1.0) = P(z \leq -2.5).$



$P(0 \leq z \leq 2.5).$ 0.4938.

$0.5 - 0.4938 \Rightarrow 0.0062 \Rightarrow 0.62\%$ replacement.

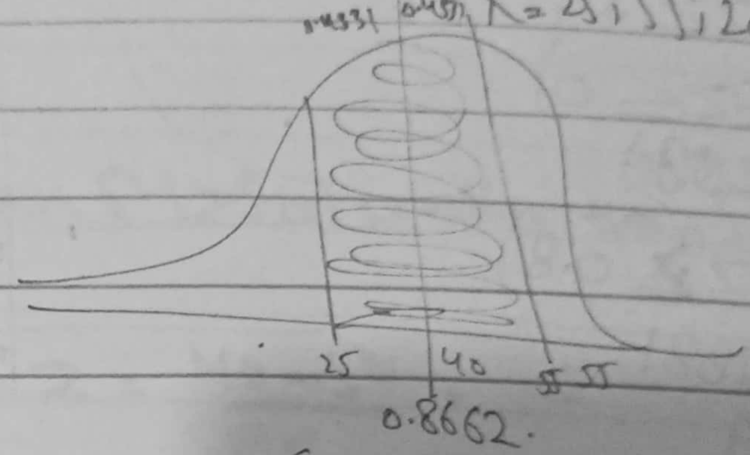
Example

In a normal dist. $\mu = 40$ & probability $P(25 \le X \le 55) = 0.8662$

find $P(20 \le X \le 60)$.

$X = 25, 55, 20, 60.$

$Z = \frac{X - \mu}{\sigma}$



$\cancel{0.8662} = \frac{25-40}{\sigma}$

$\frac{\cancel{-15}}{\sigma} =$

$(0 \le X \le 25) + ($

0.4938

$\div 0.8662$ by 2 to get the values around $\mu = 40$

$P(25 \le X \le 40) = P(40 \le X \le 55) = 0.4331)$

inverse use $\rightarrow (Z | P = 0.4331) = 1.50.$

$Z = \frac{X - \mu}{\sigma} = 1.5 = \frac{55-40}{\sigma} = 1.5 = \frac{15}{\sigma} = \sigma = \frac{15}{15} \times 10.$

$\boxed{\sigma = 10}$

$\bar{X} = \frac{X}{n}.$

$X = n\bar{X}.$

Regression & Correlation:-

The relation b/w Expected value of dependent variable & Independent variable is called Regression Relation. When we study of the dependence of variable on single Independent variable is called a simple & or two variable Regression. When the dependence of variable on two or more than two Independent variables is studied. It is called Multiple regression. furthermore, When the dependence is represented by straight line equation the Regression is said to be linear. Otherwise, it is said to be ~~curly~~ curvi linear.

$1.47 + 2.8311(102)$
$1.47 + 288.7722 = 290.2422$

Example

Compute the least Square Regression Equation of y on x for the following data. What is the Regression Coefficient & what does it mea

$\bar{X} = \frac{102}{9}$

$\bar{X} = 11.3$

$\bar{Y} = 33.5$

$X = 102$	$302\ Y$	$XY\ 3853$	$X^2 = 1308$
5	16	80	25
6	19	114	36
8	23	184	64
10	28	280	100
18	36	432	
12	41	533	144
13	44	660	169
15			225
16	45	720	256
17	50	850	289

$b = \frac{(9 \cdot 3853 - (102)(302))}{9 \cdot 1308 - (102)^2}$

$\frac{34677 - 30804}{11772 - 10404}$

$b = \frac{3873}{1368} = 2.8311$

$a = 33.5 - (2.8311)(11.5)$

$\cancel{33.5} - 99.89185$

$- \cancel{93.54}$

$33.5 - (2.8311)(11.5)$

$33.5 - 32.55$

$a = 1.47$

Estimated Regression line of Y on X is. $\hat{Y} = a + bX$

where $b = \frac{n\Sigma XY - (\Sigma X)(\Sigma Y)}{n\Sigma X^2 - (\Sigma X)^2}$

$a = \bar{Y} - b\bar{X}$.

$\frac{\Sigma Y}{n} - \frac{b\Sigma X}{n}$

$$Y = 1.47 + 2.8311\,X.$$

The estimited Reg. coefficient is 2.8311 which indicated that the values of Y increase by 2.831 units for a unit increase in X.

$$b_{yx} = \frac{n\Sigma XY - (\Sigma X)(\Sigma Y)}{n\Sigma X^2 - (\Sigma X)^2}$$

$$b_{xy} = \frac{n\Sigma YX - (\Sigma Y)(\Sigma X)}{n\Sigma Y^2 - (\Sigma Y)^2}$$

$$a = \bar{Y} - b_{yx}\bar{X}.$$

$$\hat{Y} = a + bX.$$

$$\hat{X} = a + bY$$

$$a = \bar{X} - b_{xy}\bar{Y}.$$

$$\bar{X} = \frac{\Sigma X}{n} \quad \bar{Y} = \frac{\Sigma Y}{n}.$$

Correlation:-

$$r = \frac{\Sigma(X-\bar{X})(Y-\bar{Y})}{\sqrt{\Sigma(X-\bar{X})^2\,\Sigma(Y-\bar{Y})^2}}$$

$\bar{X} = 11.3$ $\bar{Y} = 33.5$

X	Y	$(X-\bar{X})$	$(X-\bar{X})^2$	$(Y-\bar{Y})$	$(X-\bar{X})(Y-\bar{Y})$	$(Y-\bar{Y})^2$
5	16	-6.3	39.69	-17.5	110.25	306.25
6	19	-5.3	28.09	-14.5	76.85	210.25
8	23	-3.3	10.89	-10.5	346.5	110.25
10	28	-1.3	1.69	-5.5	7.15	30.25
12	36	0.7	0.49	2.5	1.75	6.25
13	41	1.7	2.89	7.5	12.75	56.25
15	44	3.7	13.69	10.5	38.85	110.25
16	45	4.7	22.09	11.5	54.05	132.25
17	50	5.7	32.49	16.5	94.05	272.25
			152.01		742.2	1234.25

$$r = \frac{742.2}{\sqrt{(152.01)(123.25)}} = \frac{742.2}{\sqrt{18735.2325}} = \frac{742.2}{136.876} = 5.4224$$