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CHAPTER 4:.

-: MEASURE OF DISPERSION:-

Dispersion:-

How much our data is dispersed/Spread:-

① **Range:.** Highest - lowest.

② **Quartile deviation (Q.D)** $= \frac{Q_3 - Q_1}{2}$

$= \frac{34.902 - 30.0171}{2} =$ **2.4425 = Q.D**

Absolute Dispersion	**Relative Dispersion:-**
Individual,	Comparison.
Same data comparision	Different data

There are 2 types of measure of dispersion. **Absolute** measure of Dispersion is one that measures the dispersion in terms of the same unit or in the sqware of unit, as the units of the data. **for example:-** If the unit of the data are Rs., meters, kg etc. The unit of the measure of dispersion is Rs, Meters, Kg etc. **A Relative Measure of Dispersion is** the one that express in the form of the ratio, coefficient, % age Ej is independent of the unit of measures of dispersion. It is useful for the comparison of data of Different Nature. A measure of ~~Central tendency~~ the main measure of Dispersion are Range, Quartile Deviation, Mean, Varians Ej Standard Deviations:

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Absolute

Range: $x_m - x_0$

Quartile Deviation:

$$\frac{Q_3 - Q_1}{2}$$

Mean Deviation:-

$$\frac{\Sigma f|x-\bar{x}|}{\Sigma f} \text{ OR } \frac{\Sigma f|x-\bar{x}|}{\Sigma f}$$

(Arithmetic Mean)

$$\frac{\Sigma f|x-\bar{x}|}{\Sigma f} = \frac{2854.88}{1000}$$

(Mean deviation from Median.) 2.85388

Variance & Standard deviation are absolute dispersion:-

$$S = h \times \sqrt{\frac{\Sigma f u^2}{\Sigma f} - \left(\frac{\Sigma f u}{\Sigma f}\right)^2}$$

Relative

$$\frac{x_m - x_0}{x_m + x_0}$$

Coefficient of Q.D:

$$\frac{Q_3 - Q_1}{Q_3 + Q_1}$$

$$\frac{34.902 - 30.0171}{34.902 + 30.0171}$$

$$\frac{4.8849}{64.9191} = 0.0752$$

Coefficient of Mean Deviation: $\frac{\text{Mean Deviation}}{\text{Mean}}$

OR

$$\frac{\text{Mean Deviation}}{\text{Median}}$$

$$\frac{2.85}{32.4909} = 0.0877$$

Co-efficient of Variance: $C.V = \frac{S}{\bar{x}} \times 100$

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4.4 :. Quartile Deviation:-

Income per week	No. of Earners..	cf	class boundries
41–50	30	30	40.5–50.5
51–60	36	66	50.5–60.5
61–70	43	109=Q1	60.5–70.5
71–80	104	213	70.5–80.5
81–90	73	286=Q3	80.5–90.5
91–100	14	300	90.5–100.5
Total.	300		

Quartile Deviation:- $\frac{Q_3 - Q_1}{2} = \frac{82.14 - 62.59}{2} = 9.7750$

* $Q_3 = l + \frac{h}{f}\left(\frac{in}{4} - c\right)$ * $Q_1 = l + \frac{h}{f}\left(\frac{in}{4} - c\right)$

$80.5 + \frac{10}{73}(225 - 213)$ $= 60.5 + \frac{10}{43}(75 - 66)$

$Q_3 = 82.14$ $Q_1 = 62.59$

4.5 :.

171	160	144	132	154	160	160	158	148	160	151	153
131	165	139	163	149	149	140	149	150	161	136	144
165	174	153	149	159	169	147	156	149	171	149	154
153	149	147	154	158	145	160	152	156	138	167	142
165	155	140	155	147	158	149	169	148	174	150	144

130 140 150 160 170.
131 149 149 153 154 160 171
139 144 149 155 158 165 169 174
147 149 153 155 160 161 171
146 147 154 156 160 166
149 150 158 163 167 174.
140 149 156 169
142 149 150 160

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class interval	frequency	cf	x	$u = \frac{x-a}{h}$	fu	fu^2
129.5 – 134.5	3	3	132.0	-3	-9	27
134.5 – 139.5	3	6	137.0	-2	-6	12
139.5 – 144.5	6	12	142	-1	-6	6
144.5 – 149.5	14 = a	26 = Q_1	147 = a	0	0	0
149.5 – 154.5	9	35 = med	152	1	9	9
154.5 – 159.5	8	43	157	2	16	32
159.5 – 164.5	7	50 = Q_3	162	3	21	63
164.5 – 169.5	6	56	167	4	24	96
169.5 – 174.5	4	60	172	5	20	100
Total	60				69	345

Median $= l + \frac{h}{f}\left(\frac{n}{2} - c\right)$

$= 149.5 + \frac{5}{9}(30 - 26)$

$= 151.7 \approx$ **152 = Median**

★ Mean $= a + \frac{\Sigma fu}{\Sigma f} \times h$

$147 + \frac{69}{60} \times 8$

Mean = 152.75

Q.D $= \frac{Q_3 - Q_1}{2} = \frac{161 - 146}{2} =$ **7.5 = Q.D**

★ $Q_3 = l + \frac{h}{f}\left(\frac{3n}{4} - c\right)$, $\frac{3n}{4} = 45$

$169.5 + \frac{5}{7}(45 - 43)$

$170.9 \approx$ **166 = Q_3**

★ $Q_1 = l + \frac{h}{f}\left(\frac{in}{4} - c\right)$, $\frac{in}{4} = 15$

$= 144.5 + \frac{5}{14}(15 - 12)$

$145.5 \approx$ **146 = Q_1**

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4.6 :: Mean Deviation from Arithmetic Mean.

Marks	No. of students.	x	fx	$x-\bar{x}$	$\lvert x-\bar{x}\rvert$	$f\lvert x-\bar{x}\rvert$
0–9	2	4.5	9	−41.67	41.67	83.34.
10–19	3	14.5	43.5	−31.69	31.69	95.01
20–29	8	24.5	196.	−21.67	21.67	173.36
30–39	24	34.5	828	−11.67	11.67	280.08
40–49	27	44.5	1201.5	−1.67.	1.67	45.09.
50–59	40	54.5	2180.	8.33	8.33	333.2
60–69	11	64.5	709.5	18.33	18.33	201.63
70–79	5.	74.5	372.5	28.33	28.33.	141.65
	120.		5540.			1353.36.

$\bar{x}$ = Mean = $\frac{\Sigma fx}{n} = \frac{5540}{120} = 46.17$.

Mean Deviation = $\frac{\Sigma f\lvert x-\bar{x}\rvert}{n}$

$= \frac{46.17}{120.} \quad \frac{1353.36}{120.}$

11.28 = Mean Deviation.

4.7 ::

Marks (birthday)	20–24	25–29	30–39	40–44	45–49	50–54	55–64.
No. of men students:	1	2	26	22	20	15	14.

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Age group	x	f	fx	$\lvert x-\bar{x}\rvert$	$f\lvert x-\bar{x}\rvert$
20–24	22	1	22	22.5	22.5
25–29	27	2	54	17.5	35
30–34	32	f_1	$32f_1$	12.5	$12.5f_1$
35–39	37	$26-f_1$	$962-37f_1$	7.5	$195-7.5f_1$
40–44	42	22	924	2.5	55
45–49	47	20	940	2.5	50
50–54	52	15	780	7.5	112.5
55–59	57	f_2	$57f_2$	12.5	$12.5f_2$
60–69	62	$14-f_2$	$868-62f_2$	17.5	$245-17.5f_2$
			$4550-5(f_1+f_2)$		$715+5(f_1-f_2)$

$$\text{Mean} = \bar{x} = \frac{\Sigma fx}{n} = \frac{4550-5(f_1+f_2)}{100}$$

$$\bar{x} = 44.5 = \frac{4550}{100} - \frac{5(f_1+f_2)}{100}$$

$$44.5 = 45.5 - \frac{f_1+f_2}{20}$$

$$\frac{f_1+f_2}{20} = 1$$

$$f_1+f_2 = 20 \longrightarrow (1) \Rightarrow f_1+f_1 = 20 \Rightarrow 2f_1 = 20 \Rightarrow \boxed{f_1 = 10}$$

Eq Mean Deviation $= \frac{1}{\Sigma f}\Sigma f\lvert x-\bar{x}\rvert$.

$$7.15 = \frac{1}{100}\,715+5(f_1-f_2)$$

$$715 = 715+5(f_1-f_2)$$

$$0 = f_1-f_2$$

$$f_1 = f_2 \Rightarrow \boxed{f_2 = 10}$$

Put in (1).

Q.8.

Group H:-

Heights:-	class boundries:-	No of persons A	No of persons B	A(cf)	B(cf)	x-Med	f|x-Med|
58	57.5-58.5	10	15	10	15	-3.35	33.50
59	58.5-59.5	18	20	28	35	-2.35	42.30
60	59.5-60.5	30	32	58	67	-1.35	40.50
61	60.5-61.5	42	35	100	102	-0.35	14.70
62	61.5-62.5	35	33	135	135	0.65	22.75
63	62.5-63.5	28	22	163	157	1.65	46.20
64	63.5-64.5	16	20	179	177	2.65	42.40
65	64.5-65.5	8	10	187	187	3.65	29.20
		187	187				271.55

For Group A:-

* Quartile Deviation:-

$$Q.D_A = \frac{Q_3 - Q_1}{2} = \frac{62.69 - 60.12}{2} \quad | \quad .628 = Q.D_A$$

$$\rightarrow Q_3 = l + \frac{h}{f}\left(\frac{in}{4} - c\right) \quad \frac{in}{4} = \frac{140.25}{}$$

$$62.5 + \frac{1}{28}(140.25 - 135)$$

$$62.69 = Q_3$$

$$\rightarrow Q_1 = l + \frac{h}{f}\left(\frac{in}{4} - c\right) \quad \frac{in}{4} = 46.75$$

$$59.5 + \frac{1}{30}(46.75 - 28)$$

$$60.12 = Q_1$$

Co-efficient of Q.D.

$$\frac{Q_3 - Q_1}{Q_3 + Q_1} = \frac{62.69 - 60.12}{62.69 + 60.12} = \frac{2.57}{122.81}$$

Coefficient of Q.D is = 0.0209

For Group B:-

$$Q.D_B = \frac{Q_3 - Q_1}{2} = \frac{62.74 - 59.87}{2} = 1.435$$

$$\rightarrow Q_3 = l + \frac{h}{f}\left(\frac{in}{4} - c\right)$$

$$62.5 + \frac{1}{22}(140.25 - 135)$$

$$Q_3 = 62.74$$

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For Group B:-

$Q_1 = l + \frac{h}{f}\left(\frac{in}{4} - c\right)$

$59.5 + \frac{1}{32}(46.75 - 35)$

$Q_1 = 59.87$

Co-efficient of Q.D:-

$\frac{Q_3 - Q_1}{Q_3 + Q_1} = \frac{2.87}{122.61}$

Co-efficient of Q.D = 0.0234

Mean Deviation:-

$\frac{\Sigma f|x - \bar{x}|}{\Sigma f}$

$\frac{271.55}{187}$

1.4521 = Mean Deviation

Co-efficient of M.D:-

Median $= l + \frac{h}{f}\left(\frac{n}{2} - c\right)$

$60.5 + \frac{1}{42}(93.5 - 58) = 61.35$

Mean Deviation / Median:-

$\frac{61.4521}{61.35} = 0.0237$

x - Med	f|x - Med|
-3.26	48.90
-2.26	45.20
-1.26	40.32
-0.26	9.10
0.74	24.42
1.74	38.28
2.74	54.80
3.74	37.40
	208.12

Mean Deviation:-

$\frac{\Sigma f|x - \bar{x}|}{\Sigma f} = \frac{208.12}{187} = 1.1129$

Median $= l + \frac{h}{f}\left(\frac{n}{2} - c\right)$

$60.5 + \frac{1}{35}(93.5 - 67)$

61.2571 = Median

Co-efficient of Mean Deviation

$\frac{M.D}{Median} = \frac{61.2571}{\left(\frac{1}{1.1129}\right)}$

$\frac{1.1129}{61.2571}$

0.0182 = M.D

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* Variance & standard deviation:-

→ Square root of variance makes standard deviation:-

* Standard deviation:-

$$S = \sqrt{\frac{\Sigma fx^2}{\Sigma f} - \left(\frac{\Sigma fx}{\Sigma f}\right)^2} \quad \because \text{variance} = S^2 :-$$

$$S = \sqrt{\frac{646414}{726} - \left(\frac{21092}{726}\right)^2}$$

$$S = \sqrt{890.4 - 844.04}$$

$$S = \sqrt{46.36}$$

S = 6.81 | **Variance = 46.36**

* Standard Deviation by Short Method:-

$$S = h \times \sqrt{\frac{\Sigma fu^2}{\Sigma f} - \left(\frac{\Sigma fu}{\Sigma f}\right)^2}$$

$$S = 20 \times \sqrt{\frac{2715}{815} - \left(\frac{-715}{815}\right)^2}$$

$$= 20 \times \sqrt{3.333 - (0.877)^2}$$

$$= 20 \times \sqrt{3.33 - 0.769129}$$

$$= 20 \times \sqrt{2.561}$$

$$= 20 \times 1.6005$$

$$= 20.6005$$

$$= 32.0100$$

variance = $h^2 \times 2.561$

400×2.561

1024.4000

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Age (years)	No. of Men (f)		x	fx	x²	fx²	u = (x−a)/h	fu	u²	fu²
							h=5, a=27			
15—19	29		17	493	289	8381	-2	-58	4	116
20—24	176		22	3872	484	85,184	-2.5	440	6.25	1100
25—29	208		27=a	5616	729	151,632	0	0	0	0
30—34	173		32	5536	1024	177,152	2.5	432.5	6.25	1081.25
35—39	82		37	3034	1369	112,258	2	164	4	328
40—44	40		42	1680	1764	79,560	3	120	9	360
45—49	15		47	705	2209	33135	4	60	16	240
50—54	3		52	156	2704	8112	5	15	25	75
S	726			21092		646,414		293.5		3300.25

lifes (hours)	No. of components (f)		u = (x−a)/h (a=1090, h=20)	fu	u²	fu²	x	
1000—1020	40		-4	-160	16	640	1010	
1020—1040	96		-3	-288	9	864	1030	
1040—1060	164		-2	-328	4	656	1050	
1060—1080	172		-1	-172	1	172	1070	
1080—1100	185		0	0	0	0	1090=a	
1100—1120	83		1	83	1	83	1110	
1120—1140	75		2	150	4	300	1130	
	815			-715		2715		

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Co-efficient of Variance

$\bar{x} = \frac{\Sigma x}{n}$.

★ Co-efficient of variance in Reliability of data:-

$\bar{x} = 18.5$

Player A Price (in Rupees) X	Player B Life (in hours) Y	x^2	y^2
30	345	900	119025
23	250	529	62500
18	180	324	32400
13	150	169	22500
8	130	64	16900
$\Sigma x = 92$	$\Sigma y = 1055$	1986	253325
$n = 5$	$n = 5$		

$S_x = \sqrt{\frac{\Sigma x^2}{n} - \left(\frac{\Sigma x}{n}\right)^2}$

$= \sqrt{\frac{1986}{5} - \left(\frac{92}{5}\right)^2}$

$= \sqrt{397.2 - (18.5)^2}$

$= \sqrt{397.2 - 342.25}$

$= \sqrt{54.95}$

$= 7.413 = S_x$

$C.V = \frac{S_x}{\bar{x}} \times 100$

$= \frac{7.413}{18.5} \times 100$

$C.V = 40.07$

$\bar{y} = 211$

$S_y = \sqrt{\frac{253325}{5} - \left(\frac{1055}{5}\right)^2}$

$\sqrt{50665 - (211)^2}$

$\sqrt{50665 - 44521}$

$\sqrt{6144}$

$S_y = 78.38$

$C.V = \frac{78.38}{211} \times 100 = 37.1469 = C.V.$

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who's C.V answer is low that data is more reliable.

Moments of mean:-

$m_i = \frac{\Sigma(x_i - \bar{x})^i}{n}$, $i = 1,2,3,4$ · grouped $m_i = \frac{\Sigma f(x_i - \bar{x})^i}{\Sigma f}$, $i = 1,2,3,4$.

x:-
32, 36, 36, 37, 39, 41, 45, 46, 48

$m_1 = \frac{\Sigma(x - \bar{x})^1}{n}$

$= \frac{0}{9} = \boxed{0 = m_1}$

$m_2 = \frac{232}{9} = \boxed{25.78 = m_2}$

$m_3 = \frac{186}{9} = \boxed{20.67 = m_3}$

$m_4 = \frac{10708}{9} = \boxed{11890 = m_4}$

x	$x_i - \bar{x}$	$(x_i - \bar{x})^2$	$(x_i - \bar{x})^3$	$(x_i - \bar{x})^4$
32	-8	64	-512	4096
36	-4	16	-64	256
36	-4	16	-64	256
37	-3	9	-27	81
39	-1	1	-1	1
41	1	1	1	1
45	5	25	125	625
46	6	36	216	1296
48	8	64	512	4096
360	$\sum x_i = 360$	232	186	10708
$\bar{x} = 40$	0			

Σfx / Σf = $\frac{\Sigma fx}{n}$ = $\frac{1318}{131}$ = $\bar{x} = 10.1$

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Weekly earning	No of Men.	$x - \bar{x}$	$(x-\bar{x})^2$	$(x-\bar{x})^3$	$(x-\bar{x})^4$	fx
5	1	-5.1	26.01	-132.651	676.5201	5
6	2	-4.1	16.81	-68.921	282.5761	12
7	5	-3.1	9.61	-29.791	92.3521	35
8	10	-2.1	4.41	-9.261	19.4481	80
9	20	-1.1	1.21	-1.331	1.4641	180
10	51	-0.1	0.01	-0.001	0.0001	510
11	22	0.9	0.81	0.729	0.6561	242
12	11	1.9	3.61	6.859	13.0321	132
13	5	2.9	8.41	24.389	70.7281	65
14	3	3.9	15.21	59.319	231.3441	42
15	1	4.9	24.01	117.649	576.4801	15
	131					1318

$f(x-\bar{x})^1$	$f(x-\bar{x})^2$	$f(x-\bar{x})^3$	$f(x-\bar{x})^4$
-5.1	26.01	-132.651	676.5201
-8.2	33.62	-137.842	565.1522
-15.5	48.05	-148.955	461.7605
-21	44.1	-92.61	194.481
-22	24.2	-26.62	29.282
-5.1	0.51	-0.051	0.0051
19.8	17.82	16.038	14.4342
20.9	39.71	75.449	143.3531
14.5	42.05	121.945	353.6405
11.7	45.63	177.957	694.0323
4.9	24.01	117.649	576.4801
-5.1	345.71	-29.691	3709.1411

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$m_1 = \frac{-5.1}{131}$

$m_1 = 0.04$.

$m_2 = \frac{345.71}{131}$

$m_2 = 2.64$

$m_3 = \frac{-29.691}{131}$

$m_3 = -0.227$ ✓ 0.5648 ✓

$m_4 = \frac{3709.1411}{131}$

$m_4 = 28.33$

m_2 = variance.

$S.D = \sqrt{m_2} = 1.6248 =$

$\beta_1 \downarrow b_1 = \frac{m_3}{S^3}$ $\frac{0.5648}{4.2891}$

0.1317.

Skewness:-

lie b/w $-2 < b_1 < 2$.

if b_1 is $0 < b_1 \longrightarrow$ +vely skewed

if $b < 0 \longrightarrow$ -vely skewed.

if $b = 0 \longrightarrow$ symetric.

Kurtosis.

$\beta_2 \downarrow b_2$ $\frac{m_4}{m_2^2}$ $\frac{28.33}{(0.5648\ 2.641)^2} = 4.6689$.

if answer is 3 or nearly 3 than distribution is normal.