APS

10/21

Ch. 6 Probability

- Probability means the chances of something happening (chances of occurances of anything)

Random Experiment

- The term experiment means a planned activity or process, whose results yield a set of data.
- A single performance of an experiment is called a trial
- The result obtained from an experiment or a trial, is called an outcome

Example: Toss a coin 5 times.

~~H, T, H, T, T~~

H, T, T, H, H

Sample Space

- A set consisting of all possible outcomes that can result from an random experiment is defined to be Sample Space.
- Denoted by 'S'

- Each possible outcome is the member of the sample space, and is called a sample point in that sample space

• We have a die, it is rolled, The possible outcomes are:

$S = \{1, 2, 3, 4, 5, 6\}$ ⇒ Sample Space for one rolling

- Two die are rollet, All possible outcomes:

S =

6	1,6	2,6	3,6	4,6	5,6	6,6
5	1,5	2,5	3,5	4,5	5,5	6,5
4	1,4	2,4	3,4	4,4	5,4	6,4
3	1,3	2,3	3,3	4,3	5,3	6,3
2	1,2	2,2	3,2	4,2	5,2	6,2
1	1,1	2,1	3,1	4,1	5,1	6,1
	1	2	3	4	5	6

$n(S) = 36$

Event

- An event is an individual outcome or any no. of outcomes of a rantom experiment/trial

Rule of Permutation and Combination

$$^nP_r = \frac{n!}{(n-r)!} \qquad\qquad ^nC_r = \frac{n!}{r!\,(n-r)!}$$

- A club consists of 4 members. ~~thom~~ How many sample points are in the sample space, when 3 officers; president, secretary and treasurer are to be chosen

  ⇒ Different roles; so order matters

  ∴ No. of sample points = $^4P_3 = 24$

- A 3 person commitee is to be formed from a list of 4 persons. How many sample points are associated with the experiment

  ⇒ Commitee's people are all equal, so any order of choosing results in equivalent result

  ∴ " " " " = $^4C_3 = 4$

A, B, C, D $\qquad$ S = {ABC, ABD, ACD, DBC}

~~A~~

Let A be an event. Then probability of event A is denoted by: P(A), is defined as:

~~P(A) = ~~

$$\boxed{P(A) = \frac{\text{no. of favourable outcomes}}{\text{Total no. of outcomes}}}$$

~~n(S)~~

$$P(A) = \frac{m}{n} = \frac{n(A)}{n(S)}$$

- Probability of an event is ≥ 0

A fair coin is tossed 3 times. What is P that at least 1 head appears

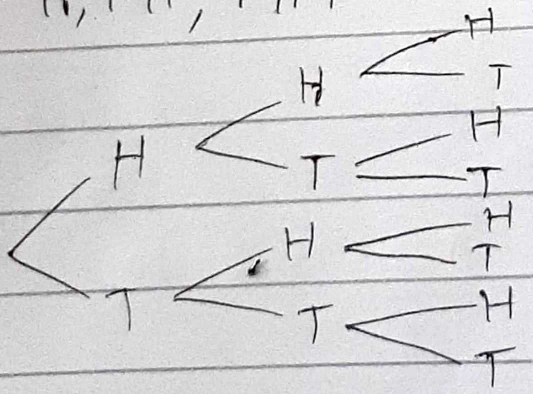
$$\cancel{P(\text{at least 1 head}) = \frac{(3!-1)}{3!} = \frac{5}{6}}$$

HHH, HHT, HTT, TTT

THH, TTH

HTH, THT

⇒ Simply make Tree diagram



S = {HHH, HHT,
HTH, HTT,
THH, THT,
TTH, TTT}

$n(S) = 8$

let A denote the event where at least 1 head appears. Then;

A = {HHH, HHT, HTH, HTT, THH, THT, TTH}

$n(A) = 7$

$$\therefore P(A) = \frac{7}{8} = \frac{n(A)}{n(S)}$$

If two fair die are thrown what is the P of getting: 1) double 6 2) Sum of 8 or more

1) $S = \{(1,1), (1,2), (1,3), (1,4), (1,5), (1,6)$

$\vdots$

$(6,1), (6,2), (6,3), (6,4), (6,5), (6,6)\}$

let A be the event where double 6 appears

$A = \{(6,6)\}$ $\therefore P(A) = \frac{1}{36}$

2) let B be event where sum of 8 or more appears

$B = \{(6,2), (5,3), (6,3), (4,4), (5,4), (6,4), \ldots\ldots (6,6)\}$

$n(B) = 15$ $\therefore P(B) = \frac{15}{36} \simeq 0.42 \simeq 42\%$

If a card is drawn from ordinary ~~set~~ deck of 52 playing cards. Find P of 1) Card is Red
2) Card is a diamond 3) Card is a 10

52

26 Red 26 Black

13 heart 13 diamond 13 club 13 spate

K
Q
J
10
$\vdots$
2
A

1) Let A be event where chosen card is red

$n(A) = 26$

$\therefore P(A) = \frac{26}{52} = \frac{1}{2}$

2) ~~Let B be the event where chosen card is a diamond~~

~~$n(B) = 13$ $\therefore P(B) = \frac{13}{52} = \frac{1}{4}$~~

3) Let C be the event where chosen card is a 10

$n(C) = 4$ $\therefore P(C) = \frac{4}{52} = \frac{1}{13}$

6 white balls and 4 black balls which are indistinguishable; apart from colour, are placed in a bag. If 6 balls are taken from the bag. Find P of their being 3 white and 3 black

$n(S) = {}^{10}C_6 = 210$

$n(A) = {}^{6}C_3 \times {}^{4}C_3 = 80$

$\therefore P(A) = \frac{8}{21} \approx 0.38$

⇒ Always assume with replacement

⇒ Without replacement will be explicitly stated

- First know n(S) or sample space
- Create event and n(A)
- Find probability.

Example 6.8, 6.9
do at home

APS | Laws of Probability

Laws of Complementation

$$A + \bar{A} = S \Rightarrow \bar{A} = S - A$$

$$P(A) + P(\bar{A}) = P(S) = 1$$

$$\Rightarrow \boxed{P(\bar{A}) = 1 - P(A)}$$

A coin is tossed 4 times in succession. What is P that at least 1 head occurs

Sol $n(S) = 2^4 = 16$

let A be event where at least 1 head occurs $\Rightarrow$ 1 2 3 4 Heads

~~n(A) = n(S) - n(~~

" $\bar{A}$ " " " where no head occurs $\Rightarrow$ 0 Heads

$\bar{A} = \{TTTT\} \Rightarrow n(\bar{A}) = 1 \Rightarrow P(\bar{A}) = \frac{1}{16}$

$\therefore P(A) = 1 - P(\bar{A}) = 1 - \frac{1}{16} = \frac{15}{16}$

A coin is biased, so that the P that it falls showing tails is 3/4 (0.75). 1) Find P of getting at least 1 head after 5 tosses. 2) How many times must the coin be tossed so that P of obtaining at least 1 head is > 0.98

1) Let A be where at least 1 head appears

$\bar{A}$ be where no head appears

~~$\bar{A} = \{TTTTT\} \Rightarrow n(\bar{A}) = 1$, $n(S) = 2^5 = 32$~~

~~$\therefore P(\bar{A}) = \frac{1}{32} \Rightarrow P(A) = 1 - \frac{1}{32}$~~

$\bar{A} = \{TTTTT\} \Rightarrow P(\bar{A}) = \left(\frac{3}{4}\right)^5 = \frac{243}{1024}$

$\therefore P(A) = 1 - \frac{243}{1024} = \frac{781}{1024} \simeq 0.763$

2) let A be " where at least 1 head appears

$\therefore P(A) \geq 0.98$

$P(\bar{A}) \leq 0.02$

Let no. of times coin is tossed be n

$\therefore \left(\frac{3}{4}\right)^n \leq 0.02$

$n \log\left(\frac{3}{4}\right) \leq \log 0.02$

$n \geq \frac{\log 0.02}{\log\left(\frac{3}{4}\right)}$

$n \geq 13.60$

$\therefore n = 14$

$\therefore$ Coin must be tossed 14 times

Addition Law

or   and

$$\boxed{P(A \cup B) = P(A) + P(B) - P(A \cap B)}$$

If one card is selected at random from 52 playing cards. What is P that card is club, face or both

$n(S) = 52$

let A be event where ~~three~~ club card

let B be event where face card

$n(A) = 13$ | $n(B) = 12$

$\Rightarrow P(A) = \frac{13}{52} = \frac{1}{4}$ | $P(B) = \frac{12}{52} = \frac{3}{13}$

$$\therefore P(A \cup B) = \frac{1}{4} + \frac{3}{13} - \frac{3}{52}$$

$$= \frac{11}{26} \simeq 0.42$$

An integer is chosen at random from first 200 integers. What is P that chosen is divisible by 6 or by 8.

$n(S) = 200$

let A be event where integer is divisible by 6

let B be event where integer is divisible by 8

~~$A = \{6, 12, 18, 24, 30, 36, 42, 48,$~~ $n(A) = \left[\frac{200}{6}\right] = 33$

Bracket used when integer part is only needed $\Leftarrow$ $n(B) = \left[\frac{200}{8}\right] = 25$

$n(A \cap B) = \left[\frac{200}{24}\right] = 8$

24 = G.C.D (H.C.F) of 8 and 6

$$\therefore P(A \cup B) = \frac{33}{200} + \frac{25}{200} - P(A \cap B)$$

$$= \frac{33}{200} + \frac{25}{200} - \frac{8}{200}$$

6.14 to yourself

3 horses A, B, C are in a race. A is twice likely to win as B. And B ~~has~~ is twice as likely to win as C. A or B wins $\Rightarrow P(A \cup B)$

Let $P(C)$ be x

$\therefore P(B) = 2x \Rightarrow P(A) = \cancel{2x}\ 4x$

$\therefore 4x + 2x + x = 1$

$x = \frac{1}{7}$

$\therefore P(C) = \frac{1}{7}, \quad P(B) = \frac{2}{7}, \quad P(A) = \frac{4}{7}$

$(A \cap B)$

$\therefore P(A \cup B) = \frac{4}{7} + \frac{2}{7} - \underbrace{0}_{P(A\cap B)} = \frac{6}{7}$

$\because$ A and B can not win together

Addition law for three events

$$\boxed{P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(A \cap C) - P(B \cap C) + P(A \cap B \cap C)}$$

6.16 to yourself

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A card is drawn at random from playing cards (52). What is P() that it is diamond / face card / King?

let, A be event where diamond is drawn
B " " face card "
C " " King "

$n(A) = 13$
$P(A) = \frac{13}{52}$

$n(B) = 12$
$P(B) = \frac{12}{52}$

$n(C) = 4$
$P(C) = \frac{4}{52}$

$n(A \cap B) = 3$
$P(A \cap B) = \frac{3}{52}$

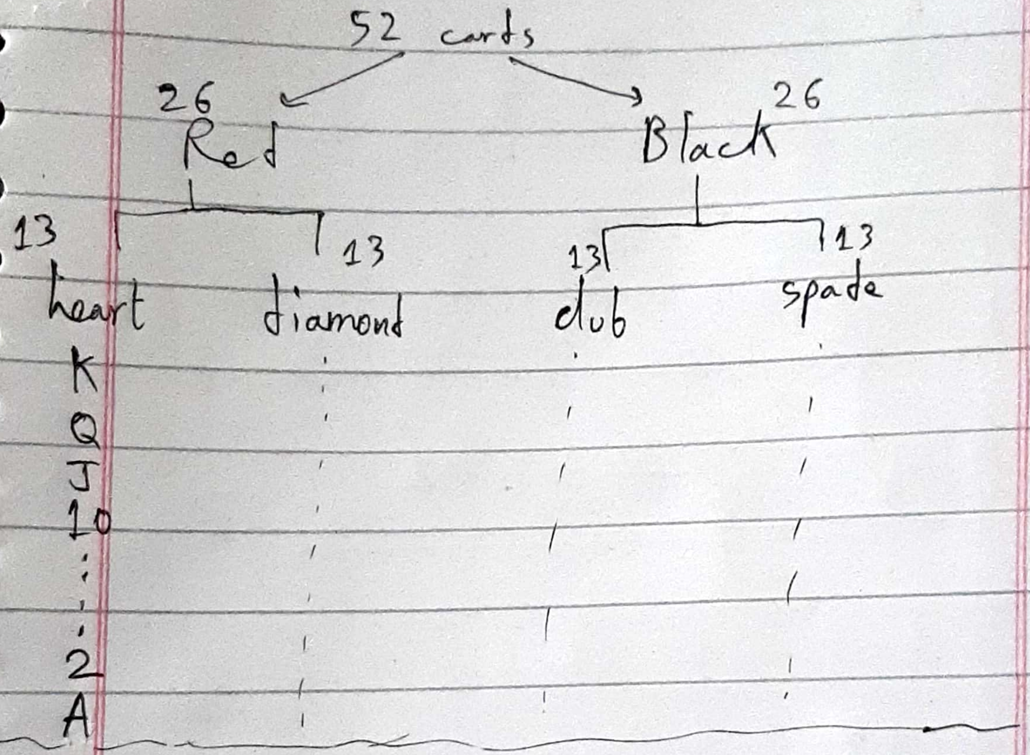
$n(A \cap C) = 1$
$P(A \cap C) = \frac{1}{52}$

$n(B \cap C) = 4$
$P(B \cap C) = \frac{4}{52}$

$$P(A\cap B\cap C) = \frac{n(A\cap B\cap C)}{n(S)} = \frac{1}{52}$$

$$P(A\cup B\cup C) = \frac{13}{52} + \frac{12}{52} + \frac{4}{52} - \frac{3}{52} - \frac{1}{52} - \frac{4}{52} + \frac{1}{52}$$

$$= \frac{22}{52} = \frac{11}{26} \simeq 0.423$$



A drawer contains 50 bolts and 150 nuts. Half the bolts and half the nuts are rusted. If one ~~of the~~ item is chosen at random, what is P that it is rusted or is a bolt

Let A be event where chosen item is rusted

B → is a bolt $\qquad n(S) = 200$

$n(A) = \frac{50}{2} + \frac{150}{2} = 100$ | $n(B) = 50$ | $n(A\cap B) = 25$

$P(A) = \frac{100}{200} = \frac{1}{2}$ | $P(B) = \frac{50}{200} = \frac{1}{4}$ | $P(A\cap B) = \frac{25}{200}$

$= \frac{1}{}$

$\therefore P(A \cup B) = \frac{1}{2} + \frac{1}{4} - \frac{1}{8}$

$= \frac{5}{8} = 0.625$

A certain carton of eggs has 3 bad eggs, 9 good eggs. An omlette is made randomly from of 3 eggs. What is P that they ~~are~~ there are 1) No bad eggs 2) At least 1 bad egg 3) Exactly two bad eggs

$n(S) = {}^{12}C_3 = 220$

1) let A be event where 3 eggs is fine

~~B → no bad egg~~

$n(A) = {}^{9}C_3 = 84$ → ${}^{9}C_3 \times {}^{3}C_0$

$\therefore P(A) = \frac{84}{220} = \frac{21}{55} \simeq 0.382$

2) let B " " at least 1 bad egg

$\bar{B}$ " " no bad egg

$\therefore P(B) + P(\bar{B}) = 1$

$P(\bar{B}) = 1 - \frac{21}{55}$

$= \frac{34}{55} \simeq 0.618$

3) let C " " exactly 2 bad eggs

$n(C) = {}^{3}C_2 \times {}^{9}C_1 = 27$

$\therefore P(C) = \frac{27}{220} \simeq 0.123$

APS

24/11

Conditional Probability → A given B

$$P(A/B) = \frac{P(A \cap B)}{P(B)} \; ; \; P(B) \neq 0$$

or

$$P(B/A) = \frac{P(A \cap B)}{P(A)} \; ; \; P(A) \neq 0$$

Let A and B be two events in sample space S. If $P(B) \neq 0$, then C.P of event A, given that event B has occured, written as $P(A/B)$, is defined by:

$$\boxed{P(A/B) = \frac{P(A \cap B)}{P(B)}}$$

$$\frac{\frac{n(A \cap B)}{n(S)}}{\frac{n(B)}{n(S)}} = \frac{n(A \cap B)}{n(S)} \times \frac{n(S)}{n(B)} = \frac{n(A \cap B)}{n(B)}$$

It should be noted that $P(A/B)$ satisfies all the basic axioms of probability. So:

(i) $0 \leq P(A/B) \leq 1$ (no shit)

(ii) $P(S/B) = 1$ ⇒ All of B is contained in S, i.e.

$$P(S/B) = \frac{P(S \cap B)}{P(B)} = \frac{P(B)}{P(B)} = 1$$

Example 6.17

Two coins are tossed. What is C.P that two heads result, given that there is at least 1 head.

Let A be event two heads

Let B be event where at least one head.

$S = \{HH, HT, TH, TT\}$

$n(S) = 4$ $P(A/B) = ?$

$n(A \cap B) = 1 \Rightarrow P(A \cap B) = \frac{1}{4}$

$n(B) = 3 \Rightarrow P(B) = \frac{3}{4}$

$\therefore P(A/B) = \frac{1/4}{3/4} = \frac{1}{3} \simeq 0.333$

A man tosses two fair dice. What is C.P that sum of the two die will be 7, given that:

(i) Sum is odd (ii) " is >6 (iii) the two dice had the same outcome

Let A be event where sum is 7

(i) Let B be event where sum is odd

$A = \{(6,1), (5,2), (4,3), (3,4), (2,5), (1,6)\}$

$n(B) = 18 \Rightarrow P(B) = \frac{18}{36} = \frac{1}{2}$

$P(A \cap B) = \frac{6}{36} = \frac{1}{6}$ $\therefore P(A/B) = \frac{1/6}{1/2} = \frac{1}{3}$

6	1,6	2,6	3,6	4,6	5,6	6,6
5	1,5	2,5	3,5	4,5	5,5	6,5
4	1,4	2,4	3,4	4,4	5,4	6,4
3	1,3	2,3	3,3	4,3	5,3	6,3
2	1,2	2,2	3,2	4,2	5,2	6,2
1	1,1	2,1	3,1	4,1	5,1	6,1
	1	2	3	4	5	6

(ii) Let C be event where sum > 6

$n(C) = \{ 21 \Rightarrow P(C) = \frac{21}{36}$

$n(A \cap C) = \cancel{\quad} 6 \Rightarrow P(A \cap C) = \frac{6}{36} = \frac{1}{6}$

$\therefore P(A/C) = \frac{1/6}{21/36} = \frac{2}{7}$

(iii) Let D be event where same outcome

$D = \{(1,1), (2,2), (3,3), (4,4), (5,5), (6,6)\} \Rightarrow P(D) = \frac{6}{36} = \frac{1}{6}$

$n(A \cap D) = 0 \Rightarrow P(A \cap D) = \frac{0}{36} = 0$

$P(A/D) = \frac{0}{1/6} = 0$ $\quad A \cap D = \{ \phi \}$