APS Assignment

DATE: ___/___/20__

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Session: 2021 , Semester: FALL 2022
Roll. no.: 2021-SE-22

Question 01: 6.6

ⓐ

$$S = \{(1,1), (1,2), (1,3), (1,4), (1,5), (1,6),$$
$$(2,1), (2,2), (2,3), (2,4), (2,5), (2,6),$$
$$(3,1), (3,2), (3,3), (3,4), (3,5), (3,6),$$
$$(4,1), (4,2), (4,3), (4,4), (4,5), (4,6),$$
$$(5,1), (5,2), (5,3), (5,4), (5,5), (5,6),$$
$$(6,1), (6,2), (6,3), (6,4), (6,5), (6,6)\}$$

A is the event that sum of scores is less than 5

$$\Rightarrow A = \{(1,1), (1,2), (1,3), (2,1), (2,2), (3,1)\}$$

B denotes the event that a 6 occurs on either die

$$\Rightarrow B = \{(1,6), (2,6), (3,6), (4,6), (5,6), (6,1),$$
$$(6,2), (6,3), (6,4), (6,5)\}$$

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(b)

$A = \{(1,2), (1,4), (1,6), (2,1), (2,3), (2,5),$
$(3,2), (3,4), (3,6), (4,1), (4,3), (4,5),$
$(5,2), (5,4), (5,6), (6,1), (6,3), (6,5)\}$

$B = \{(1,3), (2,3), (3,1), (3,2), (3,3), (3,4),$
$(3,5), (3,6), (4,3), (5,3), (6,3)\}$

$A \cup B$ = Elements belonging to A, B or both, i.e., sum of sores is odd or at least one 3 is shown

$A \cap B$ = Elements belonging to A and B, i.e., sum of scores is odd and at least one 3 is shown

$A - B$ = Elements that belong only to A and not to B. i.e, sum of scores is odd and 3 doesn't show up.

$(A \cap \bar{B}) \cup \bar{A}$ = Elements belonging to set $(A \cap \bar{B})$ or set $\bar{A}$ or both

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Question 02: 6.20

(a) Total no. of ways to draw two cards from 52 cards,

$$n(S) = {}^{52}C_2 = 1326$$

Let A be the event where 1 card drawn is a king and other is a queen.

$$\Rightarrow n(A) = {}^{4}C_1 \times {}^{4}C_1 = 16$$

$$\therefore P(A) = \frac{16}{1326}$$

$$= \frac{8}{663} \approx 0.0121$$

(b) Total no. of ways for player A and B to draw 5 and 3 cards from 8 cards respectively,

$$n(S) = {}^{8}C_5 \times {}^{3}C_3 = 56$$

Let a be the event that player A picks the joker

$$\Rightarrow n(a) = {}^{1}C_1 \times {}^{7}C_4 = 35$$

$$\therefore \cancel{P(A \text{ has the joker})} = \frac{35}{56}$$

$$P(a) = \frac{5}{8} = 0.625$$

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Question 03: 8.8 (a)

(a) - Outcome of success is the correct answer and outcome of failure is a wrong answer

- Prob. of success, $p = \frac{1}{4}$; remains same for all questions
- Answer of a question has no effect on others
- There are 15 questions

All 4 properties of a Binomial Probability Distribution are followed

$\Rightarrow$ X represents no. of correct answers, having B.P.D with $p = \frac{1}{4}$, $n = 15$, $q = \frac{3}{4}$

$$\therefore P(5 \leq X \leq 10) = P(X=5) + P(X=6) + \dots + P(X=10)$$

$$= \left[{}^{15}C_5 \cdot \left(\frac{1}{4}\right)^5 \cdot \left(\frac{3}{4}\right)^{10}\right] + \left[{}^{15}C_6 \cdot \left(\frac{1}{4}\right)^6 \cdot \left(\frac{3}{4}\right)^9\right] + \left[{}^{15}C_7 \cdot \left(\frac{1}{4}\right)^7 \cdot \left(\frac{3}{4}\right)^8\right]$$

$$+ \left[{}^{15}C_8 \cdot \left(\frac{1}{4}\right)^8 \cdot \left(\frac{3}{4}\right)^7\right] + \left[{}^{15}C_9 \cdot \left(\frac{1}{4}\right)^9 \cdot \left(\frac{3}{4}\right)^6\right] + \left[{}^{15}C_{10} \cdot \left(\frac{1}{4}\right)^{10} \cdot \left(\frac{3}{4}\right)^5\right]$$

* $P(5 \leq X \leq 10) \approx 0.313$

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Question 04: 8.28

~~- All properties of a Binomial Probability Distribution are followed~~

~~→ X represents no. of tomato cans, having B.P.D with~~ $\cancel{p=\frac{1}{2},\ n=10,\ q=\frac{1}{2}}$

$$\cancel{P(X=5) = {}^{10}C_5 \times \left(\frac{1}{2}\right)^5 \times \left(\frac{1}{2}\right)^5}$$

$$\cancel{\approx 0.246}$$

- All the properties of a Hypergeometric Distribution are followed.

Let x be no. of tomato cans found in sample.

Let X be random variable representing no. of tomato cans with, $N=10$, $n=5$, $k=5$

$$\therefore P(X=5) = \frac{{}^5C_5 \cdot {}^{10-5}C_{5-5}}{{}^{10}C_5}$$

$$= \frac{1}{252} \approx 0.00396 \approx 0.004$$

$$P(X \geq 3) = P(X=3) + P(X=4) + P(X=5)$$

$$= \frac{{}^5C_3 \times {}^5C_2}{{}^{10}C_5} + \frac{{}^5C_4 \times {}^5C_1}{{}^{10}C_5} + \frac{1}{252}$$

$$= \frac{1}{2} = 0.5$$

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Question 05: 9.26

ⓐ let X denote life of stockings

Stocking needing replacement before 35 days $= P(X \leq 35)$

For $X = 35$,

$$z = \frac{35-40}{8}$$

$$= -0.625$$

$$\Rightarrow P(X \leq 35) = P(Z \leq -0.625)$$

$$= P(Z \geq 0.625) = 0.5 - P(0 \leq Z \leq 0.625)$$

$$= 0.5 - 0.2357$$

$$= 0.2643$$

∴ For 100,000 pairs, no. of units needing replacement before 35 days $= 0.2643 \times 100,000$

$$= 26,430$$

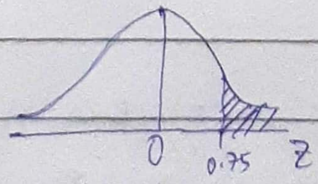
$P(X \geq 46)$ For $X = 46$; $Z = \frac{46-40}{8} = 0.75$

$= P(Z \geq 0.75)$

$= 0.5 - P(0 \leq Z \leq 0.75)$

$= 0.5 - 0.2734$

$= 0.2266$



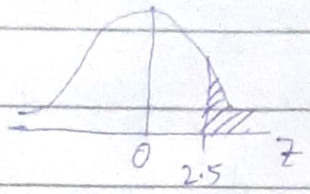
∴ For 100,000 pairs, no. of units needing replacement after 46 days $= 0.2266 \times 100,000$

$$= 22,660$$

(b) let X represent time taken to deliver milk to GOR Estate

(i) $P(X \geq 17)$ For $X=17$; $Z=\frac{17-12}{2}=2.5$

$=P(Z \geq 2.5)$



$=0.5-P(0 \leq Z \leq 2.5)$

$=0.5-0.4938=0.0062$

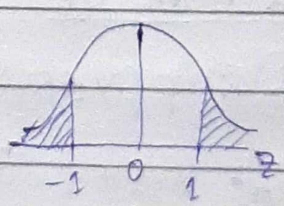
$\therefore$ Number of days where milkman takes longer than 17 minutes $=0.0062 \times 365$

≈ 2 days

(ii) $P(X \leq 10)$

For $X=10$; $Z=\frac{10-12}{2}=-1$

$P(X \leq 10)=P(Z \leq -1)$



$=P(Z \geq 1)=0.5-P(0 \leq Z \leq 1)$

$=0.5-0.3413$

$=0.1587$

$\therefore$ Number of days where milkman takes less than 10 minutes $=0.1587 \times 365$

≈ 58 days

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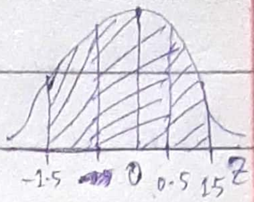
(iii) $P(9 \le X \le 13)$

For $X = 9$

$$Z = \frac{9-12}{2} = -1.5$$

For $X = 13$

$$Z = \frac{13-12}{2} = +0.5$$



$\Rightarrow P(9 \le X \le 13) = P(-1.5 \le Z \le +0.5)$

$= P(0.5 \le Z \le 1.5) = P(0 \le Z \le 1.5) + P(0 \le Z \le 0.5)$

$= 0.4332 + 0.1915$

$= 0.6247$

$\therefore$ Number of days where milkman takes between 9 and 13 minutes $= 365 \times 0.6247$

≈ 228 days

Question 06: 9.22(b)

(b) (i) $P(Y \le 54)$

For $Y = 54$; $54 = 5X + 10 \Rightarrow X = 8.8 \Rightarrow Z = \frac{8.8 - 10}{5}$

$Z = -0.24$

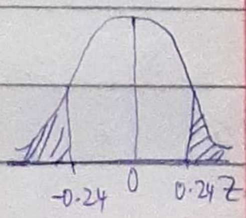
$\Rightarrow P(Y \le 54) = P(X \le 8.8)$

$= P(Z \le -0.24)$

$= 0.5 - P(0 \le Z \le 0.24)$

$= 0.5 - 0.0948$

$= 0.4052$



(ii) $P(Y \geq 68)$

For $Y = 68$; $X = \frac{68-10}{5} \Rightarrow X = 11.6$

For $X = 11.6$; $Z = \frac{11.6-10}{5} = 0.32$

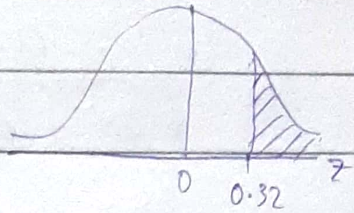
$= P(X \geq 11.6)$

$= P(Z \geq 0.32)$

$= 0.5 - P(0 \leq Z \leq 0.32)$

$= 0.5 - 0.1255$

$= 0.3745$



(iii) $P(52 \leq Y \leq 67)$

For $Y = 52$; $X = \frac{52-10}{5} \Rightarrow X = 8.4$

For $Y = 67$; $X = \frac{67-10}{5} \Rightarrow X = 11.4$

$P(52 \leq Y \leq 67) = P(8.4 \leq X \leq 11.4)$

For $X = 8.4$; $Z = \frac{8.4-10}{5} = -0.32$

For $X = 11.4$; $Z = \frac{11.4-10}{5} = 0.28$

$\Rightarrow P(52 \leq Y \leq 67) = P(-0.32 \leq Z \leq 0.28)$

$= P(-0.32 \leq Z \leq 0) + P(0 \leq Z \leq 0.28)$

$= P(0 \leq Z \leq 0.32) + P(0 \leq Z \leq 0.28)$

$= 0.1255 + 0.1103$

$= 0.2358$

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Question 07: 10.8 (a) (d)

(a)

$\bar{X}=10$; $\bar{Y}=20$; $\Sigma XY=1000$; $\Sigma X^2=2000$; $n=10$

$$b_{yx}=\frac{n(\Sigma XY)-(\Sigma X)(\Sigma Y)}{n\,\Sigma X^2-(\Sigma X)^2}$$

as $\bar{X}=\frac{\Sigma X}{n}$, and $\bar{Y}=\frac{\Sigma Y}{n}$

$\therefore$ $\Sigma X=n\cdot\bar{X}$ and $\Sigma Y=n\cdot\bar{Y}$

$$b_{yx}=\frac{n(\Sigma XY)-(n\cdot\bar{X})(n\cdot\bar{Y})}{n\,\Sigma X^2-(n\bar{X})^2}$$

$$b_{yx}=\frac{\cancel{n}[\Sigma XY-n\cdot\bar{X}\cdot\bar{Y}]}{\cancel{n}[\Sigma X^2-n\bar{X}^2]}$$

$$\therefore b_{yx}=\frac{1000-10(10)(20)}{2000-10(10)^2}=-1$$

$$a=\bar{Y}-(b_{yx})\bar{X}$$

$$a=20+10=30$$

$\therefore$ Estimated regression line is

$$\boxed{\hat{Y}=30-X}$$

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(d) $n = 10$; $\Sigma X = 1710$; $\Sigma Y = 760$; $\Sigma X^2 = 293{,}162$

$\Sigma Y^2 = 59{,}390$; $\Sigma XY = 130{,}628$

from part (a); $b_{yx} = \dfrac{\Sigma XY - n\bar{X}\cdot\bar{Y}}{\Sigma X^2 - n\bar{X}^2}$

$$\bar{X} = \frac{\Sigma X}{n} = \frac{1710}{10} = 171 \qquad \bar{Y} = \frac{\Sigma Y}{n} = \frac{760}{10} = 76$$

$$\therefore b_{yx} = \frac{130{,}628 - 10(171)(76)}{293{,}162 - 10(171)^2} = \frac{167}{188}$$

$$\simeq 0.888$$

$$a = \bar{Y} - (b_{yx})\bar{X} = 76 - \left(\frac{167}{188}\right) \times 171$$

$$a \simeq -75.9$$

$\therefore$ Estimated regression line is

$$\hat{Y} = 0.89X - 75.9$$

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Question 08: 10.16 (b)

(b) $\bar{X} = 2$, $\bar{Y} = 8$, $\Sigma X^2 = 180$, $\Sigma Y^2 = 1424$, $\Sigma XY = 404$

$n = 20$

$$r = \frac{\Sigma XY - n\bar{X}.\bar{Y}}{\sqrt{[\Sigma X^2 - n\bar{X}^2][\Sigma Y^2 - n\bar{Y}^2]}}$$

$$r = \frac{404 - 20(2)(8)}{\sqrt{[180 - 20(2^2)][1424 - 20(8)^2]}}$$

$$r = \frac{7}{10}$$

$$r = 0.7$$