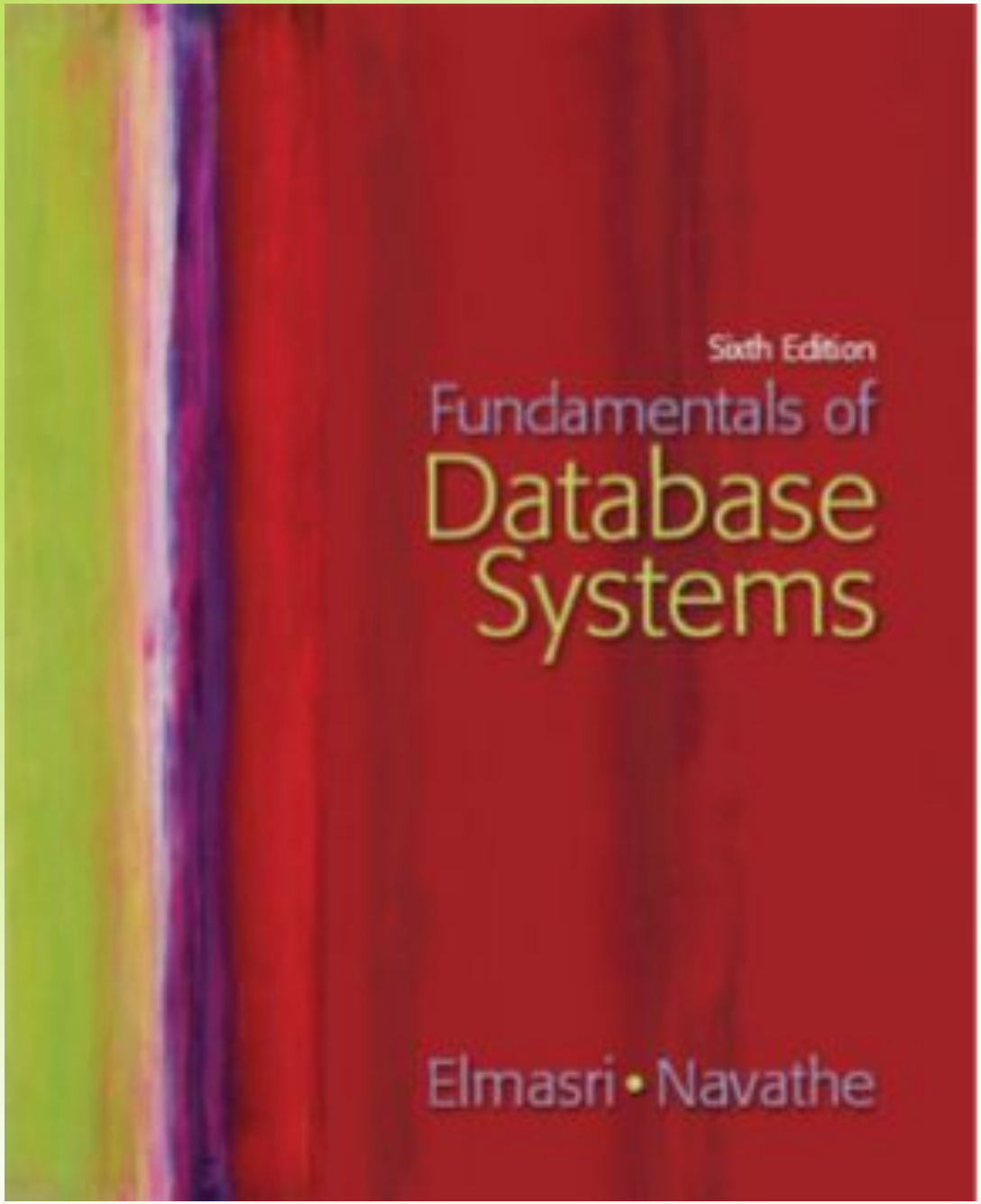
Chapter 6

The Relational Algebra and Relational Calculus



Addison-Wesley
is an imprint of
PEARSON

Chapter 6

The Relational Algebra and Relational Calculus

Addison-Wesley
is an imprint of
PEARSON

Chapter Outline

- Relational Algebra
  - Unary Relational Operations
  - Relational Algebra Operations From Set Theory
  - Binary Relational Operations
  - Additional Relational Operations
  - Examples of Queries in Relational Algebra
  - Relating SQL and Relational Algebra
- Relational Calculus
  - Tuple Relational Calculus
  - Relating Tuple Relational Calculus to SQL
  - Domain Relational Calculus

Relational Algebra Overview

- Relational algebra is the basic set of operations for the theoretical relational model presented in Chapter 3
- SQL (chapters 4 and 5) is the standard language for practical relational databases
- Relational algebra operations are used in DBMS query optimization
  - SQL is converted to relational operations for optimization and processing (see chapter 19)

Relational Algebra Overview (cont.)

- Input to a relational algebra operation is one or more relations
- These operations can specify basic retrieval requests (or queries)
- The result of an operation is a new relation, derived from the input relations
  - This property makes the algebra “closed” (all objects in relational algebra are relations)

Relational Algebra Overview (cont.)

- A query will typically require multiple operations
  - Result of one operation can be further manipulated using additional operations of the same algebra
- A sequence of relational algebra operations forms a relational algebra expression
  - Final result is a relation that represents the result of a database query (or retrieval request)

Relational Algebra Overview (cont.)

- Relational Algebra consists of several groups of operations
  - Unary Relational Operations – one input table
    - SELECT (symbol: σ (sigma))
    - PROJECT (symbol: π (pi))
    - RENAME (symbol: ρ (rho))
  - Relational Algebra Operations From Set Theory
    - UNION (∪), INTERSECTION (∩), DIFFERENCE (or MINUS, –)
    - CARTESIAN PRODUCT (x)
  - Binary Relational Operations – two input tables
    - JOIN (several variations of JOIN exist)
    - DIVISION
  - Additional Relational Operations
    - OUTER JOINS, OUTER UNION
    - AGGREGATE FUNCTIONS (These compute summary of information: for example, SUM, COUNT, AVG, MIN, MAX)
    - Other operations

Database State for COMPANY

- Examples in slides refer to the COMPANY database schema (Figure 3.7 (shown on next slide))
- Examples of results of relational operations refer to the database state in Figure 3.6 – shown after schema

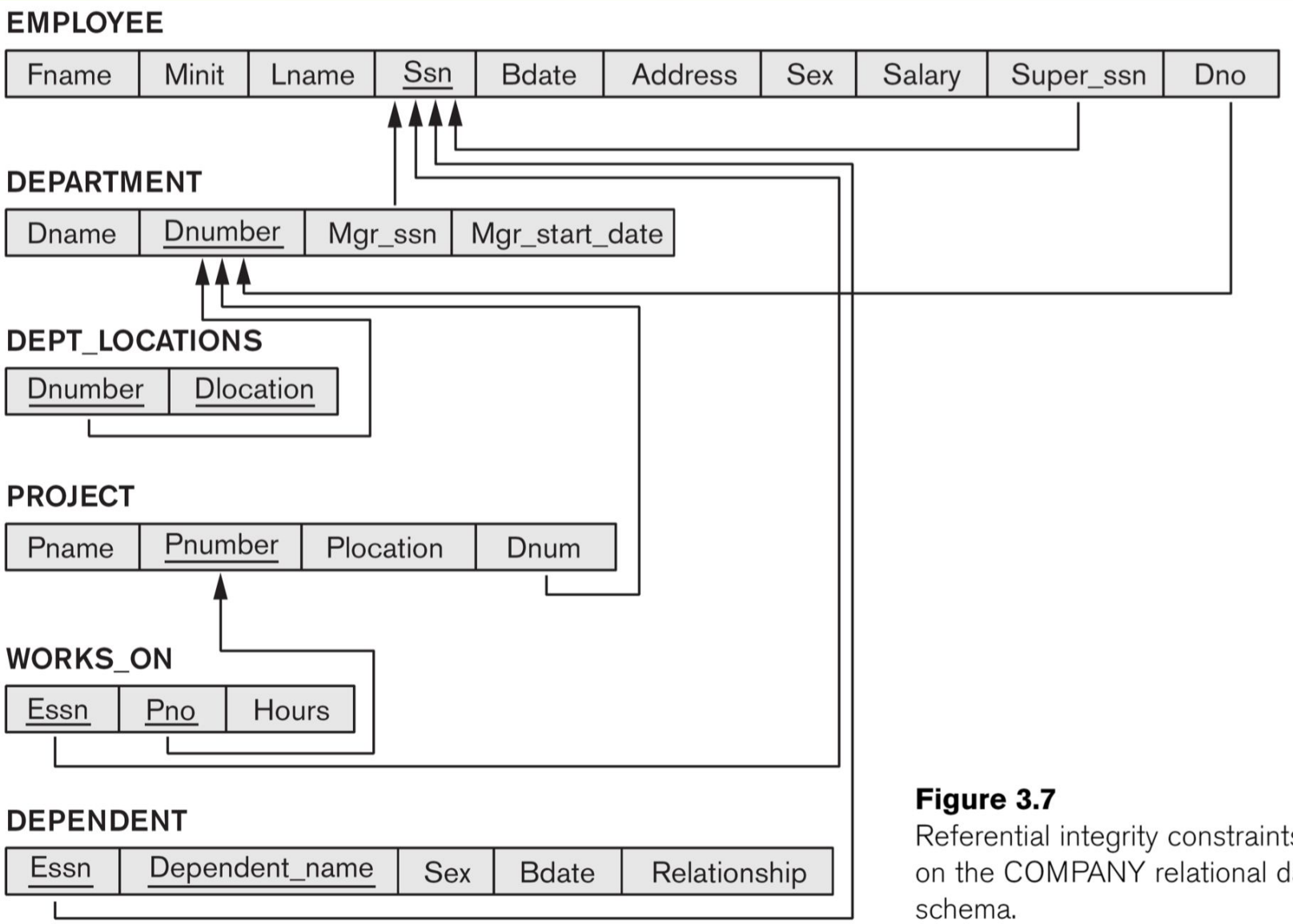


Figure 3.7
Referential integrity constraints displayed on the COMPANY relational database schema.

Figure 3.6
One possible database state for the COMPANY relational database schema.

EMPLOYEE

Fname	Minit	Lname	Ssn	Bdate	Address	Sex	Salary	Super_ssn	Dno
John	B	Smith	123456789	1965-01-09	731 Fondren, Houston, TX	M	30000	333445555	5
Franklin	T	Wong	333445555	1955-12-08	638 Voss, Houston, TX	M	40000	888665555	5
Alicia	J	Zelaya	999887777	1968-01-19	3321 Castle, Spring, TX	F	25000	987654321	4
Jennifer	S	Wallace	987654321	1941-06-20	291 Berry, Bellaire, TX	F	43000	888665555	4
Ramesh	K	Narayan	666884444	1962-09-15	975 Fire Oak, Humble, TX	M	38000	333445555	5
Joyce	A	English	453453453	1972-07-31	5631 Rice, Houston, TX	F	25000	333445555	5
Ahmad	V	Jabbar	987987987	1969-03-29	980 Dallas, Houston, TX	M	25000	987654321	4
James	E	Borg	888665555	1937-11-10	450 Stone, Houston, TX	M	55000	NULL	1

DEPARTMENT

Dname	Dnumber	Mgr_ssn	Mgr_start_date
Research	5	333445555	1988-05-22
Administration	4	987654321	1995-01-01
Headquarters	1	888665555	1981-06-19

DEPT_LOCATIONS

Dnumber	Dlocation
1	Houston
4	Stafford
5	Bellaire
5	Sugarland
5	Houston

Continued next page...

WORKS_ON

Essn	Pno	Hours
123456789	1	32.5
123456789	2	7.5
666884444	3	40.0
453453453	1	20.0
453453453	2	20.0
333445555	2	10.0
333445555	3	10.0
333445555	10	10.0
333445555	20	10.0
999887777	30	30.0
999887777	10	10.0
987987987	10	35.0
987987987	30	5.0
987654321	30	20.0
987654321	20	15.0
888665555	20	NULL

PROJECT

Pname	Pnumber	Plocation	Dnum
ProductX	1	Bellaire	5
ProductY	2	Sugarland	5
ProductZ	3	Houston	5
Computerization	10	Stafford	4
Reorganization	20	Houston	1
Newbenefits	30	Stafford	4

DEPENDENT

Essn	Dependent_name	Sex	Bdate	Relationship
333445555	Alice	F	1986-04-05	Daughter
333445555	Theodore	M	1983-10-25	Son
333445555	Joy	F	1958-05-03	Spouse
987654321	Abner	M	1942-02-28	Spouse
123456789	Michael	M	1988-01-04	Son
123456789	Alice	F	1988-12-30	Daughter
123456789	Elizabeth	F	1967-05-05	Spouse

Unary Relational Operations: SELECT

- The SELECT operation (denoted by σ (sigma)) selects a subset of the tuples from a relation based on a selection condition.
  - The selection condition acts as a filter
  - Keeps only those tuples that satisfy the qualifying condition
  - Tuples satisfying the condition are selected whereas the other tuples are discarded (filtered out)
  - Important Note: The SELECT operation is different from the SELECT-clause of SQL (presented in Chapters 3 and 4)
- Examples:
  - Select the EMPLOYEE tuples whose department number is 4:

$$\sigma_{DNO = 4} (EMPLOYEE)$$

  - Select the employee tuples whose salary is greater than $30,000:

$$\sigma_{SALARY > 30,000} (EMPLOYEE)$$

SELECT operation (cont.)

- In general, the select operation is denoted by $\sigma_{\text{<selection condition>}}(R)$ where
  - the symbol σ (sigma) is used to denote the select operator
  - the selection condition is a Boolean (conditional) expression specified on the attributes of relation R
  - tuples that make the condition true are selected
    - appear in the result of the operation
  - tuples that make the condition false are filtered out
    - discarded from the result of the operation

SELECT operation (cont.)

- SELECT Operation Properties
  - SELECT operation $\sigma_{\text{<selection condition>}}(R)$ produces a relation S that has the same schema (same attributes) as R
  - SELECT σ is commutative:
    - $\sigma_{\text{<condition1>}}(\sigma_{\text{< condition2>}}(R)) = \sigma_{\text{<condition2>}}(\sigma_{\text{< condition1>}}(R))$
  - Because of commutativity property, a cascade (sequence) of SELECT operations may be applied in any order:
    - $\sigma_{\text{<cond1>}}(\sigma_{\text{<cond2>}}(\sigma_{\text{<cond3>}}(R)) = \sigma_{\text{<cond2>}}(\sigma_{\text{<cond3>}}\sigma_{\text{<cond1>}}(R)))$
  - A cascade of SELECT operations may be replaced by a single selection with a conjunction of all the conditions:
    - $\sigma_{\text{<cond1>}}(\sigma_{\text{< cond2>}}(\sigma_{\text{<cond3>}}(R)) = \sigma_{\text{<cond1> AND < cond2> AND < cond3>}}(R)))$
  - The number of tuples in the result of a SELECT is less than (or equal to) the number of tuples in the input relation R

Unary Relational Operations: PROJECT

- PROJECT Operation is denoted by π (pi symbol)
- Keeps certain columns (attributes) from a relation and discards the other columns.
  - PROJECT creates a vertical partitioning
    - The list of specified columns (attributes) is kept in each tuple
    - The other attributes in each tuple are discarded
- Example: List each employee's first and last name and salary:

$$\pi_{\text{LNAME, FNAME,SALARY}}(\text{EMPLOYEE})$$

PROJECT operation (cont.)

- General form of project operation is:

$$\pi_{\text{<attribute list>}}(R)$$

  - π (pi) is the symbol used to represent the project operation
  - <attribute list> is the desired list of attributes from relation R.
- The operation removes any duplicate tuples
  - This is because the result of the project operation must be a set of distinct tuples as per formal relational model
    - Mathematical sets do not allow duplicate elements.

PROJECT operation (cont.)

- PROJECT Operation Properties
  - Number of tuples in result of projection $\pi_{<list>}(R)$ less or equal to the number of tuples in R
    - If the list of projected attributes includes a key of R, then number of tuples in result of PROJECT is equal to the number of tuples in R
  - PROJECT is not commutative
    - $\pi_{<list1>}(\pi_{<list2>}(R)) = \pi_{<list1>}(R)$ as long as <list2> contains the attributes in <list1>

Example

- Next slide (Figure 6.1) shows the result of some SELECT and PROJECT operations when applied to the database state in Figure 3.6

- $\pi_{\text{Sex, Salary}}$ (EMPLOYEE) correspond to

SELECT **DISTINCT** Sex, Salary
FROM EMPLOYEE

Figure 6.1

Results of SELECT and PROJECT operations. (a) $\sigma_{(Dno=4\ \textbf{AND}\ Salary>25000)\ OR\ (Dno=5\ \textbf{AND}\ Salary>30000)}$ (EMPLOYEE). (b) $\pi_{Lname,\ Fname,\ Salary}$(EMPLOYEE). (c) $\pi_{Sex,\ Salary}$(EMPLOYEE).

(a)

Fname	Minit	Lname	Ssn	Bdate	Address	Sex	Salary	Super_ssn	Dno
Franklin	T	Wong	333445555	1955-12-08	638 Voss, Houston, TX	M	40000	888665555	5
Jennifer	S	Wallace	987654321	1941-06-20	291 Berry, Bellaire, TX	F	43000	888665555	4
Ramesh	K	Narayan	666884444	1962-09-15	975 Fire Oak, Humble, TX	M	38000	333445555	5

(b)

Lname	Fname	Salary
Smith	John	30000
Wong	Franklin	40000
Zelaya	Alicia	25000
Wallace	Jennifer	43000
Narayan	Ramesh	38000
English	Joyce	25000
Jabbar	Ahmad	25000
Borg	James	55000

(c)

Sex	Salary
M	30000
M	40000
F	25000
F	43000
M	38000
M	25000
M	55000

Relational Algebra Expressions

- To apply several relational algebra operations
  - Either we write the operations as a single relational algebra expression by nesting the operations in parentheses, or
  - We apply one operation at a time and create intermediate result relations.
- In the latter case, we must give names to the relations that hold the intermediate results.

Single expression versus sequence of relational operations (Example)

- To retrieve the first name, last name, and salary of all employees who work in department number 5, we must apply a select and a project operation
- Single relational algebra expression:
  - $\pi_{\text{FNAME, LNAME, SALARY}}(\sigma_{\text{DNO=5}}(\text{EMPLOYEE}))$
- OR sequence of operations, giving a name to each intermediate relation:
  - DEP5_EMPS <- $\sigma_{\text{DNO=5}}(\text{EMPLOYEE})$
  - RESULT <- $\pi_{\text{FNAME, LNAME, SALARY}}$(DEP5_EMPS)

Relational SELECT and PROJECT Operations in SQL

- The SELECT - clause of SQL lists the projection attributes, and the WHERE - clause includes the selection conditions
- Example: The relational algebra expression:
  - $\pi_{\text{FNAME, LNAME, SALARY}}(\sigma_{\text{DNO=5}}(\text{EMPLOYEE}))$
- Corresponds to the SQL query:

```
SELECT   FNAME, LNAME, SALARY
FROM     EMPLOYEE
WHERE    DNO=5 ;
```

Unary Relational Operations: RENAME

- The RENAME operator is denoted by ρ (rho symbol)
- Used to rename the attributes of a relation or the relation name or both
  - Useful when a query requires multiple operations
  - Necessary in some cases (see JOIN operation later)

RENAME operation (cont.)

- The general RENAME operation ρ can be expressed by any of the following forms:
  - $\rho_{S(B1, B2, \ldots, Bn)}(R)$ changes both:
    - the relation name to S, and
    - the column (attribute) names to B1, B1, …, Bn
  - $\rho_{S}(R)$ changes:
    - the relation name only to S
  - $\rho_{(B1, B2, \ldots, Bn)}(R)$ changes:
    - the column (attribute) names only to B1, B1, …..Bn

RENAME operation (cont.)

- For convenience, we also use shorthand for renaming an intermediate relation:
  - If we write:
    - R(FN,LN,SAL) $\leftarrow \pi_{FNAME,LNAME,SALARY}$ (DEP5_EMPS)
    - R will rename FNAME to FN, LNAME to LN, and SALARY to SAL
  - If we write:
    - R(F,M,L,S,B,A,SX,SAL,SU,D) $\leftarrow$ EMPLOYEE
    - Then EMPLOYEE is renamed to R and the 10 attributes of EMPLOYEE are renamed to F, M, L, S, B, A, SX, SAL, SU, D, respectively

Example

- Next slide (Figure 6.2) shows the result of the following two expressions when applied to the database state in Figure 3.6
- Single relational algebra expression (Fig 6.2(a)):
  - $\pi_{\text{FNAME, LNAME, SALARY}}(\sigma_{\text{DNO=5}}(\text{EMPLOYEE}))$
- Sequence of operations, giving a name to each intermediate relation (Fig 6.2(b)):
  - $\text{TEMP} \leftarrow \sigma_{\text{DNO=5}}(\text{EMPLOYEE})$
  - $\text{R} \leftarrow \pi_{\text{FNAME, LNAME, SALARY}}(\text{TEMP})$

(a)

Fname	Lname	Salary
John	Smith	30000
Franklin	Wong	40000
Ramesh	Narayan	38000
Joyce	English	25000

Figure 6.2
Results of a sequence of operations. (a) $\pi_{Fname, Lname, Salary}(\sigma_{Dno=5}(EMPLOYEE))$. (b) Using intermediate relations and renaming of attributes.

(b)

TEMP

Fname	Minit	Lname	Ssn	Bdate	Address	Sex	Salary	Super_ssn	Dno
John	B	Smith	123456789	1965-01-09	731 Fondren, Houston,TX	M	30000	333445555	5
Franklin	T	Wong	333445555	1955-12-08	638 Voss, Houston,TX	M	40000	888665555	5
Ramesh	K	Narayan	666884444	1962-09-15	975 Fire Oak, Humble,TX	M	38000	333445555	5
Joyce	A	English	453453453	1972-07-31	5631 Rice, Houston, TX	F	25000	333445555	5

R

First_name	Last_name	Salary
John	Smith	30000
Franklin	Wong	40000
Ramesh	Narayan	38000
Joyce	English	25000

Relational RENAME Operation in SQL

- In SQL, RENAME is achieved by using AS
- Example: The relational algebra expression:

$$\rho_{FN,\ LN,\ SAL}(\pi_{FNAME,\ LNAME,\ SALARY}(\sigma_{DNO=5}(EMPLOYEE)))$$

- Corresponds to the SQL query:

```
SELECT   FNAME AS FN, LNAME AS LN, SALARY AS SAL
FROM     EMPLOYEE
WHERE    DNO=5 ;
```

Relational Algebra Operations from Set Theory: UNION

- UNION Operation
  - Binary operation, denoted by ∪
  - The result of R ∪ S, is a relation that includes all tuples that are either in R or in S or in both R and S
  - Duplicate tuples are eliminated
  - The two operand relations R and S must be "type compatible" (or "UNION compatible")
    - R and S must have same number of attributes
    - Each pair of corresponding attributes must be type compatible (have same or compatible domains)

UNION operation (cont.)

- Example:
  - To retrieve the social security numbers of all employees who either work in department 5 (RESULT1 below) or directly supervise an employee who works in department 5 (RESULT2 below)
  - We can use the UNION operation as follows:

$$DEP5\_EMPS \leftarrow \sigma_{DNO=5}(EMPLOYEE)$$
$$RESULT1 \leftarrow \pi_{SSN}(DEP5\_EMPS)$$
$$RESULT2(SSN) \leftarrow \pi_{SUPERSSN}(DEP5\_EMPS)$$
$$RESULT \leftarrow RESULT1 \cup RESULT2$$

  - The union operation produces the tuples that are in either RESULT1 or RESULT2 or both (see Fig 6.3)

RESULT1

Ssn
123456789
333445555
666884444
453453453

RESULT2

Ssn
333445555
888665555

RESULT

Ssn
123456789
333445555
666884444
453453453
888665555

Figure 6.3
Result of the UNION operation RESULT ← RESULT1 ∪ RESULT2.

Type Compatibility

- Type Compatibility of operands is required for the binary set operation UNION ∪, (also for INTERSECTION ∩, and SET DIFFERENCE –, see next slides)
- R1(A1, A2, ..., An) and R2(B1, B2, ..., Bn) are type compatible if:
  - they have the same number of attributes, and
  - the domains of corresponding attributes are type compatible (i.e. dom(Ai)=dom(Bi) for i=1, 2, ..., n).
- The resulting relation for R1 ∪ R2 (also for R1 ∩ R2, or R1–R2, see next slides) has the same attribute names as the first operand relation R1 (by convention)

Relational Algebra Operations from Set Theory: INTERSECTION

- INTERSECTION is denoted by ∩
- The result of the operation R ∩ S, is a relation that includes all tuples that are in both R and S
  - The attribute names in the result will be the same as the attribute names in R
- The two operand relations R and S must be "type compatible"

Relational Algebra Operations from Set Theory: SET DIFFERENCE

- SET DIFFERENCE (also called MINUS or EXCEPT) is denoted by –
- The result of R – S, is a relation that includes all tuples that are in R but not in S
  - The attribute names in the result will be the same as the attribute names in R
- The two operand relations R and S must be "type compatible"

Examples, Set Operations in SQL

- Figure 6.4 (next slide) shows some examples of set operations on relations
- SQL has UNION, INTERSECT, EXCEPT operations (see Chapter 4)
  - These operations work with sets of tuples (duplicate tuples are eliminated)
  - In addition, SQL has UNION ALL, INTERSECT ALL, EXCEPT ALL for multisets (duplicates are allowed)

Figure 6.4

The set operations UNION, INTERSECTION, and MINUS. (a) Two union-compatible relations. (b) STUDENT ∪ INSTRUCTOR. (c) STUDENT ∩ INSTRUCTOR. (d) STUDENT – INSTRUCTOR. (e) INSTRUCTOR – STUDENT.

(a) **STUDENT**

Fn	Ln
Susan	Yao
Ramesh	Shah
Johnny	Kohler
Barbara	Jones
Amy	Ford
Jimmy	Wang
Ernest	Gilbert

INSTRUCTOR

Fname	Lname
John	Smith
Ricardo	Browne
Susan	Yao
Francis	Johnson
Ramesh	Shah

(b)

Fn	Ln
Susan	Yao
Ramesh	Shah
Johnny	Kohler
Barbara	Jones
Amy	Ford
Jimmy	Wang
Ernest	Gilbert
John	Smith
Ricardo	Browne
Francis	Johnson

(c)

Fn	Ln
Susan	Yao
Ramesh	Shah

(d)

Fn	Ln
Johnny	Kohler
Barbara	Jones
Amy	Ford
Jimmy	Wang
Ernest	Gilbert

(e)

Fname	Lname
John	Smith
Ricardo	Browne
Francis	Johnson

Some properties of UNION, INTERSECT, and DIFFERENCE

- Both union and intersection are commutative operations; that is
  - $R \cup S = S \cup R$, and $R \cap S = S \cap R$
- Both union and intersection can be treated as n-ary operations applicable to any number of relations as both are associative operations; that is
  - $R \cup (S \cup T) = (R \cup S) \cup T$
  - $(R \cap S) \cap T = R \cap (S \cap T)$
- The minus operation is not commutative; that is, in general
  - $R - S \neq S - R$

Relational Algebra Operations from Set Theory: CARTESIAN PRODUCT

- CARTESIAN (or CROSS) PRODUCT Operation
  - Different from the other three set operations
  - Used to combine tuples from two relations in a combinatorial fashion.
  - Denoted by R(A1, A2, . . ., An) x S(B1, B2, . . ., Bm)
  - Result is a relation Q with degree n + m attributes:
    - Q(A1, A2, . . ., An, B1, B2, . . ., Bm), in that order.
  - The resulting relation state has one tuple for each combination of tuples—one from R and one from S.
  - Hence, if R has n_R tuples (denoted as $|R| = n_R$), and S has n_S tuples, then R x S will have $n_R * n_S$ tuples.
  - R and S do NOT have to be "type compatible"

CARTESIAN PRODUCT operation (cont.)

- Generally, CROSS PRODUCT is not a meaningful operation
  - Can become meaningful when followed by other operations
- Example (not meaningful):
  - FEMALE_EMPS $\leftarrow \sigma_{SEX='F'}(EMPLOYEE)$
  - EMPNAMES $\leftarrow \pi_{FNAME, LNAME, SSN}(FEMALE\_EMPS)$
  - EMP_DEPENDENTS $\leftarrow$ EMPNAMES x DEPENDENT
- EMP_DEPENDENTS will contain every combination of tuples from EMPNAMES and DEPENDENT
  - whether or not the tuples are actually related

CARTESIAN PRODUCT operation (cont.)

- To keep only combinations where the DEPENDENT is related to the EMPLOYEE by the condition ESSN=SSN, we add a SELECT operation
- Example (meaningful, see Figure 6.5, next two slides):
  - FEMALE_EMPS $\leftarrow \sigma_{SEX='F'}$(EMPLOYEE)
  - EMPNAMES $\leftarrow \pi_{FNAME, LNAME, SSN}$ (FEMALE_EMPS)
  - EMP_DEPENDENTS $\leftarrow$ EMPNAMES x DEPENDENT
  - ACTUAL_DEPS $\leftarrow \sigma_{SSN=ESSN}$(EMP_DEPENDENTS)
  - RESULT $\leftarrow \pi_{FNAME, LNAME, DEPENDENT\_NAME}$ (ACTUAL_DEPS)
- RESULT will now contain the name of female employees and their dependents

Figure 6.5
The Cartesian Product (Cross Product) operation.

FEMALE_EMPS

Fname	Minit	Lname	Ssn	Bdate	Address	Sex	Salary	Super_ssn	Dno
Alicia	J	Zelaya	999887777	1968-07-19	3321Castle, Spring, TX	F	25000	987654321	4
Jennifer	S	Wallace	987654321	1941-06-20	291Berry, Bellaire, TX	F	43000	888665555	4
Joyce	A	English	453453453	1972-07-31	5631 Rice, Houston, TX	F	25000	333445555	5

Continued next page…

EMPNAMES

Fname	Lname	Ssn
Alicia	Zelaya	999887777
Jennifer	Wallace	987654321
Joyce	English	453453453

EMP_DEPENDENTS

Fname	Lname	Ssn	Essn	Dependent_name	Sex	Bdate	...
Alicia	Zelaya	999887777	333445555	Alice	F	1986-04-05	...
Alicia	Zelaya	999887777	333445555	Theodore	M	1983-10-25	...
Alicia	Zelaya	999887777	333445555	Joy	F	1958-05-03	...
Alicia	Zelaya	999887777	987654321	Abner	M	1942-02-28	...
Alicia	Zelaya	999887777	123456789	Michael	M	1988-01-04	...
Alicia	Zelaya	999887777	123456789	Alice	F	1988-12-30	...
Alicia	Zelaya	999887777	123456789	Elizabeth	F	1967-05-05	...
Jennifer	Wallace	987654321	333445555	Alice	F	1986-04-05	...
Jennifer	Wallace	987654321	333445555	Theodore	M	1983-10-25	...
Jennifer	Wallace	987654321	333445555	Joy	F	1958-05-03	...
Jennifer	Wallace	987654321	987654321	Abner	M	1942-02-28	...
Jennifer	Wallace	987654321	123456789	Michael	M	1988-01-04	...
Jennifer	Wallace	987654321	123456789	Alice	F	1988-12-30	...
Jennifer	Wallace	987654321	123456789	Elizabeth	F	1967-05-05	...
Joyce	English	453453453	333445555	Alice	F	1986-04-05	...
Joyce	English	453453453	333445555	Theodore	M	1983-10-25	...
Joyce	English	453453453	333445555	Joy	F	1958-05-03	...
Joyce	English	453453453	987654321	Abner	M	1942-02-28	...
Joyce	English	453453453	123456789	Michael	M	1988-01-04	...
Joyce	English	453453453	123456789	Alice	F	1988-12-30	...
Joyce	English	453453453	123456789	Elizabeth	F	1967-05-05	...

ACTUAL_DEPENDENTS

Fname	Lname	Ssn	Essn	Dependent_name	Sex	Bdate	...
Jennifer	Wallace	987654321	987654321	Abner	M	1942-02-28	...

RESULT

Fname	Lname	Dependent_name
Jennifer	Wallace	Abner

CARTESIAN PRODUCT Operation in SQL

- By leaving out the WHERE-clause (Chapter 4)

```
SELECT    *
FROM    EMPLOYEE, DEPARTMENT ;
```

- OR, by using CROSS JOIN in joined tables (Chapter 5)

```
SELECT    *
FROM  (DEPARTMENT CROSS JOIN EMPLOYEE) ;
```

- In both examples, every department-employee record combination appears in the result (whether or not the employee works for the department)

Binary Relational Operations: JOIN (also called INNER JOIN)

- JOIN Operation (denoted by ⋈ (bowtie symbol))
  - The sequence of CARTESIAN PRODECT followed by SELECT can be used to identify and select related tuples from two relations
  - The JOIN operation combines this sequence into a single operation
  - JOIN is very important for any relational database with more than a single relation, because it allows to combine related tuples from various relations
  - The general form of a join operation on two relations R(A1, A2, . . ., An) and S(B1, B2, . . ., Bm) is:

$$R \bowtie_{\text{<join condition>}} S$$

  - where R and S can be base relations or any relations that result from general relational algebra expressions.

JOIN operation (cont.)

- Example: Suppose that we want to retrieve the name of the manager of each department.
  - To get the manager's name, we need to combine each DEPARTMENT tuple with the EMPLOYEE tuple whose SSN value matches the MGRSSN value in the department tuple.
  - We do this by using the join $\bowtie$ operation
  - DEPT_MGR $\leftarrow$ DEPARTMENT $\underset{\text{MGRSSN=SSN}}{\bowtie}$ EMPLOYEE
  - RESULT $\leftarrow \pi_{\text{Dname, Lname, Fname}}$(DAPT_MGR)
- MGRSSN=SSN is called the join condition
  - Combines each department record with the employee who manages the department
  - The join condition can also be specified as DEPARTMENT.MGRSSN= EMPLOYEE.SSN
  - Result in Figure 6.6 (next slide)

DEPT_MGR

Dname	Dnumber	Mgr_ssn	...	Fname	Minit	Lname	Ssn	...
Research	5	333445555	...	Franklin	T	Wong	333445555	...
Administration	4	987654321	...	Jennifer	S	Wallace	987654321	...
Headquarters	1	888665555	...	James	E	Borg	888665555	...

Figure 6.6
Result of the JOIN operation DEPT_MGR ← DEPARTMENT $\bowtie_{Mgr\_ssn=Ssn}$ EMPLOYEE.

Properties of JOIN

- Consider the following JOIN operation:
  - R(A1, A2, . . ., An) $\bowtie_{R.Ai=S.Bj}$ S(B1, B2, . . ., Bm)
  - Result is a relation Q with degree n + m attributes:
    - Q(A1, A2, . . ., An, B1, B2, . . ., Bm), in that order.
  - The resulting relation state has one tuple for each combination of tuples—r from R and s from S, but only if they satisfy the join condition r.Ai=s.Bj
  - Hence, if R has n_R tuples, and S has n_S tuples, then the join result will generally have less than $n_R * n_S$ tuples.
  - Only related tuples (based on the join condition) will appear in the result

Properties of JOIN (cont.)

- General case of JOIN operation is called a Theta-join: $R \bowtie_{theta} S$
- The join condition is called theta
- Theta can be any general boolean expression on the attributes of R and S; for example:
  - R.Ai<S.Bj AND (R.Ak=S.Bl OR R.Ap<S.Bq)
- Most join conditions involve one or more equality conditions "AND"ed together; for example:
  - R.Ai=S.Bj AND R.Ak=S.Bl AND R.Ap=S.Bq

Variations of JOIN: EQUIJOIN

- EQUIJOIN Operation is the most common use of join; involves join conditions with equality comparisons only (Ai = Bj)
  - The result of an EQUIJOIN will always have one or more pairs of attributes (whose names need not be identical) that have identical values in every tuple.
  - The JOIN in the previous example was an EQUIJOIN; every tuple in the result will have SSN=MGRSSN

Variations of JOIN: NATURAL JOIN

- NATURAL JOIN Operation
  - Another variation of JOIN called NATURAL JOIN — denoted by * — was created to get rid of the second (superfluous) attribute in an EQUIJOIN result.
    - because one of each pair of attributes with identical values is superfluous
  - The standard definition of natural join requires that the two join attributes, or each pair of corresponding join attributes, have the same name in both relations
  - If this is not the case, a renaming operation is applied first.

NATURAL JOIN (cont.)

- Example: To apply a natural join on the DNUMBER attributes of DEPARTMENT and DEPT_LOCATIONS, it is sufficient to write:
  - DEPT_LOCS ← DEPARTMENT * DEPT_LOCATIONS
- Only attribute with the same name is DNUMBER
- An implicit join condition is created based on this attribute (see Figure 6.7(b)), next slide:

  DEPARTMENT.DNUMBER=DEPT_LOCATIONS.DNUMBER

- Another example: Q ← R(A,B,C,D) * S(C,D,E)
  - The implicit join condition includes each pair of attributes with the same name, "AND"ed together:
    - R.C=S.C AND R.D.S.D
  - Result keeps only one attribute of each such pair:
    - Q(A,B,C,D,E)

(a)

PROJ_DEPT

Pname	Pnumber	Plocation	Dnum	Dname	Mgr_ssn	Mgr_start_date
ProductX	1	Bellaire	5	Research	333445555	1988-05-22
ProductY	2	Sugarland	5	Research	333445555	1988-05-22
ProductZ	3	Houston	5	Research	333445555	1988-05-22
Computerization	10	Stafford	4	Administration	987654321	1995-01-01
Reorganization	20	Houston	1	Headquarters	888665555	1981-06-19
Newbenefits	30	Stafford	4	Administration	987654321	1995-01-01

(b)

DEPT_LOCS

Dname	Dnumber	Mgr_ssn	Mgr_start_date	Location
Headquarters	1	888665555	1981-06-19	Houston
Administration	4	987654321	1995-01-01	Stafford
Research	5	333445555	1988-05-22	Bellaire
Research	5	333445555	1988-05-22	Sugarland
Research	5	333445555	1988-05-22	Houston

Figure 6.7
Results of two NATURAL JOIN operations. (a) PROJ_DEPT ← PROJECT * DEPT.
(b) DEPT_LOCS ← DEPARTMENT * DEPT_LOCATIONS.

Another example, NATURAL JOIN in SQL

- If the join attributes do not have the same name, one can be renamed before applying the natural join operation using * (see Figure 6.7(a), previous slide):

DEPT_LOCS ← (PROJECT *

$\rho_{D(DNAME, DNUM, MGR\_SSN, MGR\_START\_DATE)}$(DEPARTMENT))

- We renamed DNUMBER in DEPARTMENT to DNUM to match the join attribute name (DNUM) in PROJECT
- Implicit join condition is PROJECT.DNUM = D.DNUM
- In SQL, NATURAL JOIN can be specified in joined table:

```
SELECT  *
FROM    (PROJECT NATURAL JOIN DEPARTMENT AS
        D(DNAME, DNUM, MGR_SSN, MGR_START_DATE) ;
```

(Note: * in SQL means all the attributes, not NATURAL JOIN)

INNER JOIN versus OUTER JOIN

- All the join operations so far are known as inner joins
- A tuple in one relation that does not have any matching tuples in the other relation (based on the join condition) does not appear in the join result when using INNER JOIN
- If the user wants every tuple to appear at least once in the join result, another form of join known as OUTER JOIN can be used
- OUTER JOINs are discussed later in this chapter

Complete Set of Relational Operations

- The set of operations {SELECT σ, PROJECT π , UNION $\cup$, DIFFERENCE $-$, RENAME ρ, and CARTESIAN PRODUCT X} is called a complete set because any relational algebra expression using the operations presented so far can be as a combination of these six operations.
- For example:
  - $R \cap S = (R \cup S) - ((R - S) \cup (S - R))$
  - $R \bowtie_{\text{<join condition>}} S = \sigma_{\text{<join condition>}} (R \times S)$

Binary Relational Operations: DIVISION

- DIVISION Operation (see Figure 6.8, next slide)
  - The division operation is applied to two relations
  - $R(Z) \div S(X)$, where X subset Z. Let $Y = Z - X$ (and hence $Z = X \cup Y$); that is, let Y be the set of attributes of R that are not attributes of S.
  - The result of DIVISION is a relation T(Y) that includes a tuple t if tuples t_R appear in R with $t_R[Y] = t$, and with
    - $t_R[X] = t_s$ for every tuple t_s in S.
  - For a tuple t to appear in the result T of the DIVISION, the values in t must appear in R in combination with every tuple in S.

Figure 6.8
The DIVISION operation. (a) Dividing SSN_PNOS by SMITH_PNOS. (b) $T \leftarrow R \div S$.

(a)

SSN_PNOS

Essn	Pno
123456789	1
123456789	2
666884444	3
453453453	1
453453453	2
333445555	2
333445555	3
333445555	10
333445555	20
999887777	30
999887777	10
987987987	10
987987987	30
987654321	30
987654321	20
888665555	20

SMITH_PNOS

Pno
1
2

SSNS

Ssn
123456789
453453453

(b)

R

A	B
a1	b1
a2	b1
a3	b1
a4	b1
a1	b2
a3	b2
a2	b3
a3	b3
a4	b3
a1	b4
a2	b4
a3	b4

S

A
a1
a2
a3

T

B
b1
b4

Table 6.1 Operations of Relational Algebra

OPERATION	PURPOSE	NOTATION
SELECT	Selects all tuples that satisfy the selection condition from a relation R.	$\sigma_{\text{<selection condition>}}(R)$
PROJECT	Produces a new relation with only some of the attributes of R, and removes duplicate tuples.	$\pi_{\text{<attribute list>}}(R)$
THETA JOIN	Produces all combinations of tuples from R_1 and R_2 that satisfy the join condition.	$R_1 \bowtie_{\text{<join condition>}} R_2$
EQUIJOIN	Produces all the combinations of tuples from R_1 and R_2 that satisfy a join condition with only equality comparisons.	$R_1 \bowtie_{\text{<join condition>}} R_2$, OR $R_1 \bowtie_{(\text{<join attributes 1>}), (\text{<join attributes 2>})} R_2$
NATURAL JOIN	Same as EQUIJOIN except that the join attributes of R_2 are not included in the resulting relation; if the join attributes have the same names, they do not have to be specified at all.	$R_1 *_{\text{<join condition>}} R_2$, OR $R_1 *_{(\text{<join attributes 1>}), (\text{<join attributes 2>})} R_2$ OR $R_1 * R_2$
UNION	Produces a relation that includes all the tuples in R_1 or R_2 or both R_1 and R_2; R_1 and R_2 must be union compatible.	$R_1 \cup R_2$
INTERSECTION	Produces a relation that includes all the tuples in both R_1 and R_2; R_1 and R_2 must be union compatible.	$R_1 \cap R_2$
DIFFERENCE	Produces a relation that includes all the tuples in R_1 that are not in R_2; R_1 and R_2 must be union compatible.	$R_1 - R_2$
CARTESIAN PRODUCT	Produces a relation that has the attributes of R_1 and R_2 and includes as tuples all possible combinations of tuples from R_1 and R_2.	$R_1 \times R_2$
DIVISION	Produces a relation $R(X)$ that includes all tuples $t[X]$ in $R_1(Z)$ that appear in R_1 in combination with every tuple from $R_2(Y)$, where $Z = X \cup Y$.	$R_1(Z) \div R_2(Y)$

Query Tree Notation

- Query Tree (see Figure 6.9, next slide)
  - An internal data structure to represent a query
  - Standard technique for estimating the work involved in executing the query, the generation of intermediate results, and the optimization of execution (see Chapter 19)
  - Nodes stand for operations like selection, projection, join, renaming, division, ….
  - Leaf nodes represent base relations
  - A tree gives a good visual feel of the complexity of the query and the operations involved
  - Algebraic Query Optimization consists of rewriting the query or modifying the query tree into an equivalent tree (see Chapter 19)

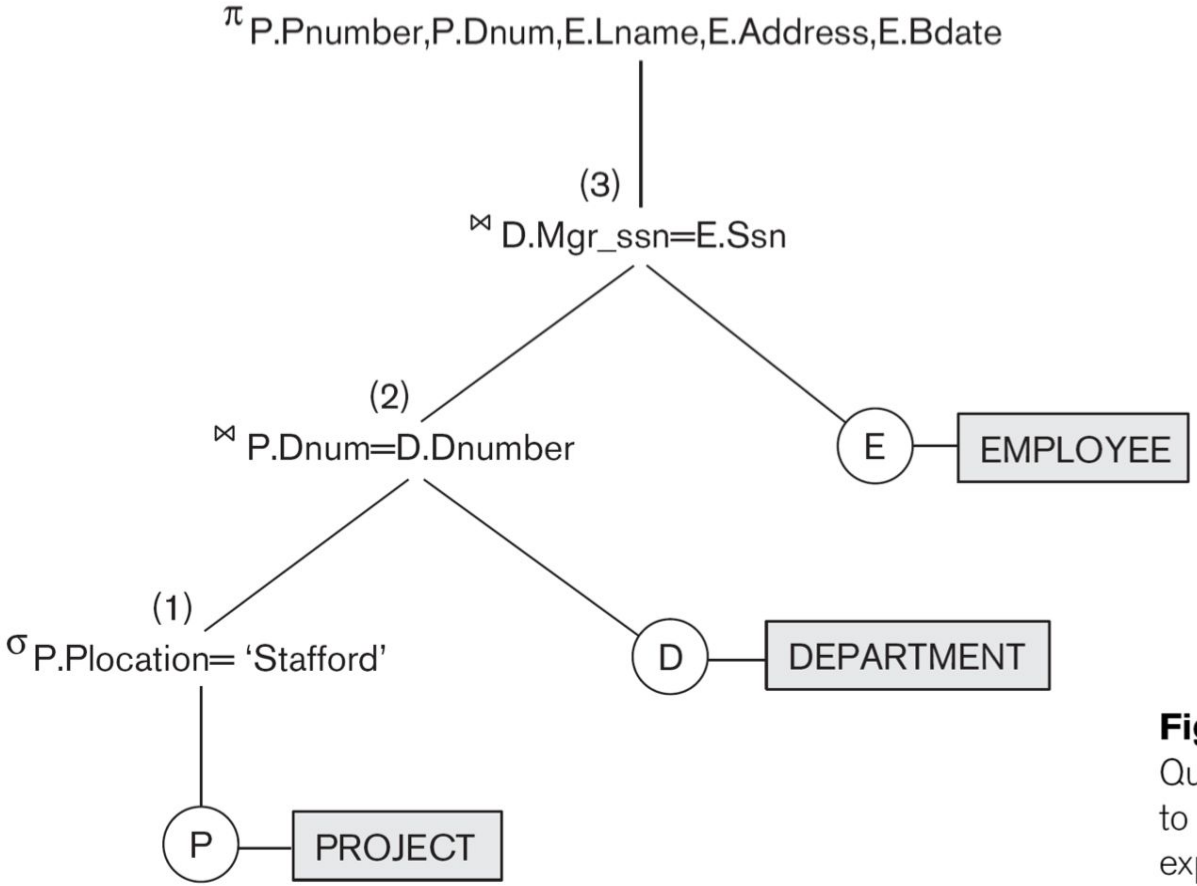


Figure 6.9
Query tree corresponding to the relational algebra expression for Q2.

For every project located in 'Stafford', list the project number, the controlling department number, and the department manager's last name, address, and birth date.

$$\pi_{\text{Pnumber, Dnum, Lname, Address, Bdate}}(((\sigma_{\text{Plocation='Stafford'}}(\text{PROJECT})) \bowtie_{\text{Dnum = Dnumber}} (\text{DEPARTMENT})) \bowtie_{\text{Mgr\_ssn=Ssn}} (\text{EMPLOYEE}))$$

Additional Relational Operations: Aggregate Functions and Grouping

- A type of query that cannot be expressed in the basic relational algebra is to specify mathematical aggregate functions on collections of values from the database.
- Examples of such functions include retrieving the average or total salary of all employees or the total number of employee tuples.
  - These functions are used in simple statistical queries that summarize information from the database tuples.
- Common functions applied to collections of numeric values include
  - SUM, AVERAGE, MAXIMUM, and MINIMUM.
- The COUNT function is used for counting tuples or values

An operation $\mathcal{F}$ (aggregate Function) can be added to the relational algebra for such queries.

Aggregate Function Operation

- Use of the Aggregate Function operation $\mathcal{F}$
  - $\mathcal{F}_{\text{MAX Salary}}$ (EMPLOYEE) retrieves the maximum salary value from the EMPLOYEE relation
  - $\mathcal{F}_{\text{MIN Salary}}$ (EMPLOYEE) retrieves the minimum Salary value from the EMPLOYEE relation
  - $\mathcal{F}_{\text{SUM Salary}}$ (EMPLOYEE) retrieves the sum of all the Salary values from the EMPLOYEE relation
  - $\mathcal{F}_{\text{COUNT SSN, AVERAGE Salary}}$ (EMPLOYEE) computes the count (number) of employees and their average salary (Figure 6.10(c))
    - Note: COUNT(*) just counts the number of rows

Using Grouping with Aggregation

- The previous examples summarized one or more attributes for a set of tuples into a single tuple
  - Maximum Salary or Count (number of) Ssn
- Grouping can be combined with Aggregate Functions
- Example: For each department, retrieve the DNO, COUNT SSN, and AVERAGE SALARY
- A variation of aggregate operation $\mathcal{F}$ allows this:
  - Grouping attribute placed to left of symbol
  - Aggregate functions to right of symbol
  - $_{\text{DNO}}\mathcal{F}_{\text{COUNT SSN, AVERAGE Salary}}$ (EMPLOYEE)
- Above operation groups employees by DNO (department number) and computes the count of employees and average salary per department (see Figure 6.10)

Figure 6.10

The aggregate function operation.

a. $\rho_{R(\text{Dno, No\_of\_employees, Average\_sal})}(_{\text{Dno}}\Im_{\text{COUNT Ssn, AVERAGE Salary}}(\text{EMPLOYEE}))$.

b. $_{\text{Dno}}\Im_{\text{COUNT Ssn, AVERAGE Salary}}(\text{EMPLOYEE})$.

c. $\Im_{\text{COUNT Ssn, AVERAGE Salary}}(\text{EMPLOYEE})$.

R

(a)

Dno	No_of_employees	Average_sal
5	4	33250
4	3	31000
1	1	55000

(b)

Dno	Count_ssn	Average_salary
5	4	33250
4	3	31000
1	1	55000

(c)

Count_ssn	Average_salary
8	35125

[8]Note that this is an arbitrary notation we are suggesting. There is no standard notation.

Additional Relational Operations (cont.)

- Recursive Closure Operations
  - Another type of operation that, in general, cannot be specified in the basic original relational algebra is recursive closure.
    - This operation is applied to a recursive relationship, between tuples of same type.
  - Example of a recursive operation is to retrieve all SUPERVISEES of an EMPLOYEE e at all levels — that is, all EMPLOYEE e' directly supervised by e; all employees e'' directly supervised by each employee e'; all employees e''' directly supervised by each employee e''; and so on.

Recursive Closure (cont.)

- Although it is possible to retrieve employees at each level and then take their union (Figure 6.11), we cannot, in general, specify a query such as “retrieve the supervisees of ‘James Borg’ at all levels” without utilizing a looping mechanism.
  - SQL standard now includes syntax for specifying recursive queries.

SUPERVISION

(Borg's Ssn is 888665555)

(Ssn)	(Super_ssn)
Ssn1	Ssn2
123456789	333445555
333445555	888665555
999887777	987654321
987654321	888665555
666884444	333445555
453453453	333445555
987987987	987654321
888665555	null

RESULT1

Ssn
333445555
987654321

(Supervised by Borg)

RESULT2

Ssn
123456789
999887777
666884444
453453453
987987987

(Supervised by Borg's subordinates)

RESULT

Ssn
123456789
999887777
666884444
453453453
987987987
333445555
987654321

(RESULT1 ∪ RESULT2)

Figure 6.11
A two-level recursive query.

Additional Relational Operations (cont.)

- The OUTER JOIN Operation
  - In NATURAL JOIN and EQUIJOIN, tuples without a matching (or related) tuple are eliminated from the join result
    - Tuples with null in the join attributes are also eliminated
    - Some user queries may want to see all tuples.
  - A set of operations, called OUTER joins, can be used when the user wants to keep all the tuples in R, or all those in S, or both in the result of the join (regardless of whether or not they have matching tuples in the other relation).

OUTER JOIN Operations (cont.)

- LEFT OUTER JOIN keeps every tuple in the first or left relation R in R ⟕ S; if no matching tuple is found in S, then the attributes of S in the join result are filled or "padded" with null values.
- RIGHT OUTER JOIN keeps every tuple in the second or right relation S in the result of R ⟖ S.
- FULL OUTER JOIN denoted by ⟗ keeps all tuples in both the left and the right relations when no matching tuples are found, padding them with null values as needed.

LEFT OUTER JOIN Example

- Retrieve the name of every employee, and if the employee manages a department, retrieve the name of that department.
- Result is shown on next slide.

$$\text{DEPT\_MGR} \leftarrow \text{EMPLOYEE} \ ⟕_{\text{SSN=MGR\_SSN}} \ \text{DEPARTMENT}$$

$$\text{RESULT} \leftarrow \pi_{\text{FNAME, MINIT, LNAME, DNAME}}(\text{DEPT\_MGR})$$

Figure 6.12
The result of a LEFT OUTER JOIN operation.

RESULT

Fname	Minit	Lname	Dname
John	B	Smith	NULL
Franklin	T	Wong	Research
Alicia	J	Zelaya	NULL
Jennifer	S	Wallace	Administration
Ramesh	K	Narayan	NULL
Joyce	A	English	NULL
Ahmad	V	Jabbar	NULL
James	E	Borg	Headquarters

Additional Relational Operations (cont.)

- OUTER UNION Operations
  - OUTER UNION takes the union of tuples from two relations that are not fully type compatible.
  - Union of tuples in two relations R(X, Y) and S(X, Z) that are partially compatible (only some of their attributes, say X, are type compatible).
  - The attributes X that are type compatible are represented only once in the result
  - Attributes that are not type compatible (Y and Z) are also kept in the result relation T(X, Y, Z).

OUTER UNION (cont.)

- Example: An outer union of two relations whose schemas are STUDENT(Name, SSN, Department, Advisor) and INSTRUCTOR(Name, SSN, Department, Rank).
  - Tuples from the two relations are matched based on having the same combination of values of the shared attributes— Name, SSN, Department.
  - If a student is also an instructor, both Advisor and Rank will have a value; otherwise, one of these two attributes will be null.
  - All the tuples from both relations are included in the result but tuples with same (name, ssn, department) combination will appear only once in the result
  - The result relation STUDENT_OR_INSTRUCTOR will have the following attributes:

STUDENT_OR_INSTRUCTOR (Name, SSN, Department, Advisor, Rank)

Examples of Queries in Relational Algebra (with Intermediate Relations)

- **Q1: Retrieve the name and address of all employees who work for the 'Research' department.**

$$RESEARCH\_DEPT \leftarrow \sigma_{DNAME='Research'}(DEPARTMENT)$$

$$RESEARCH\_EMPS \leftarrow (RESEARCH\_DEPT \bowtie_{DNUMBER=DNOEMPLOYEE} EMPLOYEE)$$

$$RESULT \leftarrow \pi_{FNAME, LNAME, ADDRESS}(RESEARCH\_EMPS)$$

- **Q6: Retrieve the names of employees who have no dependents.**

$$ALL\_EMPS \leftarrow \pi_{SSN}(EMPLOYEE)$$

$$EMPS\_WITH\_DEPS(SSN) \leftarrow \pi_{ESSN}(DEPENDENT)$$

$$EMPS\_WITHOUT\_DEPS \leftarrow (ALL\_EMPS - EMPS\_WITH\_DEPS)$$

$$RESULT \leftarrow \pi_{LNAME, FNAME}(EMPS\_WITHOUT\_DEPS * EMPLOYEE)$$

Examples of Queries in Relational Algebra (Single expressions)

As a single expression, these queries become:

- **Q1: Retrieve the name and address of all employees who work for the 'Research' department.**

$$\pi_{\text{Fname, Lname, Address}} (\sigma_{\text{Dname= 'Research'}} (\text{DEPARTMENT} \bowtie_{\text{Dnumber=Dno}} \text{EMPLOYEE}))$$

- **Q6: Retrieve the names of employees who have no dependents.**

$$\pi_{\text{Lname, Fname}}((\pi_{\text{Ssn}}(\text{EMPLOYEE}) - \rho_{\text{Ssn}}(\pi_{\text{Essn}}(\text{DEPENDENT}))) * \text{EMPLOYEE})$$

Relational Calculus

- Another query language for the formal relational model (see Chapter 3).
- Based on the branch of mathematics known as first-order predicate logic.
- Two types of relational calculus:
  - Tuple relational calculus (the SQL language (Chapters 4 and 5) is partially based on this).
  - Domain relational calculus.

Relational Calculus (Cont.)

- Relational calculus is considered to be a nonprocedural or declarative language; order of query processing not specified in query.
- This differs from relational algebra, where we must write a sequence of operations to specify a retrieval request
- In relational algebra, order of operations is specified; considered to be a procedural way of stating a query.

Relational Calculus (cont.)

- A relational calculus expression specifies a query in terms of variables that range over rows (tuples) of the database relations (in tuple calculus) or over columns (attributes) of the database relations (in domain calculus).
- Query result is a relation
- No order of operations to specify how to retrieve the query result—specifies only what information the result should contain.
  - This is the main distinguishing feature between relational algebra and relational calculus.

Tuple Relational Calculus

- Based on specifying a number of tuple variables.
- Each tuple variable usually ranges (loops) over the tuples in a particular database relation (the variable takes as its value any individual tuple from that relation).
- A simple tuple relational calculus query is of the form

$$\{t \mid COND(t)\}$$

  - where t is a tuple variable and COND (t) is a conditional expression involving t.
  - The result of such a query is the set of all tuples t that satisfy COND (t).

Tuple Relational Calculus (Cont.)

- Example: Find the first and last names of all employees whose salary is above $50,000:

$$\{t.FNAME, t.LNAME \mid EMPLOYEE(t) \text{ AND } t.SALARY>50000\}$$

- The condition EMPLOYEE(t) specifies the range relation of tuple variable t – corresponds to FROM-clause in SQL.
- The first and last name of each EMPLOYEE tuple t that satisfies the condition t.SALARY>50000 will be retrieved.
- Left of | corresponds to SELECT-clause in SQL, and to PROJECTION operation ($\pi_{FNAME, LNAME}$) in relational algebra
- Conditions to right of | correspond to WHERE-clause selection conditions in SQL, and to SELECT operation ($\sigma_{SALARY >50000}$) in relational algebra

Tuple Relational Calculus (cont.)

- Example: Retrieve the department name and manager last name for each department:

  {e.LNAME, d.DNAME | EMPLOYEE(e) AND DEPARTMENT(d) AND d.MGR_SSN=e.SSN}

- The tuple variable e ranges over EMPLOYEE tuples, and d ranges over DEPARTMENT tuples.
- The condition d.MGR_SSN=e.SSN is a join condition
- This corresponds to the SQL query:

```
SELECT    E.LNAME, D.DNAME
FROM    EMPLOYEE AS E, DEPARTMENT AS D
WHERE    D.MGR_SSN=E.SSN;
```

Query Graph Notation

- A data structure to represent a basic tuple relational calculus query expression
- Nodes in graph represent tables/aliases and constant values – edges represent selection and join conditions
- Example: Retrieve the department name and manager last name for each department:

{p.PNUMBER, d.DNUMBER, e.LNAME, e.ADDRESS, e.BDATE |
EMPLOYEE(e) AND PROJECT(p) AND DEPARTMENT(d)
AND p.PLOCATION='Stafford' AND p.DNUM=d.DNUMBER
AND d.MGR_SSN=e.SSN}

- Query graph shown on next slide

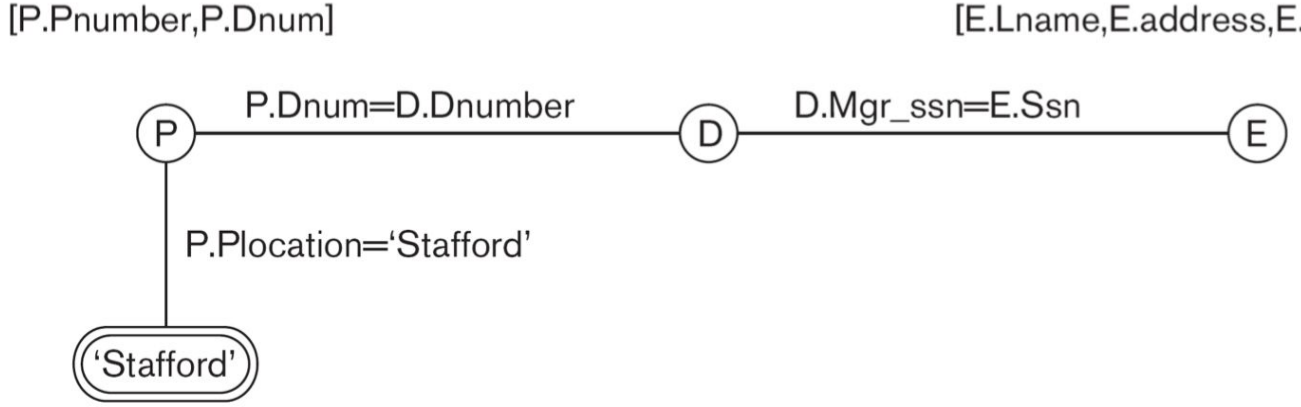


Figure 6.13
Query graph for Q2.

The Existential and Universal Quantifiers

- Two special symbols called quantifiers can appear in relational calculus formulas (conditions); these are the universal quantifier (∀) and the existential quantifier (∃).
- Informally, a tuple variable t is bound if it is quantified, meaning that it appears in an (∀ t) or (∃ t) clause; otherwise, it is free.
- If F is a formula (boolean condition), then so are (∃ t)(F) and (∀ t)(F), where t is a tuple variable.
  - The formula (∃ t)(F) is true if the formula F evaluates to true for some (at least one) tuple assigned to free occurrences of t in F; otherwise (∃ t)(F) is false.
  - The formula (∀ t)(F) is true if the formula F evaluates to true for every tuple (in the "universe") assigned to free occurrences of t in F; otherwise (∀ t)(F) is false.

Existential and Universal Quantifiers (cont.)

- ∀ is called the universal or “for all” quantifier because every tuple in “the universe of” tuples must make F true to make the quantified formula true.
- ∃ is called the existential or “there exists” quantifier because if any (at least one) tuple exists in “the universe of” tuples that makes F, then the quantified formula is true.

Example Query Using Existential Quantifier

- Retrieve the name and address of all employees who work for the 'Research' department. The query can be expressed as :

{t.FNAME, t.LNAME, t.ADDRESS | EMPLOYEE(t) and (∃ d) (DEPARTMENT(d) and d.DNAME='Research' and d.DNUMBER=t.DNO) }

- The only free tuple variables in a relational calculus expression should be those that appear to the left of the bar (|).
  - In above query, t is the only free variable; it is then bound successively to each tuple.
- If a tuple satisfies the conditions specified in the query, the attributes FNAME, LNAME, and ADDRESS are retrieved for each such tuple.
  - The conditions EMPLOYEE (t) and DEPARTMENT(d) specify the range relations for t and d.
  - The condition d.DNAME = 'Research' is a selection condition and corresponds to a SELECT operation in the relational algebra, whereas the condition d.DNUMBER = t.DNO is a JOIN condition.

Example Query Using Universal Quantifier

- Find the names of employees who work on all the projects controlled by department number 5. The query can be:

{e.LNAME, e.FNAME | EMPLOYEE(e) and ((∀ x)(not(PROJECT(x)) or not(x.DNUM=5)

OR ((∃ w)(WORKS_ON(w) and w.ESSN=e.SSN and x.PNUMBER=w.PNO))))}

- Exclude from the universal quantification all tuples that we are not interested in by making the condition true for all such tuples.
  - The first tuples to exclude (by making them evaluate automatically to true) are those that are not in the relation R of interest.
- In query above, using the expression not(PROJECT(x)) inside the universally quantified formula evaluates to true all tuples x that are not in the PROJECT relation.
  - Then we exclude the tuples we are not interested in from R itself. The expression not(x.DNUM=5) evaluates to true all tuples x that are in the project relation but are not controlled by department 5.
- Finally, we specify a condition that must hold on all the remaining tuples in R.

((∃ w)(WORKS_ON(w) and w.ESSN=e.SSN and x.PNUMBER=w.PNO)

Languages Based on Tuple Relational Calculus

- The basic queries in SQL (known as select-project-join queries) are based on tuple calculus:
  - SELECT <list of attributes>
  - FROM <list of relations>
  - WHERE <selection and join conditions>
- SELECT clause lists the attributes being projected, the FROM clause lists the relations needed in the query, and the WHERE clause lists the selection and join conditions.
  - The general SQL syntax is expanded further to accommodate other more complex queries (such as aggregate functions, outer joins, etc. - see Chapter 5).

Languages Based on Tuple Relational Calculus (Cont.)

- Another language based on tuple calculus is the early relational language QUEL, which actually uses range variables as in tuple calculus. Its syntax includes:
  - RANGE OF <variable name> IS <relation name>
- Then it uses
  - RETRIEVE <list of attributes from range variables>
  - WHERE <conditions>
- This language was proposed in the relational DBMS INGRES (system is currently still supported by Computer Associates – but the QUEL language is no longer there).

The Domain Relational Calculus

- Another variation of relational calculus called the domain relational calculus, or simply, domain calculus is equivalent to tuple calculus and to relational algebra.
- The language called QBE (Query-By-Example) that is related to domain calculus was developed almost concurrently to SQL at IBM Research, Yorktown Heights, New York.
  - Domain calculus was thought of as a way to explain what QBE does.
- Domain calculus differs from tuple calculus in the type of variables used in formulas:
  - Rather than having variables range over tuples, the variables range over single values from domains of attributes.
- To form a relation of degree n for a query result, we must have n of these domain variables— one for each attribute.

Domain Relational Calculus (Cont.)

- An expression of the domain calculus is of the form

$$\{ x_1, x_2, \ldots, x_n \mid \text{COND}(x_1, x_2, \ldots, x_n, x_{n+1}, x_{n+2}, \ldots, x_{n+m})\}$$

  - where $x_1, x_2, \ldots, x_n, x_{n+1}, x_{n+2}, \ldots, x_{n+m}$ are domain variables that range over domains (of attributes)
  - and COND is a condition or formula of the domain relational calculus.

Example Query Using Domain Calculus

Retrieve the birthdate and address of the employee whose name is ‘John B. Smith’.

- Query :

{uv | (∃ q) (∃ r) (∃ s) (∃ t) (∃ w) (∃ x) (∃ y) (∃ z)
(EMPLOYEE(qrstuvwxyz) and q=’John’ and r=’B’ and s=’Smith’)}

- Abbreviated notation EMPLOYEE(qrstuvwxyz) uses the variables without the separating commas: EMPLOYEE(q,r,s,t,u,v,w,x,y,z)

- Ten variables for the employee relation are needed, one to range over the domain of each attribute in order.
  - Of the ten variables q, r, s, . . ., z, only u and v are free.
- Specify the requested attributes, BDATE and ADDRESS, by the free domain variables u for BDATE and v for ADDRESS.
- Specify the condition for selecting a tuple following the bar (|)—
  - namely, that the sequence of values assigned to the variables qrstuvwxyz be a tuple of the employee relation and that the values for q (FNAME), r (MINIT), and s (LNAME) be ‘John’, ‘B’, and ‘Smith’, respectively.

Chapter 6 Summary

- Relational Algebra
  - Unary Relational Operations
  - Relational Algebra Operations From Set Theory
  - Binary Relational Operations
  - Additional Relational Operations
  - Examples of Queries in Relational Algebra
- Relational Calculus
  - Tuple Relational Calculus
  - Domain Relational Calculus