

# Operations Research Project Report



**Topic: Linear Programming Models**

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## Problem 1

$X_1$  = Number of 20-inch girls bicycles produced this week

$X_2$  = Number of 20-inch boys bicycles produced this week

$X_3$  = Number of 26-inch girls bicycles produced this week

$X_4$  = Number of 26-inch boys bicycles produced this week

$$\text{MAX } 27X_1 + 32X_2 + 38X_3 + 51X_4$$

S.T.

$$\begin{array}{rcll} X_1 & + & X_3 & \geq 200 & (\text{Min girls models}) \\ & X_2 & + & X_4 & \geq 200 & (\text{Min boys models}) \\ 12X_1 + 12X_2 + 9X_3 + 9X_4 & \leq & 4800 & & (\text{Production minutes}) \\ 6X_1 + 9X_2 + 12X_3 + 18X_4 & \leq & 4800 & & (\text{Assembly minutes}) \\ 2X_1 + 2X_2 & \leq & 500 & & (\text{20-inch tires}) \\ & 2X_3 + 2X_4 & \leq & 800 & (\text{26-inch tires}) \\ & \text{All } X\text{'s} & \geq & 0 & \end{array}$$

### Solution:

$$x_1 = 150.0 \quad \text{Reduced Cost} = -0.0$$

$$x_2 = 100.0 \quad \text{Reduced Cost} = -0.0$$

$$x_3 = 100.0 \quad \text{Reduced Cost} = -0.0$$

$$x_4 = 100.0 \quad \text{Reduced Cost} = -0.0$$

$$\text{Optimal Value: } 16150.0$$

### Interpretation of Results:

The optimal Value of this problem is 16150.0. Furthermore, the cost reduced per unit of decision variable is 0.0. This means that the cost won't be reduced per unit decrease the value of the decision variable.

## Problem 2

- a)  $X_1$  = Number of stoves produced weekly  
 $X_2$  = Number of washers produced weekly  
 $X_3$  = Number of electric dryers produced weekly  
 $X_4$  = Number of gas dryers produced weekly  
 $X_5$  = Number of refrigerators produced weekly

$$\text{MAX } 110X_1 + 90X_2 + 75X_3 + 80X_4 + 130X_5$$

S.T.

$$\begin{aligned} 5.5X_1 + 5.2X_2 + 5.0X_3 + 5.1X_4 + 7.5X_5 &\leq 4800 \text{ (Molding/pressing)} \\ 4.5X_1 &\leq 1200 \text{ (Stove assembly)} \\ 4.5X_2 + 4.0X_3 + 3.0X_4 &\leq 2400 \text{ (Washer/dryer assembly)} \\ 9.0X_5 &\leq 1200 \text{ (Refrigerator assembly)} \\ 4.0X_1 + 3.0X_2 + 2.5X_3 + 2.0X_4 + 4.0X_5 &\leq 3000 \text{ (Packaging)} \\ \text{All } X\text{'s} &\geq 0 \end{aligned}$$

- b) Add the following constraints:

$$\begin{aligned} X_2 - X_3 - X_4 &= 0 \text{ (Washers = Dryers)} \\ X_3 - X_4 &\leq 100 \text{ (E. Dryers } \leq \text{ G. Dryers + 100)} \\ -X_3 + X_4 &\leq 100 \text{ (G. Dryers } \leq \text{ E. Dryers + 100)} \end{aligned}$$

## Solution:

$$x_1 = 266.667 \quad \text{Reduced Cost} = 0.0$$

$$x_2 = 448.718 \quad \text{Reduced Cost} = 0.0$$

$$x_3 = 0.0 \quad \text{Reduced Cost} = -11.538$$

$$x_4 = 0.0 \quad \text{Reduced Cost} = -8.269$$

$$x_5 = 133.333 \quad \text{Reduced Cost} = 0.0$$

$$\text{Optimal Value: } 87051.2821$$

## Interpretation of Results:

The optimal Value of this problem is 87051.2821. Furthermore, the cost reduced per unit of some decision variables is non-zero. This means that the cost will be reduced per unit decrease the value of the decision variable.

### Problem 3

$X_1$  = the number of standard Z345's produced weekly

$X_2$  = the number of industrial Z345's produced weekly

$X_3$  = the number of standard W250's produced weekly

$X_4$  = the number of industrial W250's produced weekly

$X_5$  = the total number of products produced weekly

$$\text{MAX } 400X_1 + 560X_2 + 560X_3 + 700X_4$$

$$\text{S.T. } 25X_1 + 46X_2 + 16X_3 + 34X_4 \leq 2500 \quad (\text{zinc})$$

$$50X_1 + 30X_2 + 28X_3 + 12X_4 \leq 2800 \quad (\text{iron})$$

$$X_1 + X_2 \geq 20 \quad (\text{Min Z345's})$$

$$X_1 + X_2 + X_3 + X_4 - X_5 = 0 \quad (\text{X5 definition})$$

$$X_2 + X_4 - .50X_5 \geq 0 \quad (\text{Industrial min.})$$

$$X_1 + X_2 - .75X_5 \leq 0 \quad (\text{Max Z345's})$$

$$X_3 + X_4 - .75X_5 \leq 0 \quad (\text{Max W250's})$$

$$X_1, X_2, X_3, X_4, X_5 \geq 0$$

#### Solution:

$$x_1 = 22.936 \quad \text{Reduced Cost} = -0.0$$

$$x_2 = 0.0 \quad \text{Reduced Cost} = -44.954$$

$$x_3 = 22.936 \quad \text{Reduced Cost} = -0.0$$

$$x_4 = 45.872 \quad \text{Reduced Cost} = -0.0$$

$$x_5 = 91.743 \quad \text{Reduced Cost} = -0.0$$

$$\text{Optimal Value: } 54128.4408$$

#### Interpretation of Results:

The optimal Value of this problem is 54128.4408. Furthermore, the cost reduced per unit of some decision variables is non-zero. This means that the cost will be reduced per unit decrease the value of the decision variable.

## Problem 4

$X_1$  = amount invested in EAL stock  
 $X_2$  = amount invested in BRU stock  
 $X_3$  = amount invested in TAT stock  
 $X_4$  = amount invested in long term bonds  
 $X_5$  = amount invested in short term bonds  
 $X_6$  = amount invested in the tax deferred annuity  
 $X_7$  = the total amount invested in stocks only

$$\text{MAX } .15X_1 + .12X_2 + .09X_3 + .11X_4 + .085X_5 + .06X_6$$

S.T.

$$\begin{array}{rcll}
 X_1 + & X_2 + & X_3 + & X_4 + & X_5 + & X_6 & = & 50,000 \text{ (Total)} \\
 & & & & & X_6 & \geq & 10,000 \text{ (TDA)} \\
 X_1 + & X_2 + & X_3 & & & & - & X_7 & = & 0 \text{ (Stocks)} \\
 & & X_3 & & & & -.25X_7 & \geq & 0 \text{ (Min TAT)} \\
 & & & X_4 + & X_5 & & - & X_7 & \geq & 0 \text{ (Bond } \geq \text{ stock)} \\
 & & X_3 + & & X_5 + & X_6 & \leq & 12,500 \text{ (Low \%)} \\
 & & & \text{All } X\text{'s} \geq 0 & & & & & & 
 \end{array}$$

### Solution:

$$\begin{array}{ll}
 x_1 = 7500.0 & \text{Reduced Cost} = 0.0 \\
 x_2 = 0.0 & \text{Reduced Cost} = -0.03 \\
 x_3 = 2500.0 & \text{Reduced Cost} = 0.0 \\
 x_4 = 30000.0 & \text{Reduced Cost} = 0.0 \\
 x_5 = 0.0 & \text{Reduced Cost} = -0.125 \\
 x_6 = 10000.0 & \text{Reduced Cost} = 0.0 \\
 x_7 = 10000.0 & \text{Reduced Cost} = 0.0
 \end{array}$$

$$\text{Optimal Value: } 5250.0$$

### Interpretation of Results:

The optimal Value of this problem is 5250.0. Furthermore, the cost reduced per unit of some decision variables is non-zero. This means that the cost will be reduced per unit decrease the value of the decision variable.

## Problem 5

	<u>Unit Profit</u>
$X_1$ = Number of full comforters produced daily	$19 - 3(.50) - 55(.20) = 6.50$
$X_2$ = Number of queen comforters produced daily	$26 - 4(.50) - 75(.20) = 9.00$
$X_3$ = Number of king comforters produced daily	$32 - 6(.50) - 95(.20) = 10.00$

$$\text{MAX } 6.50X_1 + 9.00X_2 + 10.00X_3$$

S.T.

$$\begin{array}{rclcl} 3X_1 + & 4X_2 + & 6X_3 & \leq & 2,700 \text{ (Stuffing)} \\ 55X_1 + & 75X_2 + & 95X_3 & \leq & 48,000 \text{ (Fabric)} \\ 3X_1 + & 5X_2 + & 6X_3 & \leq & 3,000 \text{ (Cutting minutes)} \\ 5X_1 + & 6X_2 + & 8X_3 & \leq & 12,000 \text{ (Sewing minutes)} \\ & & \text{All } X\text{'s} \geq & \underline{120} & \end{array}$$

**Solution:**

$$x_1 = 240.0 \quad \text{Reduced Cost} = 0.0$$

$$x_2 = 312.0 \quad \text{Reduced Cost} = 0.0$$

$$x_3 = 120.0 \quad \text{Reduced Cost} = -1.35$$

$$\text{Optimal Value: } 5568.0$$

**Interpretation of Results:**

The optimal Value of this problem is 5568.0. Furthermore, the cost reduced per unit of some decision variables is non-zero. This means that the cost will be reduced per unit decrease the value of the decision variable.



## Problem 6

$X_1$  = number of 8-oz. portions of steak in the diet

$X_2$  = number of ounces of cheese in the diet

$X_3$  = number of apples in the diet

$X_4$  = number of 8-oz. portion of milk in the diet

$$\text{MIN} \quad 51X_1 + 9X_2 + 1X_3 + 8X_4$$

S.T.

$$692X_1 + 110X_2 + 81X_3 + 150X_4 \geq 1410 \quad (=1800-390 \text{ minimum calories})$$

$$692X_1 + 110X_2 + 81X_3 + 150X_4 \leq 1610 \quad (=2000-390 \text{ maximum calories})$$

$$57X_1 + 6X_2 + 1X_3 + 8X_4 \geq 85 \quad (=100-15 \text{ grams of protein})$$

$$1X_2 + 22X_3 + 12X_4 \geq 25 \quad (=45-20 \text{ grams of carbs.})$$

$$\text{All } X\text{'s} \geq 0$$

### Solution:

$$x_1 = 1.395 \quad \text{Reduced Cost} = 0.0$$

$$x_2 = 0.0 \quad \text{Reduced Cost} = 3.575$$

$$x_3 = 5.49 \quad \text{Reduced Cost} = 0.0$$

$$x_4 = 0.0 \quad \text{Reduced Cost} = 0.761$$

$$\text{Optimal Value: } 76.6306$$

### Interpretation of Results:

The optimal Value of this problem is 76.6306. Furthermore, the cost reduced per unit of some decision variables is non-zero. This means that the cost will be reduced per unit decrease the value of the decision variable.

## Problem 7

$X_1$  = Number of Student models produced each week

$X_2$  = Number of Plus models produced each week

$X_3$  = Number of Net models produced each week

$X_4$  = Number of Pro models produced each week

$$\text{MAX } 70X_1 + 80X_2 + 130X_3 + 150X_4$$

S.T.

$$\begin{array}{rcllcl} & & X_3 & \geq & 100 & \text{(Contract)} \\ .4X_1 + .5X_2 + .6X_3 + .8X_4 & \leq & 750 & \text{(Production Hours)} \\ X_1 + & + & X_3 & \leq & 700 & \text{(Celeron)} \\ & X_2 + & & X_4 & \leq & 550 & \text{(Pentium)} \\ X_1 + & X_2 + & X_3 + & & \leq & 800 & \text{(20gb Hard Drives)} \\ & & & X_4 & \leq & 950 & \text{(30gb Hard Drives)} \\ X_1 + & X_2 + & 2X_3 + & X_4 & \leq & 1600 & \text{(Floppy Drives)} \\ X_1 + & X_2 + & & X_4 & \leq & 1000 & \text{(Zip Drives)} \\ X_1 + & & X_3 + & X_4 & \leq & 1600 & \text{(CD R/W's)} \\ & X_2 + & X_3 + & X_4 & \leq & 900 & \text{(DVD's)} \\ X_1 + & X_2 & & & \leq & 850 & \text{(15-in. monitors)} \\ & & X_3 + & X_4 & \leq & 800 & \text{(17-in. monitors)} \\ & X_2 + & X_3 & & \leq & 1250 & \text{(Mini-tower cases)} \\ X_1 + & & & X_4 & \leq & 750 & \text{(Tower cases)} \\ & & \text{All } X\text{'s} \geq 0 & & & & \end{array}$$

### Solution:

$$x_1 = 325.0 \quad \text{Reduced Cost} = 0.0$$

$$x_2 = 100.0 \quad \text{Reduced Cost} = 0.0$$

$$x_3 = 375.0 \quad \text{Reduced Cost} = 0.0$$

$$x_4 = 425.0 \quad \text{Reduced Cost} = 0.0$$

$$\text{Optimal Value: } 143250.0$$

### Interpretation of Results:

The optimal Value of this problem is 143250.0. Furthermore, the cost reduced per unit of decision variable is 0.0. This means that the cost won't be reduced per unit decrease the value of the decision variable.

## Problem 8

$X_1$  = the number of Delta assemblies produced daily

$X_2$  = the number of Omega assemblies produced daily

$X_3$  = the number of Theta assemblies produced daily

$$\text{MAX } 800X_1 + 900X_2 + 600X_3$$

$$\begin{aligned} \text{S.T. } \quad & X_1 + X_2 + X_3 \leq 7 \quad (\text{X70686 chips}) \\ & 2X_1 + X_2 + X_3 \leq 8 \quad (\text{Production hours}) \\ & 80X_1 + 160X_2 + 80X_3 \leq 480 \quad (\text{Quality minutes}) \\ & \text{All } X\text{'s} \geq 0 \end{aligned}$$

### Solution:

$$x_1 = 2.0 \quad \text{Reduced Cost} = -0.0$$

$$x_2 = 0.0 \quad \text{Reduced Cost} = -100.0$$

$$x_3 = 4.0 \quad \text{Reduced Cost} = -0.0$$

$$\text{Optimal Value: } 4000.0$$

### Interpretation of Results:

The optimal Value of this problem is 4000.0. Furthermore, the cost reduced per unit of some decision variables is non-zero. This means that the cost will be reduced per unit decrease the value of the decision variable.

## Problem 9

$X_1$  = the number in group I contacted by telephone  
 $X_2$  = the number in group II contacted by telephone  
 $X_3$  = the number in group III contacted by telephone  
 $X_4$  = the number in group IV contacted by telephone  
 $X_5$  = the number in group I contacted in person  
 $X_6$  = the number in group II contacted in person  
 $X_7$  = the number in group III contacted in person  
 $X_8$  = the number in group IV contacted in person

$$\begin{array}{ll}
 \text{MIN} & 15X_1 + 12X_2 + 20X_3 + 18X_4 + 35X_5 + 30X_6 + 50X_7 + 40X_8 \\
 \text{S.T.} & X_1 + X_2 + X_3 + X_4 + X_5 + X_6 + X_7 + X_8 = 2000 \text{ (Total)} \\
 & X_1 + X_2 + X_5 + X_6 \geq 1000 \text{ (W\&R)} \\
 & X_5 + X_6 + X_7 + X_8 \geq 500 \text{ (In person)} \\
 & -0.5X_1 + 0.5X_5 \geq 0 \text{ (W\&R,ip)} \\
 & X_2 + X_4 + X_6 + X_8 \leq 800 \text{ (Small)} \\
 & -0.25X_2 - 0.25X_4 + 0.75X_6 + 0.75X_8 \leq 0 \text{ (Small,ip)} \\
 & X_1 + X_5 \geq 200 \text{ (Min I)} \\
 & X_2 + X_6 \geq 200 \text{ (Min II)}
 \end{array}$$

$$\begin{array}{llll}
 & X_3 & & + X_7 \geq 200 \text{ (Min III)} \\
 & & X_4 & + X_8 \geq 200 \text{ (Min IV)} \\
 X_1 & & + X_5 & \leq 1000 \text{ (Max I)} \\
 & X_2 & & + X_6 \leq 1000 \text{ (Max II)} \\
 & & X_3 & + X_7 \leq 1000 \text{ (Max III)} \\
 & & & X_4 + X_8 \leq 1000 \text{ (Max IV)} \\
 & & & \text{All } X_i \geq 0
 \end{array}$$

### Solution:

$$x_1 = 500.0 \quad \text{Reduced Cost} = 0.0$$

$$x_2 = 600.0 \quad \text{Reduced Cost} = 0.0$$

$$x_3 = 200.0 \quad \text{Reduced Cost} = 0.0$$

$$x_4 = 200.0 \quad \text{Reduced Cost} = 0.0$$

$$x_5 = 500.0 \quad \text{Reduced Cost} = 0.0$$

$$x_6 = 0.0 \quad \text{Reduced Cost} = 8.0$$

$$x_7 = 0.0 \quad \text{Reduced Cost} = 20.0$$

$$x_8 = 0.0 \quad \text{Reduced Cost} = 12.0$$

Optimal Value: 39800.0

### **Interpretation of Results:**

The optimal Value of this problem is 39800.0. Furthermore, the cost reduced per unit of some decision variables is non-zero. This means that the cost will be reduced per unit decrease the value of the decision variable.

## Problem 10

$X_1$  = the number of ounces of Multigrain Cheerios in the mixture

$X_2$  = the number of ounces of Grape Nuts in the mixture

$X_3$  = the number of ounces of Product 19 in the mixture

$X_4$  = the number of ounces of Frosted Bran in the mixture

$X_5$  = the total number of ounces in the mixture

$$\begin{array}{llllll}
 \text{MIN} & 12X_1 + 9X_2 + 9X_3 + 15X_4 & & & & \\
 \text{S.T.} & 30X_1 + 30X_2 + 20X_3 + 20X_4 & \geq 50 & \text{(Vitamin A)} & & \\
 & 25X_1 + 2X_2 + 100X_3 + 25X_4 & \geq 50 & \text{(Vitamin C)} & & \\
 & 25X_1 + 25X_2 + 25X_3 + 25X_4 & \geq 50 & \text{(Vitamin D)} & & \\
 & 25X_1 + 25X_2 + 100X_3 + 25X_4 & \geq 50 & \text{(Vitamin B6)} & & \\
 & 45X_1 + 45X_2 + 100X_3 + 25X_4 & \geq 50 & \text{(Iron)} & & \\
 & X_1 + X_2 + X_3 + X_4 - X_5 = 0 & & \text{(Total)} & & \\
 & X_1 & - .1X_5 \geq 0 & (\geq 10\% \text{ M/G Cheerios}) & & \\
 & X_2 & - .1X_5 \geq 0 & (\geq 10\% \text{ Grape Nuts}) & & \\
 & X_3 & - .1X_5 \geq 0 & (\geq 10\% \text{ Product 19}) & & \\
 & X_4 & - .1X_5 \geq 0 & (\geq 10\% \text{ Frosted Bran}) & & \\
 & \text{All } X\text{'s} \geq 0 & & & & 
 \end{array}$$

### Solution:

$$x_1 = 0.2 \quad \text{Reduced Cost} = 0.0$$

$$x_2 = 1.224 \quad \text{Reduced Cost} = 0.0$$

$$x_3 = 0.376 \quad \text{Reduced Cost} = 0.0$$

$$x_4 = 0.2 \quad \text{Reduced Cost} = 0.0$$

$$x_5 = 2.0 \quad \text{Reduced Cost} = 0.0$$

Optimal Value: 19.8

### Interpretation of Results:

The optimal Value of this problem is 19.8. Furthermore, the cost reduced per unit of decision variable is 0.0. This means that the cost won't be reduced per unit decrease the value of the decision variable.

## Problem 11

$X_1$  = Number of refrigerator/ovens produced

$X_2$  = Number of French fry makers produced

$X_3$  = Number of French toast makers produced

MIN  $140X_1 + 50X_2 + 36X_3$

S.T.

$100X_1 + 35X_2 + 27X_3$	$\geq 2,000,000$	(Min Profit)
$X_1$	$\geq 5,000$	(Min Refrig/oven)
$X_2$	$\geq 4,000$	(Min French fry maker)
$X_3$	$\geq 2,300$	(Min French toast maker)
$X_1$	$\leq 15,000$	(Max Refrig/oven)
$X_2$	$\leq 15,000$	(Max French fry maker)
$X_3$	$\leq 15,000$	(Max French toast maker)

### Solution:

$$x_1 = 14550.0 \quad \text{Reduced Cost} = 0.0$$

$$x_2 = 4000.0 \quad \text{Reduced Cost} = -0.0$$

$$x_3 = 15000.0 \quad \text{Reduced Cost} = 0.0$$

$$\text{Optimal Value: } 2777000.0$$

### Interpretation of Results:

The optimal Value of this problem is 2777000.0. Furthermore, the cost reduced per unit of decision variable is 0.0. This means that the cost won't be reduced per unit decrease the value of the decision variable.

## Problem 12

- a)  $X_1$  = Number of plates made per day  
 $X_2$  = Number of mugs made per day  
 $X_3$  = Number of steins made per day  
 $X_4$  = Total daily production

$$\text{MAX } 2.50X_1 + 3.25X_2 + 3.90X_3$$

S.T.

$$\begin{array}{rclcl} 2X_1 + & 3X_2 + & 6X_3 & \leq & 1920 & ((4)(8)(60) \text{ Molding min.}) \\ 8X_1 + & 12X_2 + & 14X_3 & \leq & 3840 & ((8)(8)(60) \text{ Finishing min.}) \\ & X_2 & & \geq & 150 & (\text{Minimum mugs}) \end{array}$$

$$\begin{array}{rclcl} -2X_1 - & 2X_2 + & X_3 & \leq & 0 & (\text{Steins} \leq 2(\text{Plates} + \text{Mugs})) \\ X_1 + & X_2 + & X_3 - X_4 & = & 0 & (\text{Total Definition}) \\ X_1 & & & - .3X_4 & \leq & 0 & (\text{Plates} \leq 30\% \text{ Total Produced}) \end{array}$$

All X's  $\geq 0$

- b) Combine the first two constraints into one:

$$10X_1 + 15X_2 + 20X_3 \leq 5760$$

### Solution:

$$x_1 = 101.803 \quad \text{Reduced Cost} = -0.0$$

$$x_2 = 150.0 \quad \text{Reduced Cost} = -0.0$$

$$x_3 = 87.541 \quad \text{Reduced Cost} = -0.0$$

$$x_4 = 339.344 \quad \text{Reduced Cost} = 0.0$$

$$\text{Optimal Value: } 1083.418$$

### Interpretation of Results:

The optimal Value of this problem is 1083.418. Furthermore, the cost reduced per unit of decision variable is 0.0. This means that the cost won't be reduced per unit decrease the value of the decision variable.



## Problem 13

$X_1$  = \$ invested in first trust deeds  
 $X_2$  = \$ invested in second trust deeds  
 $X_3$  = \$ invested in third trust deeds  
 $X_4$  = \$ invested in commercial trust deeds  
 $X_5$  = \$ invested in a savings account  
 $X_6$  = Total \$ invested in residential trust deeds  
 $X_7$  = Total \$ invested in all trust deeds

$$\text{MAX } .0775X_1 + .1125X_2 + .1425X_3 + .9875X_4 + .0445X_5$$

S.T.

$$\begin{array}{rcll}
 X_1 + & X_2 + & X_3 + & X_4 + & X_5 & = & 68,000,000 \text{ (Total)} \\
 & & & & X_5 & \geq & 5,000,000 \text{ (Save)} \\
 X_1 + & X_2 + & X_3 + & & -X_6 & = & 0 \text{ (Res Tr.)} \\
 X_1 + & X_2 + & X_3 + & X_4 & & -X_7 = & 0 \text{ (Total Tr)} \\
 & & & & X_6 - .8X_7 & \geq & 0 \text{ (80\% Res.)} \\
 X_1 & & & & -.6X_6 & \geq & 0 \text{ (60\% First)} \\
 4X_1 + & 6X_2 + & 9X_3 + & 3X_4 & & \leq & 340,000,000 (*) \\
 & & & \text{All } X\text{'s} \geq 0
 \end{array}$$

\*Average Risk Factor is found by:

$$4 \frac{X_1}{68,000,000} + 6 \frac{X_2}{68,000,000} + 9 \frac{X_3}{68,000,000} + 3 \frac{X_4}{68,000,000} + 0 \frac{X_5}{68,000,000}$$

This expression must be  $\leq 5$ . Multiplying both sides by 68,000,000 gives the above constraint.

### Solution:

$$x1 = 30240000.0 \quad \text{Reduced Cost} = -0.0$$

$$x2 = 66666.667 \quad \text{Reduced Cost} = 0.0$$

$$x3 = 20093333.0 \quad \text{Reduced Cost} = 0.0$$

$$x4 = 12600000.0 \quad \text{Reduced Cost} = -0.0$$

$$x5 = 5000000.0 \quad \text{Reduced Cost} = 0.0$$

$$x6 = 50400000.0 \quad \text{Reduced Cost} = -0.0$$

$$x7 = 63000000.0 \quad \text{Reduced Cost} = 0.0$$

$$\text{Optimal Value: } 17879399.9525$$

### **Interpretation of Results:**

The optimal Value of this problem is 17879399.9525. Furthermore, the cost reduced per unit of decision variable is 0.0. This means that the cost won't be reduced per unit decrease the value of the decision variable.

## Problem 14

$X_{JR}$  = Motor home cabinets produced in regular time in July  
 $X_{JO}$  = Motor home cabinets produced in overtime in July  
 $X_{AR}$  = Motor home cabinets produced in regular time in August  
 $X_{AO}$  = Motor home cabinets produced in overtime in August  
 $X_{JS}$  = Motor home cabinets produced in regular time in September  
 $X_{JS}$  = Motor home cabinets produced in overtime in September  
 $Y_{JR}$  = Mobile home cabinets produced in regular time in July  
 $Y_{JO}$  = Mobile home cabinets produced in overtime in July  
 $Y_{AR}$  = Mobile home cabinets produced in regular time in August  
 $Y_{AO}$  = Mobile home cabinets produced in overtime in August  
 $Y_{JS}$  = Mobile home cabinets produced in regular time in September  
 $Y_{JS}$  = Mobile home cabinets produced in overtime in September  
 $S_J$  = Motor home cabinets stored in July  
 $S_A$  = Motor home cabinets stored in August  
 $S_S$  = Motor home cabinets stored in September  
 $T_J$  = Mobile home cabinets stored in July  
 $T_A$  = Mobile home cabinets stored in August  
 $T_S$  = Mobile home cabinets stored in September

### Solution:

Mobile\_home\_cabinets\_produced\_in\_overtime\_in\_August = 131.0      Reduced Cost = 0.0  
Mobile\_home\_cabinets\_produced\_in\_overtime\_in\_July = 95.0      Reduced Cost = 0.0  
Mobile\_home\_cabinets\_produced\_in\_overtime\_in\_September = 0.0      Reduced Cost = 6.0  
Mobile\_home\_cabinets\_produced\_in\_regular\_time\_in\_August = 150.0      Reduced Cost = 0.0  
Mobile\_home\_cabinets\_produced\_in\_regular\_time\_in\_July = 285.0      Reduced Cost = 0.0  
Mobile\_home\_cabinets\_produced\_in\_regular\_time\_in\_September = 144.0      Reduced Cost = 0.0  
Mobile\_home\_cabinets\_stored\_in\_August = 281.0      Reduced Cost = 0.0  
Mobile\_home\_cabinets\_stored\_in\_July = 300.0      Reduced Cost = 0.0  
Mobile\_home\_cabinets\_stored\_in\_September = 25.0      Reduced Cost = 0.0  
Motor\_home\_cabinets\_stored\_in\_August = 0.0      Reduced Cost = 0.6  
Motor\_home\_cabinets\_stored\_in\_July = 0.0      Reduced Cost = 3.0  
Motor\_home\_cabinets\_stored\_in\_September = 10.0      Reduced Cost = 0.0  
cabinets\_produced\_in\_overtime\_in\_August = 0.0      Reduced Cost = 0.0

cabinets\_produced\_in\_overtime\_in\_July = 0.0    Reduced Cost = 0.0  
cabinets\_produced\_in\_overtime\_in\_September = 0.0    Reduced Cost = 3.6  
cabinets\_produced\_in\_regular\_in\_July = 225.0    Reduced Cost = 0.0  
cabinets\_produced\_in\_regular\_time\_in\_August = 250.0    Reduced Cost = 0.0  
cabinets\_produced\_in\_regular\_time\_in\_September = 160.0    Reduced Cost = -0.0  
Optimal Value: 367969.0

### **Interpretation of Results:**

The optimal Value of this problem is 367969.0. Furthermore, the cost reduced per unit of some decision variables is non-zero. This means that the cost will be reduced per unit decrease the value of the decision variable.

## Problem 15

$X_1$  = the number of acres of wheat planted

$X_2$  = the number of acres of corn planted

$X_3$  = the number of acres of oats planted

$X_4$  = the number of acres of soybeans planted

Profit coefficients are  $210(\$3.20) - \$50 = \$622$ ,  $300(\$2.55) - \$75 = \$690$ ,  $180(\$1.45) - \$30 = \$231$ , and  $240(\$3.10) - \$60 = \$684$  respectively.

$$\begin{array}{llllll}
 \text{MAX} & 622X_1 + 690X_2 + 231X_3 + 684X_4 & & & & \\
 \text{S.T.} & 4X_1 + 5X_2 + 3X_3 + 10X_4 & \leq & 1,800 & & \text{(Labor hours)} \\
 & 50X_1 + 75X_2 + 30X_3 + 60X_4 & \leq & 25,000 & & \text{(Expenses)} \\
 & 2X_1 + 6X_2 + X_3 + 4X_4 & \leq & 1,200 & & \text{(Water)} \\
 & 210X_1 & \geq & 30,000 & & \text{(Min. Wheat)} \\
 & 300X_2 & \geq & 30,000 & & \text{(Min. Corn)} \\
 & 180X_3 & \leq & 25,000 & & \text{(Max Oats)} \\
 & X_1 + X_2 + X_3 + X_4 & \leq & 300 & & \text{(Total acres)} \\
 & \text{All } X\text{'s} \geq 0 & & & & 
 \end{array}$$

### Solution:

$$x_1 = 142.857 \quad \text{Reduced Cost} = 0.0$$

$$x_2 = 142.857 \quad \text{Reduced Cost} = -0.0$$

$$x_3 = 0.0 \quad \text{Reduced Cost} = -444.0$$

$$x_4 = 14.286 \quad \text{Reduced Cost} = -0.0$$

$$\text{Optimal Value: } 197199.9961$$

### Interpretation of Results:

The optimal Value of this problem is 197199.9961. Furthermore, the cost reduced per unit of some decision variables is non-zero. This means that the cost will be reduced per unit decrease the value of the decision variable.