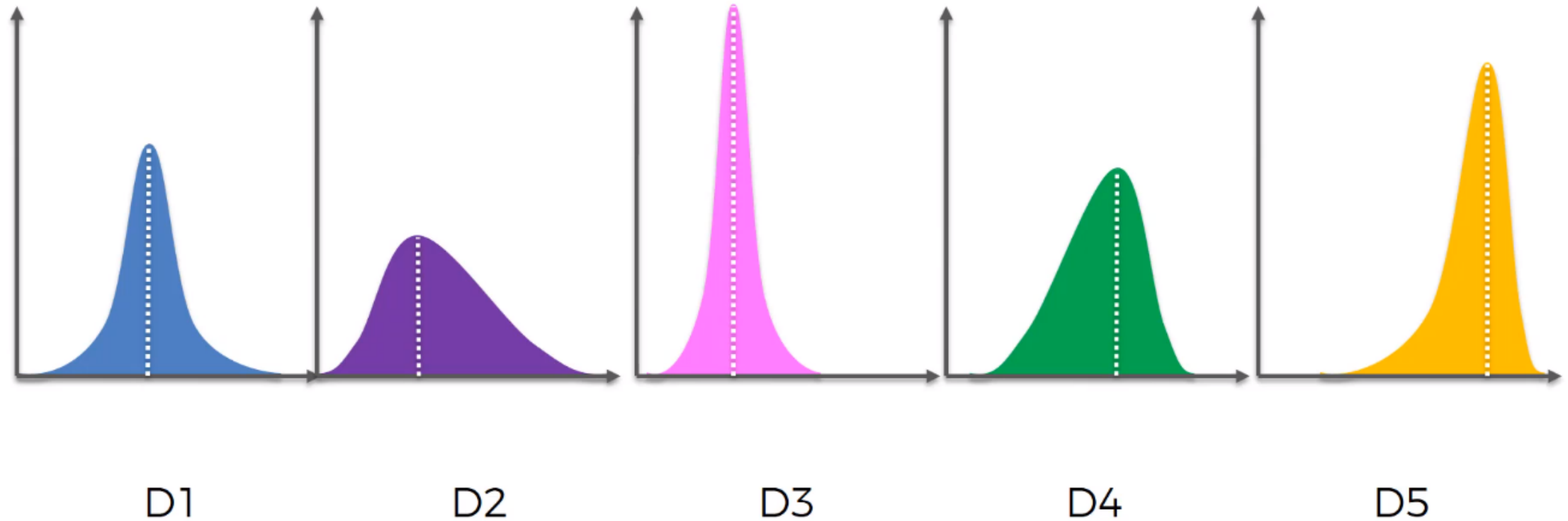


Thompson Sampling

The Multi-Armed Bandit Problem



The Multi-Armed Bandit Problem



Bayesian Inference

- Ad i gets rewards \mathbf{y} from Bernoulli distribution $p(\mathbf{y}|\theta_i) \sim \mathcal{B}(\theta_i)$.
- θ_i is unknown but we set its uncertainty by assuming it has a uniform distribution $p(\theta_i) \sim \mathcal{U}([0, 1])$, which is the prior distribution.
- Bayes Rule: we approach θ_i by the posterior distribution

$$\underbrace{p(\theta_i|\mathbf{y})}_{\text{posterior distribution}} = \frac{p(\mathbf{y}|\theta_i)p(\theta_i)}{\int p(\mathbf{y}|\theta_i)p(\theta_i)d\theta_i} \propto \underbrace{p(\mathbf{y}|\theta_i)}_{\text{likelihood function}} \times \underbrace{p(\theta_i)}_{\text{prior distribution}}$$

- We get $p(\theta_i|\mathbf{y}) \sim \beta(\text{number of successes} + 1, \text{number of failures} + 1)$
- At each round n we take a random draw $\theta_i(n)$ from this posterior distribution $p(\theta_i|\mathbf{y})$, for each ad i .
- At each round n we select the ad i that has the highest $\theta_i(n)$.

Thompson Sampling Algorithm

Step 1. At each round n , we consider two numbers for each ad i :

- $N_i^1(n)$ - the number of times the ad i got reward 1 up to round n ,
- $N_i^0(n)$ - the number of times the ad i got reward 0 up to round n .

Step 2. For each ad i , we take a random draw from the distribution below:

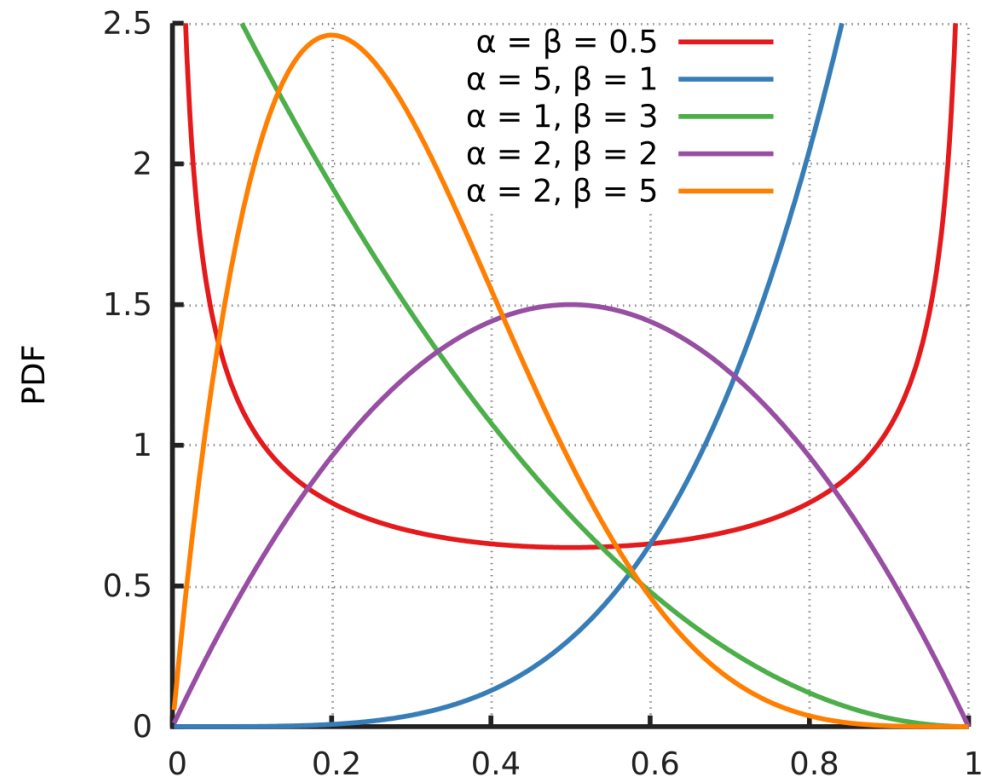
$$\theta_i(n) = \beta(N_i^1(n) + 1, N_i^0(n) + 1)$$

Step 3. We select the ad that has the highest $\theta_i(n)$.

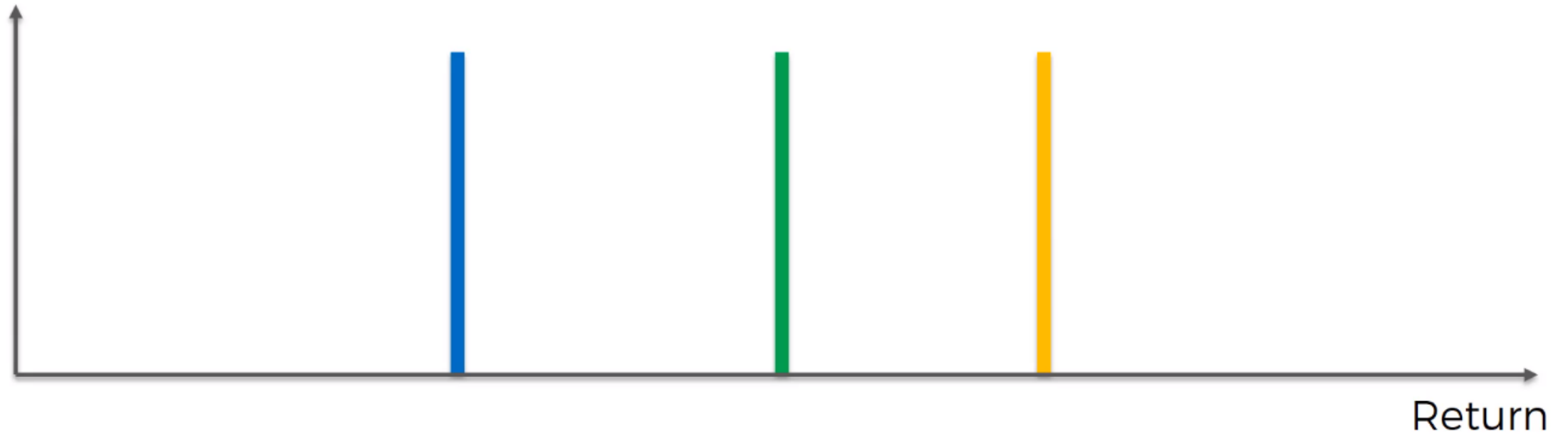
Thompson Sampling Algorithm

Step 2. For each ad i , we take a random draw from the distribution below:

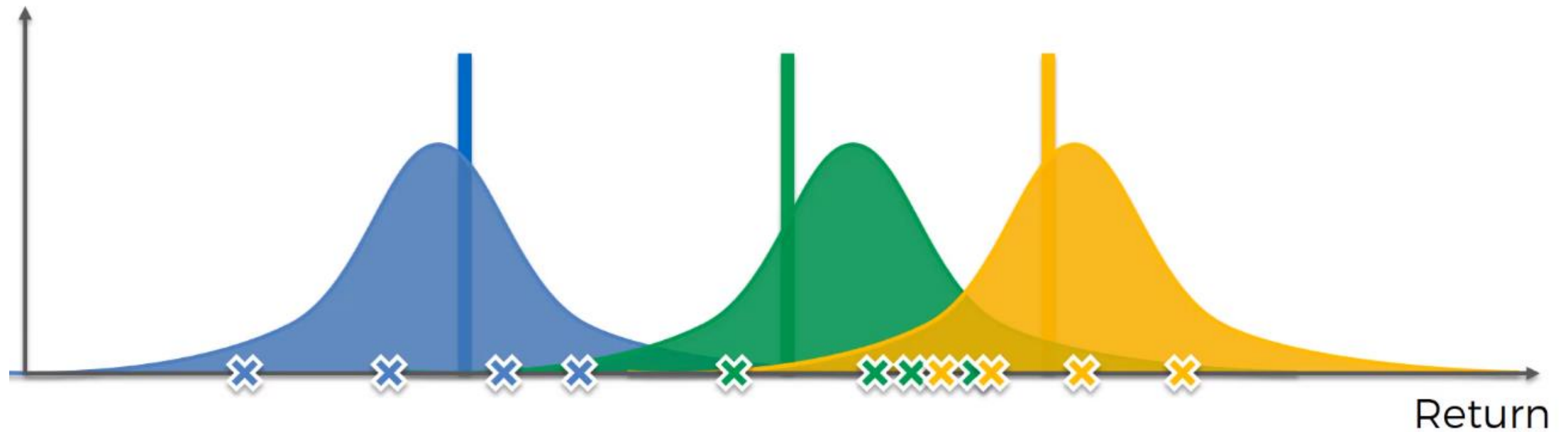
$$\theta_i(n) = \beta(N_i^1(n) + 1, N_i^0(n) + 1)$$



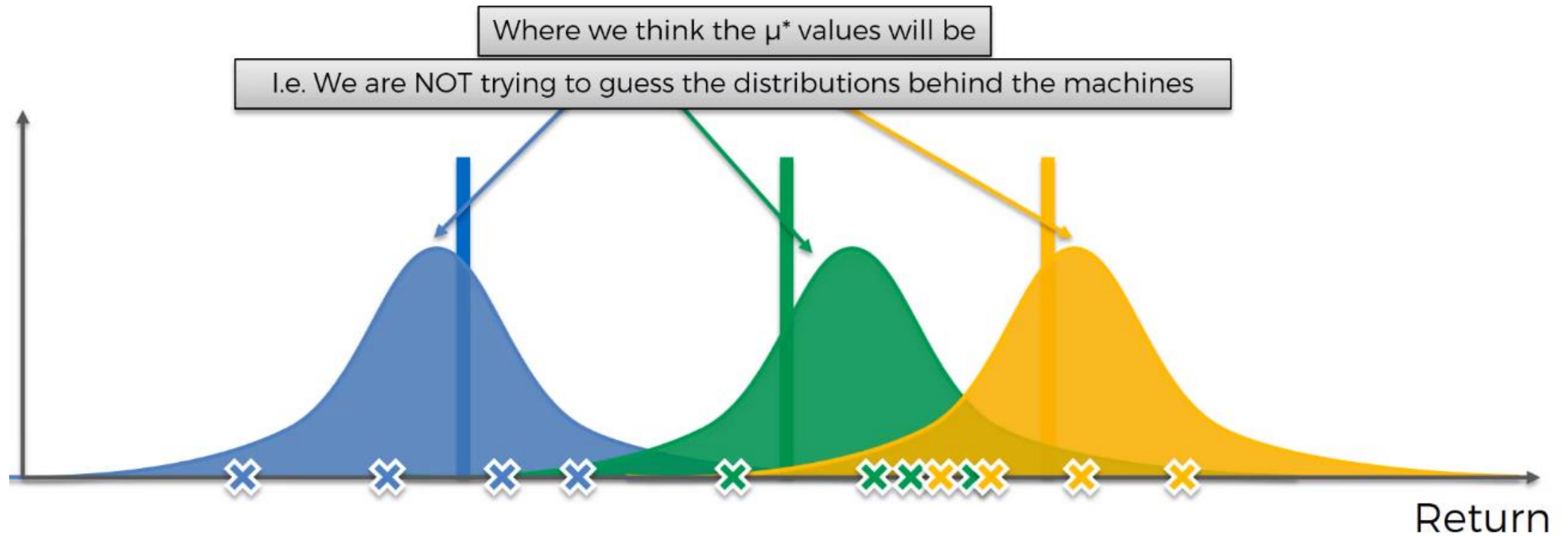
Thompson Sampling Algorithm



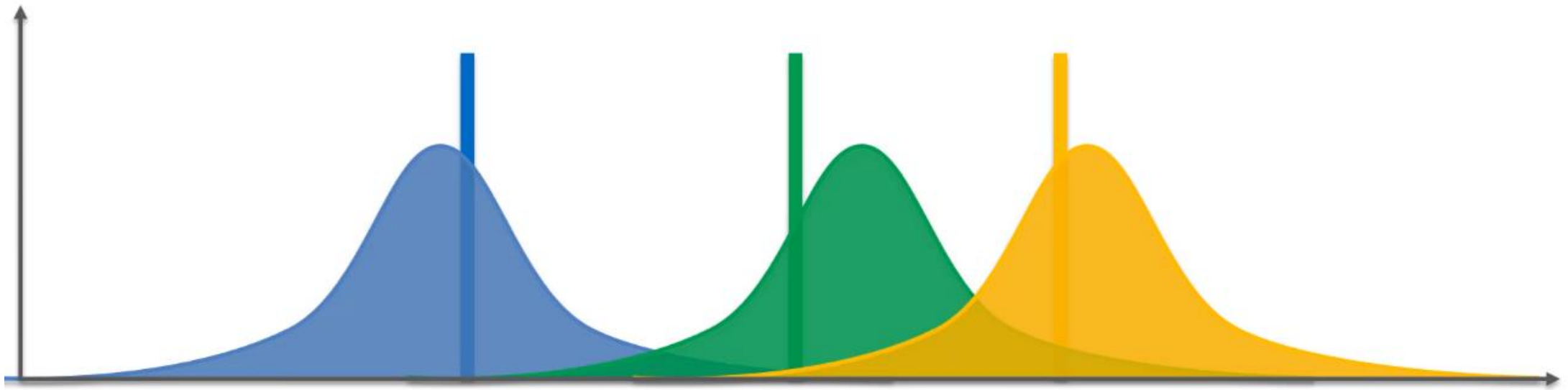
Thompson Sampling Algorithm



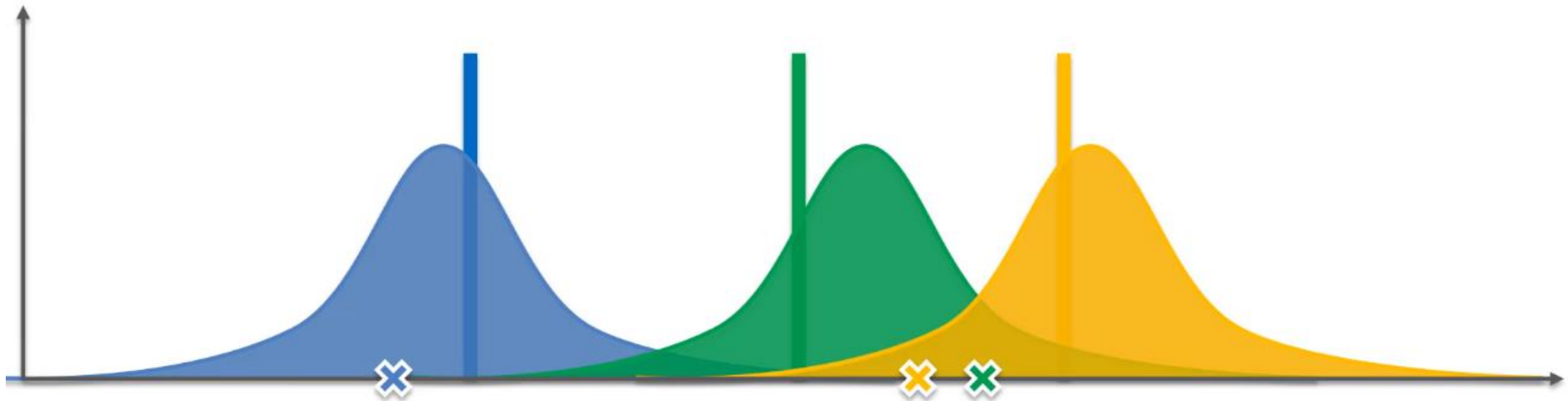
Thompson Sampling Algorithm



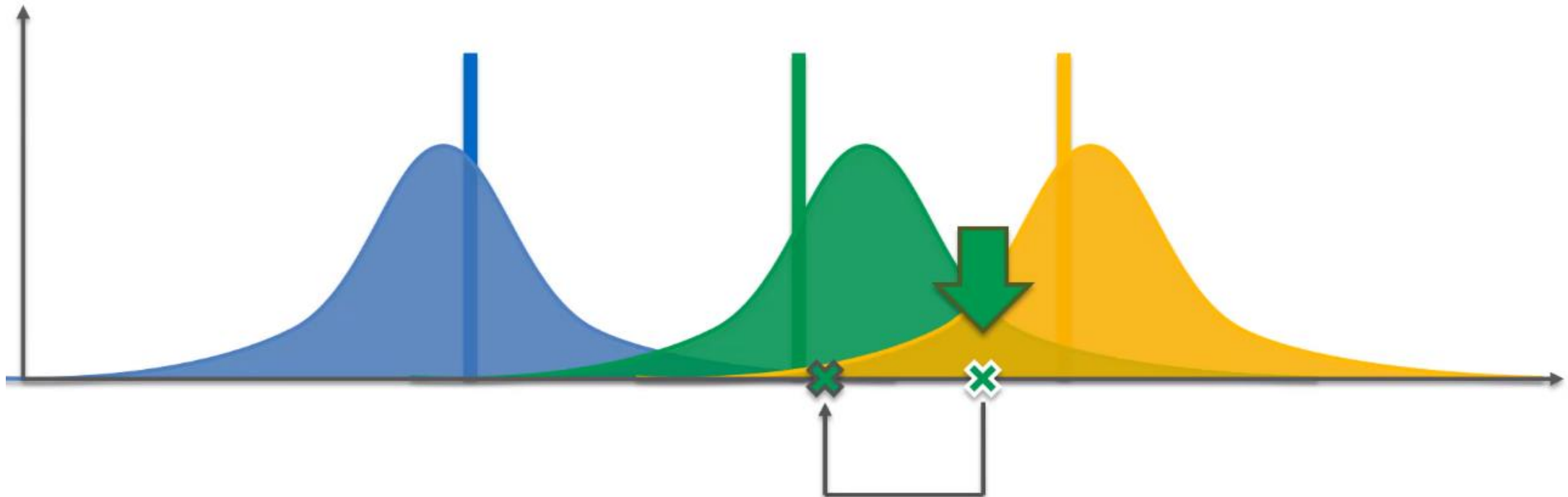
Thompson Sampling Algorithm



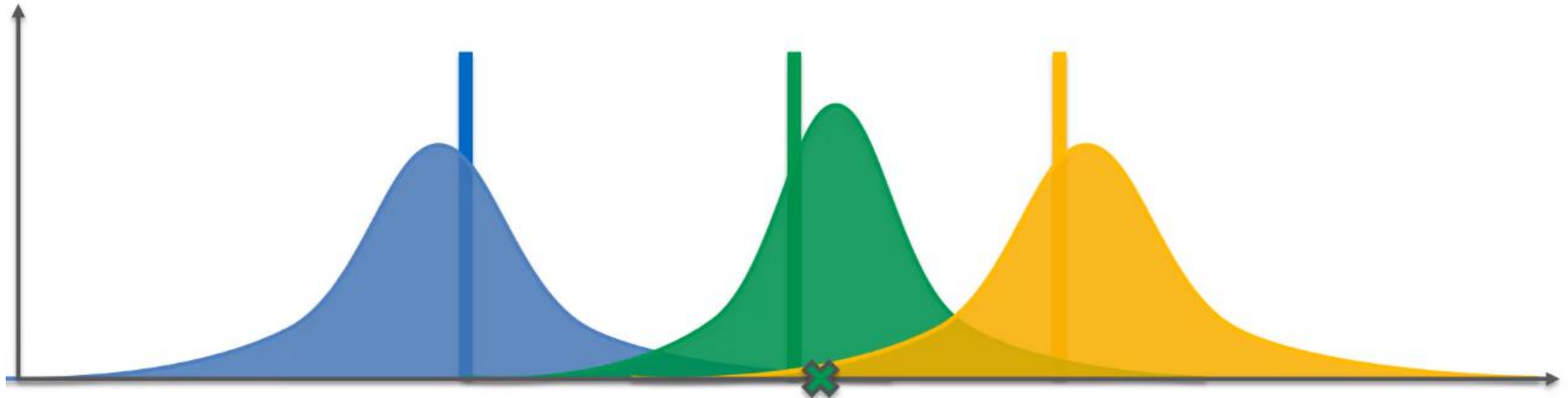
Thompson Sampling Algorithm



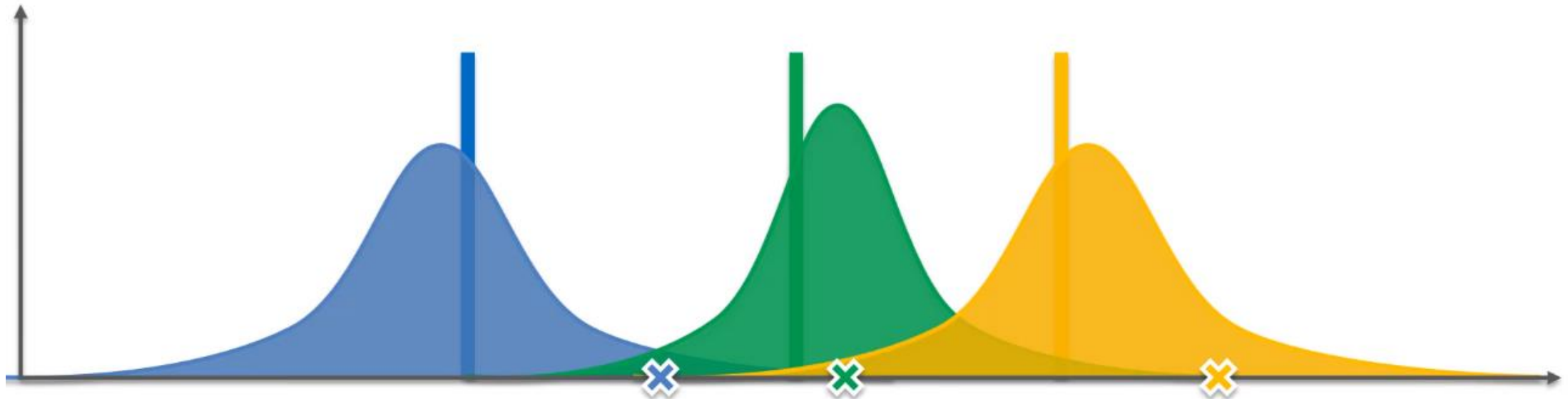
Thompson Sampling Algorithm



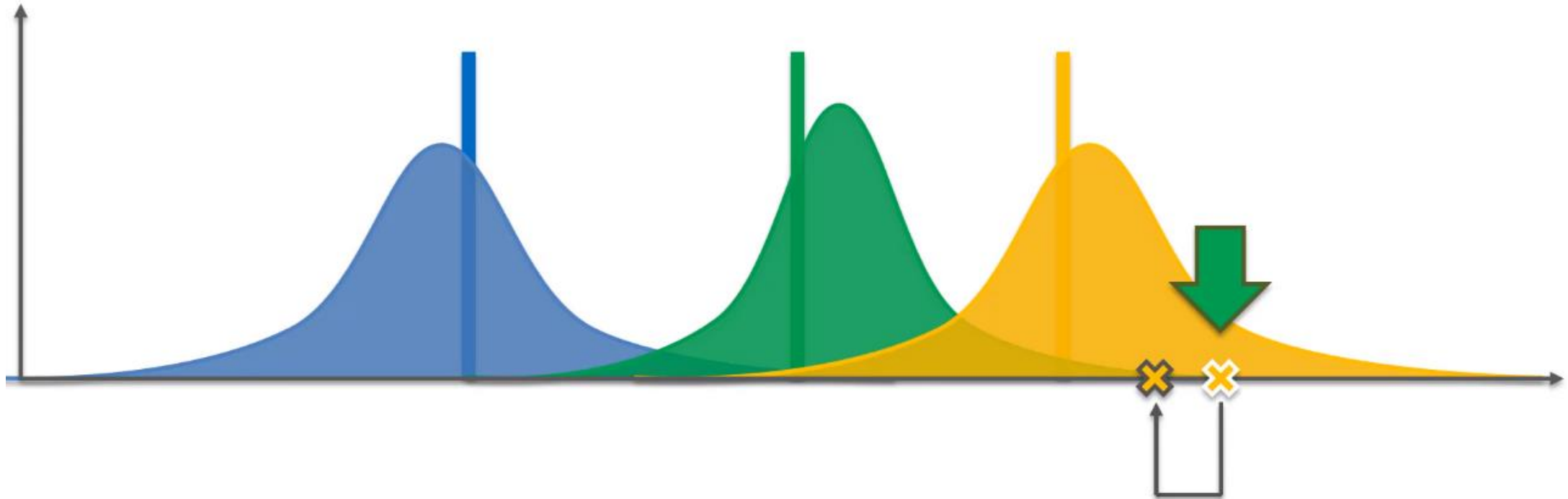
Thompson Sampling Algorithm



Thompson Sampling Algorithm



Thompson Sampling Algorithm



Thompson Sampling Algorithm

