

## 3D Autonomous Checkers

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AC Studio 2  
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# Overall Objectives

- Extend traditional 2D Chinese Checkers into a 3D version
- Have autonomous 'armies' compete with each other in reaching each other's base
- Incorporate 'ladder' behavior characteristic in checkers
- Incorporate A\* Shortest Path to make shortest possible game
- Train opponent 'armies' to outperform their rival

# Inspiration<sup>1</sup>

## THE SHORTEST GAME OF CHINESE CHECKERS AND RELATED PROBLEMS

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### Abstract

In 1979, David Fabian found a complete game of two-person Chinese Checkers in 30 moves (15 by each player) [Martin Gardner, Penrose Tiles to Trapdoor Ciphers, MAA, 1997]. This solution requires that the two players cooperate to generate a win as quickly as possible for one of them. We show, using computational search techniques, that no shorter game is possible. We also consider a solitaire version of Chinese Checkers where one player attempts to move her pieces across the board in as few moves as possible. In 1971, Octave Levenspiel found a solution in 27 moves [Ibid.]; we demonstrate that no shorter solution exists. To show optimality, we employ a variant of A\* search, as well as bidirectional search.

### A.1 Shortest games

Chinese Checkers in 30 moves (Figure 5), 10 man armies, 6-move rules (by David Fabian):

c2-d2, h8-h6, d1-d3, i6-g6, a3-c3-e3, g9-g7-g5, a2-c2-e2-e4, h7-h5-f7, e4-f4, i9-g9-g7-e7, d3-c4, g6-g4-e4-e2-c2-a2, a4-c2-c2-e4-g4-g6-e8-e6, i8-i6-g6-g4-e4-e2-c2-a4, a1-a3-c3-c5, f9-h7-h5-f5-f3-d3-d1, c5-d5, e7-e5-c5-c3-a3-a1, b2-b4-d4-d6-f6-f8-h8, i7-g9-g7-e7-e5-c5-c3-a3, c1-e1-c3-c5-e5-e7-g7, h6-f8-f6-d6-d4-b4-b2, b3-b4, g8-g6-g4-e4-e2-c2, b1-b3-b5-d3-f3-f5, g5-e5-c5-c3-c1, f5-f6, f7-f5-f3-d3-b5-b3-b1, e6-g6-g8-i8, h9-h7-f7-f5-f3-d3-b5-b3 (red wins).

Chinese Checkers in 36 moves, 15 man armies, 6-move rules: e1-c2, g8-g6, c1-e1-e3, h6-f6, e3-e4, f9-f7-f5, a1-c1-e1-e3-e5-g5-e7, g7-g5-e5-e3-e1-c1-a1, a5-b5, i7-g7-g5-e5-e3-e1-c1, c3-a5-c5, g9-i7-g7-g5-e5-e3-e1-c3-a5, a3-c3-e1-e3-e5-g5-g7-i7-g9, i9-i7-g7-g5-e5-e3-e1-c3-a3, a4-c4-c6, e9-g7-g5-e5-e3-e1-c3, c6-d6, i5-i7-g7-g5-e5-e3-e1, a2-a4-c4-c6-e6-g4, i8-g8-e8-e6-c6-c4-a4-a2, d2-f2, i6-g8-e8-e6-c6-c4-a4, b3-d3-f1-f3-d5-d7-f7-h5, f5-f7-d7-d5-f3-f1-d3-b3, d1-f1-d3, h8-h6-h4-f4-d4-d2, b1-d1-f1-f3-d5-d7-f7-f5, h9-f9-f7-d7-d5-f3-f1-d1-b1, b4-b6-d4-f4-h4-h6-h8, h7-h9-f9-f7-d7-d5-f3-f1-d1, c2-c4-c6-e6-e8, g6-e6-c6-c4-c2, b2-b4-b6-d4-f4-h4-h6, f6-f4-d4-b6-b4-b2, d3-f1-f3-d5-d7-f7-f9-h9, f8-d8-f6-f4-d4-b6-b4 (red wins).

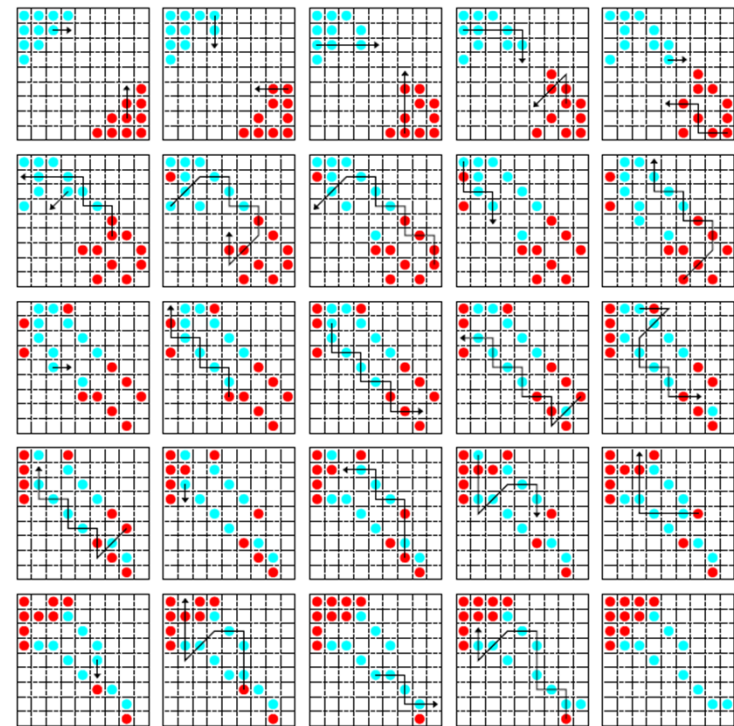


Figure 5: David Fabian's 30 move game of Chinese Checkers.

In summary, some of the properties that a short solution usually has are:

1. The winning player only jumps (on odd boards).
2. The first ladder is built by both players during the first  $\alpha$  moves.
3. The second ladder is completed by the losing player on moves after  $\alpha$ .

<sup>1</sup> Bell, George, I. 2009. The Shortest Game of Chinese Checkers and Related Problems. <https://arxiv.org/pdf/0803.1245.pdf>

# Looking Forward

- Can we exploit 'higher dimensional solution space' (game permutations existing in 3D vs 2D) to make ever-shorter checker games using very efficient laddering?
- Will the 'ladder behavior' inherent in checkers combined with higher permutation space (and an implementation of A\* shortest path as in the paper below) yield shorter games than those that have been documented below?

[The Shortest Game of Chinese Checkers and Related Problems](#)

# Simple Goal-Oriented Motion (2D)

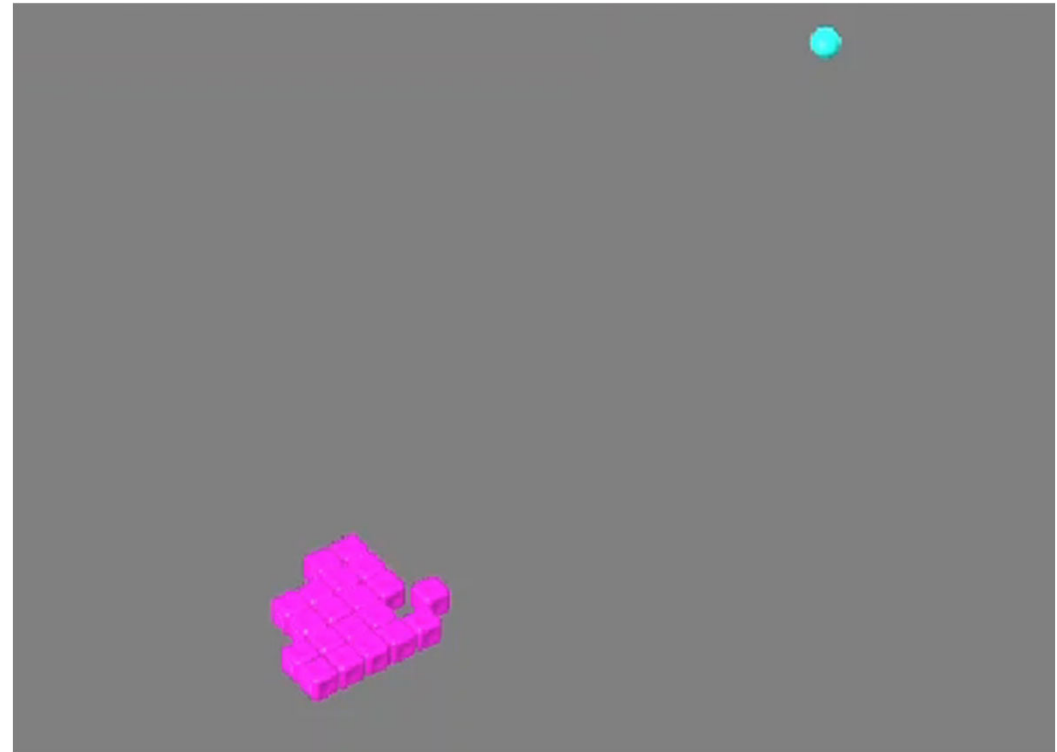
- Euclidean distance of neighbors to target.
- *Raycasts* and *box colliders* to determine neighbors and possible moves.

- One step at a time:

- Either

- 1, 0, 0
    - 0, 1, 0
    - -1, 0, 0
    - 0, -1, 0

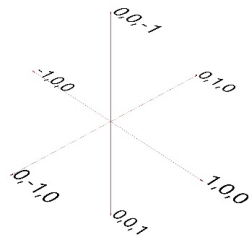
0,1,0  
-1,0,0      1,0,0  
0,-1,0  
Possible Moves/  
Neighbor  
Selection



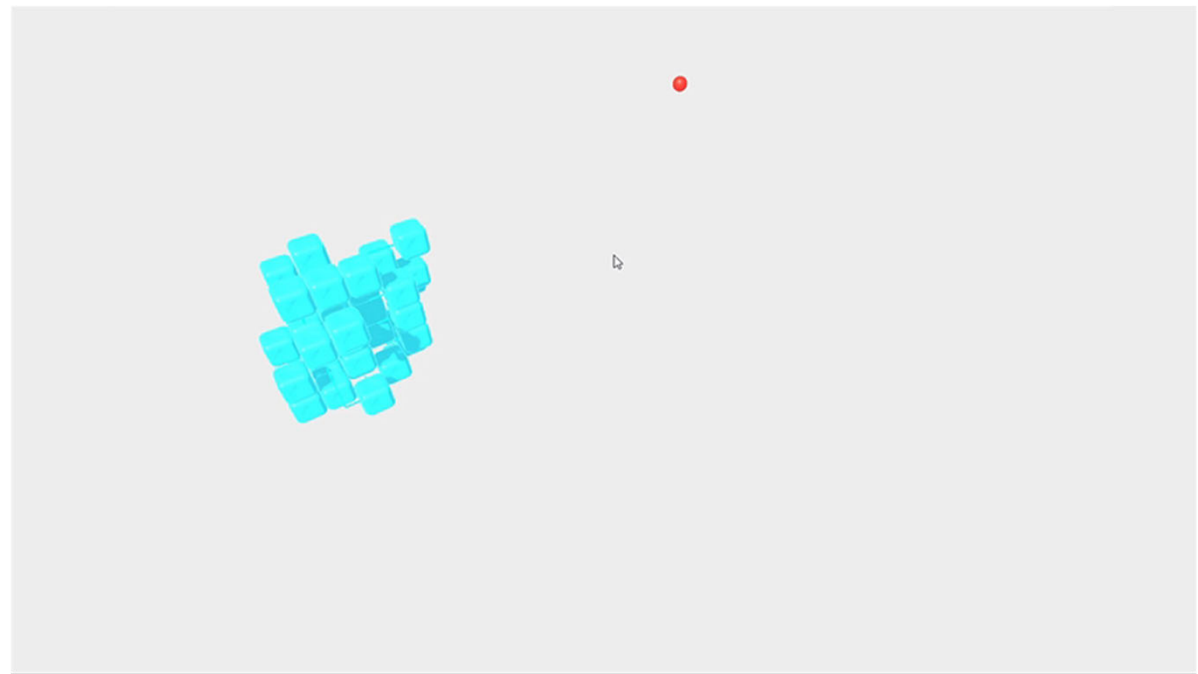
Progress Video

# Simple GOM(3D)

- Euclidean OR Manhattan distance of neighbors to target.
- *Raycasts* and *box colliders* to determine neighbors and possible moves.
- One step at a time:
  - Either
    - 1, 0, 0
    - 0, 1, 0
    - 0, 0, 1
    - -1, 0, 0
    - 0, -1, 0
    - 0, 0, -1



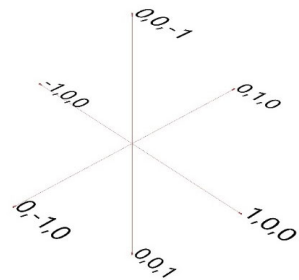
Possible Moves/  
Neighbor  
Selection



Progress Video

# Simple GOM 2 Teams(3D)

- Incorporating *LineRenderer* to visualize paths taken
- One step at a time:
  - Either
    - 1, 0, 0
    - 0, 1, 0
    - 0, 0, 1
    - -1, 0, 0
    - 0, -1, 0
    - 0, 0, -1



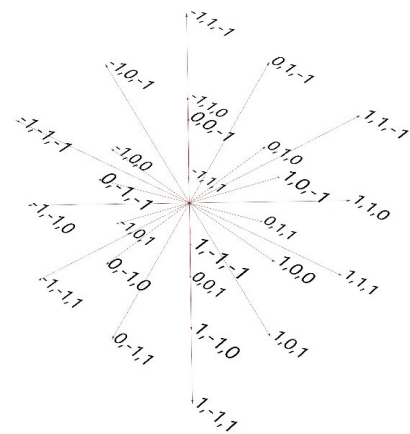
Possible Moves/  
Neighbor  
Selection



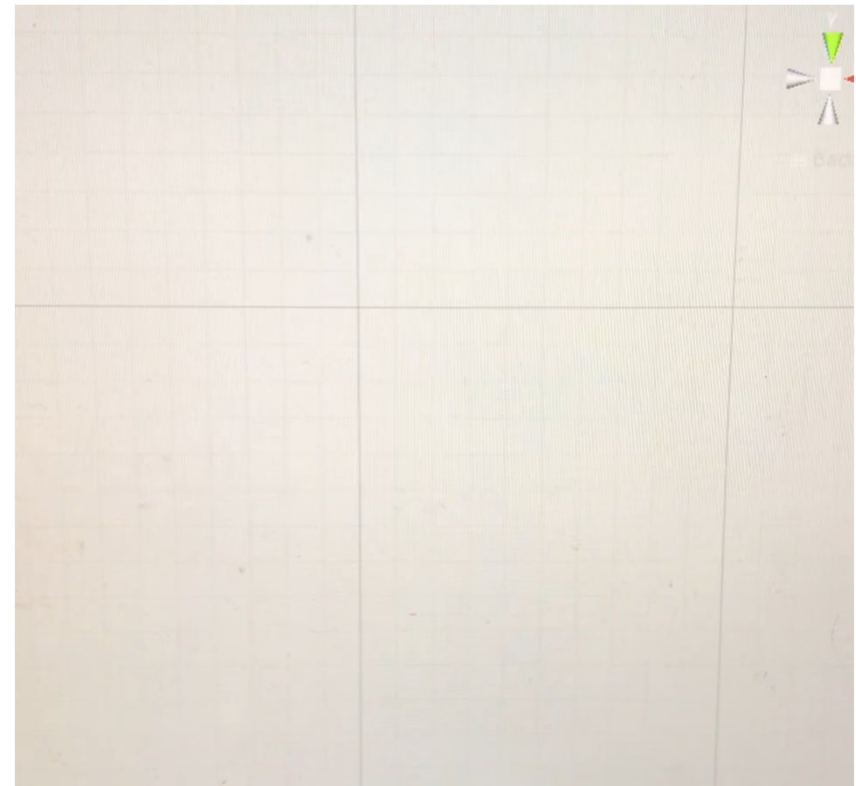
Progress Video

# Refactored GOM 2 Teams(3D)

- [,] Array with Cell states
  - Better Performance
  - Path-related querying abilities
- Incorporating *LineRenderer* to visualize paths taken
- One step at a time:
  - Either
    - 1,0,0
    - 0,1,0
    - 1,1,0
    - 1,-1,0
    - -1,1,0
    - -1,-1,0
    - -1,-1,-1
    - 1,1,-1
    - 1,-1,-1
    - 1,-1,1
    - 1,1,1
    - -1,-1,1
    - -1,1,1
    - 0,1,1
    - 0,-1,-1
    - 0,-1,1
    - 0,1,-1
    - 1,0,1
    - 1,0,-1
    - -1,0,1
    - -1,0,-1
    - 0,0,1
    - -1,0,0
    - 0,-1,0
    - 0,0,-1



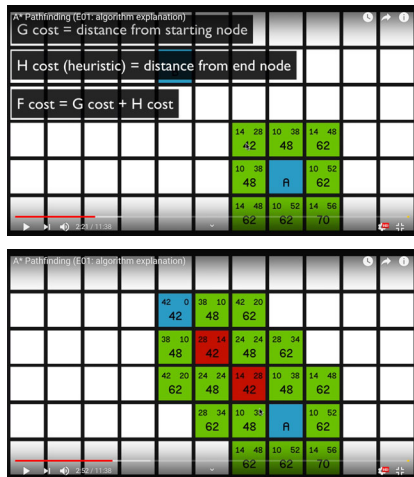
Possible Moves/  
Neighbor  
Selection



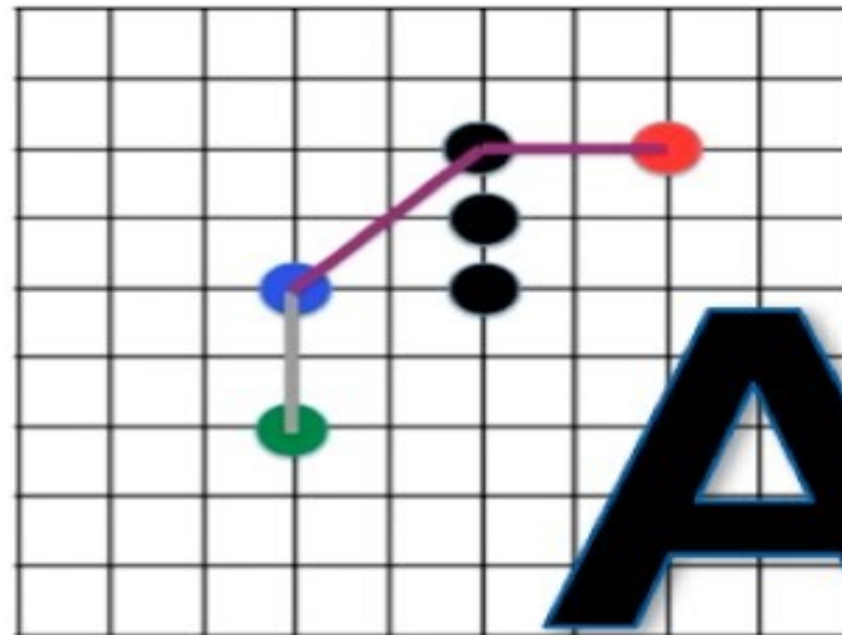
Progress Video



# A\* Shortest Path Review:



<https://www.youtube.com/watch?v=-L-WgKMFuhE>



- Node #
- X and Y Coords
- G-Cost
- H-Cost

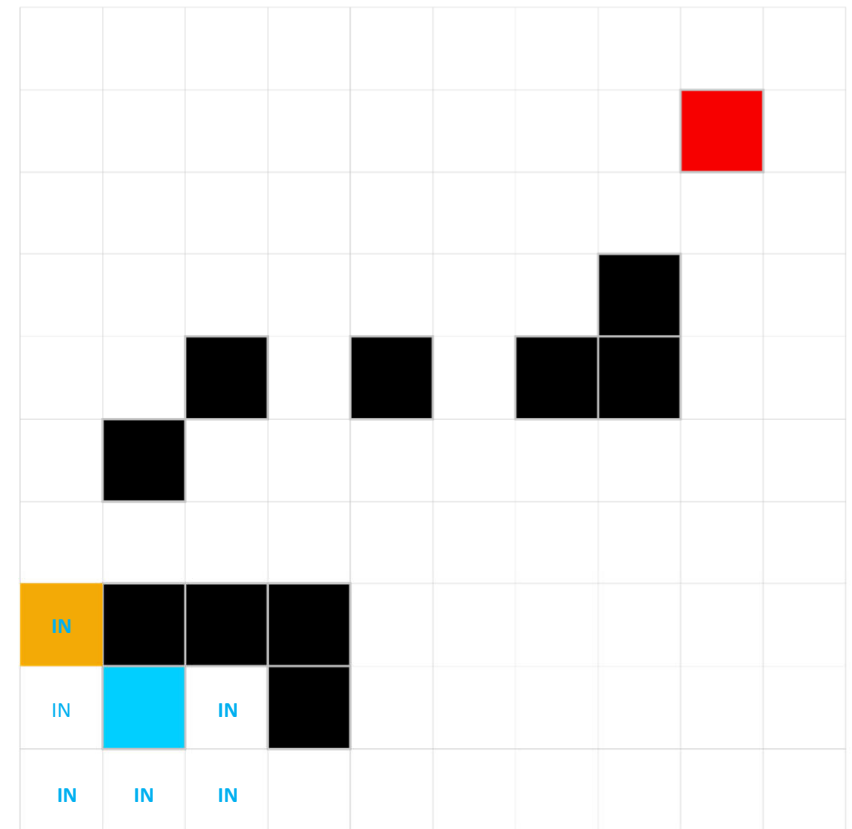
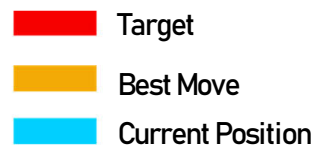


<https://www.youtube.com/watch?v=pKnV6ViDpAI>

(Go to 5: 40 for the comparison between Dijkstra's and A\* algorithms)

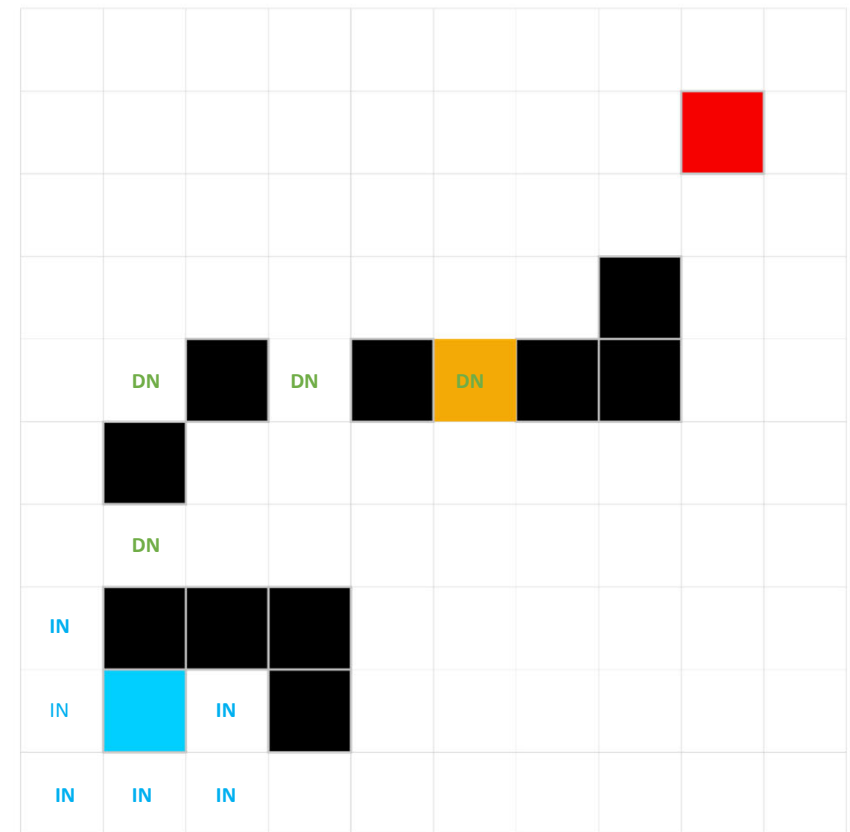
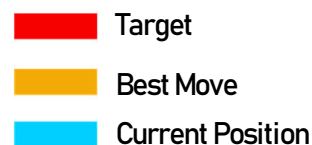
# Move Selection

1. Find Immediate Neighbors (IN) = 6
2. Sort neighbors by their Manhattan(to target) and their Euclidean(to active cell)
3. Make the best ranking move

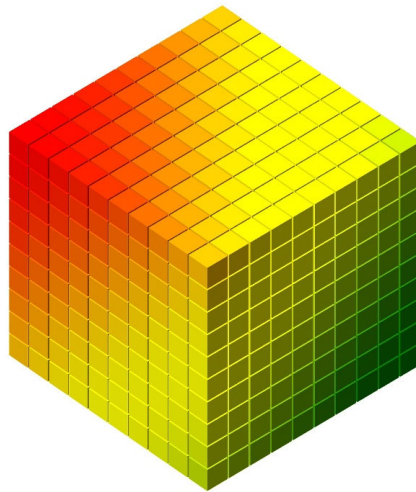


# Move Selection– Including Ladder

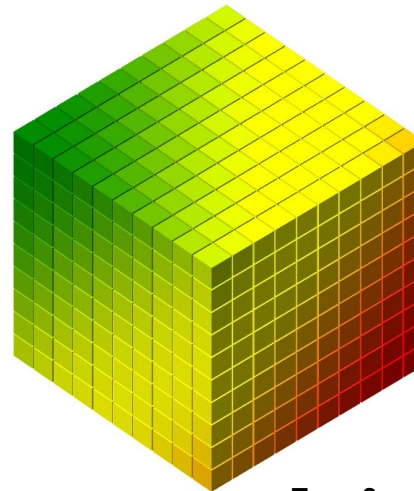
1. Find Immediate Neighbors (IN) = 6
2. Find Derived Neighbors (DN) = 4
3. Put IN and DN in one bucket = 10
4. Sort neighbors by their Manhattan(to target) and their Euclidean(to active cell)
5. Make the best ranking move



# Target Selection:



Team1



Team2

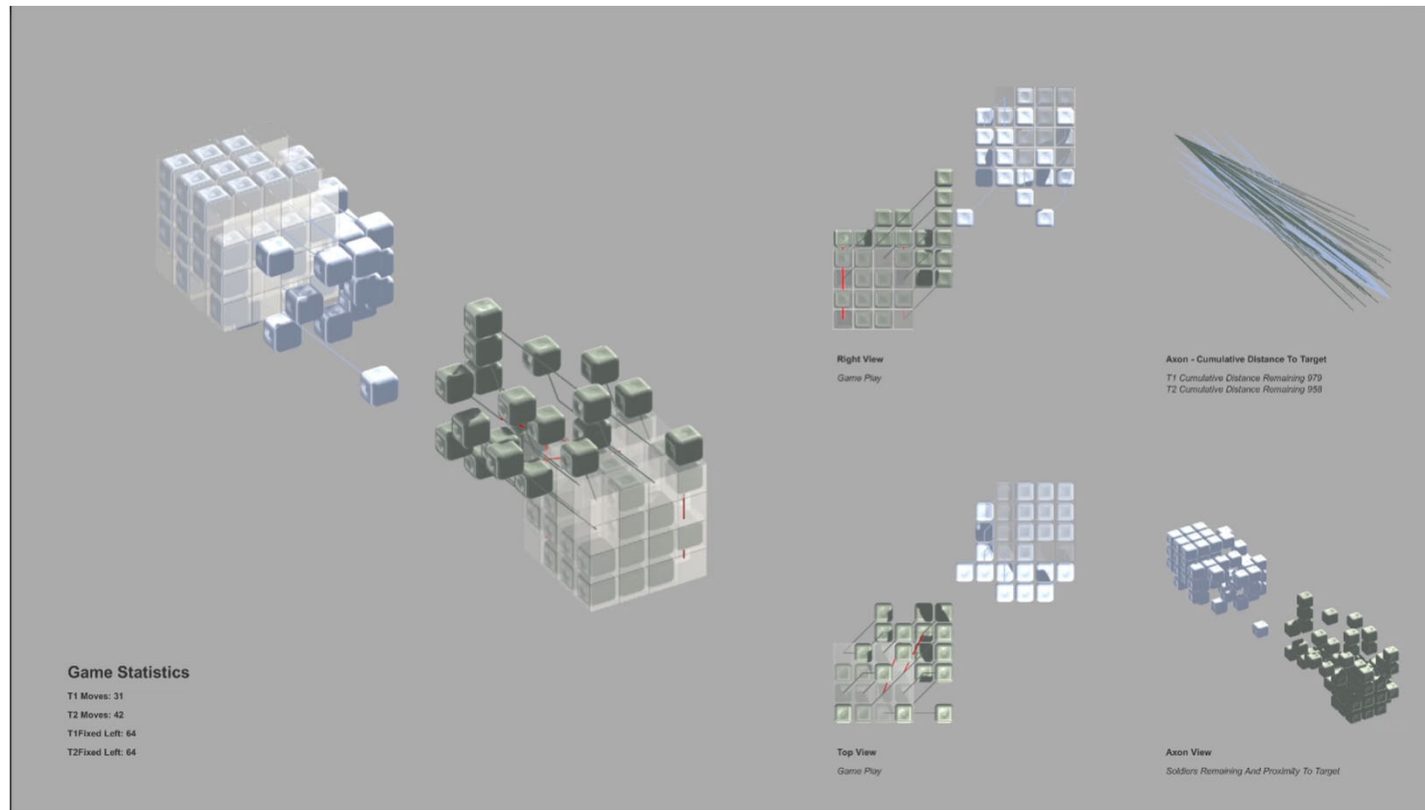
 First Targets  
 Last Targets

## Pseudocode

*(At each step):*

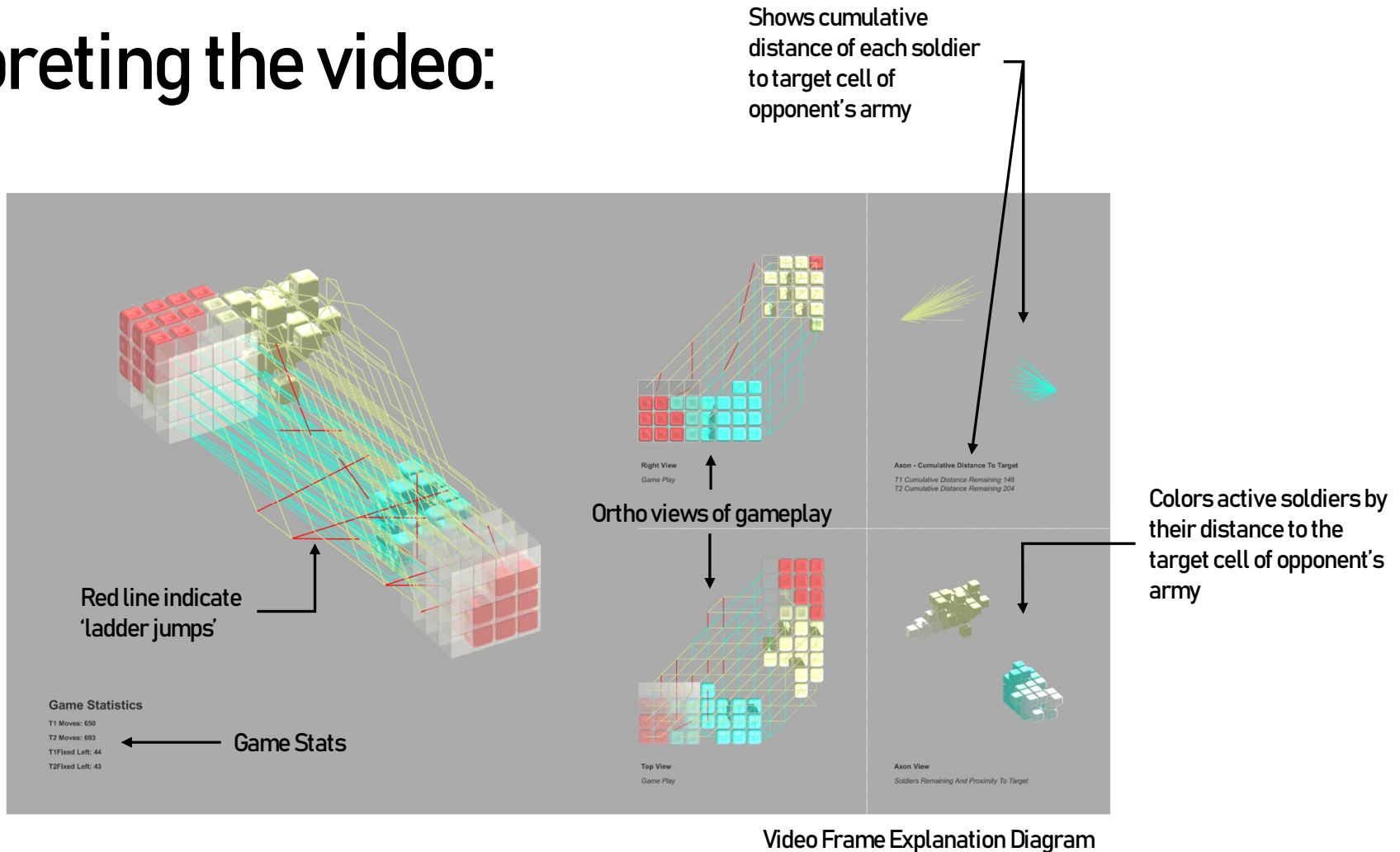
```
If(activeCell == Targets[0])  
    Targets.RemoveAt(0);
```

# Shortest Path Visualization:



Progress Video

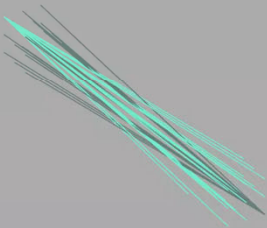
# Interpreting the video:



Final – 0:33



Right View  
*Game Play*



Axon - Cumulative Distance To Target  
*T1 Cumulative Distance Remaining 361  
T2 Cumulative Distance Remaining 360*



Top View  
*Game Play*



Axon View  
*Soldiers Remaining And Proximity To Target*

Game Statistics

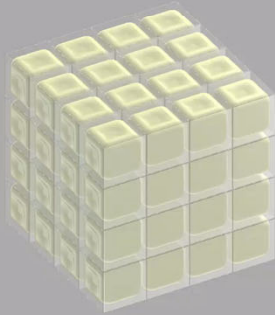
T1 Moves: 0  
T2 Moves: 1  
T1Fixed Left: 27  
T2Fixed Left: 27

Final – 2:42



Game Statistics

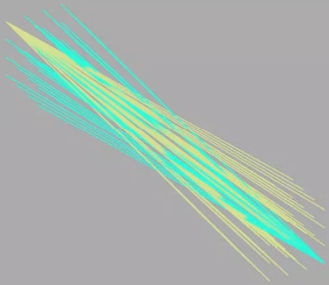
T1 Moves: 0  
T2 Moves: 0  
T1Fixed Left: 64  
T2Fixed Left: 64



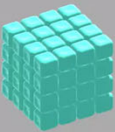
Right View  
Game Play



Top View  
Game Play



Axon - Cumulative Distance To Target  
T1 Cumulative Distance Remaining 1024  
T2 Cumulative Distance Remaining 1024

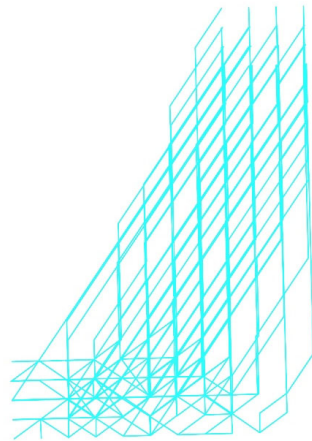


Axon View  
Soldiers Remaining And Proximity To Target

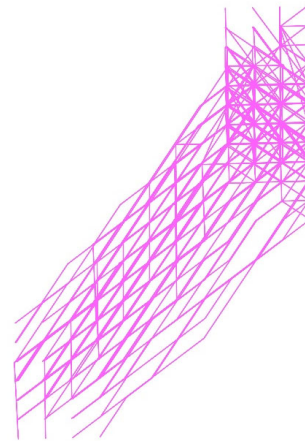


# CSV Export & Data Viz

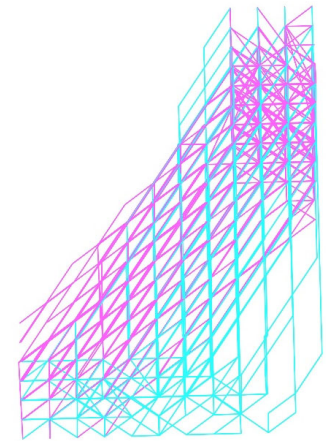
- These visualizations represent the data that might be used to train the game (positions, sequence, and number of moves).
- A hypothesis might be that the game with the smallest number of lines and the shortest sum of the lengths might be the shortest game.



Team1 Paths



Team2 Paths



Combined Paths

# Combinatorial Game Theory (CGT)

- Branch of mathematics and theoretical CS that typically studies sequential games with perfect information<sup>2</sup>
- What is a combinatorial game<sup>3</sup>:
  - Two players – *order in which they make moves is only distinguishing factor*
  - No chance (*only who gets to go first*).
  - Perfect information– *state of game fully visible by both players*
  - Turn based.
  - Absolute winner (*no ties or draws*).
  - Winning condition– *clearly defined by a 'terminal position'*
- “The type of games studied by CGT is also of interest in [artificial intelligence](#), particularly for [automated planning and scheduling](#). In CGT there has been less emphasis on refining practical [search algorithms](#) (such as the [alpha-beta pruning](#) heuristic included in most artificial intelligence textbooks), but more emphasis on descriptive theoretical results (such as measures of [game complexity](#) or proofs of optimal solution existence without necessarily specifying an algorithm, such as the [strategy-stealing argument](#)).”<sup>4</sup>

<sup>2</sup>[https://en.wikipedia.org/wiki/Combinatorial\\_game\\_theory](https://en.wikipedia.org/wiki/Combinatorial_game_theory)

<sup>3</sup>[https://is.muni.cz/th/325040/fi\\_b/Combinatorial\\_games.pdf](https://is.muni.cz/th/325040/fi_b/Combinatorial_games.pdf)

<sup>4</sup>[https://en.wikipedia.org/wiki/Combinatorial\\_game\\_theory](https://en.wikipedia.org/wiki/Combinatorial_game_theory)