Matam Kushagra 23B179

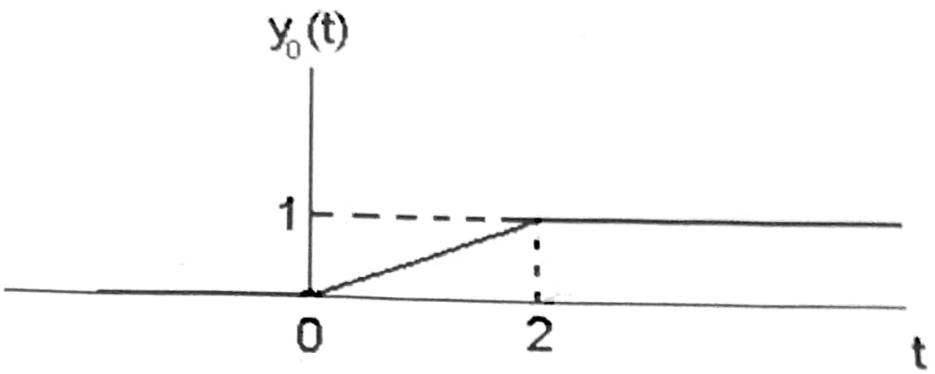
EE229: Signal Processing-1
Autumn Semester 2024-25
Instructor: Prof. Vikram Gadre

Class Test - 1
29 August 2024
Marks: 20

Question 1) We are given a certain linear time-invariant system with impulse response $h_0(t)$. We are told that when the input is $x_0(t)$, the output is $y_0(t)$, which is sketched in the figure below. We are then given the following set of inputs to linear time-invariant systems with the indicated impulse responses:

Part	Input $x(t)$	Impulse response $h(t)$
(a)	$x(t) = x_0(t) - x_0(t-2)$	$h(t) = h_0(t)$
(b)	$x(t) = x_0(-t)$	$h(t) = h_0(-t)$
(c)	$x(t) = \dot{x}_0(t)$	$h(t) = h_0(t)$

[Here $\dot{x}_0(t)$ and $\dot{h}_0(t)$ denote the first derivative of $x_0(t)$ and $h_0(t)$, respectively.]



$y_0(t)$

In each of these cases, determine whether or not we have enough information to determine output $y(t)$ when the input is $x(t)$ and the system has impulse response $h(t)$. If it is possible to determine $y(t)$, provide an accurate sketch of it with numerical values clearly indicated on the graph. **[9 marks]**

Question 2) Recall that the convolution of two integrable analog signals $x(t)$ and $h(t)$ is given by

$$y(t) = \int_{\mathbb{R}} h(\tau)x(t-\tau)d\tau.$$

Show that

$$\int_{\mathbb{R}} y(t)dt = \int_{\mathbb{R}} x(t)dt \int_{\mathbb{R}} h(t)dt.$$

[5 marks]

Question 3) Consider a periodic train of impulses given by $s(t) = \sum_{n\in\mathbb{Z}} \delta(t - nT)$.

1. Let $x(t)$ be an isosceles triangle of base-width $2T$ and height α.
2. Let $x(t)$ be an isosceles triangle of base-width T and height α.

In each of the cases above, find and plot $s(t) * x(t)$. **[6 marks]**