EE229 : Signal Processing - 1
Class test 1 Solutions

1 a) $$y(t) = x(t) * h(t)$$

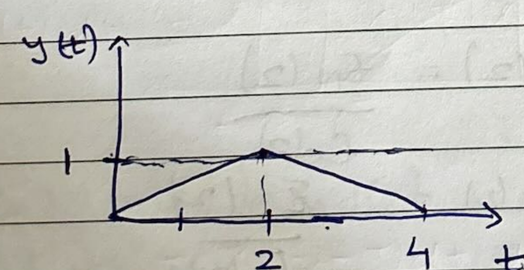
$$\Rightarrow y(t) = \int_{-\infty}^{\infty} x(\tau) h(t-\tau)\, d\tau$$

$$= \int_{-\infty}^{\infty} \left(x_0(\tau) - x_0(\tau-2)\right) h_0(t-\tau)\, d\tau$$

$$= \int_{-\infty}^{\infty} x_0(\tau) h_0(t-\tau)\, d\tau - \int_{-\infty}^{\infty} x_0(\tau-2) h_0(t-\tau)\, d\tau$$

$$= \int_{-\infty}^{\infty} x_0(\tau) h_0(t-\tau)\, d\tau - \int_{-\infty}^{\infty} x_0(\tau) h_0(t-\tau-2)\, d\tau$$

$$= y_0(t) - y_0(t-2)$$



b) $$y(t) = \int_{-\infty}^{\infty} x(\tau) h(t-\tau)\, d\tau$$

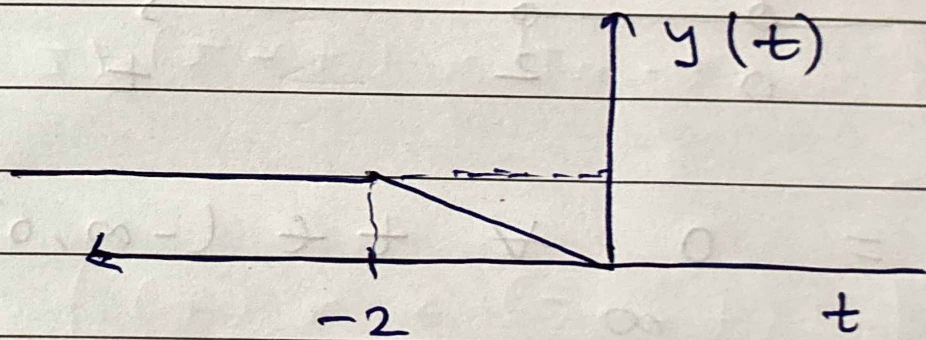
$$= \int_{-\infty}^{\infty} x_0(-\tau) h_0(\tau - t)\, d\tau$$

Substitute $\tau := -k$

$$= \int_{+\infty}^{-\infty} - x_0(k)\, h_0(-k-t)\, dk$$

$$= \int_{-\infty}^{\infty} x_0(k)\, h_0(-t-k)\, dk$$

$$= y_0(-t)$$



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c) $$y_0(t) = \int_{-\infty}^{\infty} x_0(\tau)\, h_0(t-\tau)\, d\tau$$

Taking derivative w.r.t t

$$\Rightarrow \dot{y}_0(t) = \int_{-\infty}^{\infty} x_0(\tau)\, \dot{h}_0(t-\tau)\, d\tau$$

$$\Rightarrow \dot{y}_0(t) = \int_{-\infty}^{\infty} x_0(t-\tau)\, \dot{h}_0(\tau)\, d\tau$$

Taking derivative w.r.t t

$$\Rightarrow \ddot{y}_0(t) = \int_{-\infty}^{\infty} \dot{x}_0(t-\tau)\, \dot{h}_0(\tau)\, d\tau$$

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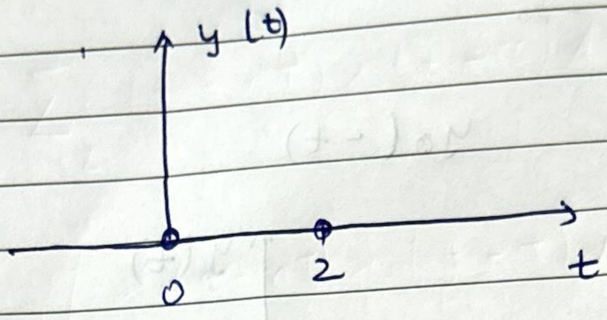
$\Rightarrow \quad \ddot{y}_0(t) = \dot{x}_0(t) * \dot{h}_0(t)$

$\Rightarrow \quad \ddot{y}_0(t) = y(t)$



$$y(t) = 0 \quad \forall \; t \in (-\infty, 0) \cup (0,2) \cup (2,\infty)$$

2 $$y(t) = \int_{-\infty}^{\infty} h(\tau)\, x(t-\tau)\, d\tau$$

To prove : $\displaystyle\int_{-\infty}^{\infty} y(t)\,dt = \int_{-\infty}^{\infty} x(t)\,dt \int_{-\infty}^{\infty} h(t)\,dt$

$$\Rightarrow \int_{-\infty}^{\infty} y(t)\,dt = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} h(\tau)\, x(t-\tau)\, d\tau\, dt$$

$$= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} h(\tau)\, x(t-\tau)\, dt\, d\tau$$

$$= \int_{-\infty}^{\infty} h(\tau) \int_{-\infty}^{\infty} x(t-\tau)\, dt\, d\tau$$

Apply Integration by parts w.r.t ~~t~~ τ

$$= \int_{-\infty}^{\infty} x(t-\tau)\,dt \int_{\cancel{t}} h(\tau)\,d\tau \,\Big|_{-\infty}^{\infty}$$

$$- \int_{-\infty}^{\infty} \frac{d}{d\tau}\left(\int_{-\infty}^{\infty} x(t-\tau)\,dt\right)\cdot\left(\int h(\tau)\,d\tau\right)d\tau$$

$$= \int_{-\infty}^{\infty} x(t)\,dt \int_{-\infty}^{\infty} h(\tau)\,d\tau$$

$$- \int_{-\infty}^{\infty}\left(-\int_{-\infty}^{\infty} x'(t-\tau)\,dt\right)\cdot\left(\int h(\tau)\,d\tau\right)d$$

$$= \int_{-\infty}^{\infty} x(t)\,dt \int_{-\infty}^{\infty} h(t)\,dt + \int_{-\infty}^{\infty}\left[x(t-\cancel{\tau})\Big|_{-\infty}^{\infty}\right]\left(\int h(\tau)\,d\tau\right)d$$

$$= \int_{-\infty}^{\infty} x(t) \int_{-\infty}^{\infty} h(t)\,dt + \int_{-\infty}^{\infty} (0)\cdot\left(\int h(\tau)\,d\tau\right)d\tau$$

$$= \int_{-\infty}^{\infty} x(t) \int_{-\infty}^{\infty} h(t)\,dt$$

Underlying Assumption $\lim_{t \to \infty} x(t) = 0$ and

$\lim_{t \to -\infty} x(t) = 0$

Q3 $\quad s(t) = \sum_{n \in Z} \delta(t - nT)$

$$s(t) * x(t) = \int_R s(\tau)\, x(t-\tau)\, d\tau$$

$$= \int_R \sum_{n \in Z} \delta(t - nT)\, x(t-\tau)\, d\tau$$

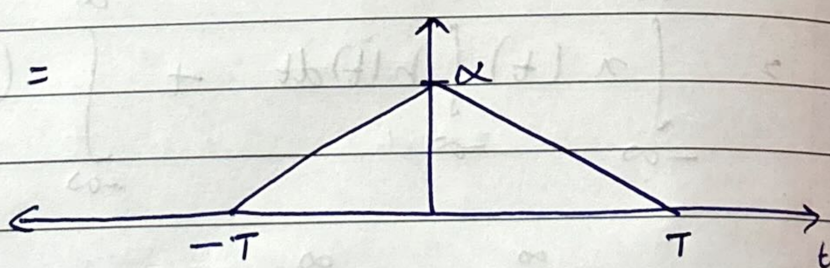
$$= \sum_{n \in Z} \int_R \delta(t - nT)\, x(t-\tau)\, d\tau$$

$$= \sum_{n \in Z} \left(\delta(t - nT) * x(t) \right)$$

Using sifting property of $\delta(t)$

$$= \sum_{n \in Z} x(t - nT)$$

a) $x(t) =$



$$\sum_{n \in Z} x(t - nT)$$

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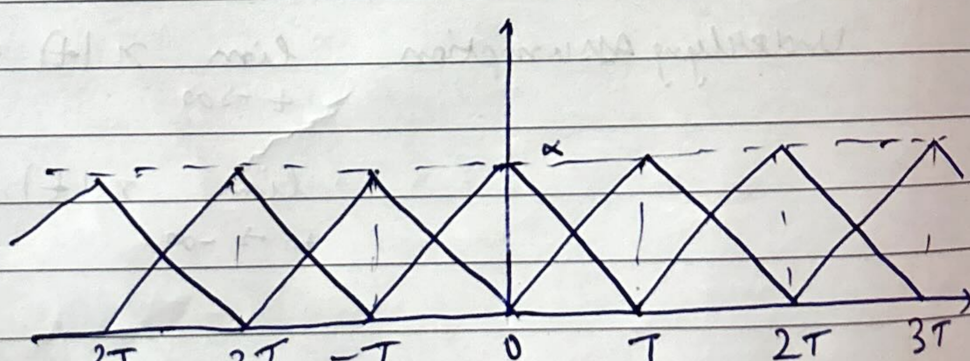

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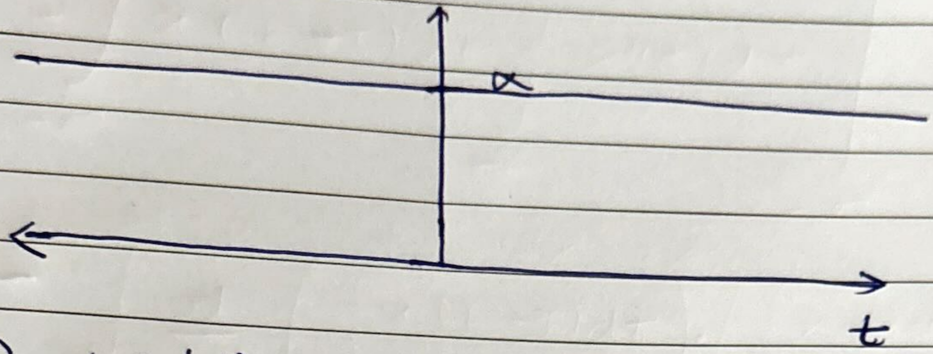
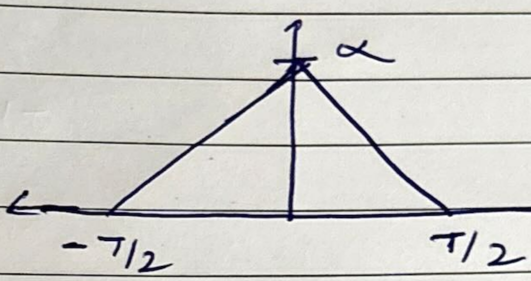


$\Rightarrow \quad s(t) * x(t) = \alpha \quad \forall\ t$

b) $\quad x(t) =$



$$\sum_{n \in \mathbb{Z}} x(t - nT)$$

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