A Crash Course on Data Compression

5. Dictionary-based Compressors

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@jermp

Overview

- LZ77
- LZSS
- LZ78
- LZW

The Dictionary-based Coding Problem

- **Problem.** We are given a list $L[1..n]$ of n symbols and we are asked to compress it in *as few as possible bits*.

- **Idea.** Assume to have a *dictionary* of m strings, $D[1..m]$.
  If the substring $L[i\,..\,j]$ is equal to the string $D[k]$, then we can represent $L[i\,..\,j]$ with k.

- Simple idea but very effective when L is *repetitive,* i.e., it has many equal substrings.

- **Q.** How to build the dictionary D such that L is compressed effectively?
  (As effectively as possible?)

LZ77

Lempel and Ziv, 1977

- **Idea.** The dictionary D is *not explicitly built*, rather it is logically represented by *all* the substrings of $L[i - W \mathinner{..} i]$, where i is the length of the prefix of L processed by the algorithm and $W > 0$ is a parameter.

- **Algorithm.**
  - At the beginning: $i = 1$.
  - At each step: determine the *longest common prefix* (lcp) between $L[i \mathinner{..} n]$ and the substring starting (at most) W positions before i but possibly ending in $L[i \mathinner{..} n]$.
  - If the lcp is found at distance d from i, the algorithm emits the triple $\langle d, |lcp|, c \rangle$, where c is the next character following the lcp.
  - Then the algorithm advances by $|lcp| + 1$ characters, that is: $i = i + |lcp| + 1$.

- The algorithm is sometimes referred to as the process of *parsing* L into *phrases*, the actual integer triples. The triples are then compressed using another coding method (e.g., a static code or Huffman).

- The parameter W (the “window” size) controls a trade-off between encoding/decoding speed and compression ratio.

LZ77 — Encoding Demo

Suppose we have $W = 8$.

$$L = \overset{0}{|}A B R A C A D A B R A B R A C A C A$$

LZ77 — Encoding Demo

Suppose we have $W = 8$.

$$L = \overset{0}{|}A\overset{1}{|}BRACADABRABRACACA$$

$$1 : \langle 0,0,A \rangle$$

LZ77 — Encoding Demo

Suppose we have $W = 8$.

$$L = \overset{0}{|}A\overset{1}{|}B\overset{2}{|}R\,A\,C\,A\,D\,A\,B\,R\,A\,B\,R\,A\,C\,A\,C\,A$$

$$1 : \langle 0,0,A \rangle$$

$$2 : \langle 0,0,B \rangle$$

LZ77 — Encoding Demo

Suppose we have $W = 8$.

$$L = \overset{0}{|}A\overset{1}{|}B\overset{2}{|}R\overset{3}{|}A\,C\,A\,D\,A\,B\,R\,A\,B\,R\,A\,C\,A\,C\,A$$

$$1 : \langle 0,0,A \rangle$$

$$2 : \langle 0,0,B \rangle$$

$$3 : \langle 0,0,R \rangle$$

LZ77 — Encoding Demo

Suppose we have $W = 8$.

$$L = \overset{0}{|}\underline{A}\overset{1}{|}B\overset{2}{|}R\overset{3}{|}A\,C\,A\,D\,A\,B\,R\,A\,B\,R\,A\,C\,A\,C\,A$$

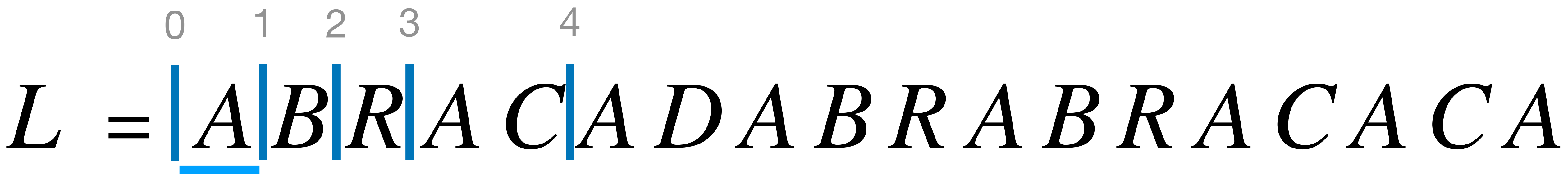
$$1 : \langle 0,0,A \rangle$$

$$2 : \langle 0,0,B \rangle$$

$$3 : \langle 0,0,R \rangle$$

LZ77 — Encoding Demo

Suppose we have $W = 8$.



$$L = |\underline{A}|B|R|A\ C|A\ D\ A\ B\ R\ A\ B\ R\ A\ C\ A\ C\ A$$

$$1 : \langle 0,0,A \rangle$$

$$2 : \langle 0,0,B \rangle$$

$$3 : \langle 0,0,R \rangle$$

$$4 : \langle 3,1,C \rangle$$

LZ77 — Encoding Demo

Suppose we have $W = 8$.

0 1 2 3 4

$$L = |\underline{A}|B|R|\underline{A}\ C|A\ D\ A\ B\ R\ A\ B\ R\ A\ C\ A\ C\ A$$

$1 : \langle 0,0,A \rangle$

$2 : \langle 0,0,B \rangle$

$3 : \langle 0,0,R \rangle$

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LZ77 — Encoding Demo

Suppose we have $W = 8$.

$$L = \overset{0}{|}\underline{A}\overset{1}{|}B\overset{2}{|}R\overset{3}{|}\underline{A}\,C\overset{4}{|}A\,D\overset{5}{|}A\,B\,R\,A\,B\,R\,A\,C\,A\,C\,A$$

$$1 : \langle 0,0,A \rangle$$

$$2 : \langle 0,0,B \rangle$$

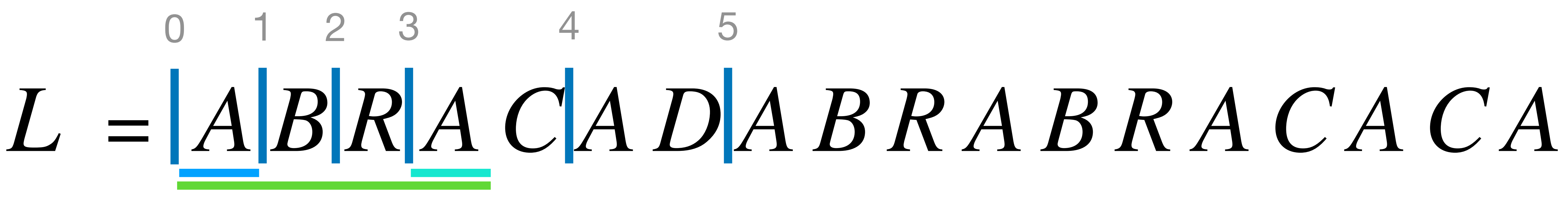
$$3 : \langle 0,0,R \rangle$$

$$4 : \langle 3,1,C \rangle$$

$$5 : \langle 2,1,D \rangle$$

LZ77 — Encoding Demo

Suppose we have $W = 8$.



$L = |A|B|R|A\,C|A\,D|A\,B\,R\,A\,B\,R\,A\,C\,A\,C\,A$

$1 : \langle 0,0,A \rangle$

$2 : \langle 0,0,B \rangle$

$3 : \langle 0,0,R \rangle$

$4 : \langle 3,1,C \rangle$

$5 : \langle 2,1,D \rangle$

LZ77 — Encoding Demo

Suppose we have $W = 8$.

$$L = \overset{0}{|}\underline{\underline{A}\overset{1}{|}B\overset{2}{|}R\overset{3}{|}\underline{A}}\,C\overset{4}{|}A\,D\overset{5}{|}A\,B\,R\,A\,B\overset{6}{|}R\,A\,C\,A\,C\,A$$

$1 : \langle 0,0,A \rangle$

$2 : \langle 0,0,B \rangle$

$3 : \langle 0,0,R \rangle$

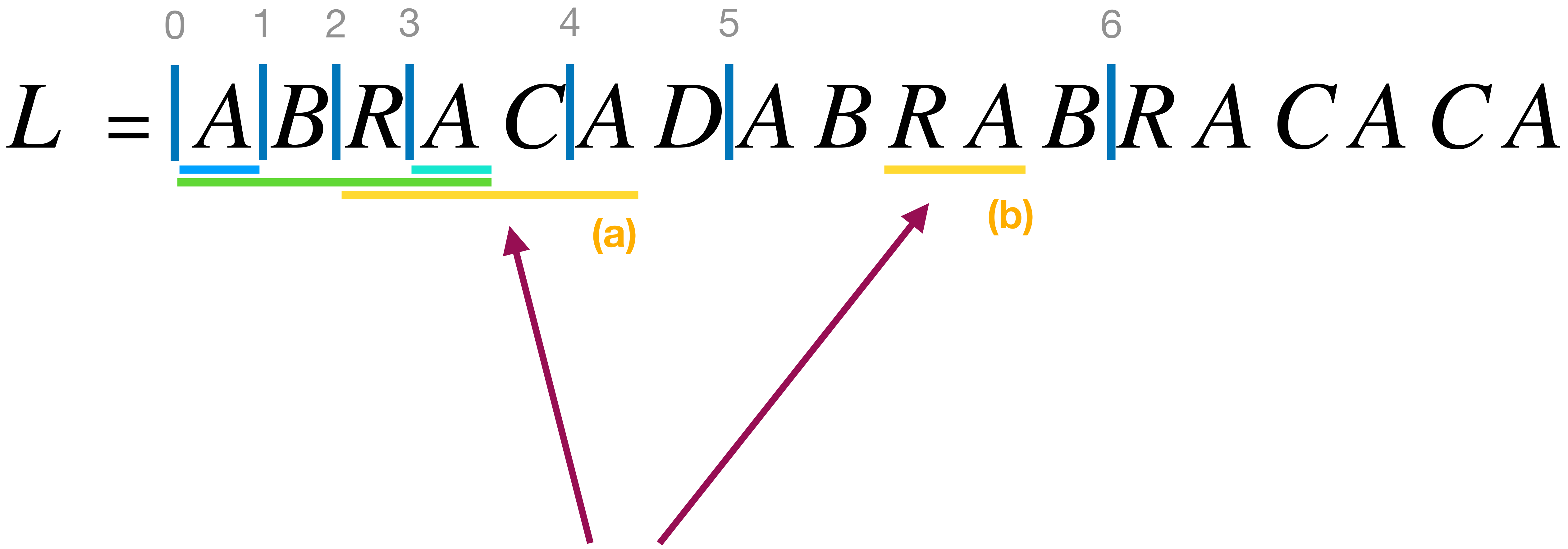
$4 : \langle 3,1,C \rangle$

$5 : \langle 2,1,D \rangle$

$6 : \langle 7,4,B \rangle$

LZ77 — Encoding Demo

Suppose we have $W = 8$.



$$L = |A|B|R|A\ C|A\ D|A\ B\ R\ A\ B|R\ A\ C\ A\ C\ A$$

The match **(a)** is at distance $d = 10$, so it is **outside** the window of size $W = 8$ (i.e., $d > W$). Therefore we have to discard it and take the match **(b)** at distance $d = 3$.

$1 : \langle 0,0,A \rangle$

$2 : \langle 0,0,B \rangle$

$3 : \langle 0,0,R \rangle$

$4 : \langle 3,1,C \rangle$

$5 : \langle 2,1,D \rangle$

$6 : \langle 7,4,B \rangle$

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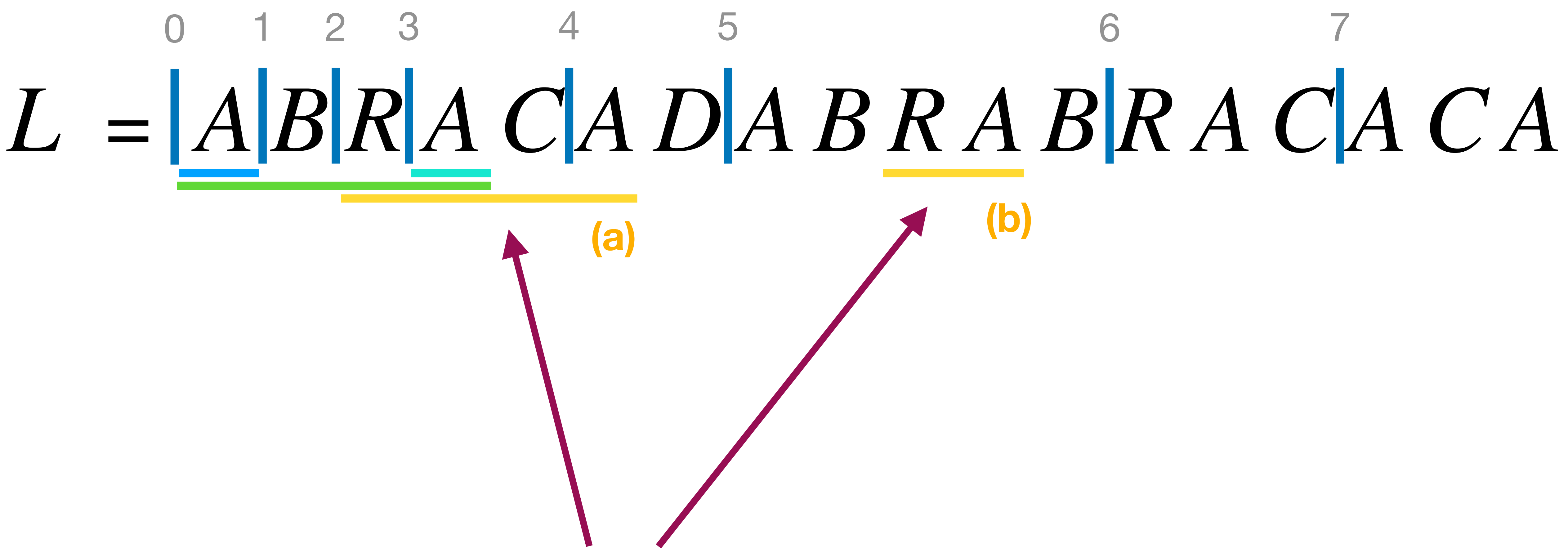

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$2 : \langle 0,0,B \rangle$

$3 : \langle 0,0,R \rangle$

$4 : \langle 3,1,C \rangle$

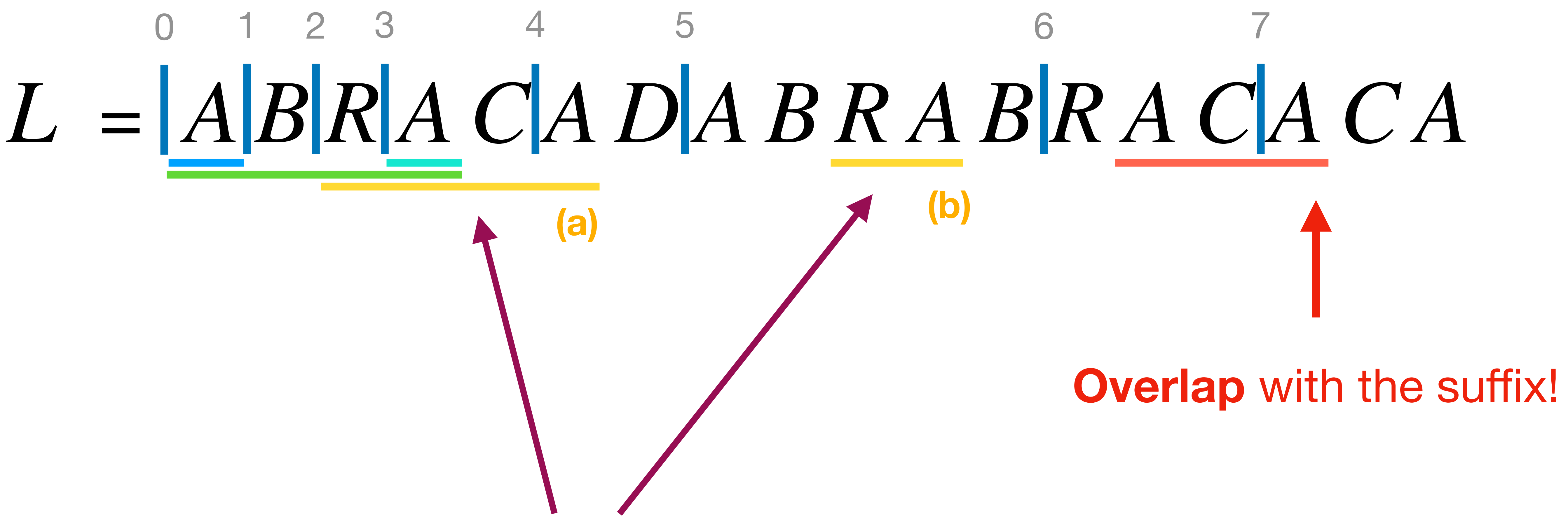
$5 : \langle 2,1,D \rangle$

$6 : \langle 7,4,B \rangle$

$7 : \langle 3,2,C \rangle$

LZ77 — Encoding Demo

Suppose we have $W = 8$.


0 1 2 3 4 5 6 7
$L = |A|B|R|A\ C|A\ D|A\ B\ R\ A\ B|R\ A\ C|A\ C\ A$
(a)
(b)
Overlap with the suffix!

The match **(a)** is at distance $d = 10$, so it is **outside** the window of size $W = 8$ (i.e., $d > W$). Therefore we have to discard it and take the match **(b)** at distance $d = 3$.

$1 : \langle 0,0,A \rangle$

$2 : \langle 0,0,B \rangle$

$3 : \langle 0,0,R \rangle$

$4 : \langle 3,1,C \rangle$

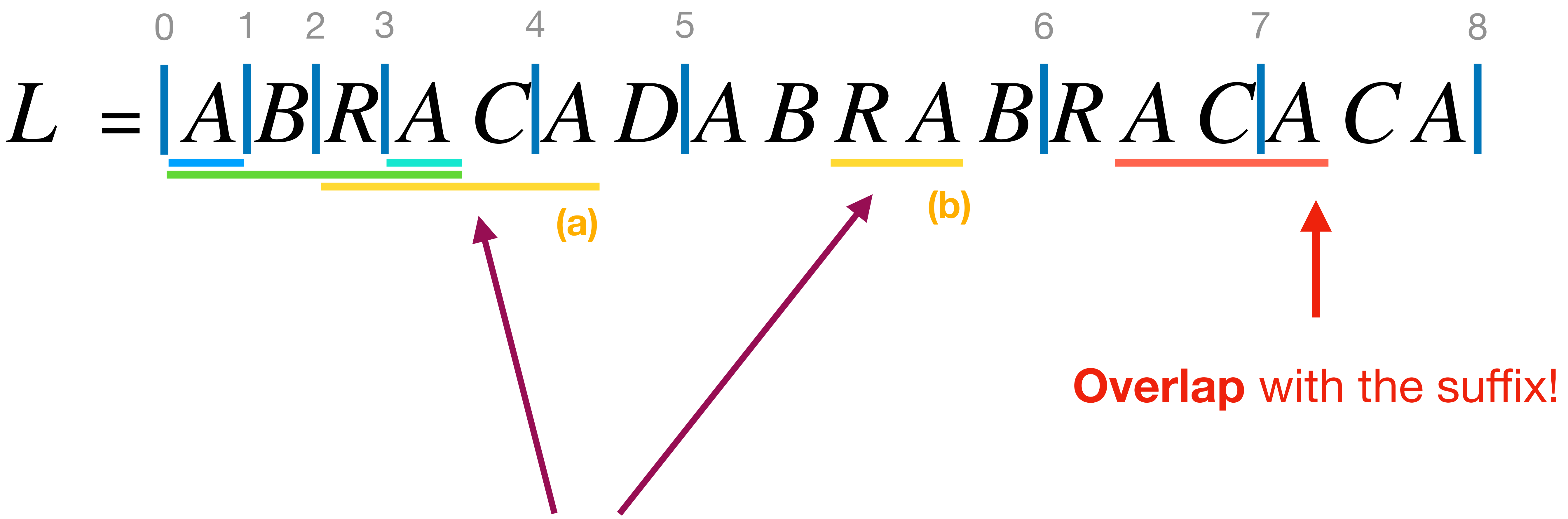
$5 : \langle 2,1,D \rangle$

$6 : \langle 7,4,B \rangle$

$7 : \langle 3,2,C \rangle$

LZ77 — Encoding Demo

Suppose we have $W = 8$.



Overlap with the suffix!

The match **(a)** is at distance $d = 10$, so it is **outside** the window of size $W = 8$ (i.e., $d > W$). Therefore we have to discard it and take the match **(b)** at distance $d = 3$.

$1 : \langle 0,0,A \rangle$

$2 : \langle 0,0,B \rangle$

$3 : \langle 0,0,R \rangle$

$4 : \langle 3,1,C \rangle$

$5 : \langle 2,1,D \rangle$

$6 : \langle 7,4,B \rangle$

$7 : \langle 3,2,C \rangle$

$8 : \langle 2,3,\text{EOF} \rangle$

LZ77 — Decoding Demo

$L = ?$

$1 : \langle 0,0,A \rangle$

$2 : \langle 0,0,B \rangle$

$3 : \langle 0,0,R \rangle$

$4 : \langle 3,1,C \rangle$

$5 : \langle 2,1,D \rangle$

$6 : \langle 7,4,B \rangle$

$7 : \langle 3,2,C \rangle$

$8 : \langle 2,3,\text{EOF} \rangle$

LZ77 — Decoding Demo

$L = ?$

$1 : L = A$

$1 : \langle 0,0,A \rangle$

$2 : \langle 0,0,B \rangle$

$3 : \langle 0,0,R \rangle$

$4 : \langle 3,1,C \rangle$

$5 : \langle 2,1,D \rangle$

$6 : \langle 7,4,B \rangle$

$7 : \langle 3,2,C \rangle$

$8 : \langle 2,3,\text{EOF} \rangle$

LZ77 — Decoding Demo

$L = ?$

$1 : L = A$

$2 : L = A\,B$

$1 : \langle 0,0,A \rangle$

$2 : \langle 0,0,B \rangle$

$3 : \langle 0,0,R \rangle$

$4 : \langle 3,1,C \rangle$

$5 : \langle 2,1,D \rangle$

$6 : \langle 7,4,B \rangle$

$7 : \langle 3,2,C \rangle$

$8 : \langle 2,3,\text{EOF} \rangle$

LZ77 — Decoding Demo

$L = ?$

$1 : L = A$

$2 : L = A\,B$

$3 : L = A\,B\,R$

$1 : \langle 0,0,A \rangle$

$2 : \langle 0,0,B \rangle$

$3 : \langle 0,0,R \rangle$

$4 : \langle 3,1,C \rangle$

$5 : \langle 2,1,D \rangle$

$6 : \langle 7,4,B \rangle$

$7 : \langle 3,2,C \rangle$

$8 : \langle 2,3,\text{EOF} \rangle$

LZ77 — Decoding Demo

$L = ?$

$1 : L = A$

$2 : L = A\,B$

$3 : L = A\,B\,R$

$4 : L = A\,B\,R\,A\,C$

$1 : \langle 0,0,A \rangle$

$2 : \langle 0,0,B \rangle$

$3 : \langle 0,0,R \rangle$

$4 : \langle 3,1,C \rangle$

$5 : \langle 2,1,D \rangle$

$6 : \langle 7,4,B \rangle$

$7 : \langle 3,2,C \rangle$

$8 : \langle 2,3,\text{EOF} \rangle$

LZ77 — Decoding Demo

$L = ?$

$1 : L = A$

$2 : L = A\,B$

$3 : L = A\,B\,R$

$4 : L = A\,B\,R\,A\,C$

$5 : L = A\,B\,R\,A\,C\,A\,D$

$1 : \langle 0,0,A \rangle$

$2 : \langle 0,0,B \rangle$

$3 : \langle 0,0,R \rangle$

$4 : \langle 3,1,C \rangle$

$5 : \langle 2,1,D \rangle$

$6 : \langle 7,4,B \rangle$

$7 : \langle 3,2,C \rangle$

$8 : \langle 2,3,\text{EOF} \rangle$

LZ77 — Decoding Demo

$$L = ?$$

$$1 : L = A$$

$$2 : L = A\,B$$

$$3 : L = A\,B\,R$$

$$4 : L = A\,B\,R\,A\,C$$

$$5 : L = A\,B\,R\,A\,C\,A\,D$$

$$6 : L = A\,B\,R\,A\,C\,A\,D\,A\,B\,R\,A\,B$$

$$1 : \langle 0{,}0{,}A \rangle$$

$$2 : \langle 0{,}0{,}B \rangle$$

$$3 : \langle 0{,}0{,}R \rangle$$

$$4 : \langle 3{,}1{,}C \rangle$$

$$5 : \langle 2{,}1{,}D \rangle$$

$$6 : \langle 7{,}4{,}B \rangle$$

$$7 : \langle 3{,}2{,}C \rangle$$

$$8 : \langle 2{,}3{,}\mathsf{EOF} \rangle$$

LZ77 — Decoding Demo

$L = ?$	
$1 : L = A$	$1 : \langle 0,0,A \rangle$
$2 : L = A\,B$	$2 : \langle 0,0,B \rangle$
$3 : L = A\,B\,R$	$3 : \langle 0,0,R \rangle$
$4 : L = A\,B\,R\,A\,C$	$4 : \langle 3,1,C \rangle$
$5 : L = A\,B\,R\,A\,C\,A\,D$	$5 : \langle 2,1,D \rangle$
$6 : L = A\,B\,R\,A\,C\,A\,D\,A\,B\,R\,A\,B$	$6 : \langle 7,4,B \rangle$
$7 : L = A\,B\,R\,A\,C\,A\,D\,A\,B\,R\,A\,B\,R\,A\,C$	$7 : \langle 3,2,C \rangle$
	$8 : \langle 2,3,\text{EOF} \rangle$

LZ77 — Decoding Demo

$$L = ?$$

$$1 : L = A$$

$$2 : L = A\,B$$

$$3 : L = A\,B\,R$$

$$4 : L = A\,B\,R\,A\,C$$

$$5 : L = A\,B\,R\,A\,C\,A\,D$$

$$6 : L = A\,B\,R\,A\,C\,A\,D\,A\,B\,R\,A\,B$$

$$7 : L = A\,B\,R\,A\,C\,A\,D\,A\,B\,R\,A\,B\,R\boxed{A\,C\ ?}$$

$$1 : \langle 0,0,A \rangle$$

$$2 : \langle 0,0,B \rangle$$

$$3 : \langle 0,0,R \rangle$$

$$4 : \langle 3,1,C \rangle$$

$$5 : \langle 2,1,D \rangle$$

$$6 : \langle 7,4,B \rangle$$

$$7 : \langle 3,2,C \rangle$$

$$8 : \langle 2,3,\text{EOF} \rangle$$

LZ77 — Decoding Demo

$L = \ ?$

$1 : L = A$

$2 : L = A\,B$

$3 : L = A\,B\,R$

$4 : L = A\,B\,R\,A\,C$

$5 : L = A\,B\,R\,A\,C\,A\,D$

$6 : L = A\,B\,R\,A\,C\,A\,D\,A\,B\,R\,A\,B$

$7 : L = A\,B\,R\,A\,C\,A\,D\,A\,B\,R\,A\,B\,R\boxed{A\,C\ ?}$

$8 : L = A\,B\,R\,A\,C\,A\,D\,A\,B\,R\,A\,B\,R\,A\,C$

$1 : \langle 0,0,A\rangle$

$2 : \langle 0,0,B\rangle$

$3 : \langle 0,0,R\rangle$

$4 : \langle 3,1,C\rangle$

$5 : \langle 2,1,D\rangle$

$6 : \langle 7,4,B\rangle$

$7 : \langle 3,2,C\rangle$

$8 : \langle 2,3,\text{EOF}\rangle$

LZ77 — Decoding Demo

$L = ?$	
$1 : L = A$	$1 : \langle 0,0,A \rangle$
$2 : L = A\,B$	$2 : \langle 0,0,B \rangle$
$3 : L = A\,B\,R$	$3 : \langle 0,0,R \rangle$
$4 : L = A\,B\,R\,A\,C$	$4 : \langle 3,1,C \rangle$
$5 : L = A\,B\,R\,A\,C\,A\,D$	$5 : \langle 2,1,D \rangle$
$6 : L = A\,B\,R\,A\,C\,A\,D\,A\,B\,R\,A\,B$	$6 : \langle 7,4,B \rangle$
$7 : L = A\,B\,R\,A\,C\,A\,D\,A\,B\,R\,A\,B\,R\boxed{A\,C\,?}$	$7 : \langle 3,2,C \rangle$
$8 : L = A\,B\,R\,A\,C\,A\,D\,A\,B\,R\,A\,B\,R\,A\,C\boxed{A\,C\,?}$	$8 : \langle 2,3,\text{EOF} \rangle$

LZ77 — Decoding Demo

$L = ?$	
$1 : L = A$	$1 : \langle 0,0,A \rangle$
$2 : L = A\,B$	$2 : \langle 0,0,B \rangle$
$3 : L = A\,B\,R$	$3 : \langle 0,0,R \rangle$
$4 : L = A\,B\,R\,A\,C$	$4 : \langle 3,1,C \rangle$
$5 : L = A\,B\,R\,A\,C\,A\,D$	$5 : \langle 2,1,D \rangle$
$6 : L = A\,B\,R\,A\,C\,A\,D\,A\,B\,R\,A\,B$	$6 : \langle 7,4,B \rangle$
$7 : L = A\,B\,R\,A\,C\,A\,D\,A\,B\,R\,A\,B\,R\,\boxed{A\,C\,\boxed{?}}$	$7 : \langle 3,2,C \rangle$
$8 : L = A\,B\,R\,A\,C\,A\,D\,A\,B\,R\,A\,B\,R\,A\,C\,\boxed{A}\boxed{C\,?}$	$8 : \langle 2,3,\text{EOF} \rangle$
$8 : L = A\,B\,R\,A\,C\,A\,D\,A\,B\,R\,A\,B\,R\,A\,C\,A\,C\,A$	

LZSS

Storer and Szymanski, 1982

- **Idea.** Output *pairs*, not triples.
- **Observation 1.**
When the lcp has size 0, we always repeat the pair $\langle d = 0, |lcp| = 0 \rangle$. We can directly emit $\langle 0,c \rangle$.
- **Observation 2.**
When $|lcp| \neq 0$, we can avoid specifying the next character c, but just emit the pair $\langle d, |lcp| \rangle$ and made the algorithm advance by $|lcp|$ characters.

LZSS

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$L = ABRACADABRABRACACA$

$1 : \langle 0,0,A \rangle$

$2 : \langle 0,0,B \rangle$

$3 : \langle 0,0,R \rangle$

$4 : \langle 3,1,C \rangle$ →

$5 : \langle 2,1,D \rangle$

$6 : \langle 7,4,B \rangle$

$7 : \langle 3,2,C \rangle$

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[LZ77]

LZSS

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  We can directly emit $\langle 0,c \rangle$.

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  When $|lcp| \neq 0$, we can avoid specifying the next character c, but just emit the pair $\langle d, |lcp| \rangle$ and made the algorithm advance by $|lcp|$ characters.

$L = |ABRACADABRABRACACA$

1

$1 : \langle 0,0,A \rangle$

$2 : \langle 0,0,B \rangle$

$3 : \langle 0,0,R \rangle$

$4 : \langle 3,1,C \rangle$ →

$5 : \langle 2,1,D \rangle$

$6 : \langle 7,4,B \rangle$

$7 : \langle 3,2,C \rangle$

$8 : \langle 2,3,\text{EOF} \rangle$

[LZ77]

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LZSS

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$L = |A|BRACADABRABRACACA$

1 2

$1 : \langle 0,0,A \rangle$

$2 : \langle 0,0,B \rangle$

$3 : \langle 0,0,R \rangle$

$4 : \langle 3,1,C \rangle$

$5 : \langle 2,1,D \rangle$

$6 : \langle 7,4,B \rangle$

$7 : \langle 3,2,C \rangle$

$8 : \langle 2,3,\text{EOF} \rangle$

[LZ77]

→

$1 : \langle 0,A \rangle$

$2 : \langle 0,B \rangle$

LZSS

Storer and Szymanski, 1982

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$L = |A|B|R\,A\,C\,A\,D\,A\,B\,R\,A\,B\,R\,A\,C\,A\,C\,A$

1 2 3

$1 : \langle 0,0,A \rangle$

$2 : \langle 0,0,B \rangle$

$3 : \langle 0,0,R \rangle$

$4 : \langle 3,1,C \rangle$ →

$5 : \langle 2,1,D \rangle$

$6 : \langle 7,4,B \rangle$

$7 : \langle 3,2,C \rangle$

$8 : \langle 2,3,\text{EOF} \rangle$

[LZ77]

$1 : \langle 0,A \rangle$

$2 : \langle 0,B \rangle$

$3 : \langle 0,R \rangle$

LZSS

Storer and Szymanski, 1982

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$L = |A|B|R|A\,C\,A\,D\,A\,B\,R\,A\,B\,R\,A\,C\,A\,C\,A$

1 2 3 4

$1 : \langle 0,0,A \rangle$

$2 : \langle 0,0,B \rangle$

$3 : \langle 0,0,R \rangle$

$4 : \langle 3,1,C \rangle$ →

$5 : \langle 2,1,D \rangle$

$6 : \langle 7,4,B \rangle$

$7 : \langle 3,2,C \rangle$

$8 : \langle 2,3,\text{EOF} \rangle$

[LZ77]

$1 : \langle 0,A \rangle$

$2 : \langle 0,B \rangle$

$3 : \langle 0,R \rangle$

$4 : \langle 3,1 \rangle$

LZSS

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$$L = |A|B|R|A|C\,A\,D\,A\,B\,R\,A\,B\,R\,A\,C\,A\,C\,A$$

1 2 3 4 5

$1 : \langle 0,0,A \rangle$

$2 : \langle 0,0,B \rangle$

$3 : \langle 0,0,R \rangle$

$4 : \langle 3,1,C \rangle$

$5 : \langle 2,1,D \rangle$

$6 : \langle 7,4,B \rangle$

$7 : \langle 3,2,C \rangle$

$8 : \langle 2,3,\text{EOF} \rangle$

[LZ77]

→

$1 : \langle 0,A \rangle$

$2 : \langle 0,B \rangle$

$3 : \langle 0,R \rangle$

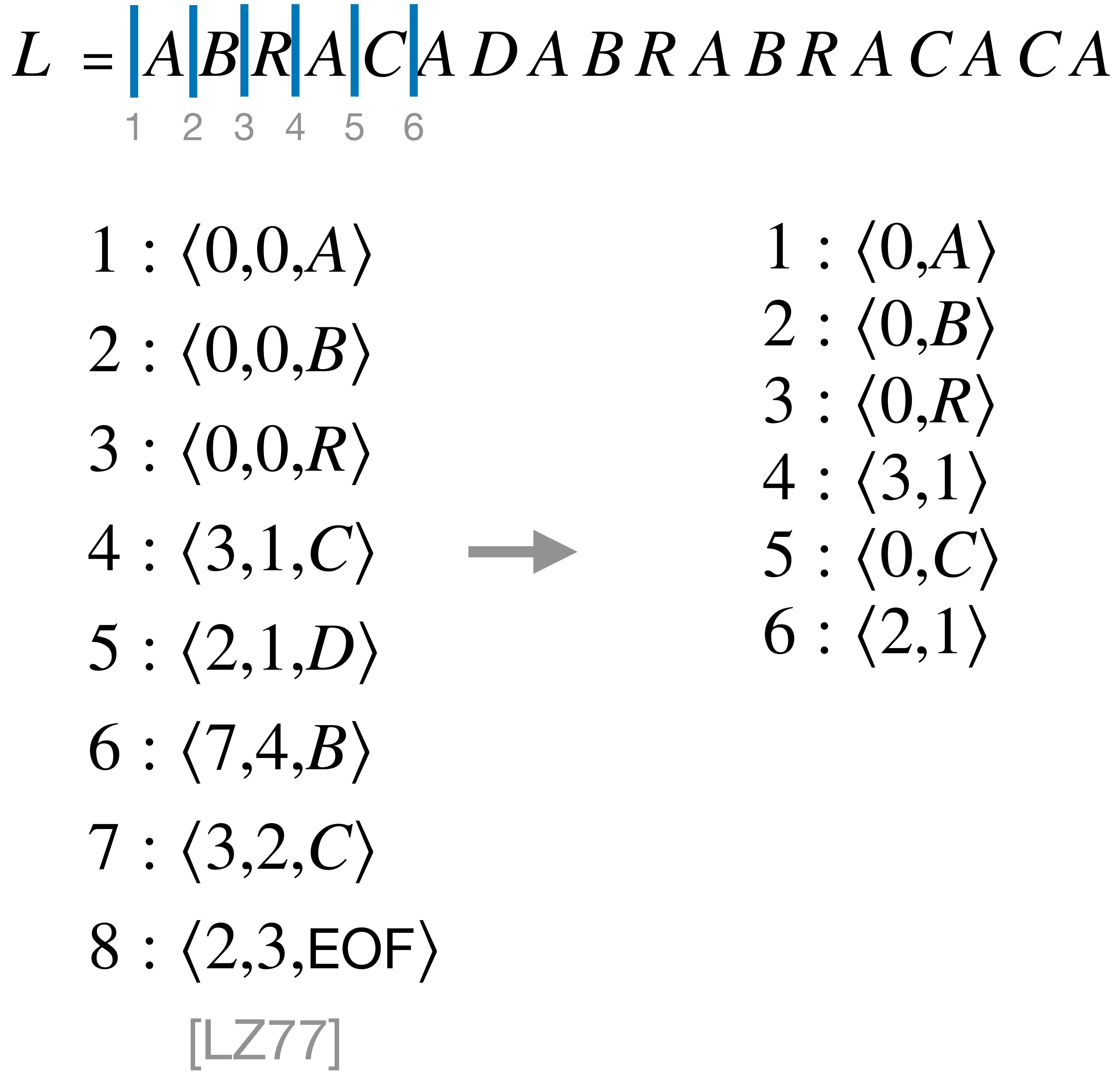
$4 : \langle 3,1 \rangle$

$5 : \langle 0,C \rangle$

LZSS

Storer and Szymanski, 1982

- **Idea.** Output *pairs*, not triples.
- **Observation 1.**
  When the lcp has size 0, we always repeat the pair $\langle d = 0, |lcp| = 0\rangle$. We can directly emit $\langle 0,c\rangle$.
- **Observation 2.**
  When $|lcp| \neq 0$, we can avoid specifying the next character c, but just emit the pair $\langle d, |lcp|\rangle$ and made the algorithm advance by $|lcp|$ characters.



$L = |A|B|R|A|C|A\,D\,A\,B\,R\,A\,B\,R\,A\,C\,A\,C\,A$

1 2 3 4 5 6

$1 : \langle 0,0,A\rangle$

$2 : \langle 0,0,B\rangle$

$3 : \langle 0,0,R\rangle$

$4 : \langle 3,1,C\rangle$

$5 : \langle 2,1,D\rangle$

$6 : \langle 7,4,B\rangle$

$7 : \langle 3,2,C\rangle$

$8 : \langle 2,3,\text{EOF}\rangle$

[LZ77]

→

$1 : \langle 0,A\rangle$

$2 : \langle 0,B\rangle$

$3 : \langle 0,R\rangle$

$4 : \langle 3,1\rangle$

$5 : \langle 0,C\rangle$

$6 : \langle 2,1\rangle$

LZSS

Storer and Szymanski, 1982

- **Idea.** Output *pairs*, not triples.

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  When the lcp has size 0, we always repeat the pair $\langle d = 0, |lcp| = 0 \rangle$. We can directly emit $\langle 0, c \rangle$.

- **Observation 2.**
  When $|lcp| \neq 0$, we can avoid specifying the next character c, but just emit the pair $\langle d, |lcp| \rangle$ and made the algorithm advance by $|lcp|$ characters.

$L = |A|B|R|A|C|A|D\,A\,B\,R\,A\,B\,R\,A\,C\,A\,C\,A$

1 2 3 4 5 6 7

$1 : \langle 0,0,A \rangle$

$2 : \langle 0,0,B \rangle$

$3 : \langle 0,0,R \rangle$

$4 : \langle 3,1,C \rangle$

$5 : \langle 2,1,D \rangle$

$6 : \langle 7,4,B \rangle$

$7 : \langle 3,2,C \rangle$

$8 : \langle 2,3,\text{EOF} \rangle$

[LZ77]

→

$1 : \langle 0,A \rangle$

$2 : \langle 0,B \rangle$

$3 : \langle 0,R \rangle$

$4 : \langle 3,1 \rangle$

$5 : \langle 0,C \rangle$

$6 : \langle 2,1 \rangle$

$7 : \langle 0,D \rangle$

LZSS

Storer and Szymanski, 1982

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- **Observation 2.**
  When $|lcp| \neq 0$, we can avoid specifying the next character c, but just emit the pair $\langle d, |lcp|\rangle$ and made the algorithm advance by $|lcp|$ characters.

$L = |A|B|R|A|C|A|D|A\,B\,R\,A\,B\,R\,A\,C\,A\,C\,A$

1 2 3 4 5 6 7 8

$1 : \langle 0,0,A\rangle$

$2 : \langle 0,0,B\rangle$

$3 : \langle 0,0,R\rangle$

$4 : \langle 3,1,C\rangle$

$5 : \langle 2,1,D\rangle$

$6 : \langle 7,4,B\rangle$

$7 : \langle 3,2,C\rangle$

$8 : \langle 2,3,\text{EOF}\rangle$

[LZ77]

→

$1 : \langle 0,A\rangle$

$2 : \langle 0,B\rangle$

$3 : \langle 0,R\rangle$

$4 : \langle 3,1\rangle$

$5 : \langle 0,C\rangle$

$6 : \langle 2,1\rangle$

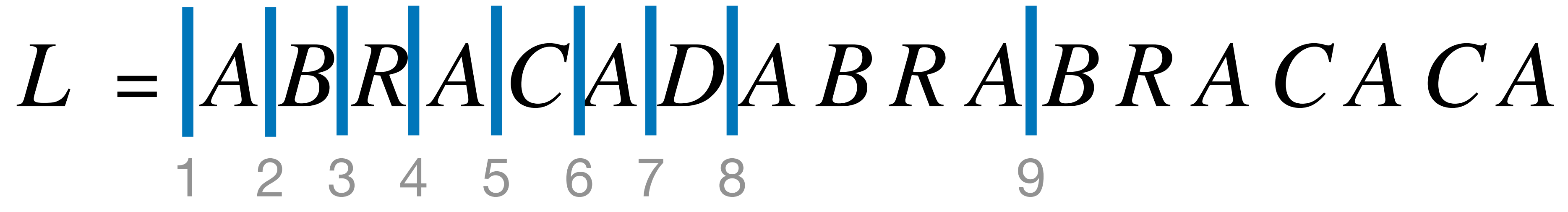
$7 : \langle 0,D\rangle$

$8 : \langle 7,4\rangle$

LZSS

Storer and Szymanski, 1982

- **Idea.** Output *pairs*, not triples.
- **Observation 1.** When the lcp has size 0, we always repeat the pair $\langle d = 0, |lcp| = 0 \rangle$. We can directly emit $\langle 0,c \rangle$.
- **Observation 2.** When $|lcp| \neq 0$, we can avoid specifying the next character c, but just emit the pair $\langle d, |lcp| \rangle$ and made the algorithm advance by $|lcp|$ characters.



$L = |A|B|R|A|C|A|D|A\,B\,R\,A|B\,R\,A\,C\,A\,C\,A$

[LZ77]		
$1 : \langle 0,0,A \rangle$	→	$1 : \langle 0,A \rangle$
$2 : \langle 0,0,B \rangle$		$2 : \langle 0,B \rangle$
$3 : \langle 0,0,R \rangle$		$3 : \langle 0,R \rangle$
$4 : \langle 3,1,C \rangle$		$4 : \langle 3,1 \rangle$
$5 : \langle 2,1,D \rangle$		$5 : \langle 0,C \rangle$
$6 : \langle 7,4,B \rangle$		$6 : \langle 2,1 \rangle$
$7 : \langle 3,2,C \rangle$		$7 : \langle 0,D \rangle$
$8 : \langle 2,3,\text{EOF} \rangle$		$8 : \langle 7,4 \rangle$
		$9 : \langle 3,3 \rangle$

LZSS

Storer and Szymanski, 1982

- **Idea.** Output *pairs*, not triples.
- **Observation 1.**
  When the lcp has size 0, we always repeat the pair $\langle d = 0, |lcp| = 0 \rangle$. We can directly emit $\langle 0, c \rangle$.
- **Observation 2.**
  When $|lcp| \neq 0$, we can avoid specifying the next character c, but just emit the pair $\langle d, |lcp| \rangle$ and made the algorithm advance by $|lcp|$ characters.

$L = |A|B|R|A|C|A|D|A\,B\,R\,A|B\,R\,A|C\,A\,C\,A$

1 2 3 4 5 6 7 8 9 10

$1 : \langle 0,0,A \rangle$

$2 : \langle 0,0,B \rangle$

$3 : \langle 0,0,R \rangle$

$4 : \langle 3,1,C \rangle$

$5 : \langle 2,1,D \rangle$

$6 : \langle 7,4,B \rangle$

$7 : \langle 3,2,C \rangle$

$8 : \langle 2,3,\text{EOF} \rangle$

[LZ77]

→

$1 : \langle 0,A \rangle$

$2 : \langle 0,B \rangle$

$3 : \langle 0,R \rangle$

$4 : \langle 3,1 \rangle$

$5 : \langle 0,C \rangle$

$6 : \langle 2,1 \rangle$

$7 : \langle 0,D \rangle$

$8 : \langle 7,4 \rangle$

$9 : \langle 3,3 \rangle$

$10 : \langle 10,2 \rangle$

LZSS

Storer and Szymanski, 1982

- **Idea.** Output *pairs*, not triples.
- **Observation 1.**
  When the lcp has size 0, we always repeat the pair $\langle d = 0, |lcp| = 0 \rangle$. We can directly emit $\langle 0,c \rangle$.
- **Observation 2.**
  When $|lcp| \neq 0$, we can avoid specifying the next character c, but just emit the pair $\langle d, |lcp| \rangle$ and made the algorithm advance by $|lcp|$ characters.

$L = |A|B|R|A|C|A|D|A\,B\,R\,A|B\,R\,A|C\,A|C\,A$

1 2 3 4 5 6 7 8 9 10 11

$1 : \langle 0,0,A \rangle$

$2 : \langle 0,0,B \rangle$

$3 : \langle 0,0,R \rangle$

$4 : \langle 3,1,C \rangle$

$5 : \langle 2,1,D \rangle$

$6 : \langle 7,4,B \rangle$

$7 : \langle 3,2,C \rangle$

$8 : \langle 2,3,\text{EOF} \rangle$

[LZ77]

→

$1 : \langle 0,A \rangle$

$2 : \langle 0,B \rangle$

$3 : \langle 0,R \rangle$

$4 : \langle 3,1 \rangle$

$5 : \langle 0,C \rangle$

$6 : \langle 2,1 \rangle$

$7 : \langle 0,D \rangle$

$8 : \langle 7,4 \rangle$

$9 : \langle 3,3 \rangle$

$10 : \langle 10,2 \rangle$

$11 : \langle 2,2 \rangle$

LZSS

Storer and Szymanski, 1982

- **Idea.** Output *pairs*, not triples.

- **Observation 1.**
  When the lcp has size 0, we always repeat the pair $\langle d = 0, |lcp| = 0 \rangle$.
  We can directly emit $\langle 0,c \rangle$.

- **Observation 2.**
  When $|lcp| \neq 0$, we can avoid specifying the next character c, but just emit the pair $\langle d, |lcp| \rangle$ and made the algorithm advance by $|lcp|$ characters.

$$L = |A|B|R|A|C|A|D|A\,B\,R\,A|B\,R\,A|C\,A|C\,A|$$

1 2 3 4 5 6 7 8 9 10 11 12

$1 : \langle 0,0,A \rangle$

$2 : \langle 0,0,B \rangle$

$3 : \langle 0,0,R \rangle$

$4 : \langle 3,1,C \rangle$

$5 : \langle 2,1,D \rangle$

$6 : \langle 7,4,B \rangle$

$7 : \langle 3,2,C \rangle$

$8 : \langle 2,3,\text{EOF} \rangle$

[LZ77]

→

$1 : \langle 0,A \rangle$

$2 : \langle 0,B \rangle$

$3 : \langle 0,R \rangle$

$4 : \langle 3,1 \rangle$

$5 : \langle 0,C \rangle$

$6 : \langle 2,1 \rangle$

$7 : \langle 0,D \rangle$

$8 : \langle 7,4 \rangle$

$9 : \langle 3,3 \rangle$

$10 : \langle 10,2 \rangle$

$11 : \langle 2,2 \rangle$

$12 : \langle 0,\text{EOF} \rangle$

[LZSS]

LZ77 in practice: `gzip`

http://www.gzip.org
https://zlib.net

Gailly and Adler, 1995

- **Q.** How do we determine efficiently the lcp string between the suffix $L[i\,..\,n]$ and the preceding W characters?
  A. Use a hash table of q-grams (strings of q characters). Usually q is small, say, $q = 3$. Insert all the q-grams of the window into the hash table: the q-gram is the key, the value is the list of *positions* of all the occurrences of the q-gram in the window.

- **Idea.** Use the q-grams' positions as "pointers" for the actual determination of the lcp string.

- At each step: search for $L[i\,..\,i+q]$ in the hash table. If not found, the pair $\langle 0, L[i] \rangle$ is emitted (and the algorithm advances by 1 character); otherwise we determine the list R of all the occurrences of $L[i\,..\,i+q]$. For each position p in R, compare $L[p\,..\,n]$ and $L[i\,..\,n]$ to determine the lcp string.

- If p_{lcp} is the position at which the lcp string is found, then the pair $\langle i - p_{lcp}, |\,lcp\,| \rangle$ is emitted and the algorithm advances by $|\,lcp\,|$ characters. All the q-grams starting in $window[1..|\,lcp\,|]$ are *deleted* from the hash table, and all the q-grams starting in $L[i\,..\,i + |\,lcp\,|]$ are *added* to the hash table.

- `gzip` has 9 different compression levels, specified with the options `-1`, `-2`, … , `-9`, and corresponding to $W =$ 100, 200, …, 900 KiB.

- The integer pairs are compressed using Huffman.

LZ78

Lempel and Ziv, 1978

- **Idea.** Differently from LZ77, the dictionary D *is explicitly built* during the encoding of L and *not limited* by the window size W.

- **Algorithm.**
  - At the beginning: $i = 1$.
  - At each step: determine the longest string of D, lcp, that is a prefix of $L[i\,..\,n]$.
  - If $index(lcp)$ is the index of lcp in D, the algorithm emits the pair $\langle index(lcp), c\rangle$, where c is the next character following lcp in $L[i\,..\,n]$.
  - The concatenation of lcp and c, the string $lcp \cdot c$, is added to D.
  - Then the algorithm advances by $|\,lcp\,| + 1$ characters: $i = i + |\,lcp\,| + 1$.

- The stream of integer pairs is compressed using another coding method.

LZ78 — Encoding Demo

$L = |A\,B\,R\,A\,C\,A\,D\,A\,B\,R\,A\,B\,R\,A\,B\,A\,R\,A$

0

D

$0 : \varepsilon$ (empty string)

LZ78 — Encoding Demo

$L = \vert A \vert B R A C A D A B R A B R A B A R A$

0 1

D

$0 : \varepsilon$ (empty string)

$1 : A$

$\langle 0, A \rangle$

1

LZ78 — Encoding Demo

$$L = \underset{0}{|}A\underset{1}{|}B\underset{2}{|}RACADABRABRABARA$$

D

$0 : \varepsilon$ (empty string)

$1 : A$

$2 : B$

$$\underset{1}{\langle 0,A\rangle}\underset{2}{\langle 0,B\rangle}$$

LZ78 — Encoding Demo

$$L = |A|B|R|A\;C\;A\;D\;A\;B\;R\;A\;B\;R\;A\;B\;A\;R\;A$$

0 1 2 3

D

$0 : \varepsilon$ (empty string)

$1 : A$

$2 : B$

$3 : R$

$$\langle 0,A\rangle\langle 0,B\rangle\langle 0,R\rangle$$

1 2 3

LZ78 — Encoding Demo

$$L = \underset{0}{|}A\underset{1}{|}B\underset{2}{|}R\underset{3}{|}A\,C\underset{4}{|}A\,D\,A\,B\,R\,A\,B\,R\,A\,B\,A\,R\,A$$

$$\underset{1}{\langle 0,A\rangle}\underset{2}{\langle 0,B\rangle}\underset{3}{\langle 0,R\rangle}\underset{4}{\langle 1,C\rangle}$$

D

$0 : \varepsilon$ (empty string)

$1 : A$

$2 : B$

$3 : R$

$4 : AC$

LZ78 — Encoding Demo

$$L = |A|B|R|A\,C|A\,D|A\,B\,R\,A\,B\,R\,A\,B\,A\,R\,A$$

0 1 2 3 4 5

$$\langle 0,A\rangle\langle 0,B\rangle\langle 0,R\rangle\langle 1,C\rangle\langle 1,D\rangle$$

1 2 3 4 5

D

$0 : \varepsilon$ (empty string)

$1 : A$

$2 : B$

$3 : R$

$4 : AC$

$5 : AD$

LZ78 — Encoding Demo

$L = |A|B|R|A\ C|A\ D|A\ B|R\ A\ B\ R\ A\ B\ A\ R\ A$

0 1 2 3 4 5 6

$\langle 0,A\rangle$	$\langle 0,B\rangle$	$\langle 0,R\rangle$	$\langle 1,C\rangle$	$\langle 1,D\rangle$	$\langle 1,B\rangle$
1	2	3	4	5	6

D

$0 :$	ε	(empty string)
$1 :$	A	
$2 :$	B	
$3 :$	R	
$4 :$	AC	
$5 :$	AD	
$6 :$	AB	

LZ78 — Encoding Demo

$$L = |A|B|R|A\ C|A\ D|A\ B|R\ A|B\ R\ A\ B\ A\ R\ A$$

0 1 2 3 4 5 6 7

$$\langle 0,A\rangle\langle 0,B\rangle\langle 0,R\rangle\langle 1,C\rangle\langle 1,D\rangle\langle 1,B\rangle\langle 3,A\rangle$$

1 2 3 4 5 6 7

D

0	ε	(empty string)
1	A	
2	B	
3	R	
4	AC	
5	AD	
6	AB	
7	RA	

LZ78 — Encoding Demo

$$L = |A|B|R|A\ C|A\ D|A\ B|R\ A|B\ R|A\ B\ A\ R\ A$$

0 1 2 3 4 5 6 7 8

$$\langle 0,A\rangle\langle 0,B\rangle\langle 0,R\rangle\langle 1,C\rangle\langle 1,D\rangle\langle 1,B\rangle\langle 3,A\rangle\langle 2,R\rangle$$

1 2 3 4 5 6 7 8

	D	
0	ε	(empty string)
1	A	
2	B	
3	R	
4	AC	
5	AD	
6	AB	
7	RA	
8	BR	

LZ78 — Encoding Demo

$$L = |A|B|R|A\ C|A\ D|A\ B|R\ A|B\ R|A\ B\ A|R\ A$$

0 1 2 3 4 5 6 7 8 9

$$\langle 0,A\rangle\langle 0,B\rangle\langle 0,R\rangle\langle 1,C\rangle\langle 1,D\rangle\langle 1,B\rangle\langle 3,A\rangle\langle 2,R\rangle\langle 6,A\rangle$$

1 2 3 4 5 6 7 8 9

D

$0 : \varepsilon$ (empty string)

$1 : A$

$2 : B$

$3 : R$

$4 : AC$

$5 : AD$

$6 : AB$

$7 : RA$

$8 : BR$

$9 : ABA$

LZ78 — Encoding Demo

$$L = |A|B|R|A\ C|A\ D|A\ B|R\ A|B\ R|A\ B\ A|R\ A|$$

0 1 2 3 4 5 6 7 8 9 10

$$\langle 0,A\rangle\langle 0,B\rangle\langle 0,R\rangle\langle 1,C\rangle\langle 1,D\rangle\langle 1,B\rangle\langle 3,A\rangle\langle 2,R\rangle\langle 6,A\rangle\langle 7,\text{EOF}\rangle$$

1 2 3 4 5 6 7 8 9 10

D

$0 :$	ε	(empty string)
$1 :$	A	
$2 :$	B	
$3 :$	R	
$4 :$	AC	
$5 :$	AD	
$6 :$	AB	
$7 :$	RA	
$8 :$	BR	
$9 :$	ABA	

LZ78 — Decoding Demo

D

$L =$

$0 : \varepsilon$ (empty string)

$\langle 0,A\rangle\langle 0,B\rangle\langle 0,R\rangle\langle 1,C\rangle\langle 1,D\rangle\langle 1,B\rangle\langle 3,A\rangle\langle 2,R\rangle\langle 6,A\rangle\langle 7,\text{EOF}\rangle$

LZ78 — Decoding Demo

$L = A$

D

$0 : \varepsilon$ (empty string)

$1 : A$

$\langle 0,A\rangle\langle 0,B\rangle\langle 0,R\rangle\langle 1,C\rangle\langle 1,D\rangle\langle 1,B\rangle\langle 3,A\rangle\langle 2,R\rangle\langle 6,A\rangle\langle 7,\text{EOF}\rangle$

1

LZ78 — Decoding Demo

$L = A\,B$

D

$0 : \varepsilon$ (empty string)

$1 : A$

$2 : B$

$$\underset{1}{\langle 0,A\rangle}\underset{2}{\langle 0,B\rangle}\langle 0,R\rangle\langle 1,C\rangle\langle 1,D\rangle\langle 1,B\rangle\langle 3,A\rangle\langle 2,R\rangle\langle 6,A\rangle\langle 7,\text{EOF}\rangle$$

LZ78 — Decoding Demo

$L = A\,B\,R$

D

$0 : \varepsilon$ (empty string)

$1 : A$

$2 : B$

$3 : R$

$$\underset{1}{\langle 0,A\rangle}\underset{2}{\langle 0,B\rangle}\underset{3}{\langle 0,R\rangle}\langle 1,C\rangle\langle 1,D\rangle\langle 1,B\rangle\langle 3,A\rangle\langle 2,R\rangle\langle 6,A\rangle\langle 7,\text{EOF}\rangle$$

LZ78 — Decoding Demo

$L = A\,B\,R\,AC$

D

$0 : \varepsilon$ (empty string)

$1 : A$

$2 : B$

$3 : R$

$4 : AC$

$\langle 0,A\rangle\langle 0,B\rangle\langle 0,R\rangle\langle 1,C\rangle\langle 1,D\rangle\langle 1,B\rangle\langle 3,A\rangle\langle 2,R\rangle\langle 6,A\rangle\langle 7,\text{EOF}\rangle$

1 2 3 4

LZ78 — Decoding Demo

$L = A\,B\,R\,AC\,AD$

$$\underset{1}{\langle 0,A\rangle}\underset{2}{\langle 0,B\rangle}\underset{3}{\langle 0,R\rangle}\underset{4}{\langle 1,C\rangle}\underset{5}{\langle 1,D\rangle}\langle 1,B\rangle\langle 3,A\rangle\langle 2,R\rangle\langle 6,A\rangle\langle 7,\text{EOF}\rangle$$

D

$0 : \varepsilon$ (empty string)

$1 : A$

$2 : B$

$3 : R$

$4 : AC$

$5 : AD$

LZ78 — Decoding Demo

$L = A\,B\,R\,AC\,AD\,AB$

$\langle 0,A\rangle\langle 0,B\rangle\langle 0,R\rangle\langle 1,C\rangle\langle 1,D\rangle\langle 1,B\rangle\langle 3,A\rangle\langle 2,R\rangle\langle 6,A\rangle\langle 7,\text{EOF}\rangle$

1 2 3 4 5 6

D

$0 : \varepsilon$ (empty string)

$1 : A$

$2 : B$

$3 : R$

$4 : AC$

$5 : AD$

$6 : AB$

LZ78 — Decoding Demo

$L = A\,B\,R\;AC\;AD\;\;AB\;RA$

$\langle 0,A\rangle\langle 0,B\rangle\langle 0,R\rangle\langle 1,C\rangle\langle 1,D\rangle\langle 1,B\rangle\langle 3,A\rangle\langle 2,R\rangle\langle 6,A\rangle\langle 7,\text{EOF}\rangle$

1 2 3 4 5 6 7

D

$0 : \varepsilon$ (empty string)

$1 : A$

$2 : B$

$3 : R$

$4 : AC$

$5 : AD$

$6 : AB$

$7 : RA$

LZ78 — Decoding Demo

$$L = A\,B\,R\;AC\;AD\;\;AB\;RA\;\;BR$$

$$\underset{1}{\langle 0,A\rangle}\underset{2}{\langle 0,B\rangle}\underset{3}{\langle 0,R\rangle}\underset{4}{\langle 1,C\rangle}\underset{5}{\langle 1,D\rangle}\underset{6}{\langle 1,B\rangle}\underset{7}{\langle 3,A\rangle}\underset{8}{\langle 2,R\rangle}\langle 6,A\rangle\langle 7,\text{EOF}\rangle$$

D

$0 : \varepsilon$ (empty string)

$1 : A$

$2 : B$

$3 : R$

$4 : AC$

$5 : AD$

$6 : AB$

$7 : RA$

$8 : BR$

LZ78 — Decoding Demo

$$L = A\,B\,R\;AC\;AD\;AB\;RA\;BR\;ABA$$

$$\underset{1}{\langle 0,A\rangle}\underset{2}{\langle 0,B\rangle}\underset{3}{\langle 0,R\rangle}\underset{4}{\langle 1,C\rangle}\underset{5}{\langle 1,D\rangle}\underset{6}{\langle 1,B\rangle}\underset{7}{\langle 3,A\rangle}\underset{8}{\langle 2,R\rangle}\underset{9}{\langle 6,A\rangle}\langle 7,\text{EOF}\rangle$$

D

$0 : \varepsilon$ (empty string)

$1 : A$

$2 : B$

$3 : R$

$4 : AC$

$5 : AD$

$6 : AB$

$7 : RA$

$8 : BR$

$9 : ABA$

LZ78 — Decoding Demo

$$L = A\,B\,R\,AC\,AD\;\;AB\;RA\;\;BR\;\;ABA\;\;\;RA$$

$$\underset{1}{\langle 0,A\rangle}\underset{2}{\langle 0,B\rangle}\underset{3}{\langle 0,R\rangle}\underset{4}{\langle 1,C\rangle}\underset{5}{\langle 1,D\rangle}\underset{6}{\langle 1,B\rangle}\underset{7}{\langle 3,A\rangle}\underset{8}{\langle 2,R\rangle}\underset{9}{\langle 6,A\rangle}\underset{10}{\langle 7,\text{EOF}\rangle}$$

D

$0 : \varepsilon$ (empty string)

$1 : A$

$2 : B$

$3 : R$

$4 : AC$

$5 : AD$

$6 : AB$

$7 : RA$

$8 : BR$

$9 : ABA$

LZW

Welch, 1984

- **Idea.** Try to avoid the second component — the "next" character — of the pairs emitted by LZ78. Since it does not emit the character, the parsing and the building of the dictionary are *misaligned*, which may induce a tricky decoding case (actually the same that can happen in LZ77).

- **Algorithm.**
  - At the beginning: pre-fill the dictionary with all possible characters. If there are m of these, the assigned indexes are $[0..m-1]$, so we set the next available $index$ into D to $index = m$. Set $i = 1$. (For ASCII, all the $m = 256$ characters correspond to the indexes from 0 to 255.)
  - At each step: determine the longest string lcp of D that is a prefix of $L[i\,..\,n]$.
  - If $index(lcp)$ is the index of lcp in D and c is the next character following lcp in $L[i\,..\,n]$, the algorithm emits the single integer $index(lcp)$.
  - The string $lcp \cdot c$ is added to D and it takes the next available $index$, that is: $index = index + 1$.
  - Then the algorithm advances by $|\,lcp\,|$ characters, that is: $i = i + |\,lcp\,|$.

- The stream of integers is compressed using another coding method.

LZW — Encoding Demo

$L = | A\,B\,R\,A\,C\,A\,D\,A\,B\,R\,A\,B\,R\,A\,B\,A\,B\,A$

0

D

…

$65 : A$

$66 : B$

$67 : C$

$68 : D$

…

$82 : R$

…

LZW — Encoding Demo

$L = |A|B\,R\,A\,C\,A\,D\,A\,B\,R\,A\,B\,R\,A\,B\,A\,B\,A$

0 1

65

1

D

…

$65 : A$ $256 : AB$

$66 : B$

$67 : C$

$68 : D$

…

$82 : R$

…

LZW — Encoding Demo

$L = |A|B|R\,A\,C\,A\,D\,A\,B\,R\,A\,B\,R\,A\,B\,A\,B\,A$

0 1 2

$65 \quad 66$

1 2

D

…	
$65 : A$	$256 : AB$
$66 : B$	$257 : BR$
$67 : C$	
$68 : D$	
…	
$82 : R$	
…	

LZW — Encoding Demo

$$L = |A|B|R|A\ C\ A\ D\ A\ B\ R\ A\ B\ R\ A\ B\ A\ B\ A$$

0 1 2 3

$$65 \quad 66 \quad 82$$

1 2 3

D

$\dots$	
$65 : A$	$256 : AB$
$66 : B$	$257 : BR$
$67 : C$	$258 : RA$
$68 : D$	
$\dots$	
$82 : R$	
$\dots$	

LZW — Encoding Demo

$$L = |A|B|R|A|C\,A\,D\,A\,B\,R\,A\,B\,R\,A\,B\,A\,B\,A$$

0 1 2 3 4

65	66	82	65
1	2	3	4

D

…	
$65 : A$	$256 : AB$
$66 : B$	$257 : BR$
$67 : C$	$258 : RA$
$68 : D$	$259 : AC$
…	
$82 : R$	
…	

LZW — Encoding Demo

$L = |A|B|R|A|C|A\,D\,A\,B\,R\,A\,B\,R\,A\,B\,A\,B\,A$

0 1 2 3 4 5

65	66	82	65	67
1	2	3	4	5

D

…

$65 : A$

$66 : B$

$67 : C$

$68 : D$

…

$82 : R$

…

$256 : AB$

$257 : BR$

$258 : RA$

$259 : AC$

$260 : CA$

LZW — Encoding Demo

$$L = |A|B|R|A|C|A|D\,A\,B\,R\,A\,B\,R\,A\,B\,A\,B\,A$$

0 1 2 3 4 5 6

65	66	82	65	67	65
1	2	3	4	5	6

D

…	
$65 : A$	$256 : AB$
$66 : B$	$257 : BR$
$67 : C$	$258 : RA$
$68 : D$	$259 : AC$
…	$260 : CA$
$82 : R$	$261 : AD$
…	

LZW — Encoding Demo

$$L = \underset{0}{|}A\underset{1}{|}B\underset{2}{|}R\underset{3}{|}A\underset{4}{|}C\underset{5}{|}A\underset{6}{|}D\underset{7}{|}A\,B\,R\,A\,B\,R\,A\,B\,A\,B\,A$$

65	66	82	65	67	65	68
1	2	3	4	5	6	7

D

…	
65 : A	256 : AB
66 : B	257 : BR
67 : C	258 : RA
68 : D	259 : AC
…	260 : CA
82 : R	261 : AD
…	262 : DA

LZW — Encoding Demo

$L = |A|B|R|A|C|A|D|A\ B|R\ A\ B\ R\ A\ B\ A\ B\ A$

0 1 2 3 4 5 6 7 8

65	66	82	65	67	65	68	256
1	2	3	4	5	6	7	8

D

…	
$65 : A$	$256 : AB$
$66 : B$	$257 : BR$
$67 : C$	$258 : RA$
$68 : D$	$259 : AC$
…	$260 : CA$
$82 : R$	$261 : AD$
…	$262 : DA$
	$263 : ABR$

LZW — Encoding Demo

$$L = |A|B|R|A|C|A|D|A\ B|R\ A|B\ R\ A\ B\ A\ B\ A$$

0 1 2 3 4 5 6 7 8 9

65	66	82	65	67	65	68	256	258
1	2	3	4	5	6	7	8	9

D

…	
$65 : A$	$256 : AB$
$66 : B$	$257 : BR$
$67 : C$	$258 : RA$
$68 : D$	$259 : AC$
…	$260 : CA$
$82 : R$	$261 : AD$
…	$262 : DA$
	$263 : ABR$
	$263 : RAB$

LZW — Encoding Demo

$$L = |A|B|R|A|C|A|D|A\ B|R\ A|B\ R|A\ B\ A\ B\ A$$

0 1 2 3 4 5 6 7 8 9 10

65	66	82	65	67	65	68	256	258	257
1	2	3	4	5	6	7	8	9	10

D

…

$65 : A$

$66 : B$

$67 : C$

$68 : D$

…

$82 : R$

…

$256 : AB$

$257 : BR$

$258 : RA$

$259 : AC$

$260 : CA$

$261 : AD$

$262 : DA$

$263 : ABR$

$263 : RAB$

$264 : BRA$

LZW — Encoding Demo

$L = |A|B|R|A|C|A|D|A\ B|R\ A|B\ R|A\ B|A\ B\ A$

0 1 2 3 4 5 6 7 8 9 10 11

65	66	82	65	67	65	68	256	258	257	256
1	2	3	4	5	6	7	8	9	10	11

D

…

65 : A

66 : B

67 : C

68 : D

…

82 : R

…

256 : AB

257 : BR

258 : RA

259 : AC

260 : CA

261 : AD

262 : DA

263 : ABR

263 : RAB

264 : BRA

265 : ABA

LZW — Encoding Demo

$L = |A|B|R|A|C|A|D|A\ B|R\ A|B\ R|A\ B|A\ B\ A|$

0 1 2 3 4 5 6 7 8 9 10 11 12

65	66	82	65	67	65	68	256	258	257	256	265
1	2	3	4	5	6	7	8	9	10	11	12

D

…			
65	A	256	AB
66	B	257	BR
67	C	258	RA
68	D	259	AC
…		260	CA
82	R	261	AD
…		262	DA
		263	ABR
		263	RAB
		264	BRA
		265	ABA

LZW — Decoding Demo

$L =$

65 66 82 65 67 65 68 256 258 257 256 265

D

…

$65 : A$

$66 : B$

$67 : C$

$68 : D$

…

$82 : R$

…

LZW — Decoding Demo

D

…

$65 : A$

$66 : B$

$67 : C$

$68 : D$

…

$82 : R$

…

$L = A$

65 66 82 65 67 65 68 256 258 257 256 265

1

LZW — Decoding Demo

$L = A\ B$

65 66 82 65 67 65 68 256 258 257 256 265

1 2

D

…

$65 : A$

$66 : B$

$67 : C$

$68 : D$

…

$82 : R$

…

$256 : AB$

LZW — Decoding Demo

$L = A\ B\ R$

65	66	82	65	67	65	68	256	258	257	256	265
1	2	3									

D

…

$65 : A$

$66 : B$

$67 : C$

$68 : D$

…

$82 : R$

…

$256 : AB$

$257 : BR$

LZW — Decoding Demo

$L = A\ B\ R\ A$

65	66	82	65	67	65	68	256	258	257	256	265
1	2	3	4								

D

…	
$65 : A$	$256 : AB$
$66 : B$	$257 : BR$
$67 : C$	$258 : RA$
$68 : D$	
…	
$82 : R$	
…	

LZW — Decoding Demo

$L = A\ B\ R\ A\ C$

65	66	82	65	67	65	68	256	258	257	256	265
1	2	3	4	5							

D

…	
$65 : A$	$256 : AB$
$66 : B$	$257 : BR$
$67 : C$	$258 : RA$
$68 : D$	$259 : AC$
…	
$82 : R$	
…	

LZW — Decoding Demo

$L = A\ B\ R\ A\ C\ A$

65	66	82	65	67	65	68	256	258	257	256	265
1	2	3	4	5	6						

D

…

$65 : A$

$66 : B$

$67 : C$

$68 : D$

…

$82 : R$

…

$256 : AB$

$257 : BR$

$258 : RA$

$259 : AC$

$260 : CA$

LZW — Decoding Demo

$L = A\ B\ R\ A\ C\ A\ D$

65	66	82	65	67	65	68	256	258	257	256	265
1	2	3	4	5	6	7					

D

…	
$65 : A$	$256 : AB$
$66 : B$	$257 : BR$
$67 : C$	$258 : RA$
$68 : D$	$259 : AC$
…	$260 : CA$
$82 : R$	$261 : AD$
…	

LZW — Decoding Demo

$L = A\ B\ R\ A\ C\ A\ D\ AB$

65	66	82	65	67	65	68	256	258	257	256	265
1	2	3	4	5	6	7	8				

D

…	
$65 : A$	$256 : AB$
$66 : B$	$257 : BR$
$67 : C$	$258 : RA$
$68 : D$	$259 : AC$
…	$260 : CA$
$82 : R$	$261 : AD$
…	$262 : DA$

LZW — Decoding Demo

$L = A\ B\ R\ A\ C\ A\ D\ AB\ RA$

65	66	82	65	67	65	68	256	258	257	256	265
1	2	3	4	5	6	7	8	9			

D

…	
65 : A	256 : AB
66 : B	257 : BR
67 : C	258 : RA
68 : D	259 : AC
…	260 : CA
82 : R	261 : AD
…	262 : DA
	263 : ABR

LZW — Decoding Demo

$L = A\ B\ R\ A\ C\ A\ D\ AB\ RA\ BR$

65	66	82	65	67	65	68	256	258	257	256	265
1	2	3	4	5	6	7	8	9	10		

D

…

$65 : A$

$66 : B$

$67 : C$

$68 : D$

…

$82 : R$

…

$256 : AB$

$257 : BR$

$258 : RA$

$259 : AC$

$260 : CA$

$261 : AD$

$262 : DA$

$263 : ABR$

$263 : RAB$

LZW — Decoding Demo

$L = A\ B\ R\ A\ C\ A\ D\ AB\ RA\ BR\ \ AB$

65	66	82	65	67	65	68	256	258	257	256	265
1	2	3	4	5	6	7	8	9	10	11	

D

…

$65 : A$

$66 : B$

$67 : C$

$68 : D$

…

$82 : R$

…

$256 : AB$

$257 : BR$

$258 : RA$

$259 : AC$

$260 : CA$

$261 : AD$

$262 : DA$

$263 : ABR$

$263 : RAB$

$264 : BRA$

LZW — Decoding Demo

$$L = A\ B\ R\ A\ C\ A\ D\ AB\ RA\ BR\ \ AB\ AB\ ?$$

65	66	82	65	67	65	68	256	258	257	256	265
1	2	3	4	5	6	7	8	9	10	11	12

D

…

$65 : A$

$66 : B$

$67 : C$

$68 : D$

…

$82 : R$

…

$256 : AB$

$257 : BR$

$258 : RA$

$259 : AC$

$260 : CA$

$261 : AD$

$262 : DA$

$263 : ABR$

$263 : RAB$

$264 : BRA$

Not in the dictionary yet! →

LZW — Decoding Demo

$$L = A\ B\ R\ A\ C\ A\ D\ AB\ RA\ BR\ \boxed{AB\ A}B\ ?$$

65	66	82	65	67	65	68	256	258	257	256	265
1	2	3	4	5	6	7	8	9	10	11	12

D

…	
$65 : A$	$256 : AB$
$66 : B$	$257 : BR$
$67 : C$	$258 : RA$
$68 : D$	$259 : AC$
…	$260 : CA$
$82 : R$	$261 : AD$
…	$262 : DA$
	$263 : ABR$
	$263 : RAB$
	$264 : BRA$

Not in the dictionary yet! →

LZW — Decoding Demo

$L = A\ B\ R\ A\ C\ A\ D\ AB\ RA\ BR\ \boxed{AB\ A}B\ A$

65	66	82	65	67	65	68	256	258	257	256	265
1	2	3	4	5	6	7	8	9	10	11	12

D

…

$65 : A$

$66 : B$

$67 : C$

$68 : D$

…

$82 : R$

…

$256 : AB$

$257 : BR$

$258 : RA$

$259 : AC$

$260 : CA$

$261 : AD$

$262 : DA$

$263 : ABR$

$263 : RAB$

$264 : BRA$

Not in the dictionary yet! → $265 : ABA$

Variants

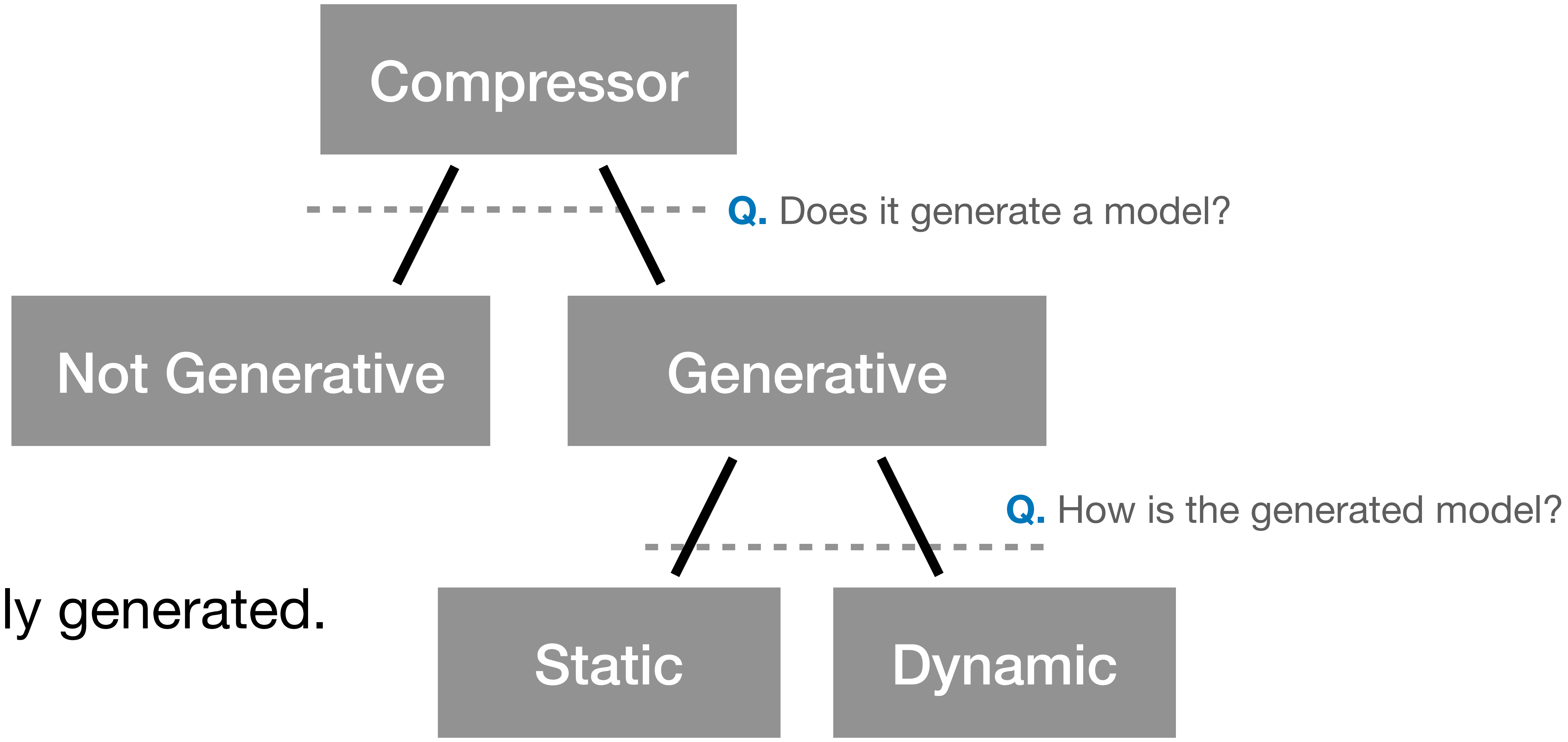
- **LZ4.** https://en.wikipedia.org/wiki/LZ4_(compression_algorithm))
- **Zstd.** http://facebook.github.io/zstd
- **LZMA.** https://en.wikipedia.org/wiki/Lempel-Ziv-Markov_chain_algorithm
- **LZO.** https://en.wikipedia.org/wiki/Lempel-Ziv-Oberhumer

Overview of Compressors

- **Not Generative.** No model generated.
  - Fast to encode/decode.
  - Moderately effective.
  - Example: Delta, Gamma, Golomb, Elias-Fano.
  - [Module 2 and 3]

- **Generative/Static.** The model is not updated.
  - First pass to actually generate the model.
  - Moderately fast to encode/decode.
  - Entropy-optimal.
  - Must transmit the model.
  - Example: Huffman, Arithmetic Coding.
  - [Module 4]

- **Generative/Dynamic.** The model is progressively generated.
  - No need to transmit the model.
  - Decoding is only sequential.
  - Very effective.
  - Example: LZ77, LZ78, LZW.
  - [Module 5]



Further Readings

- Chapter 5.5 (pages 839-845) of:
  Robert Sedgewick and Kevin Wayne. 2011. *Algorithms*. 4-th Edition.
  Addison-Wesley Professional, ISBN 0-321-57351-X.

- https://ethw.org/History_of_Lossless_Data_Compression_Algorithms

- Jacob Ziv and Abraham Lempel. 1977. *A universal algorithm for sequential data compression.* IEEE Transactions on Information Theory, IT-23(3):337-343.

- Jacob Ziv and Abraham Lempel. 1978. Compression of individual sequences via variable-rate coding. IEEE Transactions on Information Theory, IT-24(5):530-536.

- Terry A. Welch. 1984. *A technique for high-performance data compression*. Computer, pages 8-19.