A Crash Course on Data Compression

1.1 Warmup

Giulio Ermanno Pibiri

ISTI-CNR, giulio.ermanno.pibiri@isti.cnr.it

@giulio_pibiri

@jermp

Overview

- Warmup 1: Bit-packing, read/write binary data
- Warmup 2: Run-length encoding

Bit-Packing

Option 1. Use built-in data types.
Option 2. Bit-pack the quantities.

```
struct small_record {
    uint8_t weight;
    uint8_t height;
    uint8_t day;
    uint8_t month;
    uint16_t year;
};
```

- `weight`: assume 256 Kg are enough, so we need 8 bits
- `height`: assume 256 cm are enough, so we need 8 bits
- `day`: at most 31 days, so we need $\lceil \log_2 31 \rceil = 5$ bits
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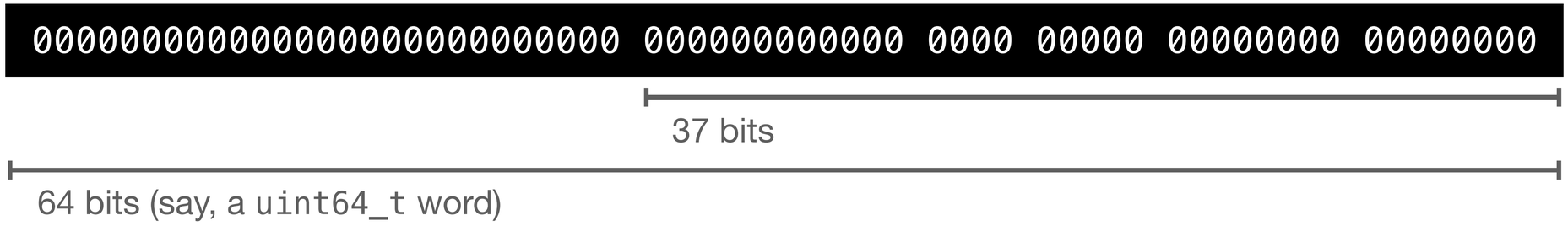

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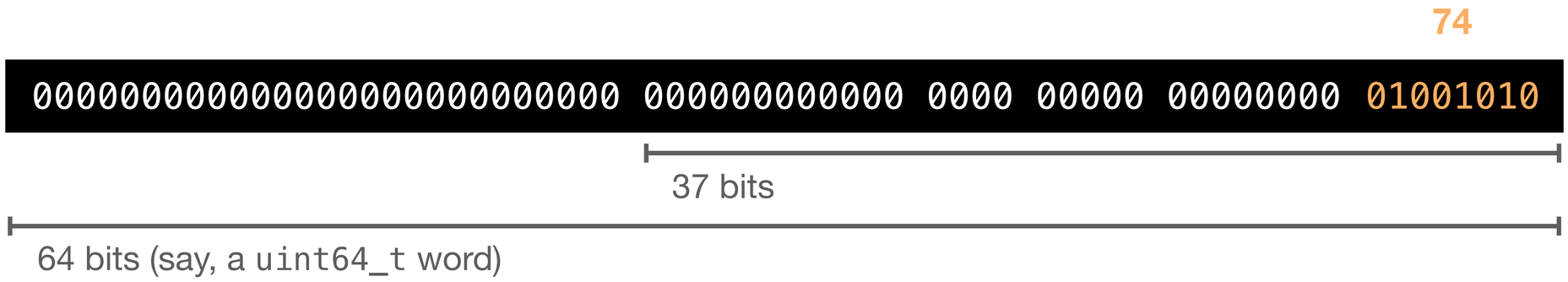

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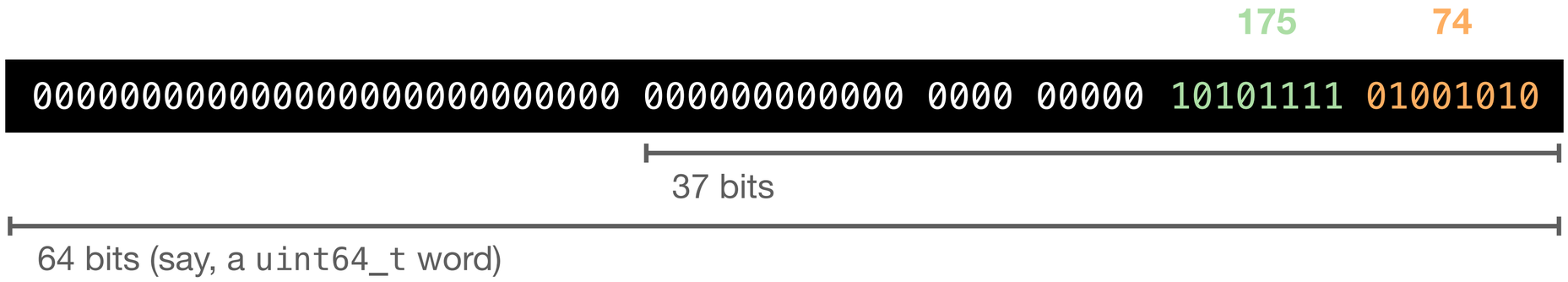

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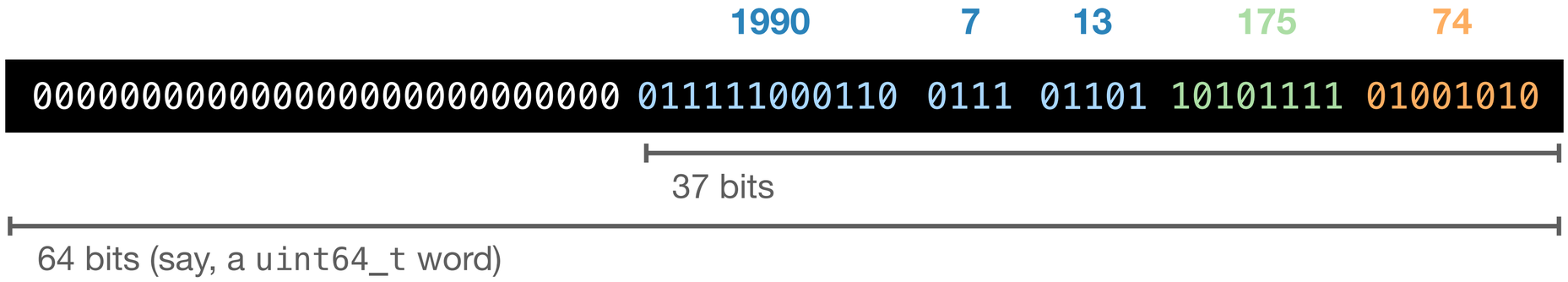

Bit-Packing

- In general: given a list of n integers, we can identify the largest one, say max, and encode the list using $n\lceil \log_2(max + 1) \rceil$ bits.
- Or perhaps: we already know that each integer is less then a given quantity U — the *universe* of representation — and we encode the list in $n\lceil \log_2 U \rceil$ bits. (Tight if U is close to max.)

Bit-Packing

- **Q.** But how do we actually read/write the bits?
- **A.** Using bit-wise operators + some logic.

C/C++ Bit-wise operators:
& (and), **|** (or), **^** (xor), **<<** (left shift), **>>** (right shift).

x	y	x & y
0	0	**0**
0	1	**0**
1	0	**0**
1	1	**1**

x	y	x | y
0	0	**0**
0	1	**1**
1	0	**1**
1	1	**1**

x	y	x ^ y
0	0	**0**
0	1	**1**
1	0	**1**
1	1	**0**

Bit-wise Operations

Shifting.

AND (masking).

OR (adding).

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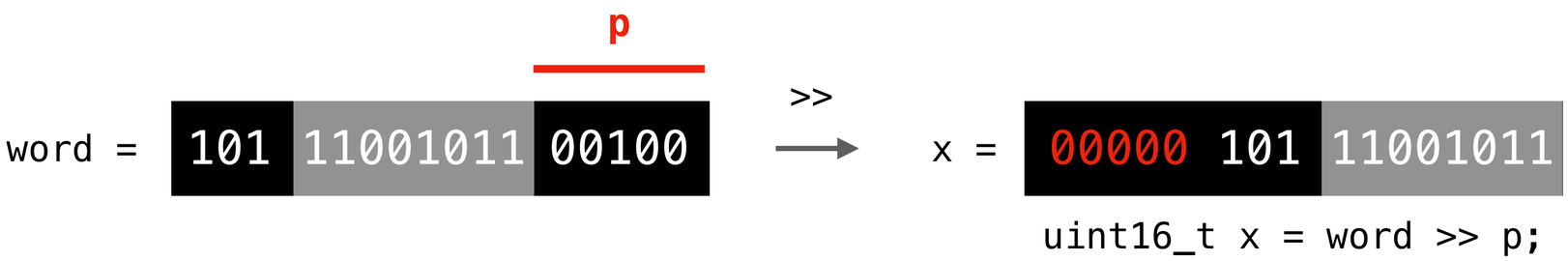


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uint16_t x = word >> p;
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AND (masking).

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Bit-wise Operations

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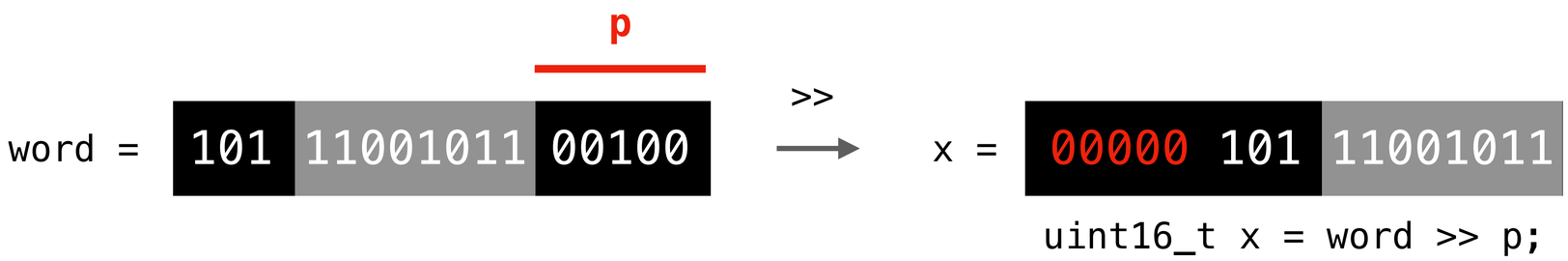


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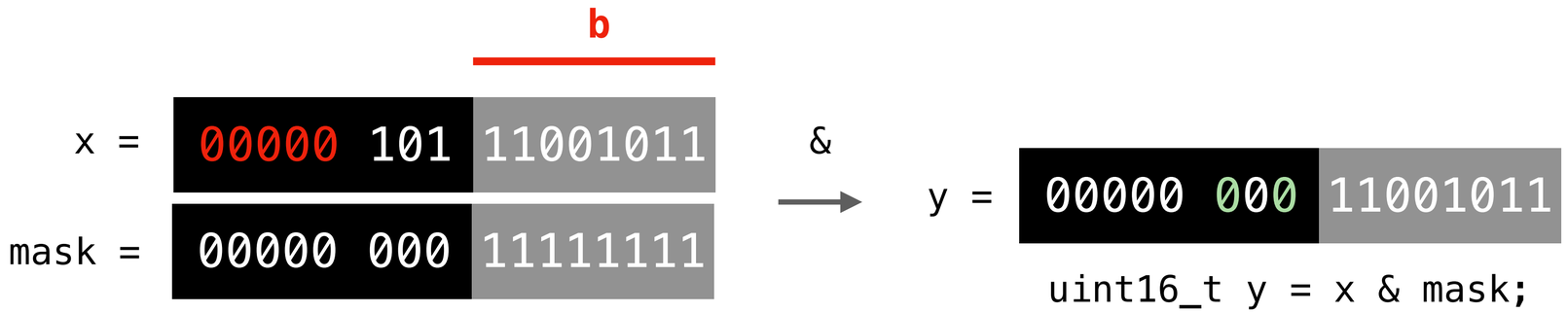


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uint16_t mask = (1 << b) - 1;
```

```
uint16_t y = x & mask;
```

OR (adding).

Bit-wise Operations

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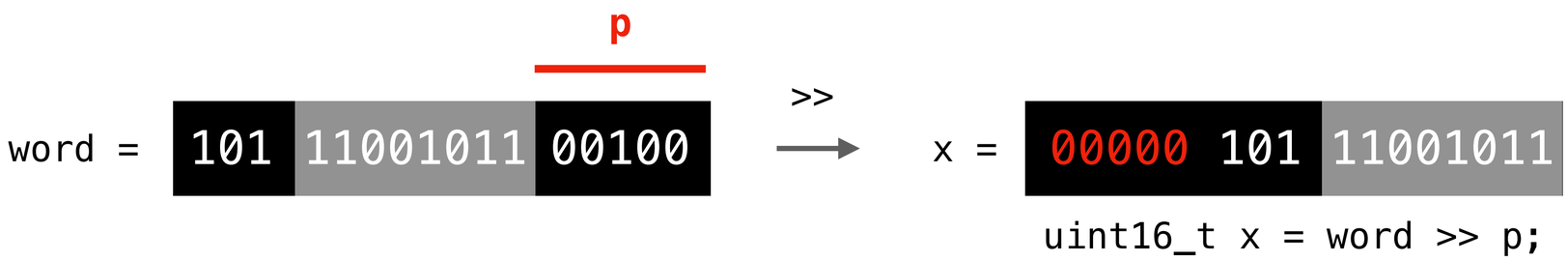


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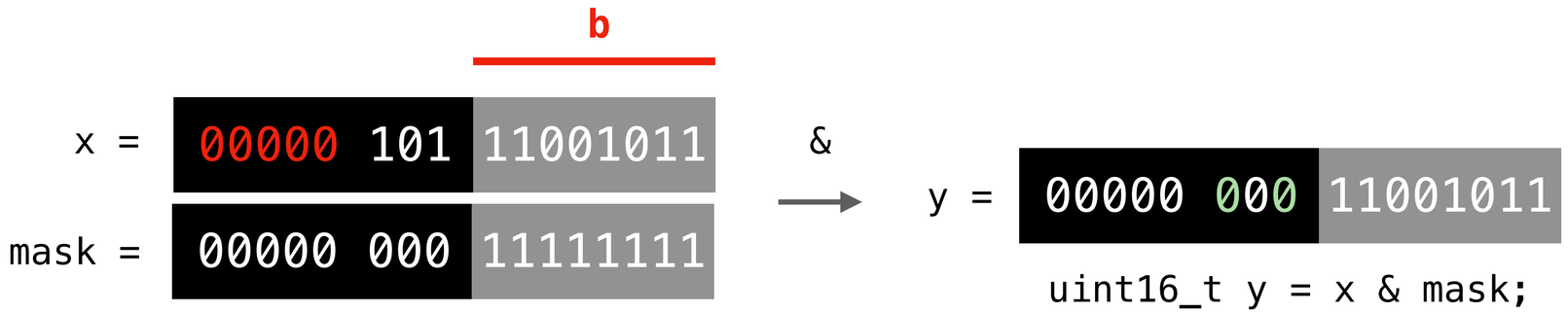


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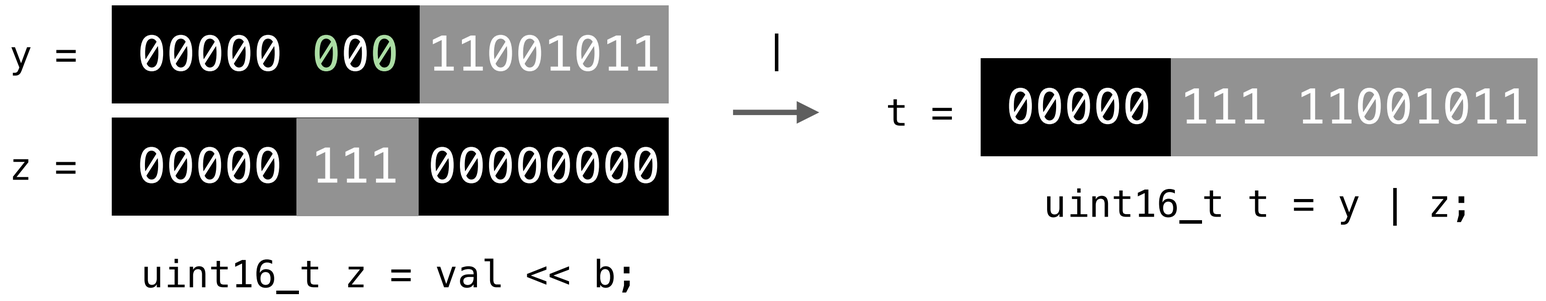


```
uint16_t z = val << b;
```

```
uint16_t t = y | z;
```

Read/Write Binary Data

- **Q.** What if you have more than 64 bits of data to read/write?

  For example, you want to store 40 bit-packed records, and so you would need $37 \cdot 40 = 1480$ bits.

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- **A.** Allocate a `std::vector<uint64_t>`.
  If the vector has size n, then it holds $n \cdot 64$ bits.

  So for 1480 bits we are going to need a vector of $\lceil 1480/64 \rceil = 24$ words.

- Armed with these basic tools (bit-wise operations and `std::vector<uint64_t>`), we can read/write *any* amount of bits.

Read/Write Binary Data

```
struct bit_vector_builder {
    bit_vector_builder() : m_num_bits(0) {}

    (...)

    void build(bit_vector& bv) {
        std::swap(m_num_bits, bv.m_num_bits);
        m_bits.swap(bv.m_bits);
    }

    void append_bits(uint64_t x, uint64_t len) {
        if (len == 0) return;
        uint64_t pos_in_word = m_num_bits % 64;
        m_num_bits += len;
        if (pos_in_word == 0) {
            m_bits.push_back(x);
        } else {
            *m_cur_word |= x << pos_in_word;
            if (len > 64 - pos_in_word) {
                m_bits.push_back(x >> (64 - pos_in_word));
            }
        }
        m_cur_word = &m_bits.back();
    }

private:
    uint64_t m_num_bits;
    std::vector<uint64_t> m_bits;
    uint64_t* m_cur_word;
};
```

build the `bit_vector` by “stealing” (i.e., swapping) the bits

appends the `len` **least significant** bits of `x` to the current end of the bit-vector (we assume `len <= 64`)

keep track of the number of written bits

write

split the write if necessary

backed data

```
struct bit_vector {
    bit_vector() : m_num_bits(0) {}

    (...)

    uint64_t get_bits(uint64_t pos, uint64_t len) const {
        if (len == 0) return 0;
        uint64_t block = pos / 64;
        uint64_t shift = pos % 64;
        uint64_t mask = -(len == 64) | ((1ULL << len) - 1);
        if (shift + len <= 64) return m_bits[block] >> shift & mask;
        return (m_bits[block] >> shift) |
               (m_bits[block + 1] << (64 - shift) & mask);
    }

    friend struct bit_vector_builder;

private:
    uint64_t m_num_bits;
    std::vector<uint64_t> m_bits;
};
```

read `len` bits starting from position `pos` and return them (we assume `len <= 64`)

read

split the read if necessary

`bit_vector_builder` is granted access to private members

Example

```cpp
#include <iostream>
#include <vector>

#include "bit_vector.hpp"

/* Definition of struct record. */
(...)

int main() {
    constexpr uint64_t n = 10000;

    std::vector<record> records;
    records.reserve(n);

    /* Initialize the vector of records. */
    (...)

    bit_vector_builder builder;
    builder.reserve(37 * n);

    for (auto r : records) {
        builder.append_bits(r.weight, 8);
        builder.append_bits(r.height, 8);
        builder.append_bits(r.day, 5);
        builder.append_bits(r.month, 4);
        builder.append_bits(r.year, 12);
    }

    bit_vector packed_records;
    builder.build(packed_records);

    return 0;
}
```

some useful library includes

include the “`bit_vector.hpp`” file that contains the implementation of the classes `bit_vector` and `bit_vector_builder`

create an object a `std::vector` of records and reserve space for n records

create an object `bit_vector_builder` and reserve space for $37n$ bits

for each record: bit-pack its quantities

create a `bit_vector` object and “steal” (i.e., swap) the bits from the `bit_vector_builder`

Compile with:

```
g++ -std=c++11 packed_records.cpp -o packed_records
```

Run with:

```
./packed_records
```

Run-Length Encoding

- Observation: sometimes data features *long runs* of equal symbols.

Example 1:
`111.000000000000000000000000.11111111111111111.000000000000.111111111111111111111111111.000000000000.1.000000000000000000000000.1111111111.000000000000000.11111111.00000000000000000000000000000000000000000000`

Example 2: this slide! If you model a pixel with a bit —colored pixel = `1`; white pixel = `0` — then most bits are `0`.

- Encode the *lengths* of the runs.

Example 1 — continued:
$[3,26,18,13,27,12,1,25,10,15,8,45] \rightarrow (12 \cdot 8) = 96$ bits vs. 203 bits (assuming a run is shorter than 256)

Example 2 — more general:
aaaaaaa.bbbb.aaaaaaaaaaaaaaaa.ccccccccccccc.aaaaaaaaa → (a,7) (b,4) (a,16) (c,12) (a,9)