1)

$x[n] = a^{n-2} u[-n+1]$ shift 2

الف) $-a^n u[-n-1] \xleftrightarrow{z} \frac{+1}{1-az^{-1}} \Rightarrow a^{n-2} u[-n+1] \xleftrightarrow{z} \frac{-z^{-2}}{1-az^{-1}}$

$n \to n-2$

و داریم

$$-a^{n-2} u[-n+1] \xleftrightarrow{z} \frac{z^{-2}}{1-az^{-1}} \qquad |z| < |a|$$

$$a^{n-2} u[-n+1] \xleftrightarrow{z} \frac{-z^{-2}}{1-az^{-1}}$$

ب)

$x[n] = \underline{a^{2n} u[2n]}$ $\qquad u[2n] = \begin{cases} 1 & 2n \geq 0 \\ 0 & \text{o.w} \end{cases} = u[n]$

$\hookrightarrow x[n] = (a^2)^n u[n]$

$$\xrightarrow{z} X(z) = \frac{1}{1-a^2 z^{-1}} \qquad |z| > |a^2|$$

Subject:
Date

c) 1 sol

$$y_1[n] = e^{-an} x[n] = \alpha^n x[n]$$

$$\boxed{\alpha = e^{-a}}$$

$$Y_1(z) = X\left(\frac{z}{\alpha}\right) = \boxed{X(e^{a} z)}$$

b) $$y_2[n] = \begin{cases} x\left[\frac{n}{2}\right] & n \text{ is even} \quad X_1[n] \\ x\left[\frac{n-1}{2}\right] & n \text{ is odd} \quad X_2[n] \end{cases}$$

$$X_1[n] = \begin{cases} x\left[\frac{n}{2}\right] & n \text{ is even} \\ 0 & \text{o.w.} \end{cases} \quad \& \quad X_2[n-1] = \begin{cases} x\left[\frac{n-1}{2}\right] & (n-1) \text{ is even} \\ 0 & \end{cases}$$

$$x_1[n-1] = x_2[n-1]$$

$$\hookrightarrow x_1[n] = x_2[n]$$

so when $n-1$ is even $\Rightarrow$ n is odd

$$y_2[n] = x_1[n] + x_2[n-1]$$

$$= x_1[n] + x_1[n-1]$$

$$\xrightarrow{Z} X(z^{+2}) \qquad \xrightarrow{Z} z^{-1} X(z^2)$$

$$Y_2[n] = (1 + z^{-1}) X(z^2)$$

Subject:
Date

2) j)

$$y_3[n] = \begin{cases} 0 & n \text{ is even} \\ x\left[\frac{n-1}{2}\right] & n \text{ is odd} \end{cases}$$

Like last part

$$y_3[n] = x_2[n-1] \Rightarrow \boxed{Y_3(z) = z^{-1}X(z^2)}$$

3) اگر سیگنال یک طرفه باشد:

$$X(z) = \frac{1}{(1-az^{-1})(1-bz^{-1})}$$

$x[1] = 1.5$

$x[2] = 1.75$

$$x(z) = \frac{1}{(1-az^{-1})(1-bz^{-1})} = \frac{A}{(1-az^{-1})} + \frac{B}{(1-bz^{-1})} = \frac{\frac{1}{(1-\frac{b}{a})}}{1-az^{-1}} + \frac{\frac{1}{(1-\frac{a}{b})}}{1-bz^{-1}}$$

$$x(k) = \frac{1}{(1-\frac{b}{a})}\, a^k u[k] + b^k u[k] \frac{1}{(1-\frac{a}{b})}$$

$$x[1] = \frac{a}{1-\frac{b}{a}} + \frac{b}{1-\frac{a}{b}} = 1.5 \quad , \quad \frac{a^2}{1-\frac{b}{a}} + \frac{b^2}{1-\frac{a}{b}} = 1.75$$

$$\left.\begin{array}{l} a+b = \frac{3}{2} \\ a^2+ba+b^2 = \frac{7}{4} \end{array}\right\} \Rightarrow \left\{\begin{array}{l} a = 1 \\ b = \frac{1}{2} \end{array}\right. \Rightarrow |a-b| = \frac{1}{2} \Bigg\} \boxed{\frac{1}{2}}$$

$$\left.\begin{array}{l} b = 1 \\ a = \frac{1}{2} \end{array}\right. \Rightarrow |a-b| = \frac{1}{2}$$

قیام ۱۷ شهریور و کشتار جمعی از مردم به دست ماموران ستم شاهی پهلوی (۱۳۵۷ هـ ش)

الف) 4

چهارشنبه ۶
28 Wednesday
۲۹

$$\frac{z^2(z+1)}{(z-1)(z^2-z+0.5)} = \frac{A}{(z-1)} + \frac{B}{(z^2-z+0.5)}$$

$$= \frac{4}{z-1} + \frac{z^2-2z+2}{z^2-z+0.5} = \underset{①}{\frac{4}{z-1}} + \underset{②}{1} + \underset{③}{\frac{-z+\frac{3}{2}}{z^2-z+0.5}}$$

① $4u[n-1]$

② $\delta[n]$

پنجشنبه ۷
29 Thursday
۱

③ $X_3(z) = \frac{-z+\frac{3}{2}}{z^2-z+0.5}$ $z > X_3\left(\frac{z}{\sqrt{2}}\right) = \frac{\frac{-z}{\sqrt{2}}+\frac{3}{2}}{\frac{z^2}{2}-\frac{z}{\sqrt{2}}+\frac{1}{2}}$

$$= \frac{-z^{-1}\sqrt{2}+3z^{-2}}{1-\frac{2\sqrt{2}}{2}z^{-1}+z^{-2}} = \frac{-z^{-1}\sqrt{2}+z^{-2}+2z^{-2}}{1-\frac{2\sqrt{2}}{2}z^{-1}+z^{-2}}$$

ولادت حضرت معصومه (س) (۱۷۳ هـ ق) - روز آتش نشانی و ایمنی

جمعه ۸
30 Friday
۲

داشد در صفحه بعد

روز بزرگداشت مولوی - روز جهانی ناشنوایان - روز جهانی دریانوردی

$$\frac{-z^{-1}\sqrt{2}\left(1-\frac{\sqrt{2}}{2}z^{-1}\right)}{1-\frac{2\sqrt{2}}{2}z^{-1}+z^{-2}} \quad , \quad \frac{\left(z^{-1}2\sqrt{2}\right)\frac{\sqrt{2}}{2}\sqrt{2}z^{-1}}{1-\frac{2\sqrt{2}}{2}z^{-1}+z^{-2}}$$

$$\Rightarrow (\sqrt{2})^n x_3[n] \Rightarrow -\sqrt{2}\cos\left[\frac{\pi}{4}(n-1)\right]u[n-1] + 2\sqrt{2}\sin\left[\frac{\pi}{4}(n-1)\right]u[n-1]$$

$$x_3[n] \Rightarrow -(\sqrt{2})^{1-n}\cos\left[\frac{\pi}{4}(n-1)\right]u[n-1] + 2(\sqrt{2})^{1-n}\sin\left[\frac{\pi}{4}(n-1)\right]u[n-1]$$

$$x[n] = x_1[n] + x_2[n] + x_3[n]$$

که هر کدام جداگانه حساب کنید

4 - س

$$X(z) = \frac{z^2}{(z-1)(z-\frac{1}{2})^2} = \frac{A}{z-1} + \frac{B}{z-\frac{1}{2}} + \frac{C}{(z-\frac{1}{2})^2}$$

$$= \frac{4}{z-1} + \frac{-3}{z-\frac{1}{2}} + \frac{\frac{-1}{2}}{(z-\frac{1}{2})^2} = \frac{4z^{-1}}{1-z^{-1}} + \frac{-3z^{-1}}{1-\frac{1}{2}z^{-1}} + \frac{\frac{-1}{2}z^{-1}}{(1-\frac{1}{2}z^{-1})^2}$$

$$= 4u[n-1] - 3\left(\frac{1}{2}\right)^{n-1} u[n-1] - \left(\frac{1}{2}\right)^{n-1}(n-1)u[n-1]$$

$$X(z) = \frac{z(z+1)}{(z-1)^3}$$

$$\frac{z(z+1)z^{k-1}}{(z-1)^3} = \frac{z^k(z+1)}{(z-1)^3}$$

$$k \geq 0 : \operatorname{Res}\{z=1\} = \lim_{z\to 1} \frac{1}{2!}\left(z^k(z+1)\right)''$$

$$= \lim_{z\to 1} \frac{1}{2}\left(k^2 z^{k-1} + \cancel{k(k-1)z^{k-2}} + k z^{k-1}\right)$$

$$= k^2 \quad \Rightarrow \boxed{n[n] \Rightarrow k^2 u[k]}$$

Subject:
Date

6) $X(z) = \log(1+az^{-1}) \quad |z| > |a|$

خاصیت مشتق :

$$\frac{dX(z)}{dz} = \frac{\frac{-a}{z^2}}{1+\frac{a}{z}} = \frac{-az^{-2}}{1+az^{-1}}$$

$x(-z)$

$$\frac{-az^{-1}}{1+az^{-1}} \xrightarrow{z^{-1}} n\,x[n] = a(-a)^{n-1}u[n-1]$$

$$\Rightarrow x[n] = \frac{(-1)^{n+1}a^n u[n-1]}{n}$$

روش بسط

$X(z) = \ln(1+az^{-1})$

$\ln(1+u) = u - \frac{u^2}{2} + \frac{u^3}{3} - \frac{u^4}{4} + \cdots$

$$X(z) = \ln(1+az^{-1}) = az^{-1} + \frac{a^2}{2}z^{-2} - \frac{a^3}{3}z^{-3} + \frac{a^4}{4}z^{-4} \cdots$$

$$\xrightarrow{z^{-1}} a\delta[n-1] - \frac{a^2}{2}\delta[n-2] - \frac{a^3}{3}\delta[n-3] \cdots$$

$$\Rightarrow x[n] = \frac{(-1)^{n+1}a^n u[n-1]}{n}$$

7)

با در نظر گرفتن شرایط اولیه

$y[n]-3y[n-1]=x[n]$

$\hookrightarrow 8u[n]$ و $y[-1]=1$

$\xrightarrow{z^{-1}} Y(z)-3(z^{-1}Y(z)+y[-1])=\frac{8}{1-z^{-1}}$

$\Rightarrow Y(z)[1-3z^{-1}]-3=\frac{8}{1-z^{-1}}$

$Y(z)[1-3z^{-1}]=\frac{8}{1-z^{-1}}+3$

$Y(z)=\underbrace{\frac{8}{(1-z^{-1})(1-3z^{-1})}}_{①}+\underbrace{\frac{3}{1-3z^{-1}}}_{②}$

$① \xrightarrow{z^{-1}} 12(3)^n u(n)-4u[n]$

$② \Rightarrow 3(3)^n u(n)$

$y[n]=12(3)^n u[n]-4u[n]+3(3)^n u[n]$

$y[0]=12-4+3=11$ ✓

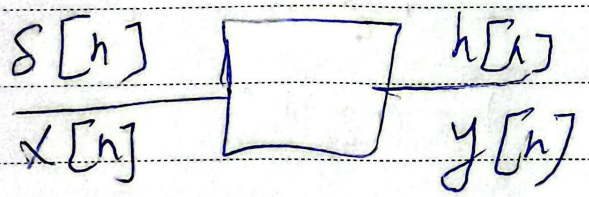
$y[1]=36-4+9=41$ ✓

Subject :
Date

8)

$$\left.\begin{aligned} x[n] &= a^n u[n] \\ y[n] &= b^n u[n] + c^n u[n] \end{aligned}\right\} \Rightarrow$$

$$y[n] = \left(\frac{b}{a}\right)^n u[n] + \left(\frac{c}{a}\right)^n u[n]$$



چون پاسخ ضربه سیستم LTI را داشته و سیستم

$$h(n) = \left(\frac{b}{a}\right)^n \delta[n] + \left(\frac{c}{a}\right)^n \delta[n]$$

$$= \delta[n]\left(\left(\frac{b}{a}\right)^n + \left(\frac{c}{a}\right)^n\right)$$