1-1) $f(t) = (t-\pi)\sin(3t)e^{2t}u(t-\pi)$

$= f(t) = (t-\pi)u(t-\pi)\sin(3t - \cancel{3}3\pi)e^{2t-2\pi} \quad e^{2\pi} \times (-1)$

$L\{\sin(3t)\} = \frac{3}{s^2+9} \Rightarrow L\{t\,u(t)\sin(3t)\} = \frac{6s}{(s^2+9)^2}$

$\Rightarrow L\{e^{2t} \times t\,u(t) \cdot \sin(3t)\} = \frac{6(s-2)}{((s-2)^2+9)^2}$

$L\{e^{2(t-\pi)} \cdot (t-\pi)\sin(3(t-\pi))u(t-\pi)\} = e^{-\pi s}\frac{6(s-2)}{((s-2)^2+9)^2}$

$L\{-e^{2(t-\pi)} \cdot (t-\pi)u(t-\pi)\sin(3(t-\pi))\} = -e^{\pi s}\frac{6(s-2)}{((s-2)^2+9)^2}$

1.2) $f(t) = \int_t^\infty \frac{e^{-x}}{x}\, dx \xrightarrow{\frac{d}{dx}} f'(t) = -\frac{e^{-t}}{t}$

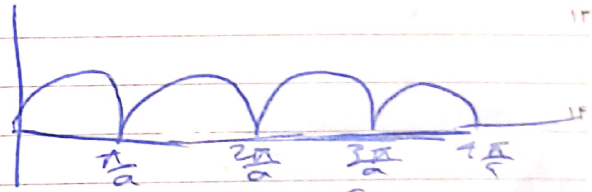
$\mathcal{L}\{t f'(t)\} = -\mathcal{L}\{e^{-t}\} = \frac{1}{s+1} = \frac{d F(s)}{ds}$

$\Rightarrow \mathcal{L}(f'(t)) = \ln(s+1) = s\,\mathcal{L}\{f(t)\} - f(0^-) \Rightarrow$

$s<0 \Rightarrow 0$

$\Rightarrow \mathcal{L}\{f(t)\} = \frac{\ln(s+1)}{s}$

1.3) $f(t) = |\sin(at)| \Rightarrow T = |\frac{\pi}{a}|$



$\mathcal{L}\{f(t)\} = \frac{F_T(s)}{1-e^{-Ts}}$ $\quad F_T(s) = \int_0^\infty f_T(t) e^{-st}\, dt = \int_0^{\frac{\pi}{a}} \sin(at) e^{-st}\, dt$

$= \frac{a}{a^2+s^2}\left(1+e^{-\frac{\pi s}{a}}\right)$

$\mathcal{L}\{f(t)\} = \frac{a}{a^2+s^2} \frac{\left(1+e^{-\frac{\pi s}{a}}\right)}{\left(1-e^{-\frac{\pi s}{a}}\right)} = \frac{a}{a^2+s^2} \cdot \frac{\cancel{e^{\frac{\pi s}{2a}}}}{\cancel{e^{-\frac{\pi s}{2a}}}} \underbrace{\frac{e^{\frac{\pi s}{2a}} + e^{-\frac{\pi s}{2a}}}{e^{\frac{\pi s}{2a}} - e^{-\frac{\pi s}{2a}}}}_{\coth\left(\frac{\pi s}{2a}\right)}$

$= \frac{a}{a^2+s^2} \coth\left(\frac{\pi s}{2a}\right)$

$1-4)\quad F(s) = \underbrace{\left(s \times \tan^{-1}\left(\frac{1}{s}\right)\right)}_{①} \underbrace{-1}_{②}$

$\mathcal{L}^{-1}\{②\} = \delta(t)$

$\frac{d①}{ds} = \tan^{-1}\left(\frac{1}{s}\right) - \frac{s}{s^2+1}$ ☆

$\frac{d}{ds}\left(\tan^{-1}\left(\frac{1}{s}\right)\right) = \frac{-1}{s^2+1} \Rightarrow \mathcal{L}^{-1}\left(\frac{-1}{s^2+1}\right) = t f_2(t) = \sin t \Rightarrow$

$-\frac{d \tan^{-1}\left(\frac{1}{s}\right)}{ds}$

$\boxed{\mathcal{L}\left\{\tan^{-1}\left(\frac{1}{s}\right)\right\} = \frac{\sin(t)}{t}}$

☆ $\Rightarrow \mathcal{L}^{-1}\left\{\frac{d①}{ds}\right\} = -t f(t) = \frac{\sin t}{t} - \cos t \Rightarrow$

☆☆

$f(t) = \frac{\cos t}{t} - \frac{\sin t}{t^2} - \delta(t)$

1-5) $L = \frac{s^2-2}{s^4+4} = \frac{s^2-2}{s^4+4\underbrace{-4s+4s}} = \frac{A_1 s+B_1}{s^2+2s+2} + \frac{A_2 s+B_2}{s^2-2s+2}$

$\xRightarrow{\times\, s^4+4} s^2-2 = (A_1 s+B_1)\underbrace{(s^2-2s+2)}_{(s-1)^2+1} + (A_2 s+B_2)\underbrace{(s^2+2s+2)}_{(s+1)^2+1}$

if $s=-1+j \Rightarrow -2(j+1) = (A_1 j - A_1 + B_1)(-4(+j-1))$

$\Rightarrow \frac{j}{2}+\frac{1}{2} = \cancel{A_1} - A_1 j - B_1 - A_1 j - \cancel{A_1} + B_1 j \Rightarrow \boxed{B_1 = \frac{-1}{2}} \quad \boxed{A_1 = \frac{-1}{2}}$

if $s=1+j = \frac{2(j-1)}{2} = (A_2 j + A_2 + B_2)(\cancel{4+4j})$ $(1+j)$ $\Rightarrow \{ A_2 =$

$A_2 j + \cancel{A_2} + B_2 + \cancel{A_2} + A_2 j + B_2 j = \frac{j}{2} - 1$

$2A_2 j + B_2 j = \frac{j}{2}$

$B_2 = \frac{-1}{2}$

$\Rightarrow \boxed{A_2 = \frac{1}{2}}$

$\boxed{B_2 = \frac{-1}{2}}$

$B_1 = -\frac{1}{2}$ $A_1 = \frac{-1}{2}$ $B_1 = \frac{-1}{2}$ $A_2 = \frac{1}{2}$

$$X(s) = \frac{-\frac{1}{2}(s+1)}{(s+1)^2+1} + \frac{\frac{1}{2}(s-1)}{(s-1)^2+1} = \frac{-\frac{1}{2}s}{s^2+1} + \frac{\frac{1}{2}s}{s^2\ 1}$$

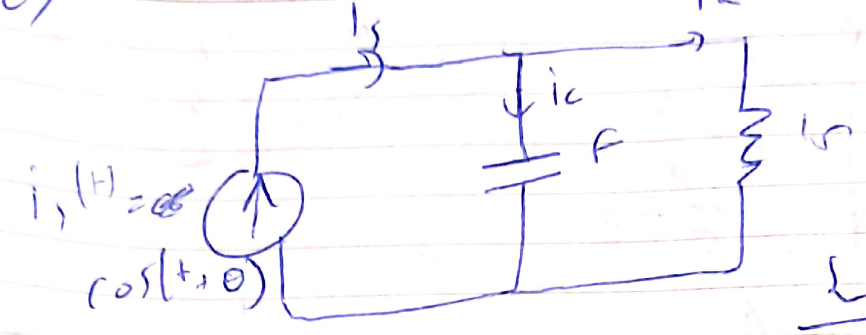
$\downarrow$ e^{-t} $\qquad$ $\downarrow$ e^{t}

$\mathcal{L}^{-1}$

$$\Rightarrow x(t) = \frac{1}{2}\left(e^{t}\cos t - e^{-t}\cos t\right) = \sinh(t)\cos t = x(t)$$

$$\frac{e^{t} - e^{-t}}{2} = \sinh(t)$$

2)



$$i_s = i_R + i_c = \frac{V_c}{R} + C\frac{dV_c}{dt}$$

$$\xrightarrow{\mathcal{L}} I_s(s) = sV_c(s) - \cancel{V_c(0)} + V_c(s)$$

$$\boxed{I_s(s) = V_c(s+1)} \quad *$$

$$i_s(t) = \cos(t+\theta) \xrightarrow{\text{گسترش}} \cos\theta\cos t - \sin t\sin\theta$$

$$\xrightarrow{\mathcal{L}} \boxed{I_s(s) = \frac{s\cos\theta}{s^2+1} - \frac{\sin\theta}{s^2+1}} \quad **$$

$$\Rightarrow V_c = \frac{I_s(s)}{s+1} = \frac{s\cos\theta}{(s^2+1)(s+1)} - \frac{\sin\theta}{(s+1)(s^2+1)}$$

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$$\frac{s}{(s^2+1)(s+1)} = \frac{As+B}{s^2+1} + \frac{C}{(s+1)} \xrightarrow{\times(s^2+1)(s+1)} s = (As+B)(s+1) + C(s^2+1)$$

$$\boxed{s=-1 \Rightarrow \frac{-1}{2} = C} \qquad \boxed{s=j} \Rightarrow j = (Aj+B)(1+j) + 0$$

$$Aj + B - A + Bj = j$$

$$B - A = 0 \Rightarrow \boxed{B = \frac{1}{2}}$$

$$A + B = 1 \qquad \boxed{A = \frac{1}{2}}$$

$$\frac{1}{(s+1)(s^2+1)} = \frac{A_2 s + B_2}{s^2+1} + \frac{C_2}{s+1} \xrightarrow{\times(s^2+1)(s+1)} 1 = (As+B)(s+1) + C(s^2+1)$$

$$s = (-1) \Rightarrow C = \frac{1}{2}$$

$$s = j \Rightarrow 1 = Aj + Bj + B - A \Rightarrow B - A = 1$$

$$A + B = 0$$

$$\boxed{B = \frac{1}{2}} \quad \boxed{A = -\frac{1}{2}}$$

$$V_c = \cos\theta\left(\frac{\frac{1}{2}s+\frac{1}{2}}{(s^2+1)} + \frac{-\frac{1}{2}}{s+1}\right) + \sin\theta\left(\frac{-\frac{1}{2}s+\frac{1}{2}}{(s^2+1)} + \frac{\frac{1}{2}}{s+1}\right)$$

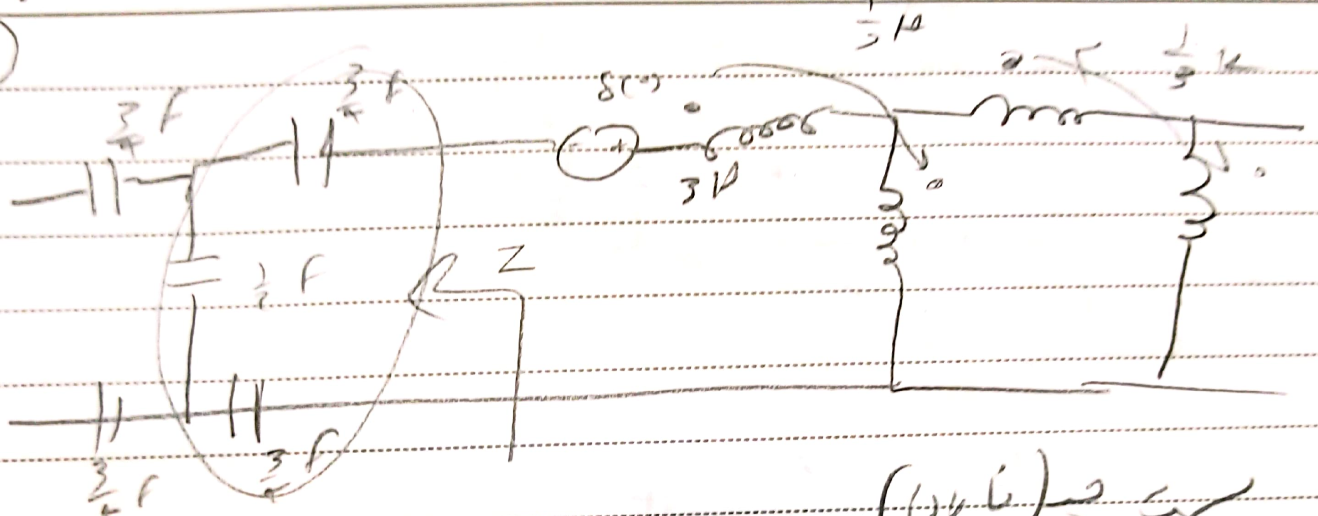
$$\xrightarrow{\mathcal{L}^{-1}} v_c = \cos\theta\left(\frac{1}{2}\cos t + \frac{1}{2}\sin t \underbrace{-\frac{1}{2}e^{-t}}_{\text{پاسخ گذرا}}\right) - \sin\theta\left(\frac{1}{2}\cos t + \frac{1}{2}\sin t \underbrace{+\frac{1}{2}e^{-t}}_{\text{پاسخ گذرا}}\right.$$

$$\cos\theta\left(-\frac{1}{2}e^{-t}\right) - \sin\theta\left(\frac{1}{2}e^{-t}\right) = 0$$

$$-e^{-t}\left(\cos\theta\cancel{\tfrac{1}{2}} + \sin\theta\right) = 0 \Rightarrow \boxed{\theta = \frac{3\pi}{4}, \frac{7\pi}{4}, \dots}$$

Subject : Date :

3)



$$Z = 0.75 \,||\, (0.5 + Z) \,||\, 0.75 \Rightarrow Z = 0.375 \,||\, (0.5 + Z)$$

$$Z = \frac{0.375(0.5 + Z)}{Z + 0.875} = \frac{\frac{3}{8}(\frac{1}{2} + Z)}{Z + \frac{7}{8}} = Z \Rightarrow$$

$$\frac{3 + 6Z}{14 + 16Z} = Z$$

$$14Z + 16Z^2 = 3 + 6Z \Rightarrow 16Z^2 + 8Z - 3 = 0 \begin{cases} \boxed{\frac{-8+16}{32}} \\ \frac{-8-16}{32} \end{cases}$$

$$Z = \frac{1}{4} \Rightarrow C_1 = \frac{1}{4} F$$

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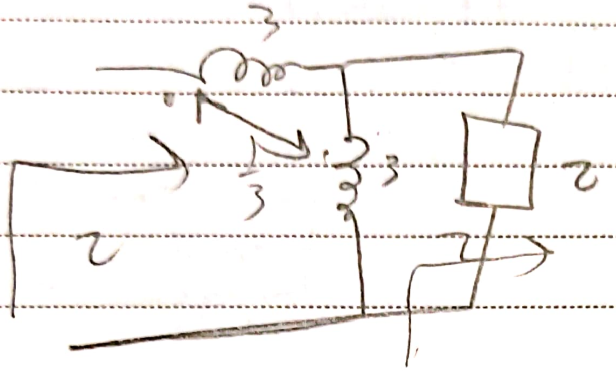

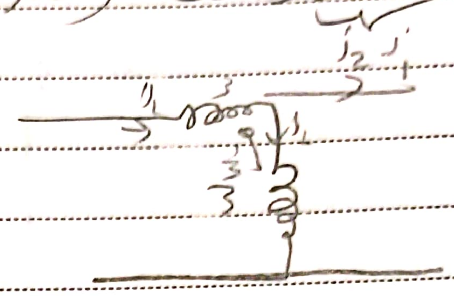

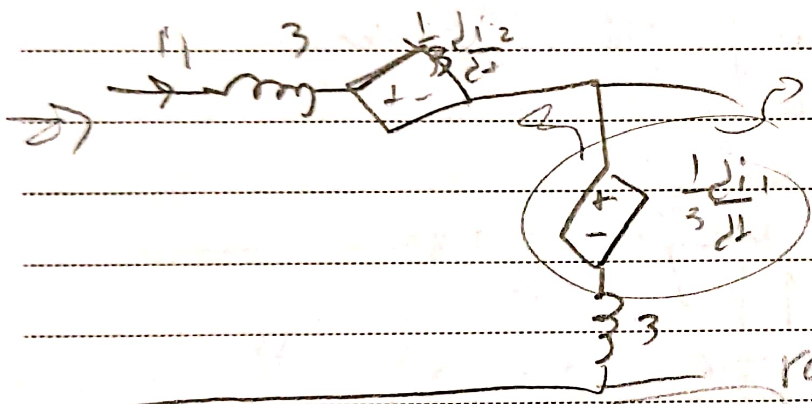

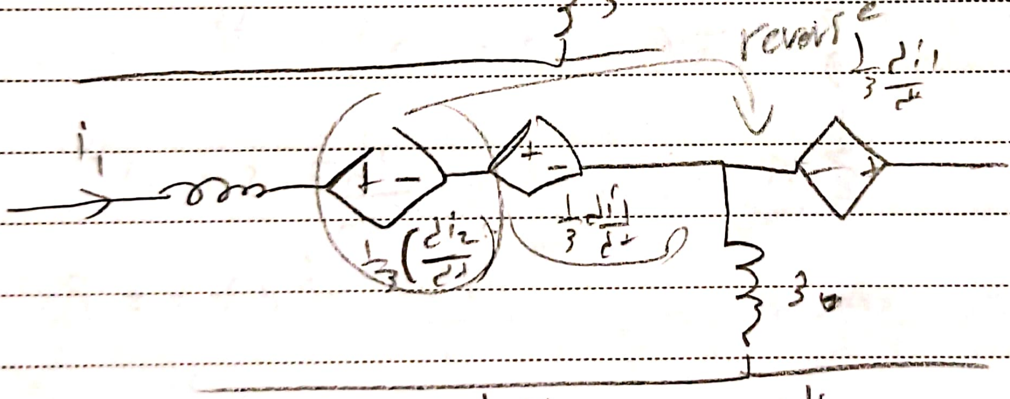

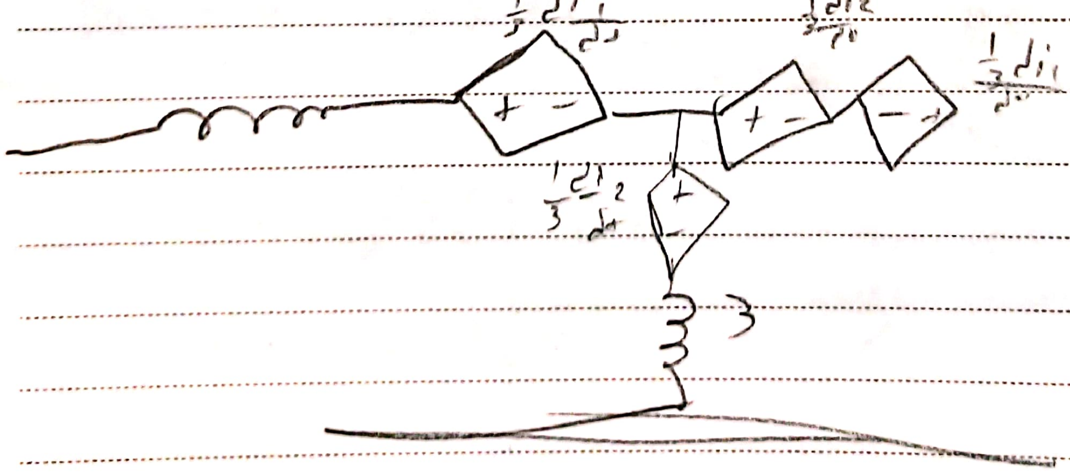

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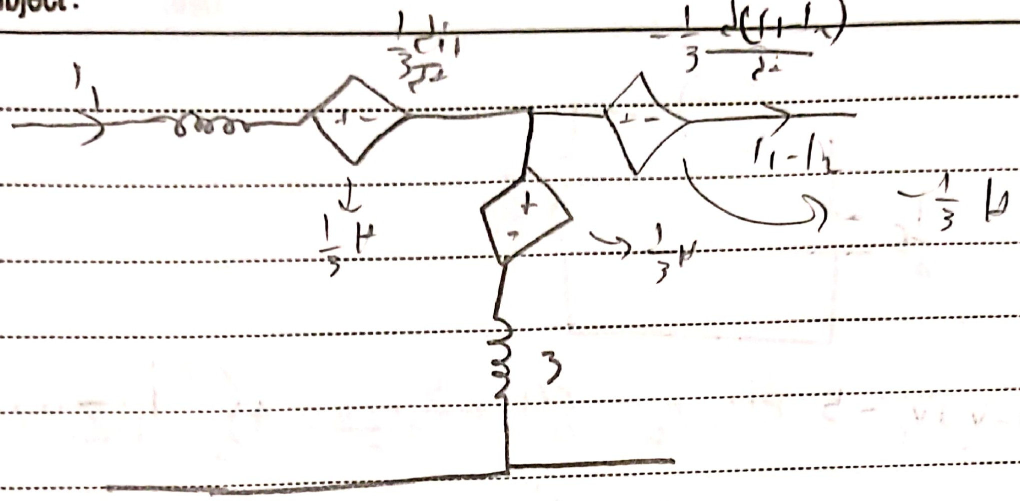

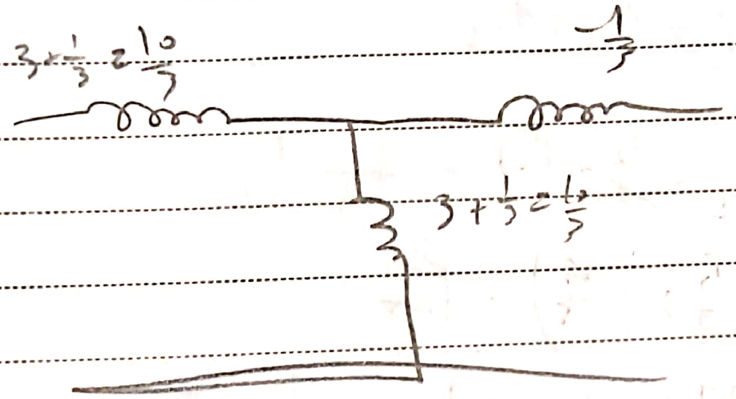

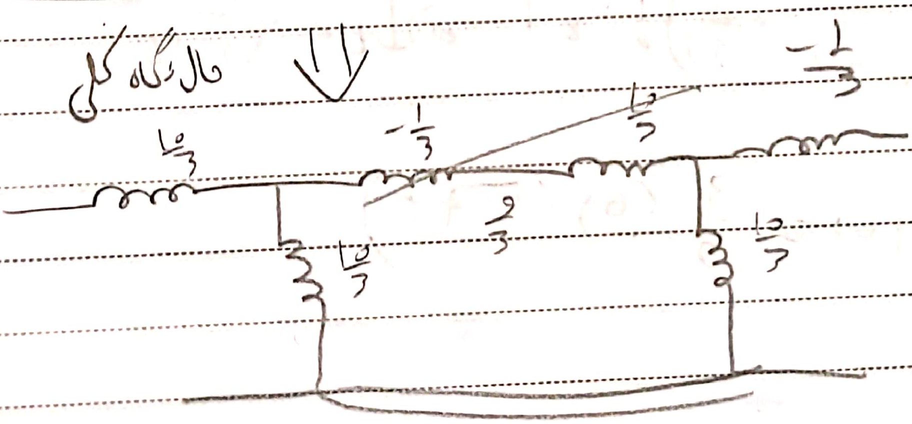

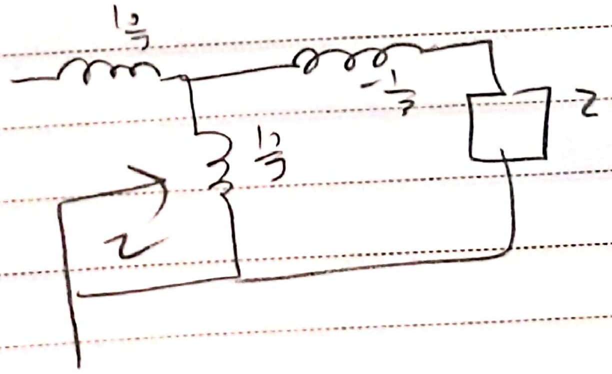

$$Z = \frac{10}{3} + \left(\frac{10}{3} \parallel \left(-\frac{1}{3} + Z\right)\right)$$

$$Z = \frac{10}{3} + \frac{\frac{10}{3}\left(Z - \frac{1}{3}\right)}{\frac{10}{3} + Z - \frac{1}{3}}$$

$$\Rightarrow Z = \frac{30}{18} = \frac{5}{3}\,H \qquad L_t = \frac{5}{3}$$

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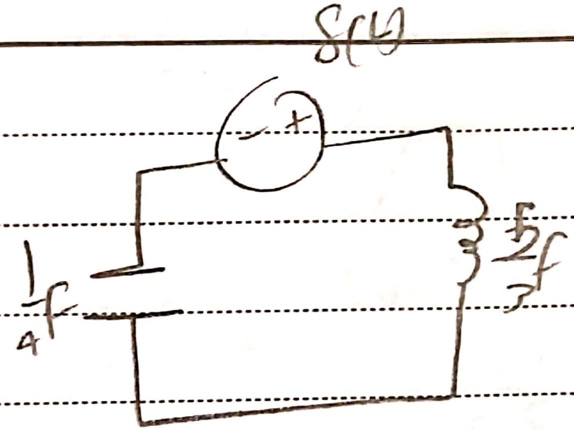


$$\delta(t) = V_L + V_c = \frac{5}{3}\frac{di_L}{dt} + V_c = \delta(t) = \frac{5}{3}\frac{di_L}{dt} + 4\int_0^t i_L(\tau)\,d\tau$$

$$V_c = 4i_c = 4i_L$$

$$\xrightarrow{\mathcal{L}} 1 = \frac{5}{3} s I_L - \cancel{\frac{5}{3} i_L(0)} + \frac{4}{s} I_L$$

$$0$$

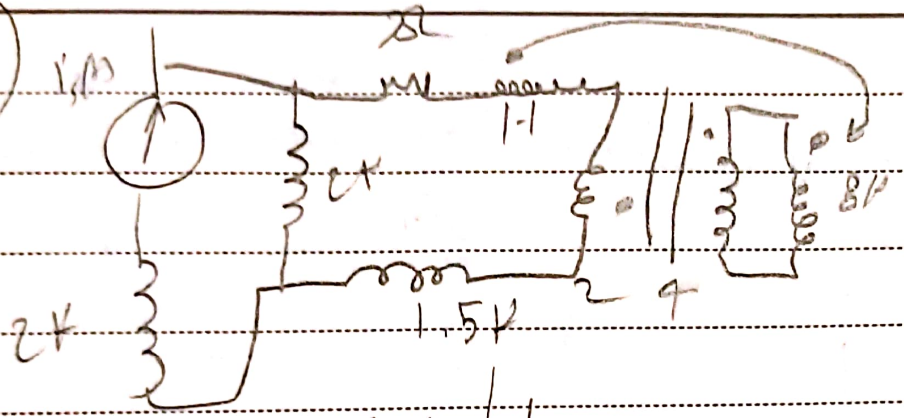
$$1 = I_L\left(\frac{5}{3}s + \frac{4}{s}\right) =$$

$$\left(\frac{5s^2}{3s} + \frac{12}{3s}\right) I_L = 1 \Rightarrow I_L = \frac{3s}{5s^2+12}$$

$$\xrightarrow{\mathcal{L}^{-1}} i_L(t) = \frac{3}{5}\cos\left(\sqrt{2.4}\,t\right)$$

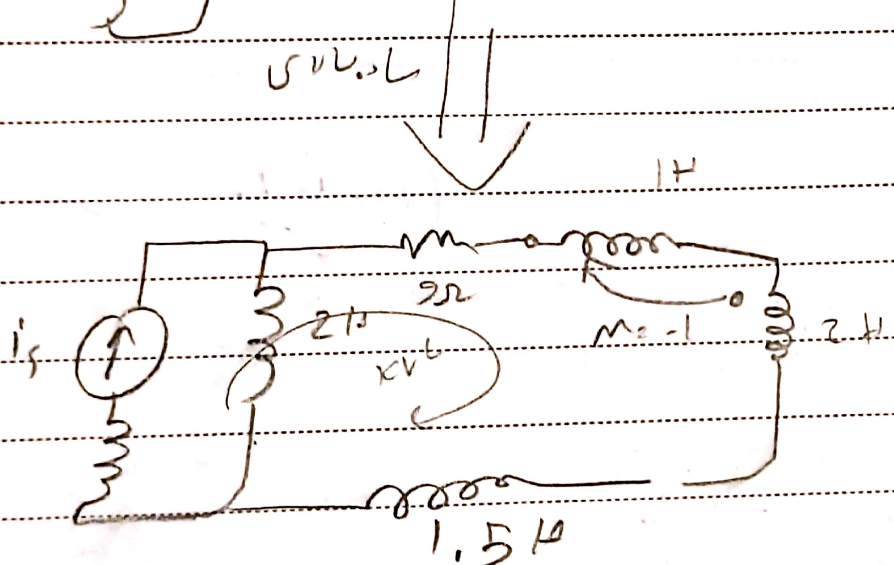
Subject :   Date :

4)



$8 \times \left(\frac{2}{4}\right)^2 = 2H$

$2 \times \left(\frac{2}{4}\right) = 1H$



$$9i + \cancel{\frac{di}{dt}} - \cancel{\frac{di}{dt}} + 2\frac{di}{dt} - \cancel{1\frac{di}{dt}} + 1.5\frac{di}{dt} = \frac{2di_s}{dt} - 2\frac{di}{dt}$$

$$9i_1 + 2.5\frac{di}{dt} = \frac{2di_s}{dt} - 2\frac{di}{dt}$$

$$9i_1 + 4.5\frac{di}{dt} = \frac{2di_s}{dt} \xrightarrow{\mathcal{L}} 4.5(sI(s) - \cancel{i(0)}) + 9I(s) = 2s$$

$$I(s)(4.5s + 9) = 2s \Rightarrow I_s = \frac{2s}{4.5(s+2)} = \frac{4}{9}\frac{(s+2)}{s+2}$$

$$\xrightarrow{\mathcal{L}^{-1}} i(t) = \frac{4}{9}\left(\delta(t) - 2e^{-2t}\right)$$

$$V_o(t) = R\,i(t) = 4\left(\delta(t) - 2e^{-2t}\right)$$

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$t<0$