Game Theory
Motivating Examples

By

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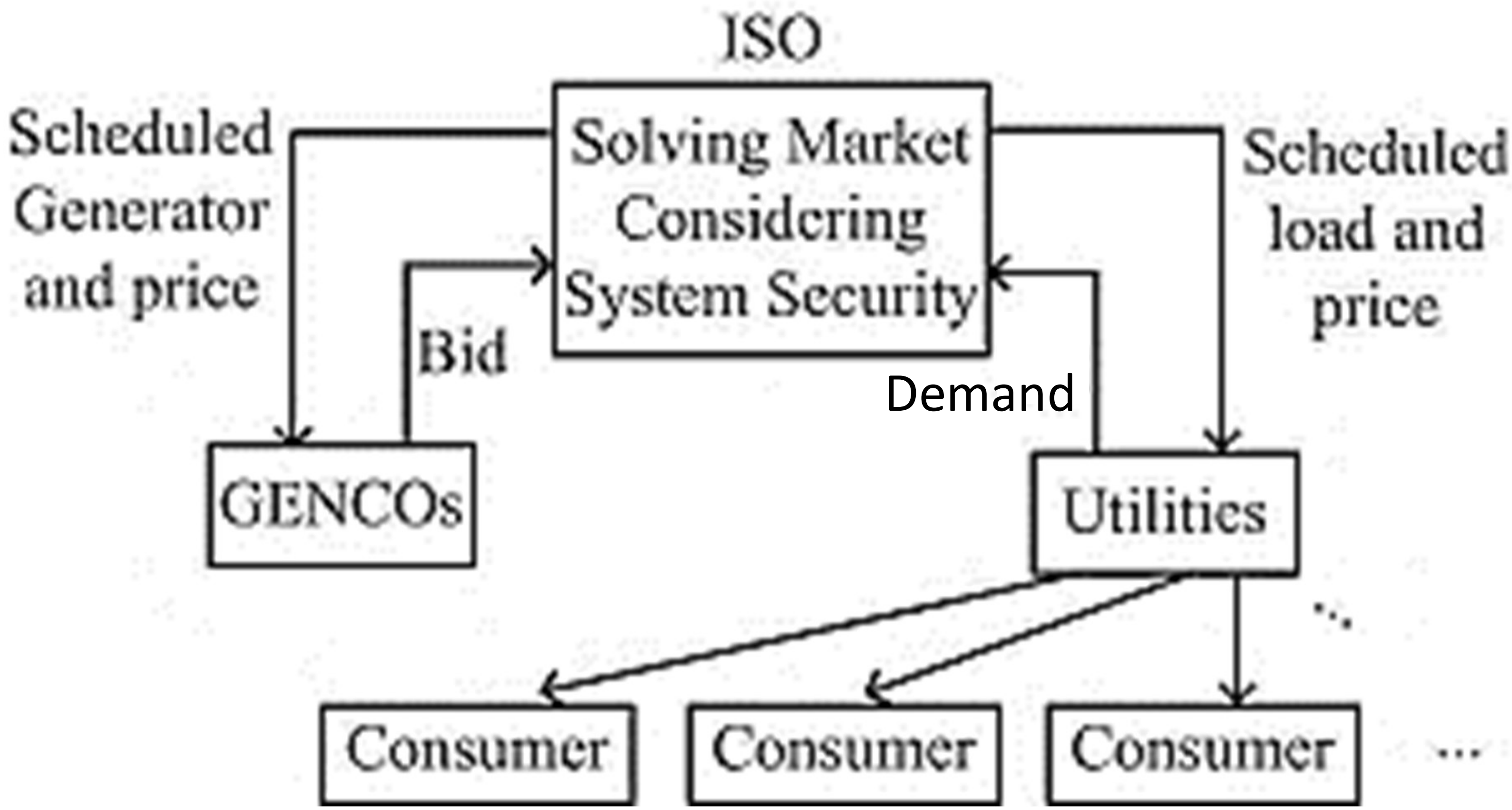
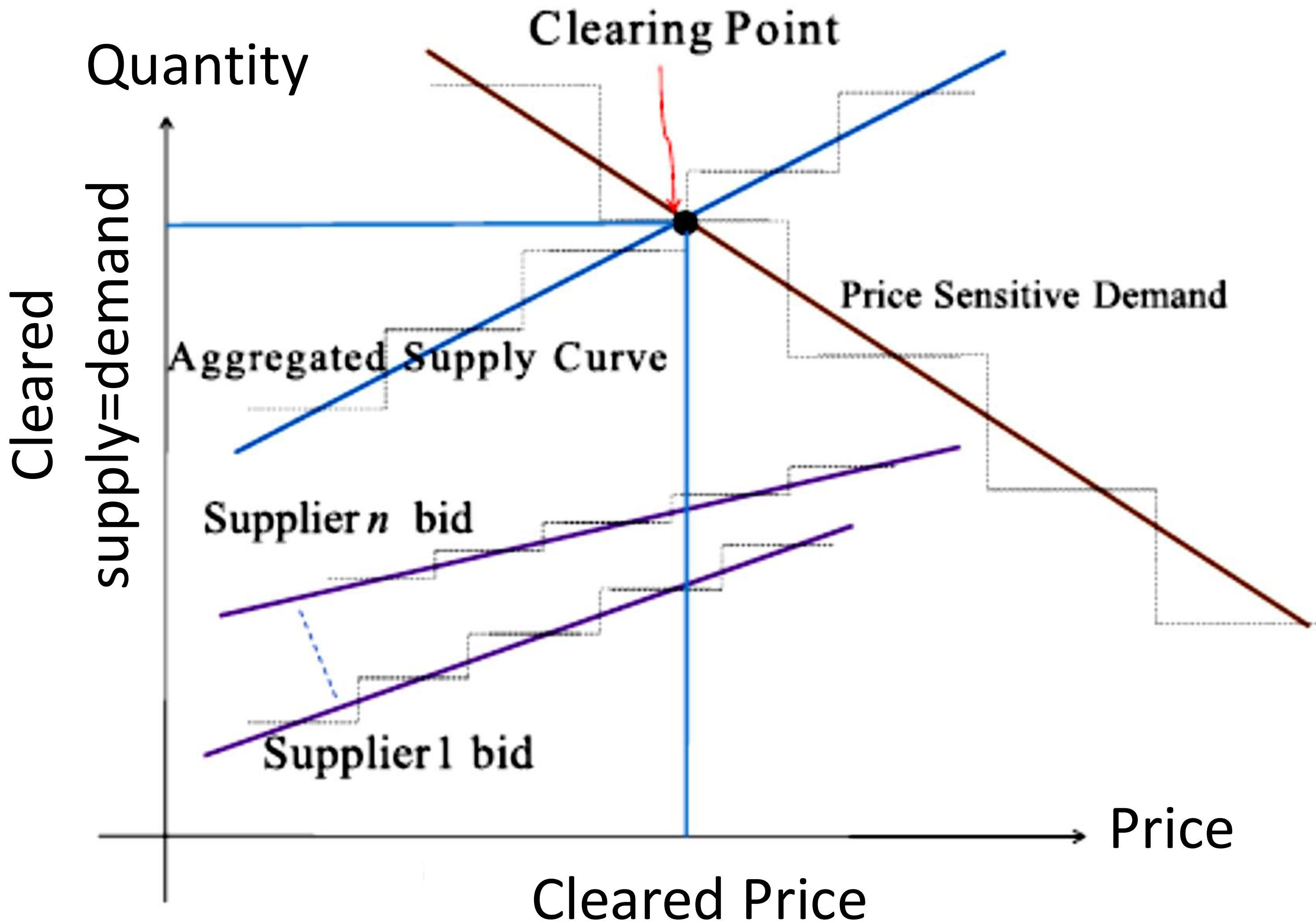
Outline: Application of the game theory in:

- Market (Electricity Market)
- Blockchain Network
- Wireless Networks: CDMA-OFDMA
- Smart Grids
- Intelligent Transportation systems
- Social Networks
- Biology: Cancer tumor–stroma interactions
- Deep Learning
- Cognitive Neuroscience

- Game Model:
  - Players (decision makers)
  - Objectives
  - Actions (Strategies)
  - Coupling term

Electricity Market




Clearing Point
Quantity
Cleared supply=demand
Price Sensitive Demand
Aggregated Supply Curve
Supplier n bid
Supplier 1 bid
Price
Cleared Price

Electricity Market

Decision variables of generator i:

$$Q_i = \eta_i(\lambda_i - \beta_i), \qquad i = 1, 2, \ldots, n$$

$$\text{Demand} = \sum_{i=1}^{n} Q_i$$

$$D_0 - \vartheta\lambda = \sum_{i=1}^{n} \eta_i(\lambda - \beta_i) = \lambda \sum_{i=1}^{n} \eta_i - \sum_{i=1}^{n} \eta_i \beta_i.$$

$$\lambda = \frac{D_0 + \sum_i \eta_i \beta_i}{\vartheta + \sum_i \eta_i}.$$

How to bid?

$$\pi_i = Q_i^c \lambda - C_i(Q_i^c) \quad i = 1,2,\dots,n$$

- Where

$$\lambda = \frac{D_0 + \sum_i \eta_i \beta_i}{\vartheta + \sum_i \eta_i}. \qquad Q_i^c = \eta_i(\lambda - \beta_i)$$

$$C_i(Q_i) = a_i Q_i^2 + b_i Q_i + c_i$$

- Baldick, Ross, Ryan Grant, and Edward Kahn. "Theory and application of linear supply function equilibrium in electricity markets." *Journal of regulatory economics* 25.2 (2004): 143-167.
- Kebriaei, Hamed, and Luigi Glielmo. "Estimation, learning, and stability analysis of supply function equilibrium game for generation companies." *IEEE Systems Journal* 12.3 (2016): 2577-2588.

Blockchain (Bitcoin) Network

Chance of winning the mining game for a player

Computational effort

$$P_i^{win} = \frac{x_i}{\sum_{j \in \mathcal{N}} x_j},$$

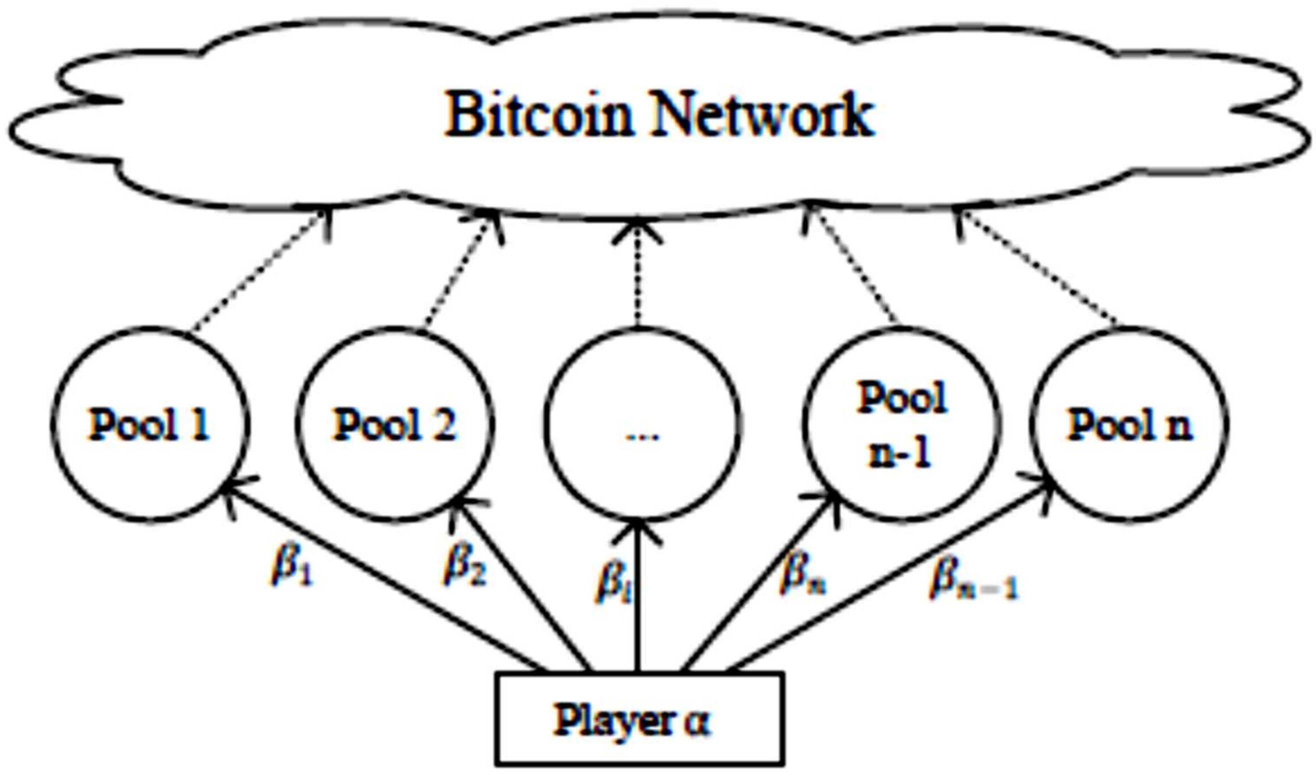
Expected utility of player *i* in a pool:

$$A_i \frac{x_i}{\sum_{k \in \mathcal{N}_i} x_k} \times \frac{\sum_{k \in \mathcal{N}_i} x_k}{\sum_{j \in \mathcal{N}} x_j},$$

Payoff function of player i

$$J_i(x) = \begin{cases} A_i \frac{x_i}{\sum_{j \in \mathcal{N}} x_j} - p_i x_i, , & \sum_{j \in \mathcal{N}} x_j > 0, \\ 0 & \sum_{j \in \mathcal{N}} x_j = 0, \end{cases}$$

- Taghizadeh, Amirheckmat, Hamed Kebriaei, and Dusit Niyato. "Mean field game for equilibrium analysis of mining computational power in blockchains." *IEEE Internet of Things Journal* 7.8 (2020): 7625-7635.



Game in BWH (block withholding) Attack in Blockchain

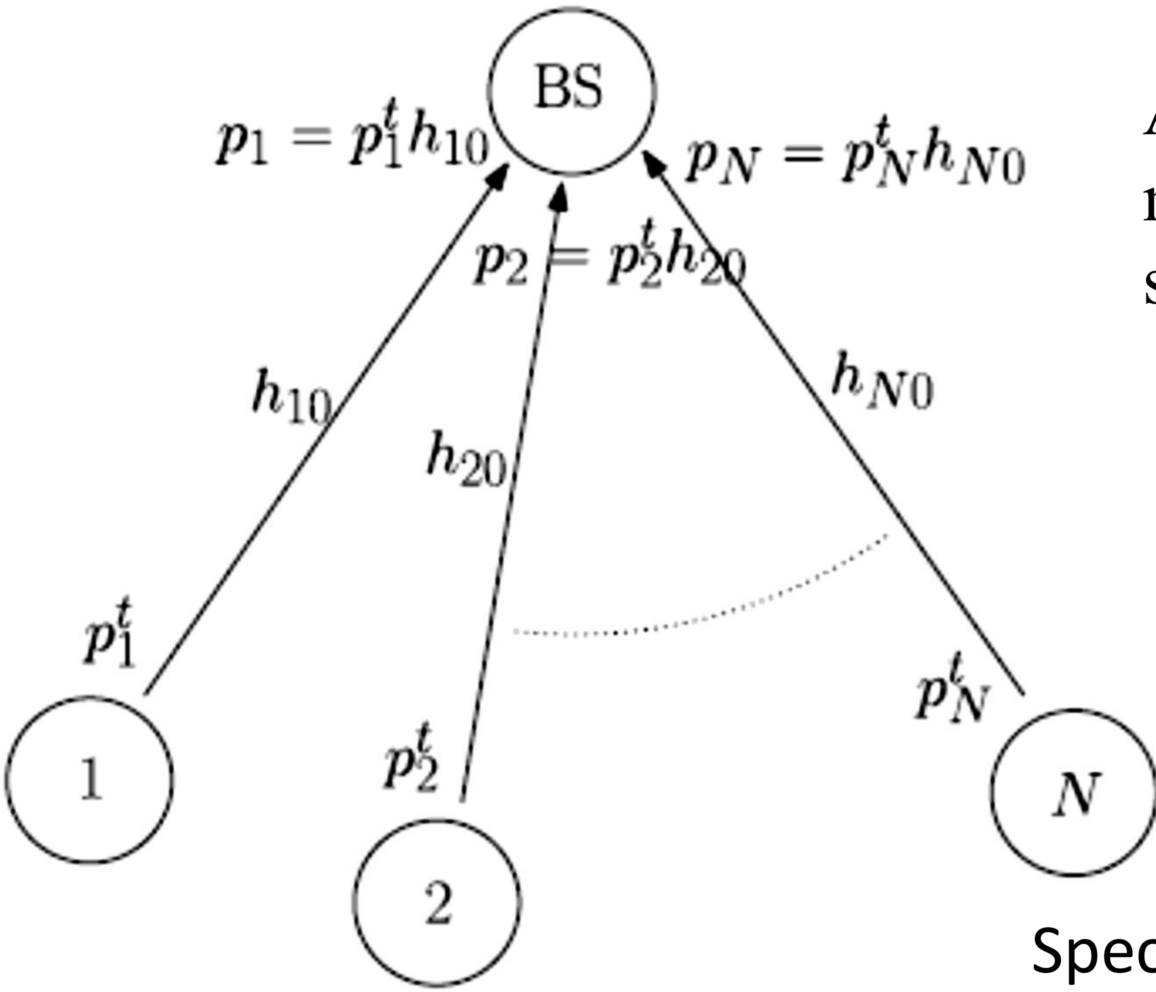
A Summary of Game Theoretical Applications for Security in Blockchains

	Ref.	Game Model	Player	Action	Strategy	Payoff	Solution
Selfish Mining Attack	[35]	Non-cooperative game	Mining pools	Infiltrate other pools to launch BWH attack	Determination of the infiltration rate	Mining rewards minus cost	Nash equilibrium
	[55]	Splitting game	One miner and pools	Distribute mining power for selfish mining	Determination of the power distribution	Mining rewards minus cost	Mixed strategy Nash equilibrium
	[56]	Mean-payoff game	Mining pools	Migrate to other pools to launch PBWH attack	Determination of the migration rate	Mean-payoff	Mean-payoff objective
	[50]	Stochastic game	Miners	Block withholding (BWH) attack	Selection between honest mining and selfish mining	Social welfare	Zero-Determinant strategy
	[57]	Non-cooperative game	Miners	Selfish propagation attack	Selection of identity duplication and transactions relaying	Mining rewards	Nash equilibrium
	[33]	Non-cooperative game	Miners	Fork chain	Selection of fork to mine	Transaction fees	Nash equilibrium
	[58]	Non-cooperative game	Miners	Delay submitting shares	Decision of the proper time to submit shares	Mining rewards	Nash equilibrium
	[28]	Non-cooperative game	Miners	Select or create a chain to mine	Selection of the chain to mine	Mining rewards	Nash equilibrium
	[28]	Stochastic game	Miners	BWH attack	Decision of the proper time to release the block	Mining rewards	Nash equilibrium
				Post smart contract			

- Liu, Ziyao, et al. "A survey on applications of game theory in blockchain." *arXiv preprint arXiv:1902.10865* (2019).

Single-cell CDMA networks

Several transmitters can send information simultaneously over a single communication channel



An uplink network with *N* mobile users and one base station

SINR of user i:

$$\gamma_i = \frac{h_i p_i}{\sum_{j \neq i} h_j p_j + \sigma^2}.$$

Spectral efficiency= $Log(1 + \gamma_i)$

Cost of User *i*

$$c_i(p_i, \mathbf{p}_{-i}) = \lambda_i p_i - \alpha_i \log(1+\gamma_i), \; p_i \geq 0, \forall i,$$

$$\min_{p} \sum_{i} c_i(p_i, p_{-i})$$

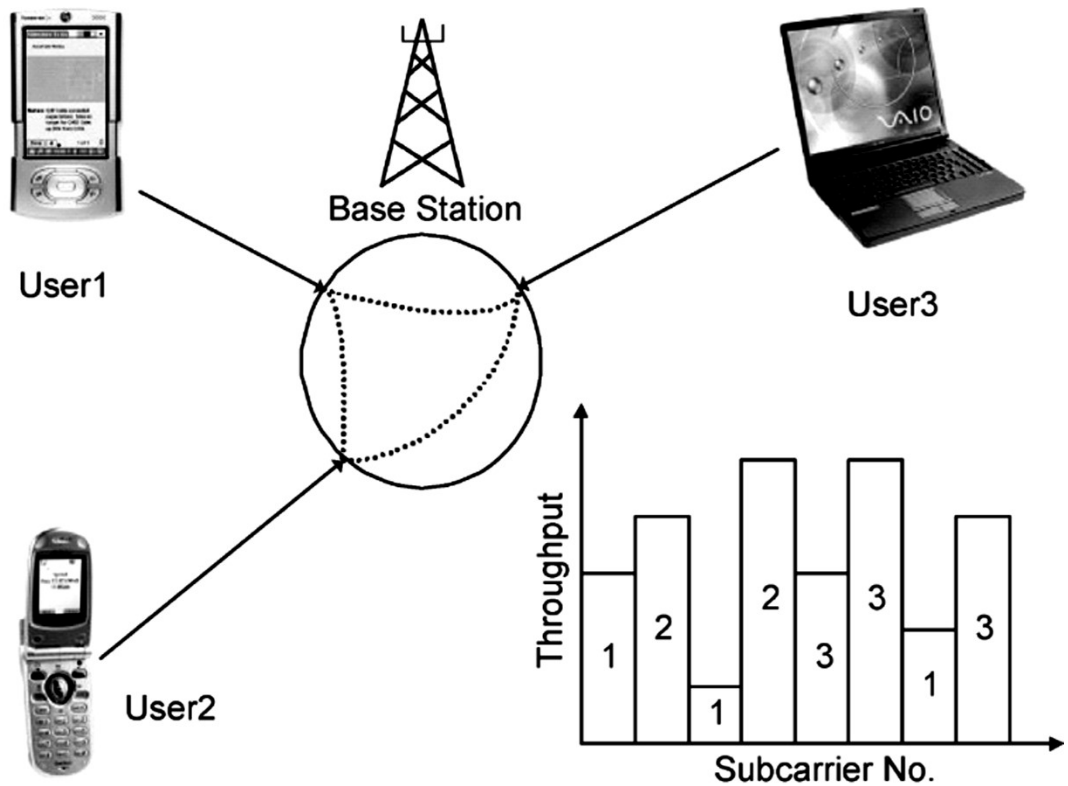
Cooperation

$$\min_{p_i} c_i(p_i, p_{-i}), i = 1,2, \ldots N$$

Competition

- Han, Zhu, et al. *Game theory in wireless and communication networks: theory, models, and applications*. Cambridge university press, 2012.

OFDMA resource-allocation model



- An uplink scenario of a single-cell multiuser system.
- There are, in total, N users randomly located within the cell.
- The users want to share their transmissions among K different subcarriers/channels.

subcarrier assignment matrix A with $[A]_{ij} = a_{ij}$

$$a_{ij} = \begin{cases} 1, & \text{if } r_{ij} > 0, \\ 0, & \text{otherwise.} \end{cases}$$

$P = [p_{ij}]$ **power-allocation matrix**

user *i*'s transmission rate R_i is allocated to different subcarriers *j*

$$r_{ij} = W \log_2 \left(1 + \frac{p_{ij} h_{ij} c_3}{\sigma^2}\right)$$

$$R_i = \sum_{j=1}^{K} r_{ij}$$

$$\max_{A,P} \; U = \prod_{i=1}^{N} \left(R_i - R_{\min}^{i}\right)$$

$$\text{s.t.} \begin{cases} \sum_{i=1}^{N} a_{ij} = 1, \forall j \in \mathcal{K}, \\ R_i \geq R_{\min}^{i}, \forall i, \\ \sum_{j=1}^{K} p_{ij} \leq p_{\max}, \forall i \in \mathcal{N}. \end{cases}$$

- Han, Zhu, Zhu Ji, and KJ Ray Liu. "Fair multiuser channel allocation for OFDMA networks using Nash bargaining solutions and coalitions." *IEEE Transactions on Communications* 53.8 (2005): 1366-1376.

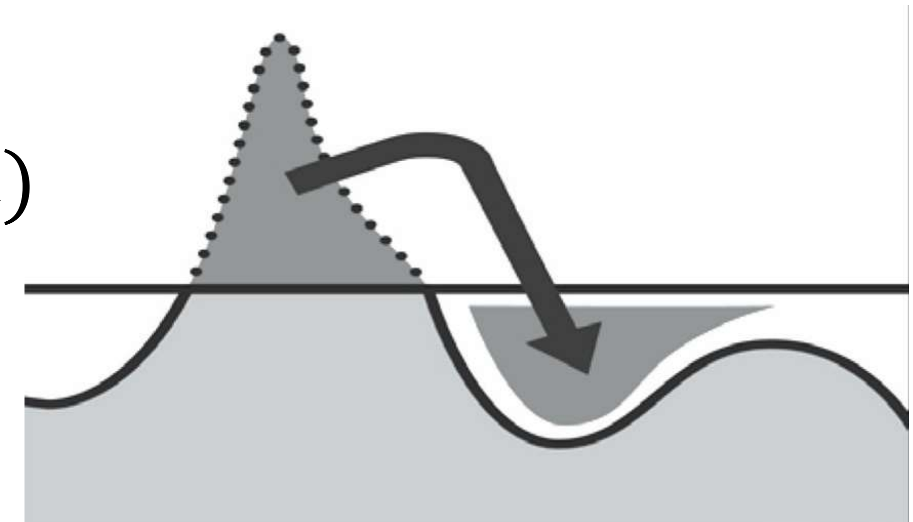
Demand Response in Smart Grids

Price of energy at each hour (n) as a function of total demand

$$p(n) = f\left(\sum_{i=1}^{N} x_i(n)\right)$$

Total demand of user i at hour n

Example: $p(n) = a(n) + b\sum_{i=1}^{N} x_i(n)$

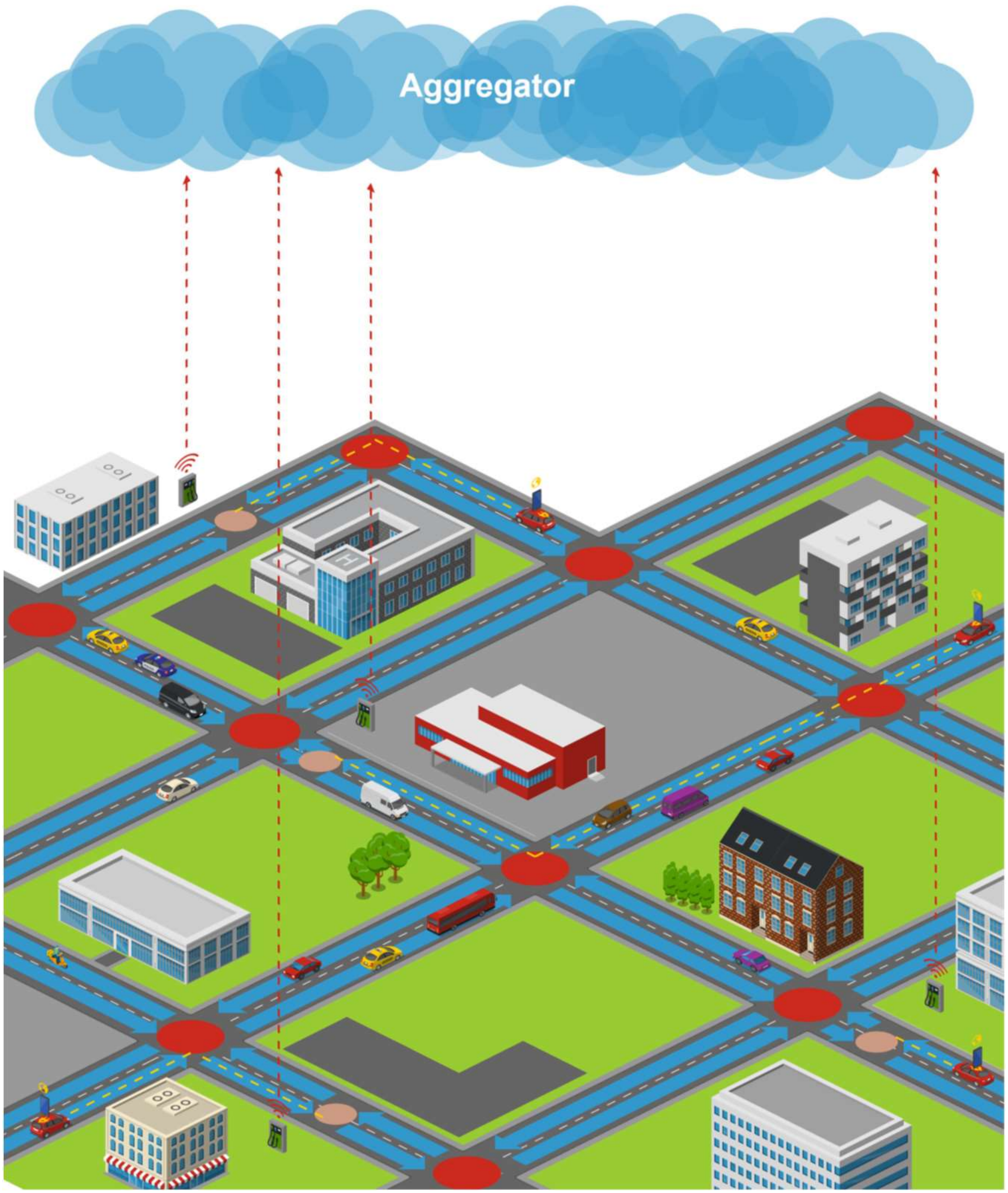


$u_i(x_i)$ Utility function of user i

Payoff function of user i

$$g_i(x_i, x_{-i}) = \sum_{n} \left(u_i(x_i(n)) - x_i(n)p(n)\right)$$

- Samadi, Pedram, et al. "Advanced demand side management for the future smart grid using mechanism design." *IEEE Transactions on Smart Grid* 3.3 (2012): 1170-1180.



Joint Routing and Destination Planning of Electric Vehicles

Players

A transportation network with N EV driving users and D charging stations

Transportation network

modeled as a strongly connected directed graph

$$\mathcal{V} = \{1, \ldots, V\} \qquad \mathcal{E} = \{1, \ldots, E\}$$

Strategies

$$t_i = [t_i^d]_{d=1}^D \qquad r_i = [r_i^e]_{e=1}^E$$

probability of choosing destination $d \in \mathcal{D}$ by user i.

probability of choosing road $e \in \mathcal{E}$ by user i

$$x_i = \mathrm{col}(r_i, t_i) \in \mathbb{R}_{\geq 0}^{E+D}$$

Payoffs

$$J_i(r_i, t_i, \sigma(r), \varphi(t)) = U_i(r_i, t_i) + \omega_i C_i^{travel} + C_i^{service}$$

traffic congestion

$$C_i^{travel}\left(r_i, \sigma(r)\right) = \sum_{e=1}^{E} l_e\left(\sigma_e\left(r^e\right)\right) r_i^e$$

$$l_e(\sigma_e(r^e)) = a_e\left(1 + \theta\left(\frac{\sigma_e(r^e)}{b_e}\right)^{\xi}\right) \qquad \sigma_e(r^e) = s_e + \sum_{i=1}^{N} r_i^e$$

Energy demand

$$C_i^{service}\left(t_i, \varphi_d(t)\right) = \sum_{d=1}^{D}\left[q_i p_d(\varphi_d(t^d)) + \rho_d\right] t_i^d$$

per-unit cost of charging

once paid fixed cost of parking

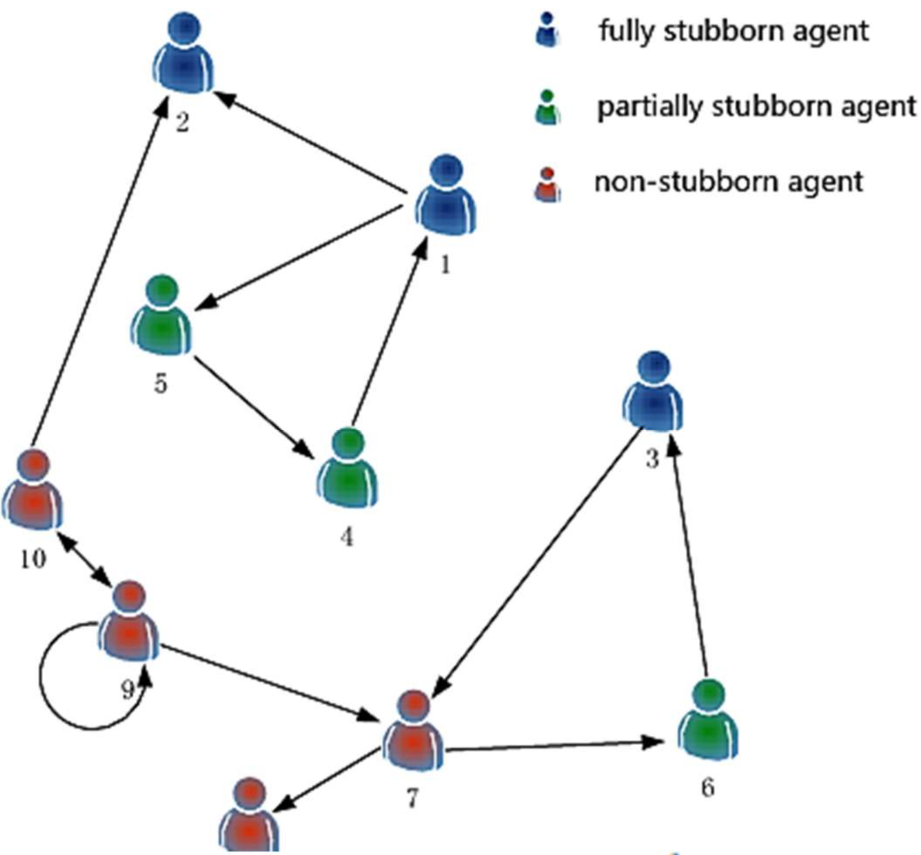
$$p_d(\varphi_d(t^d)) = \delta_d\left(\frac{\varphi_d\left(t^d\right)}{\kappa_d}\right) \qquad \varphi_d(t^d) = \sum_{i=1}^{N} \tilde{q}_i^d \qquad \tilde{q}_i^d = q_i t_i^d$$

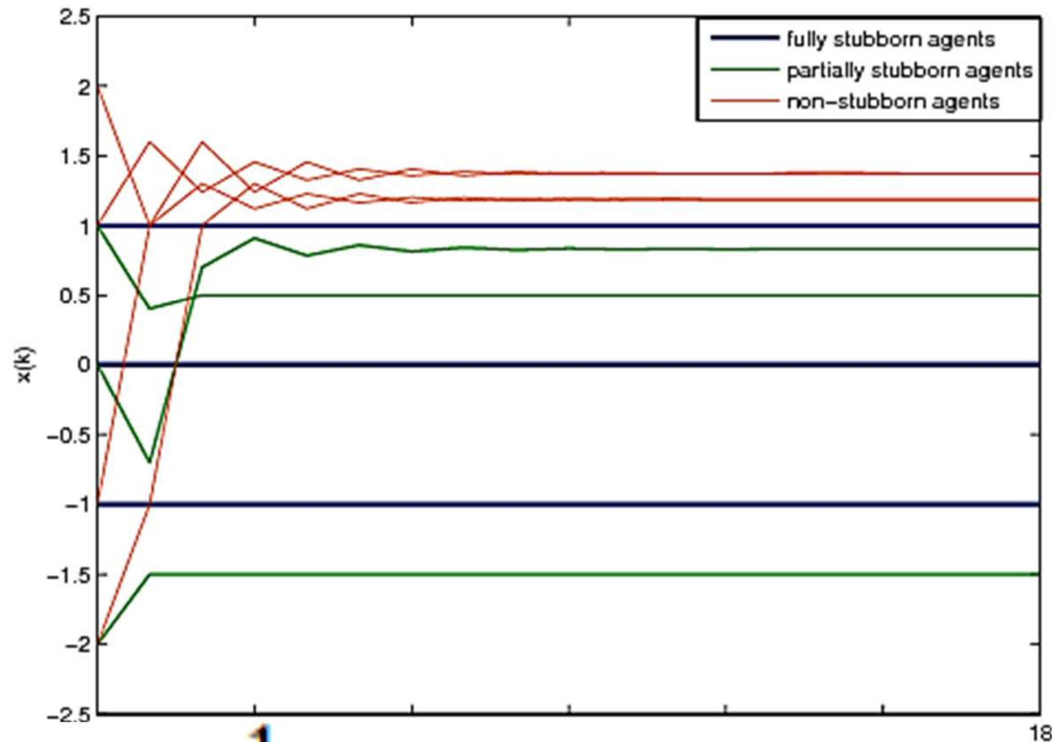
expected total demand

- Bakhshayesh, Babak Ghaffarzadeh, and Hamed Kebriaei. "Decentralized Equilibrium Seeking of Joint Routing and Destination Planning of Electric Vehicles: A Constrained Aggregative Game Approach." *IEEE Transactions on Intelligent Transportation Systems* (2021).

Opinion Dynamics in Social Networks

$$x_i(t+1) = \frac{1}{d_i + K_i} \sum_{j \in \partial_i} x_j(t) + \frac{K_i}{d_i + K_i} x_i(0)$$





$$J_i(x_i, x_{\partial_i}) = \frac{1}{2} \sum_{j \in \partial_i} (x_i - x_j)^2 + \frac{1}{2} K_i (x_i - x_i(0))^2,$$

- Ghaderi, Javad, and Rayadurgam Srikant. "Opinion dynamics in social networks: A local interaction game with stubborn agents." *2013 American Control Conference*. IEEE, 2013.

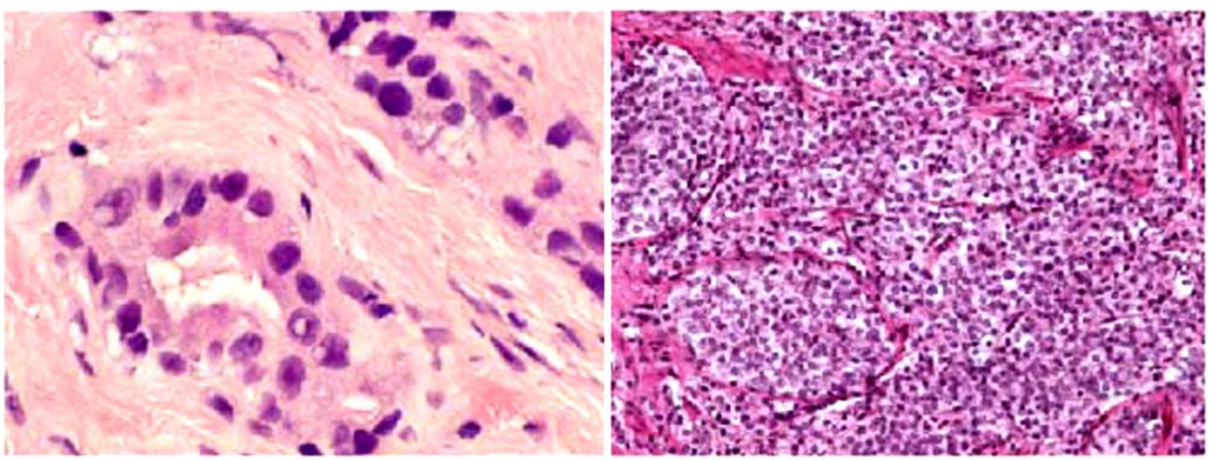
- Pagan, Nicolò, and Florian Dörfler. "Game theoretical inference of human behavior in social networks." *Nature Communications* 10.1 (2019): 1-12.

prostate cancer tumor–stroma interactions: insights from an evolutionary game

tumor phenotypes

Table 1 Payoff table that represents the interactions between the three cell types considered in the model

	S	**D**	**I**
S	0	α	0
D	$1+\alpha-\beta$	$1-2\beta$	$1-\beta+\rho$
I	$1-\gamma$	$1-\gamma$	$1-\gamma$

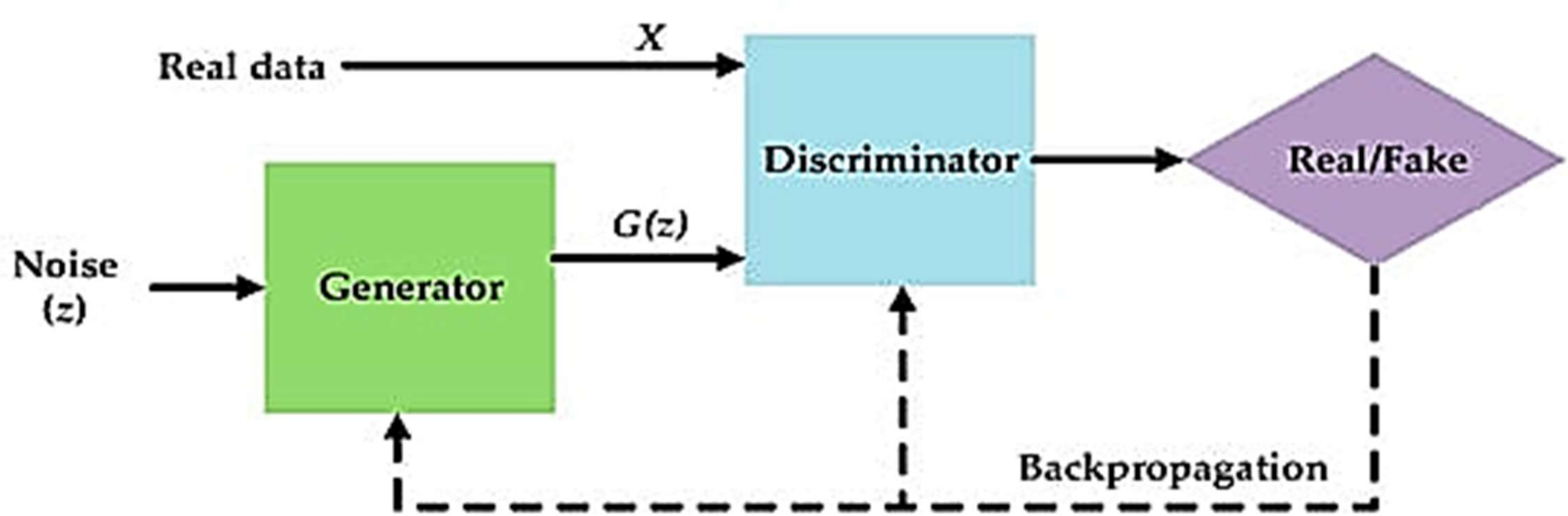


$$p_{t+1}^{\mathrm{I}} = p_t^{\mathrm{I}} \frac{W(\mathrm{I})}{\bar{W}},$$

$$p_{t+1}^{\mathrm{D}} = p_t^{\mathrm{D}} \frac{W(\mathrm{D})}{\bar{W}}.$$

- Basanta, David, et al. "Investigating prostate cancer tumour–stroma interactions: clinical and biological insights from an evolutionary game." *British journal of cancer* 106.1 (2012): 174-181.

GAN: Deep Learning and Game Theory



GANGs: Generative Adversarial Network Games

- GANs view the learning problem as a zero-sum game between the following two players:
- 1) generator G aiming to generate real-like samples from a random noise input
- 2) discriminator D trying to distinguish G's generated samples from real training data.

- This game is commonly formulated through a minimax game problem as follows

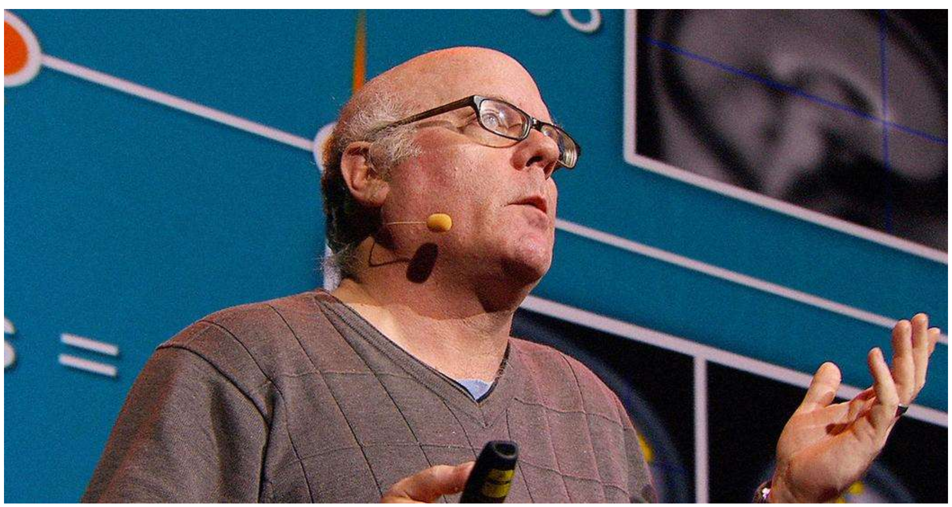
$$\min_{G \in \mathcal{G}} \max_{D \in \mathcal{D}} V(G, D)$$

- *V* (*G*, *D*) denotes the minimax objective for generator G and discriminator D capturing how dissimilar G's produced samples and real training data are.

- Farnia, Farzan, and Asuman Ozdaglar. "Do GANs always have Nash equilibria? " *International Conference on Machine Learning*. PMLR, 2020.

Game Theory and Cognitive Neuroscience

Colin Camerer TED Talk



Link to the Video

https://www.ted.com/talks/colin_camerer_when_you_re_making_a_deal_what_s_going_on_in_your_brain?language=en