Instrumentation

Force and Torque Measurement

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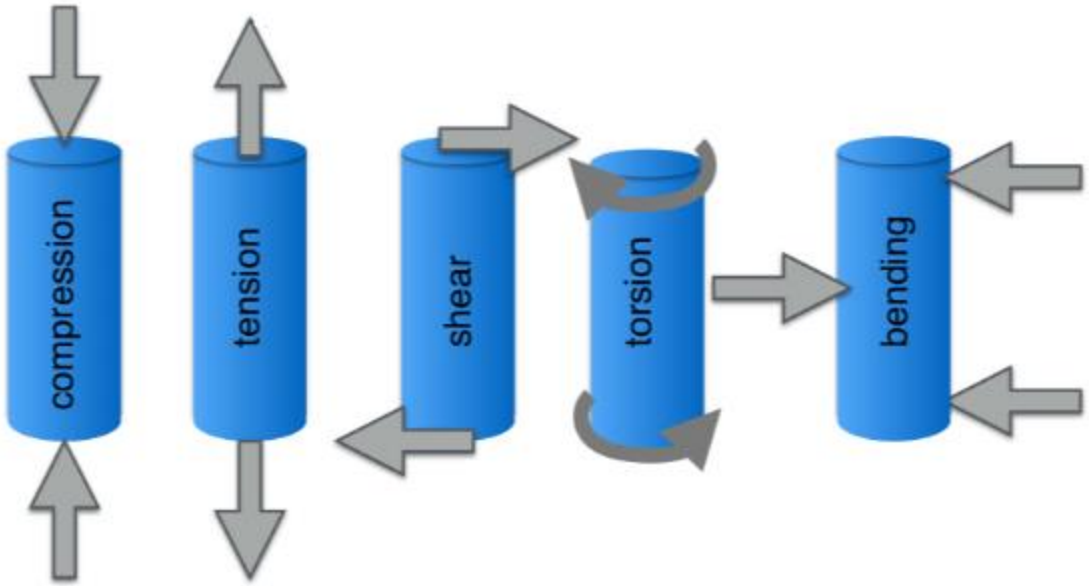
Stress and strain

- Every component in a linear motion system experiences some form of loading due to applied forces or motion
- The component’s reactions to these loads are described by its mechanical properties.
- There are five fundamental types of loading: *compression*, *tension*, *shear*, *torsion*, and *bending*.



Stress and strain

- ***Stress*** is the force applied to a material, divided by the material's cross-sectional area.

$$\sigma = \frac{F}{A_0}$$

σ = stress (N/m², Pa)

F = force (N)

A_0 = original cross-sectional area (m²)

- ***Strain*** is the deformation or displacement of material that results from an applied stress.

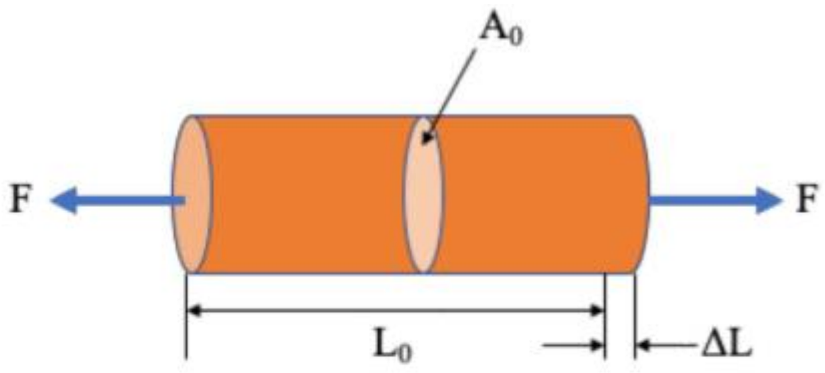
$$\varepsilon = \frac{L - L_0}{L_0}$$

ε = strain

L = length after load is applied (mm)

L_0 = original length (mm)

Note: A material's change in length ($L - L_0$) is sometimes represented as δ.

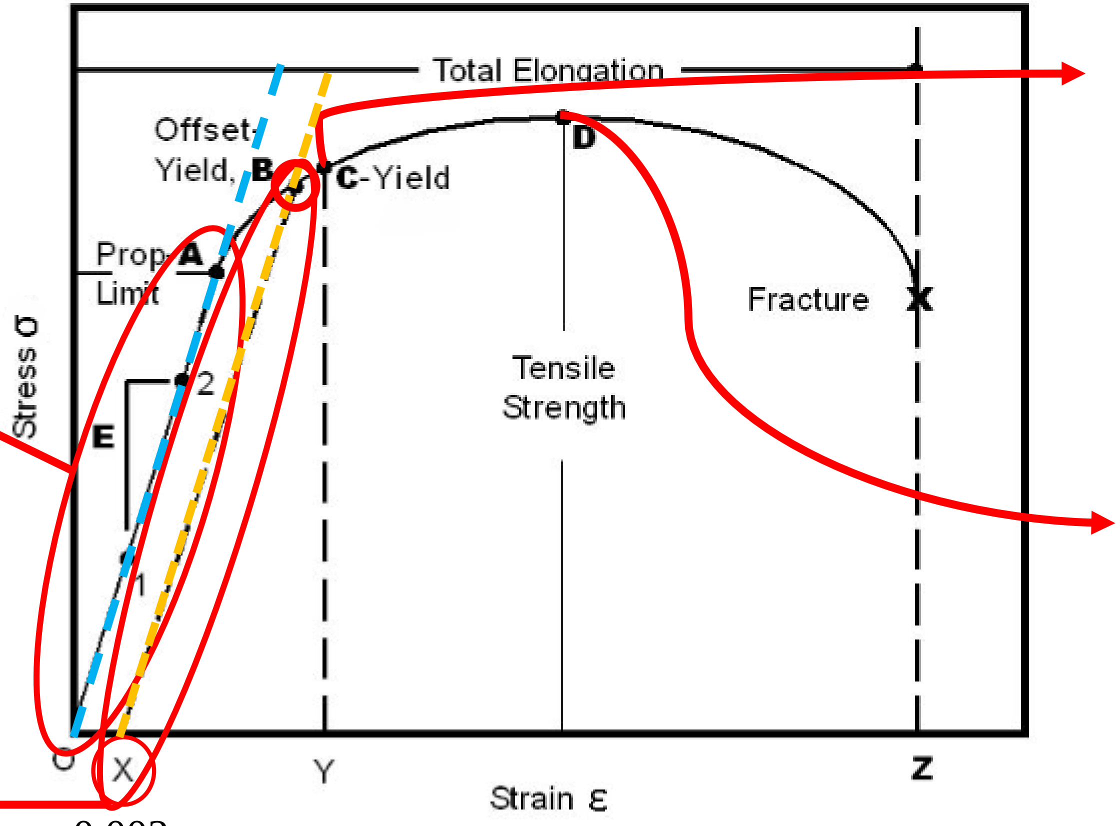


Stress and strain

- The most common way to analyze the relationship between stress and strain for a particular material is with a ***stress-strain diagram***.

A. This stress-strain relationship is known as *Hooke's Law*, and in this region, the slope of the stress-strain curve is referred to as the modulus of elasticity (aka Young's modulus), denoted E.

$$E = \frac{\sigma}{\varepsilon}$$



"C," is the point where strain increases faster than stress and the material experiences some amount of permanent deformation. (Yield point)

"D," where the curve begins to fall, the material's ultimate tensile strength has been reached. This point denotes the maximum stress that can be applied to a material in tension before failure occurs.

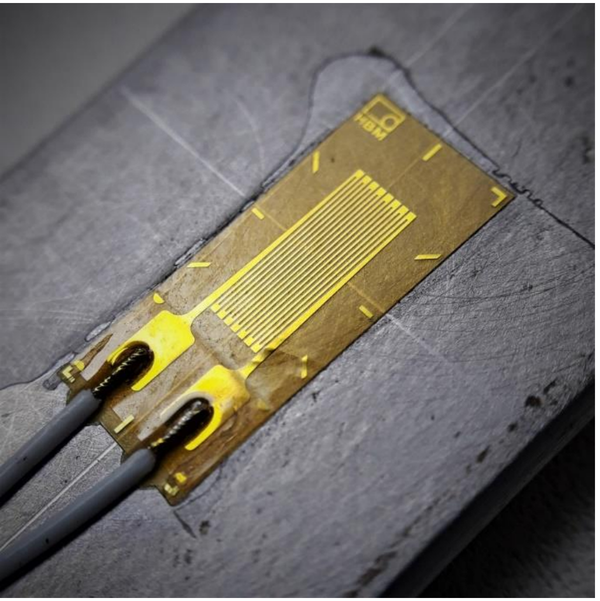
For materials that do not have a well-defined yield point, or whose yield point is difficult to determine, an offset yield strength — shown here as point "B" — is used. Offset yield strength is the stress that will cause a specified amount of permanent strain (typically 0.2 percent).

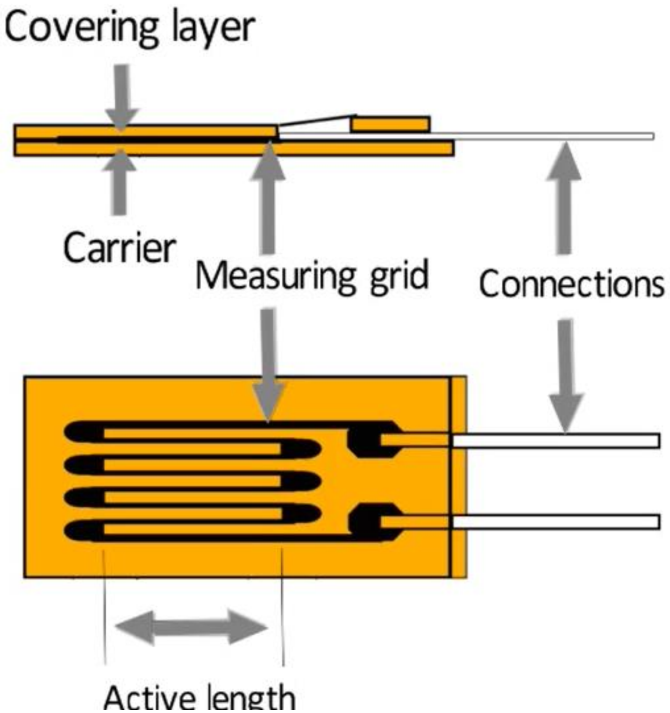
0.002

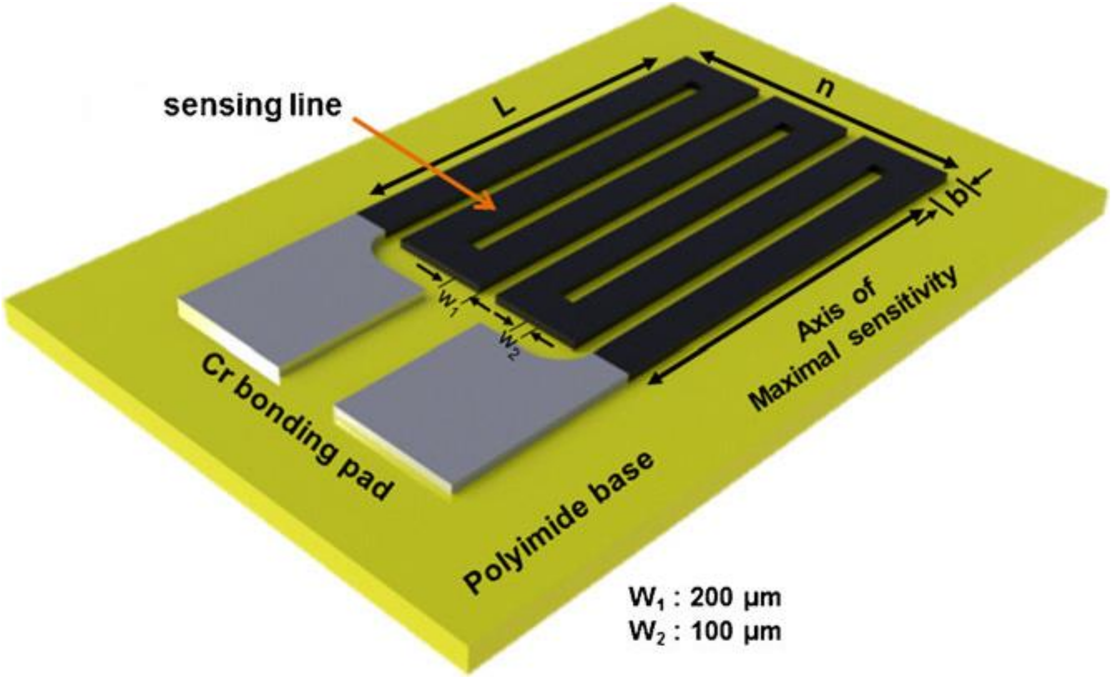
$$\sigma = \frac{F}{A_0} \rightarrow F = \sigma.A_0 = E\varepsilon.A_0 \rightarrow F = (E.A_0)\varepsilon \rightarrow \boldsymbol{F = K\varepsilon}$$

Stress and strain

Strain gauge







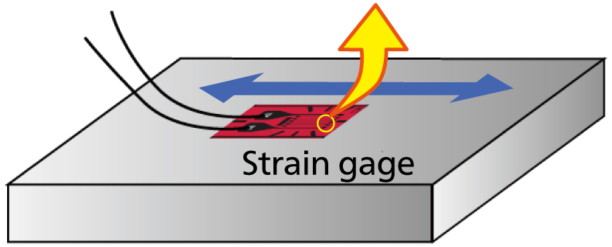
Strain gauge

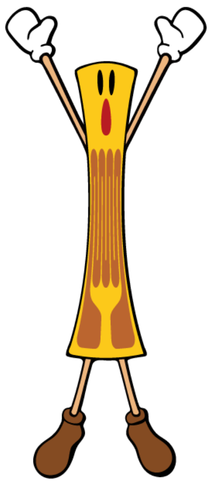
Tensile force

reduce the cross-sectional area → increase the resistance

Compression force

increase the cross-sectional area → reduce the resistance

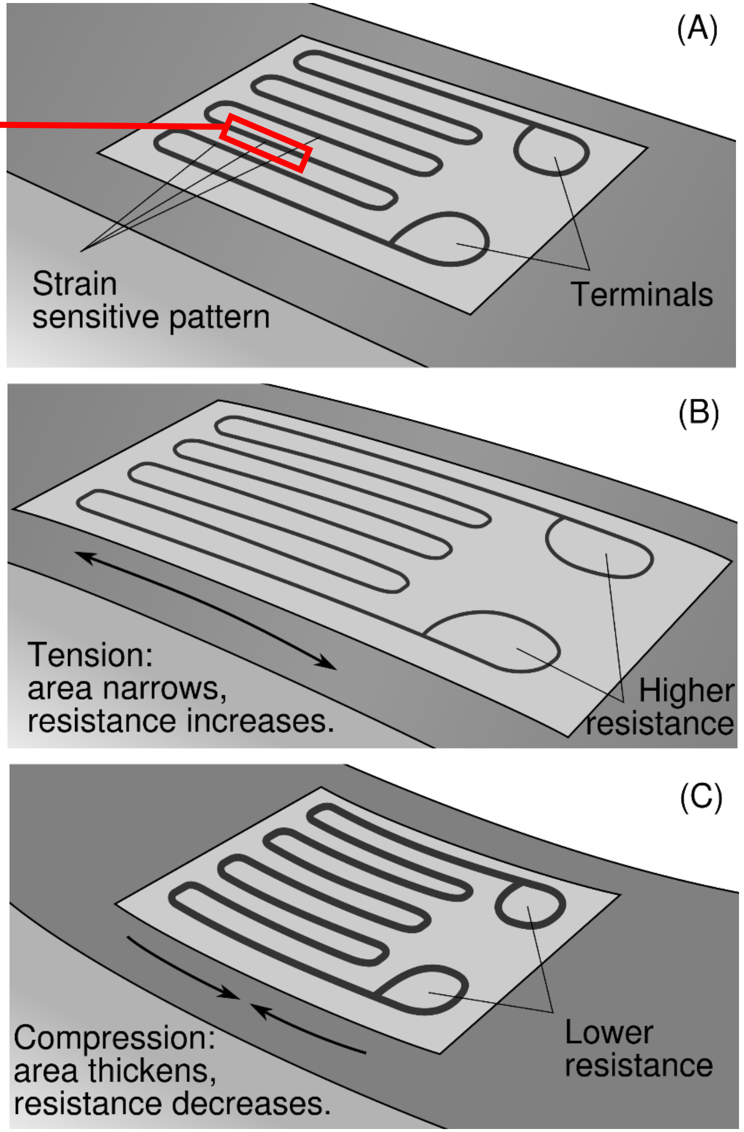




$$R = \rho \frac{L}{A}$$

$$\uparrow R = \rho \frac{L \uparrow}{A \downarrow}$$

$$\downarrow R = \rho \frac{L \downarrow}{A \uparrow}$$



Strain gauge

Taylor series: (Ignore high order terms)

$$R = R_0 + \Delta R \qquad R_0 = \rho \frac{L_0}{A_0}$$

$$\Delta R = \frac{\partial R}{\partial L}\Delta L + \frac{\partial R}{\partial A}\Delta A + \frac{\partial R}{\partial \rho}\Delta\rho$$

$$R = \rho \frac{L}{A}$$

$$\rightarrow \Delta R = \frac{\rho}{A}\Delta L - \frac{\rho L}{A^2}\Delta A + \frac{L}{A}\Delta\rho$$

$$\rightarrow \frac{\Delta R}{R} = \frac{\Delta L}{L} - \frac{\Delta A}{A} + \frac{\Delta\rho}{\rho}$$

$$\frac{\Delta L}{L} = \varepsilon \ \ (\varepsilon{:}\, strain)$$

$$\frac{\Delta A}{A} = -2\mu\varepsilon \ \ (\mu{:}\, poission\ ratio)$$

$$\frac{\Delta R}{R} = (1+2\mu)\varepsilon + \frac{\Delta\rho}{\rho}$$

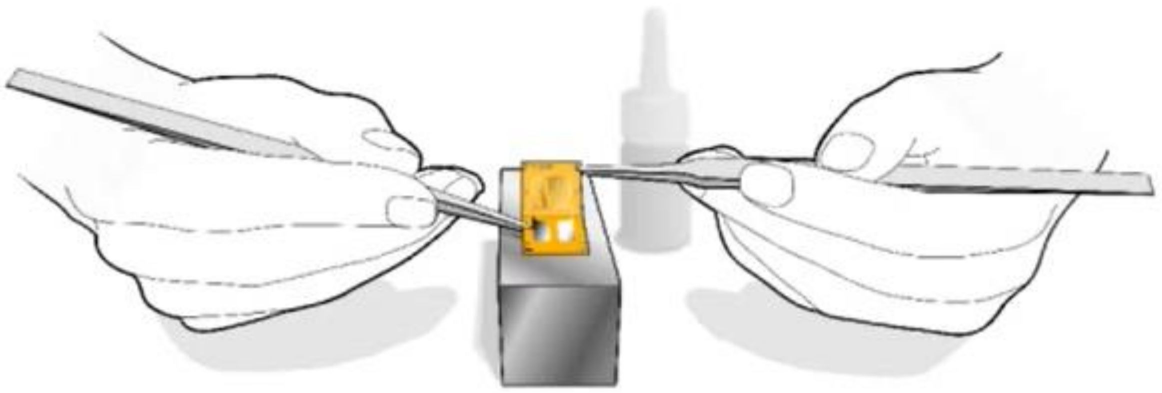
$$\boldsymbol{Gauge\ Factor(GF)} = \frac{\frac{\boldsymbol{\Delta R}}{\boldsymbol{R}}}{\boldsymbol{\varepsilon}}$$

$$\rightarrow GF = (1+2\mu) + \frac{\frac{\Delta\rho}{\rho}}{\varepsilon}$$

$$if\ \Delta\rho = 0 \rightarrow \boldsymbol{GF = 1 + 2\mu}$$

??? : (Hint: Assume the cross section of a wire is a circle!)

Strain gauge



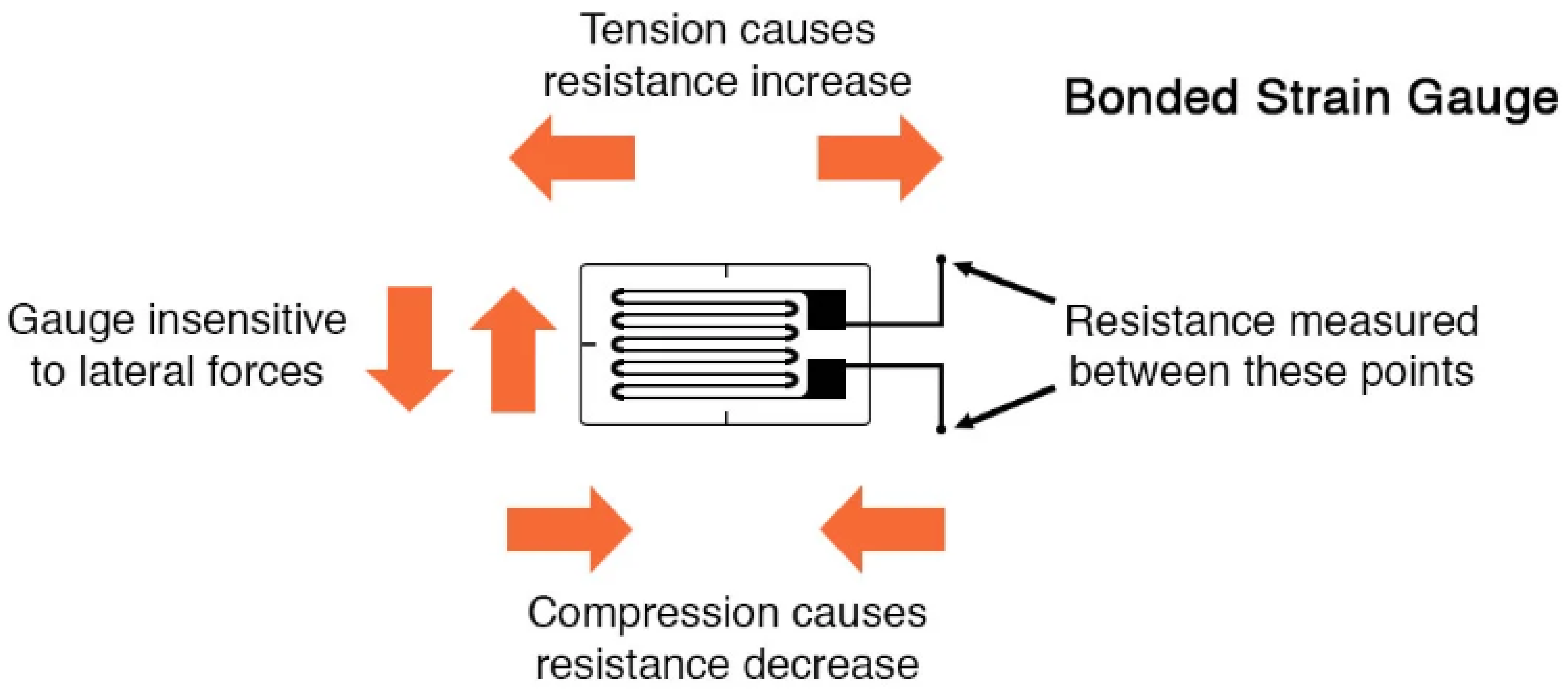
Manufacturing of transducers

- Force transducers
- Load cells
- Torque and pressure transducers
- ...

Experimental stress analysis

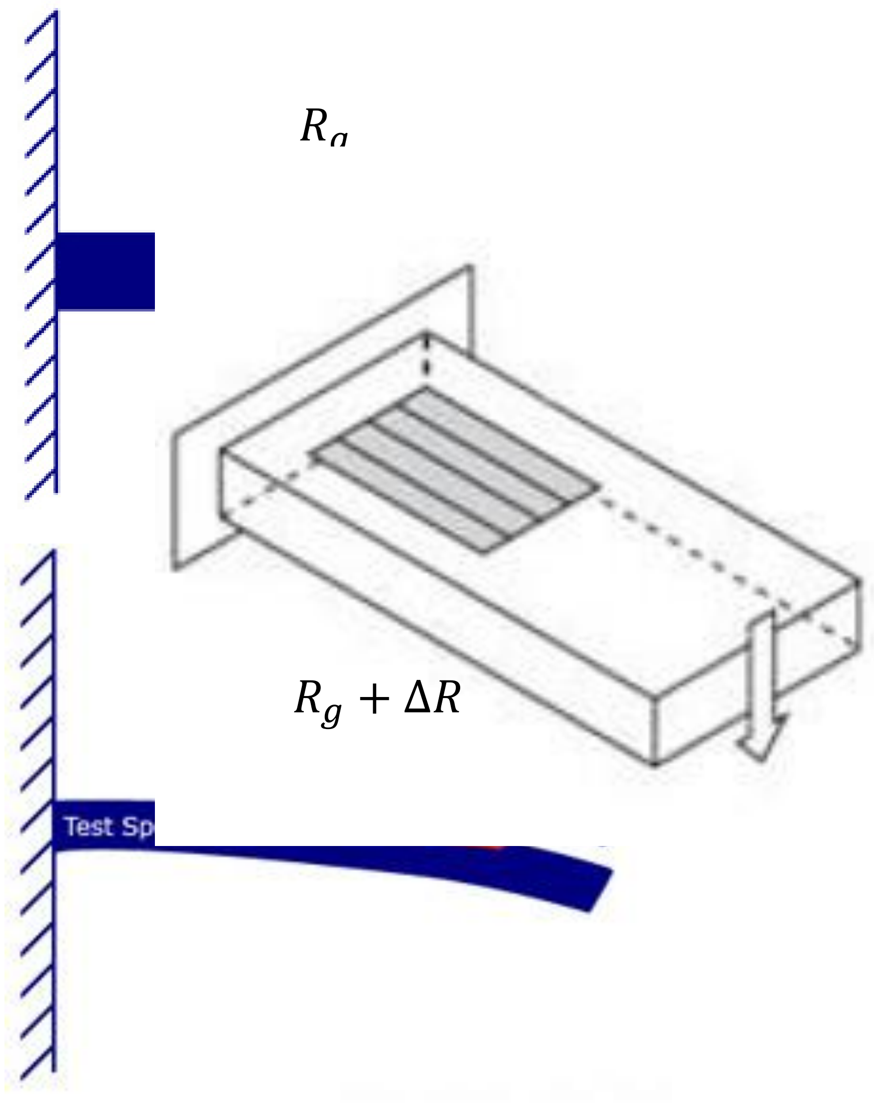
- Determination of the absolute value and direction of mechanical stresses
- Fatigue life analysis
- Residual stress analysis

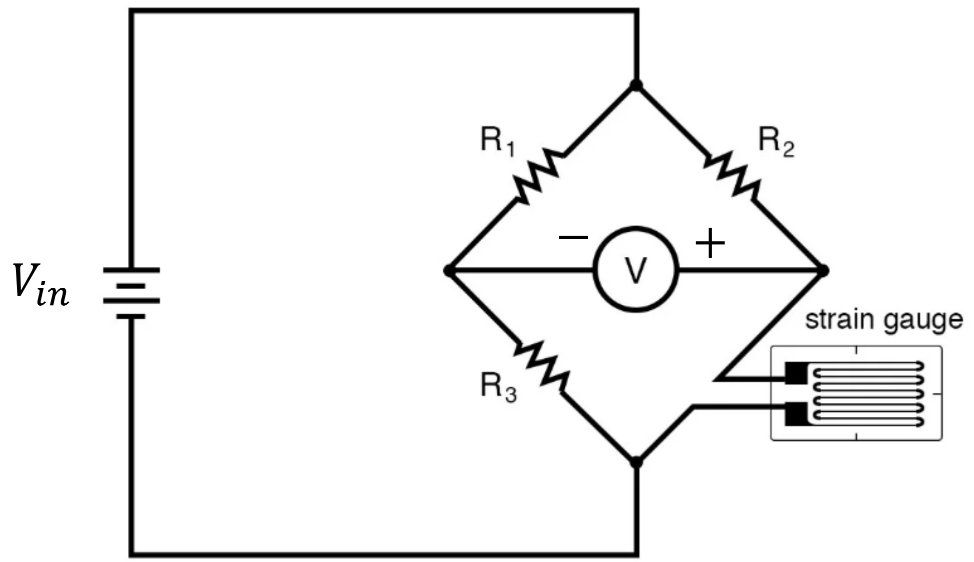
Strain gauge



Strain gauge

Quarter – Bridge Circuit





$$R_1 = R_3 \quad , \qquad R_2 = R_g(No\ load)$$

$$No\ load: \quad V = 0$$

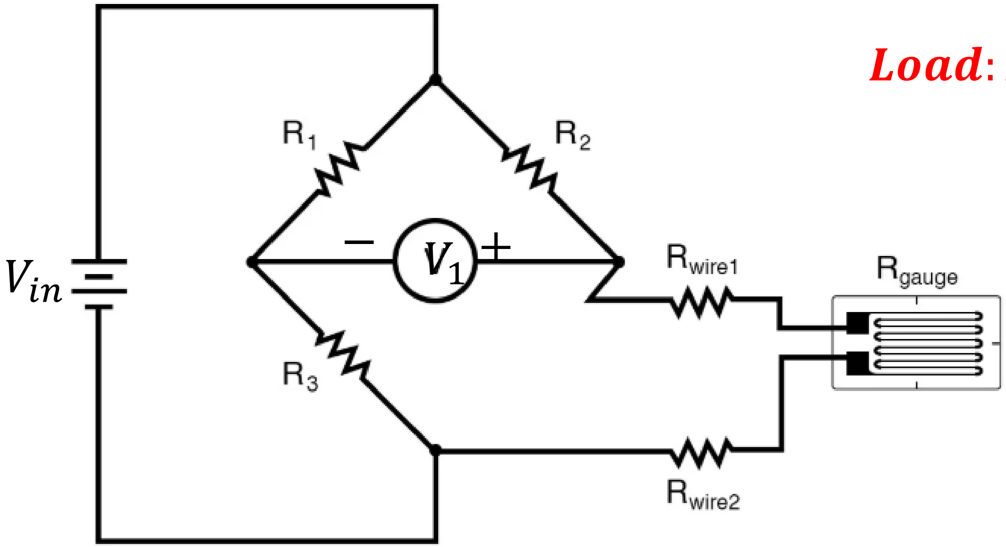
$$Load: \quad V = \left[\frac{R_g + \Delta R}{R_2 + R_g + \Delta R} - \frac{R_3}{R_1 + R_3}\right] V_{in} = \left[\frac{R_2 + \Delta R}{2R_2 + \Delta R} - \frac{1}{2}\right] V_{in}$$

$$R_2 \gg \Delta R \quad \rightarrow \boldsymbol{V \cong \left[\frac{\Delta R}{4R_2}\right] V_{in}}$$

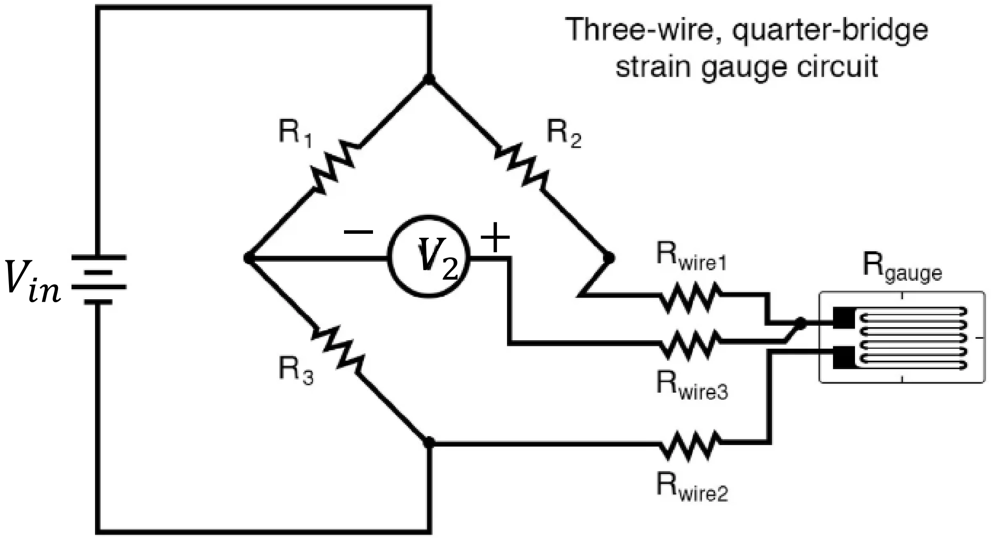
Strain gauge

Quarter – Bridge Circuit

Wire Resistances



$$\boldsymbol{Load: R_{gauge} = R_g + \Delta R \quad ??}$$



$$R_1 = R_3\,, R_2 = R_g (No\ load),$$
$$R_{wire1} = R_{wire2} = R_w$$

$$No\ load: V_1 = \left[\frac{R_g + 2R_w}{R_2 + R_g + 2R_w} - \frac{R_3}{R_1 + R_3}\right] V_{in}$$

$$\rightarrow \boldsymbol{V_1 = \frac{R_w}{2(R_g + R_w)} V_{in}}$$

$$R_1 = R_3\,, R_2 = R_g (No\ load),$$
$$R_{wire1} = R_{wire2} = R_{wire3} = R_w$$

$$No\ load: V_2 = \left[\frac{R_g + R_w}{R_2 + R_w + R_g + R_w} - \frac{R_3}{R_1 + R_3}\right] V_{in}$$

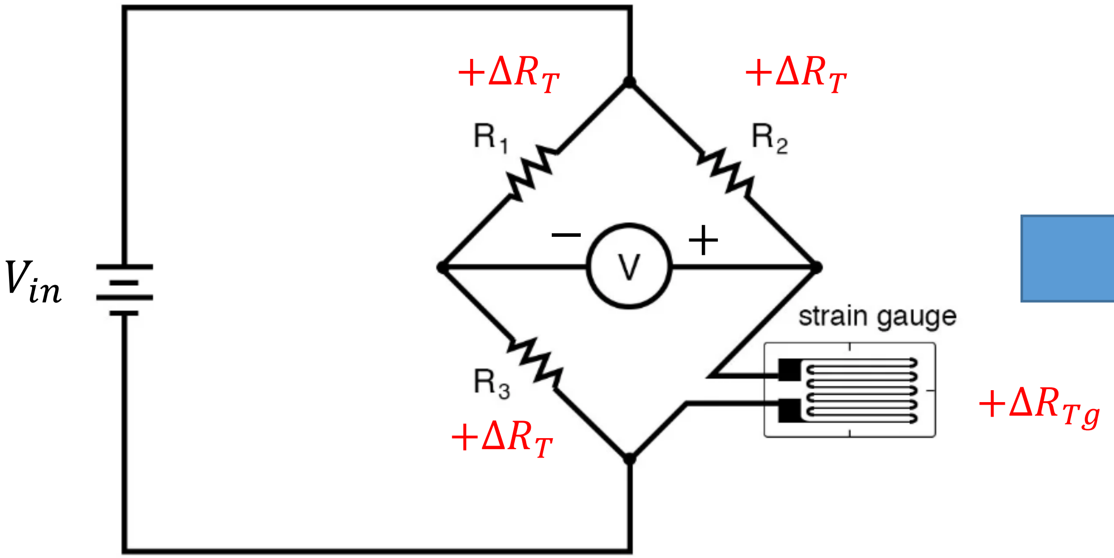
$$\rightarrow \boldsymbol{V_2 = 0}$$

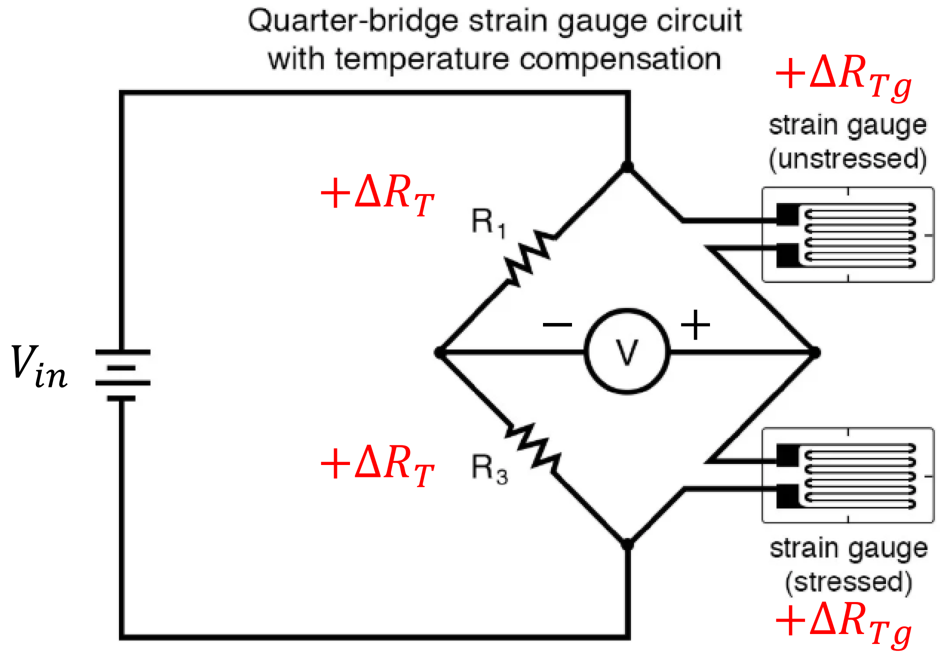
Strain gauge

Quarter – Bridge Circuit

Resistance Change in Temperature

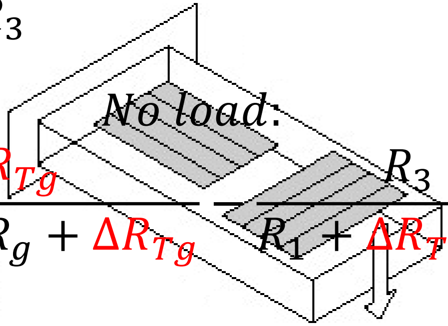
$\boldsymbol{Load: R_{gauge} = R_g + \Delta R\ ??}$





$R_1 = R_3 \quad , \qquad R_2 = R_g(No\ load)$

$R_1 = R_3$



$No\ load:$

$$V = \left[\frac{R_g + \Delta R_{Tg}}{R_2 + \Delta R_T + R_g + \Delta R_{Tg}} - \frac{R_3 + \Delta R_T}{R_1 + \Delta R_T + R_3 + \Delta R_T}\right] V_{in}$$

$$\rightarrow \boldsymbol{V \neq 0}$$

$No\ load:$

$$V = \left[\frac{R_g + \Delta R_{Tg}}{R_g + \Delta R_{Tg} + R_g + \Delta R_{Tg}} - \frac{R_3 + \Delta R_T}{R_1 + \Delta R_T + R_3 + \Delta R_T}\right] V_{in}$$

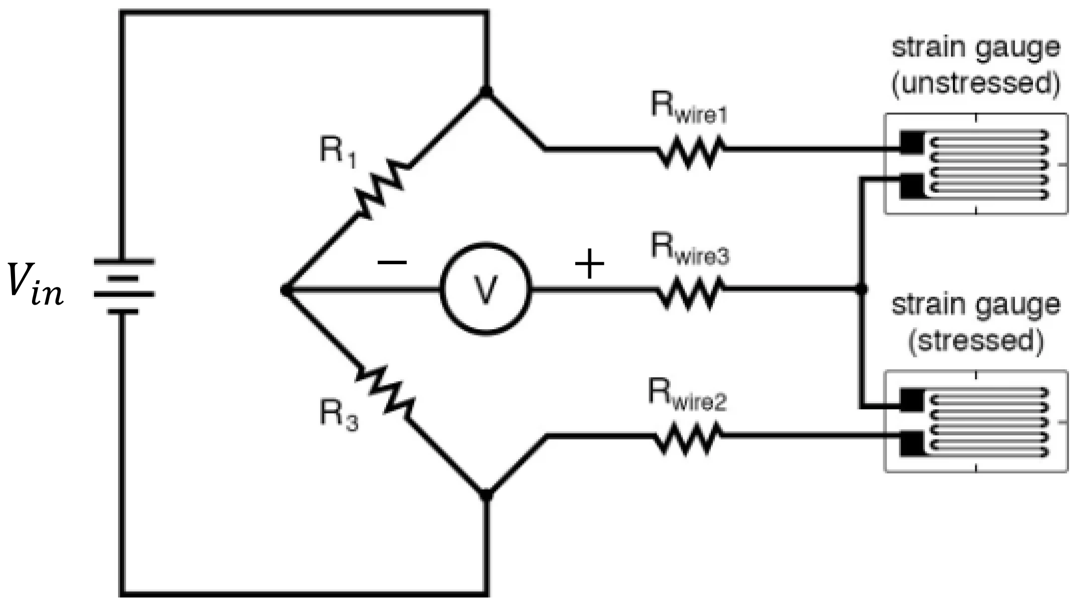
$$\rightarrow \boldsymbol{V = 0}$$

Strain gauge

Quarter – Bridge Circuit

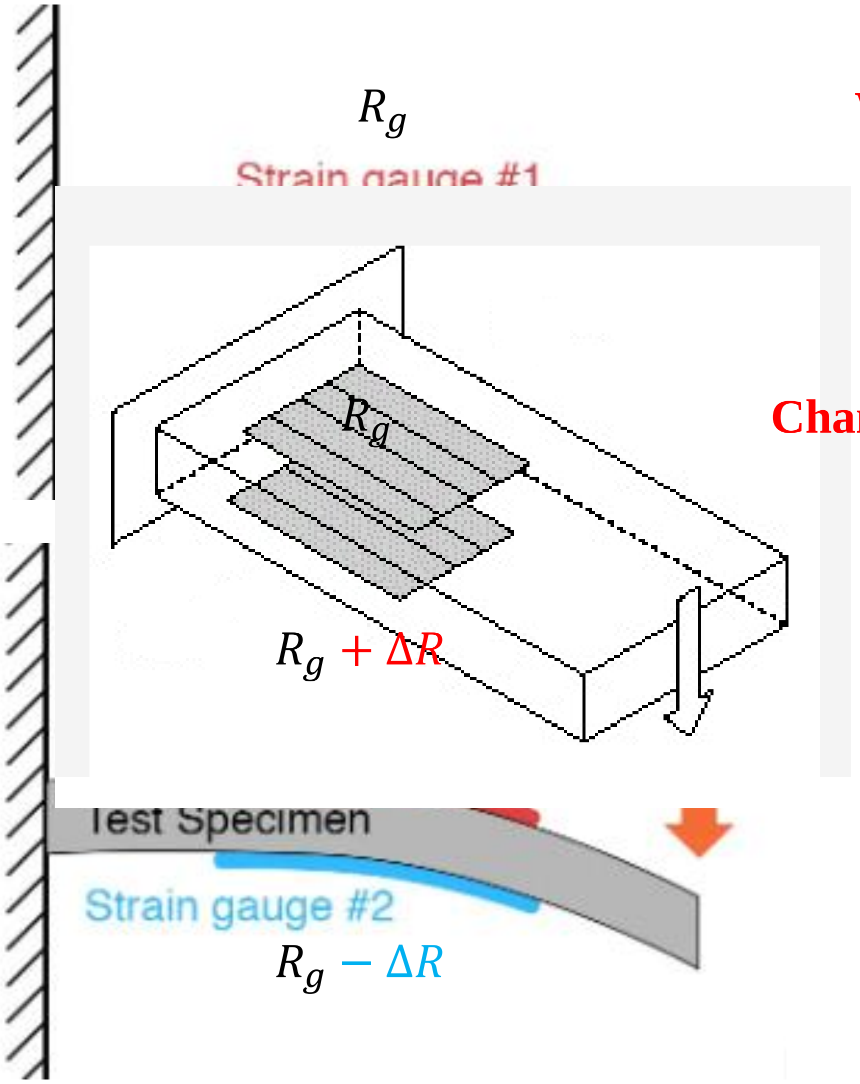
Resistance Change in Temperature

Wire Resistances ??



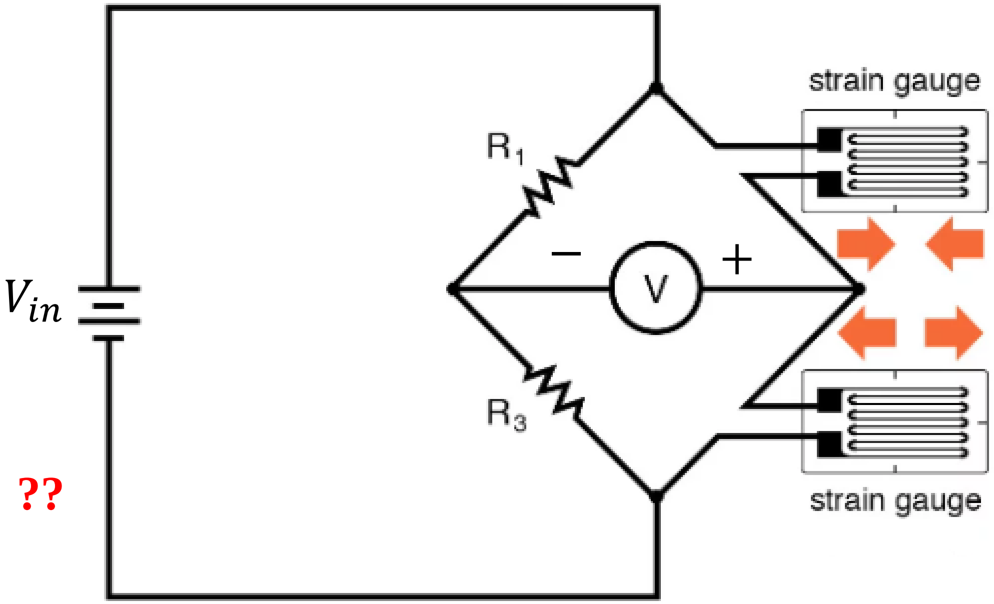
Strain gauge

Half – Bridge Circuit



Wire Resistances ??

Change in Temperature ??



$$R_1 = R_3$$

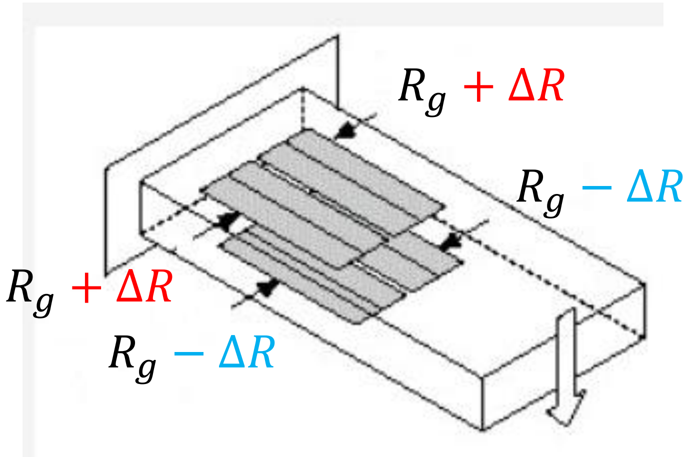
$$No\ load: \quad V = 0$$

$$Load: \quad V = \left[\frac{R_g + \Delta R}{R_g + \Delta R + R_g - \Delta R} - \frac{R_3}{R_1 + R_3}\right] V_{in}$$

$$\rightarrow \boldsymbol{V = \left[\frac{\Delta R}{2R_g}\right] V_{in}}$$

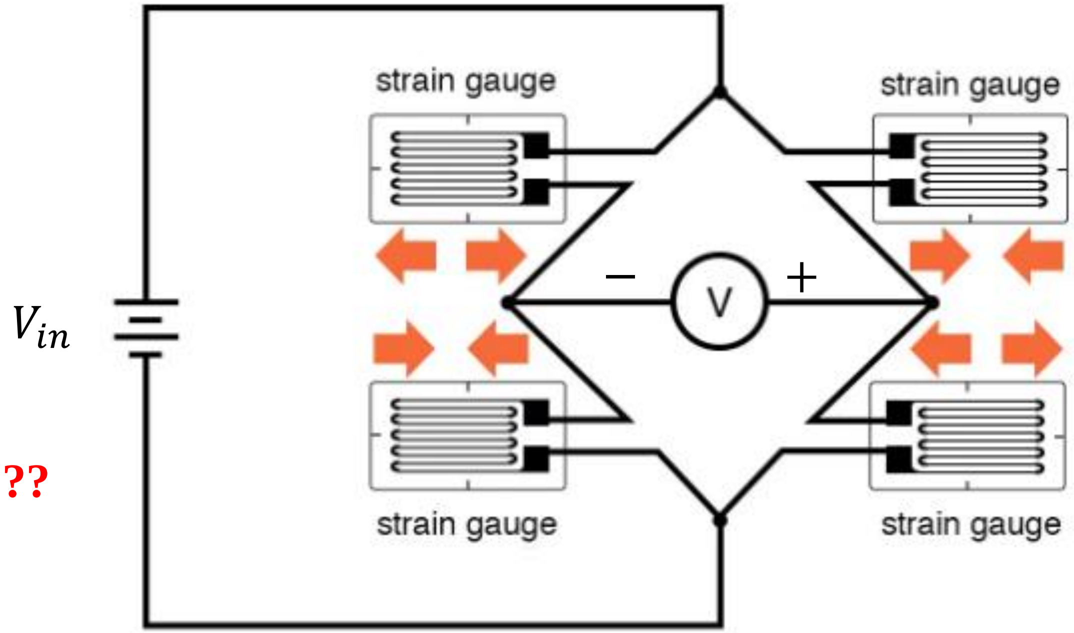
Strain gauge

Full – Bridge Circuit



Wire Resistances ??

Change in Temperature ??

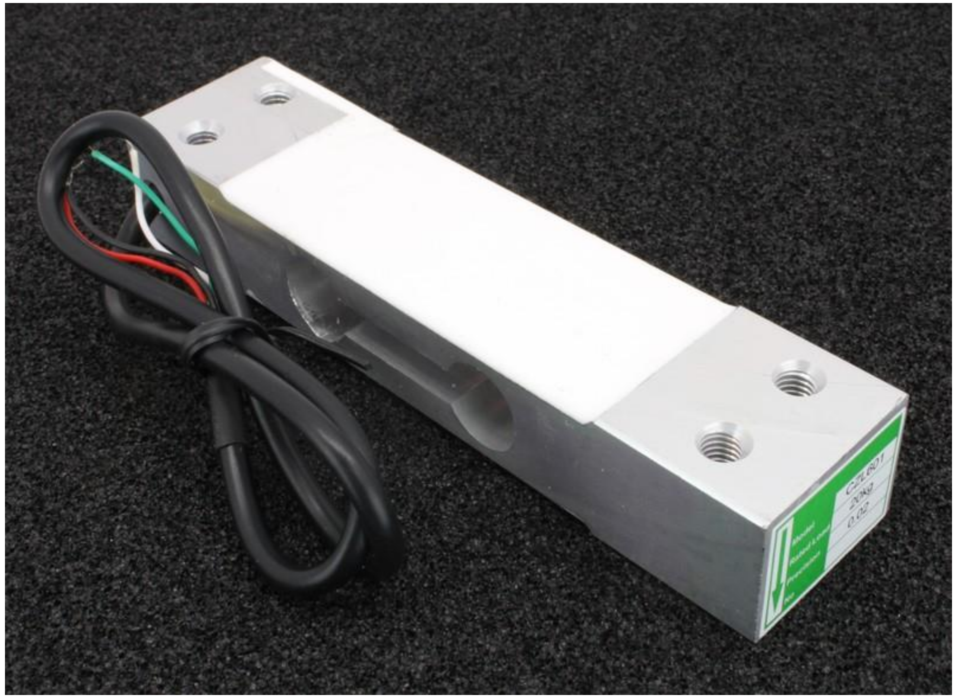


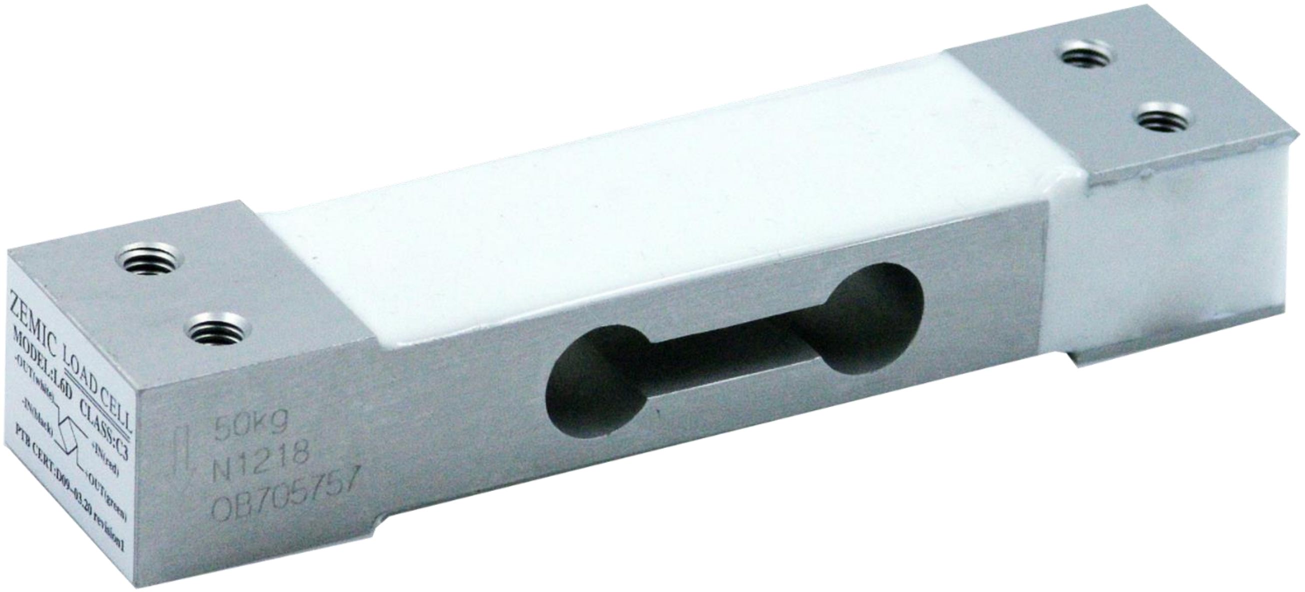
$$No\ load: \quad V = 0$$

$$Load: \quad V = \left[\frac{R_g + \Delta R}{R_g + \Delta R + R_g - \Delta R} - \frac{R_g - \Delta R}{R_g + \Delta R + R_g - \Delta R}\right] V_{in}$$

$$\rightarrow \boldsymbol{V = \left[\frac{\Delta R}{R_g}\right] V_{in}} \qquad \boldsymbol{V_{Half-Bridge} = \left[\frac{\Delta R}{2R_g}\right] V_{in}}$$

Load cell





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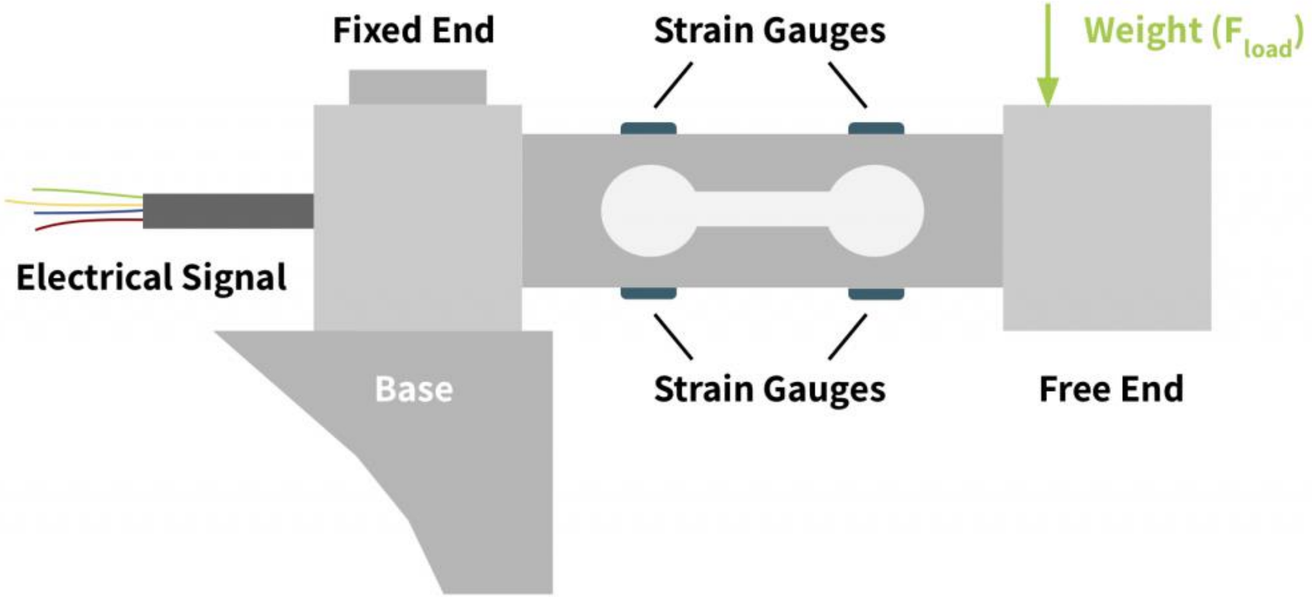
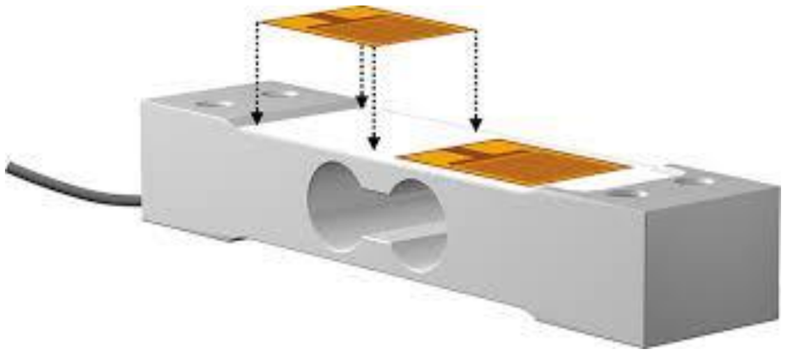
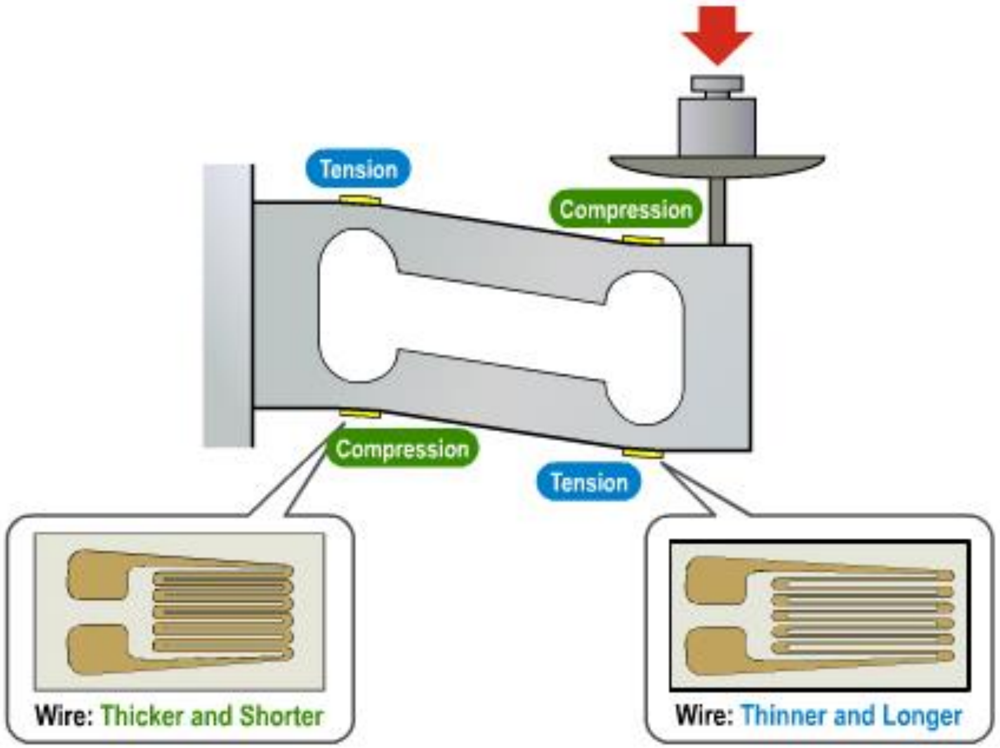
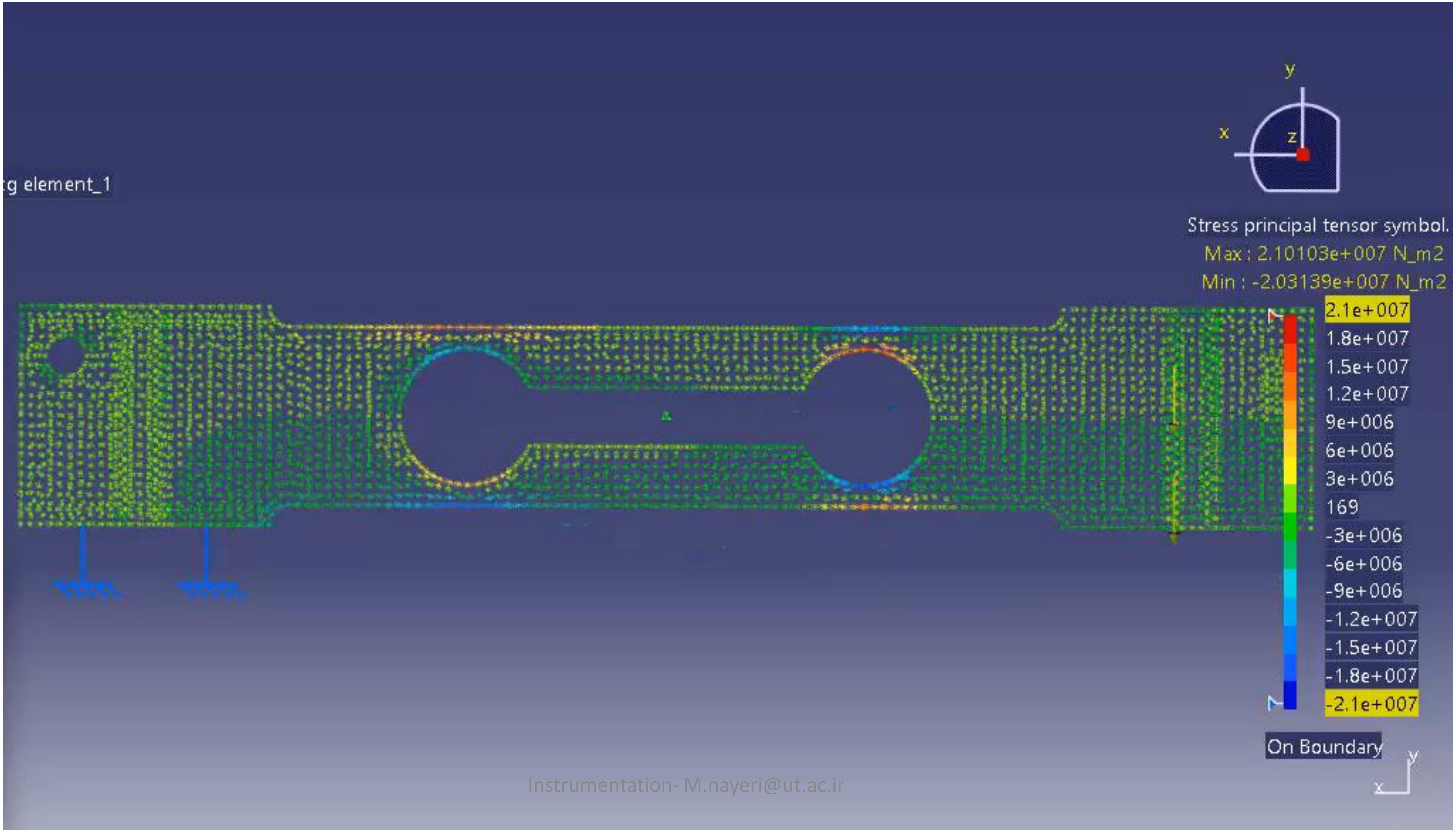

Fixed End
Strain Gauges
Weight (F_{load})
Electrical Signal
Base
Strain Gauges
Free End



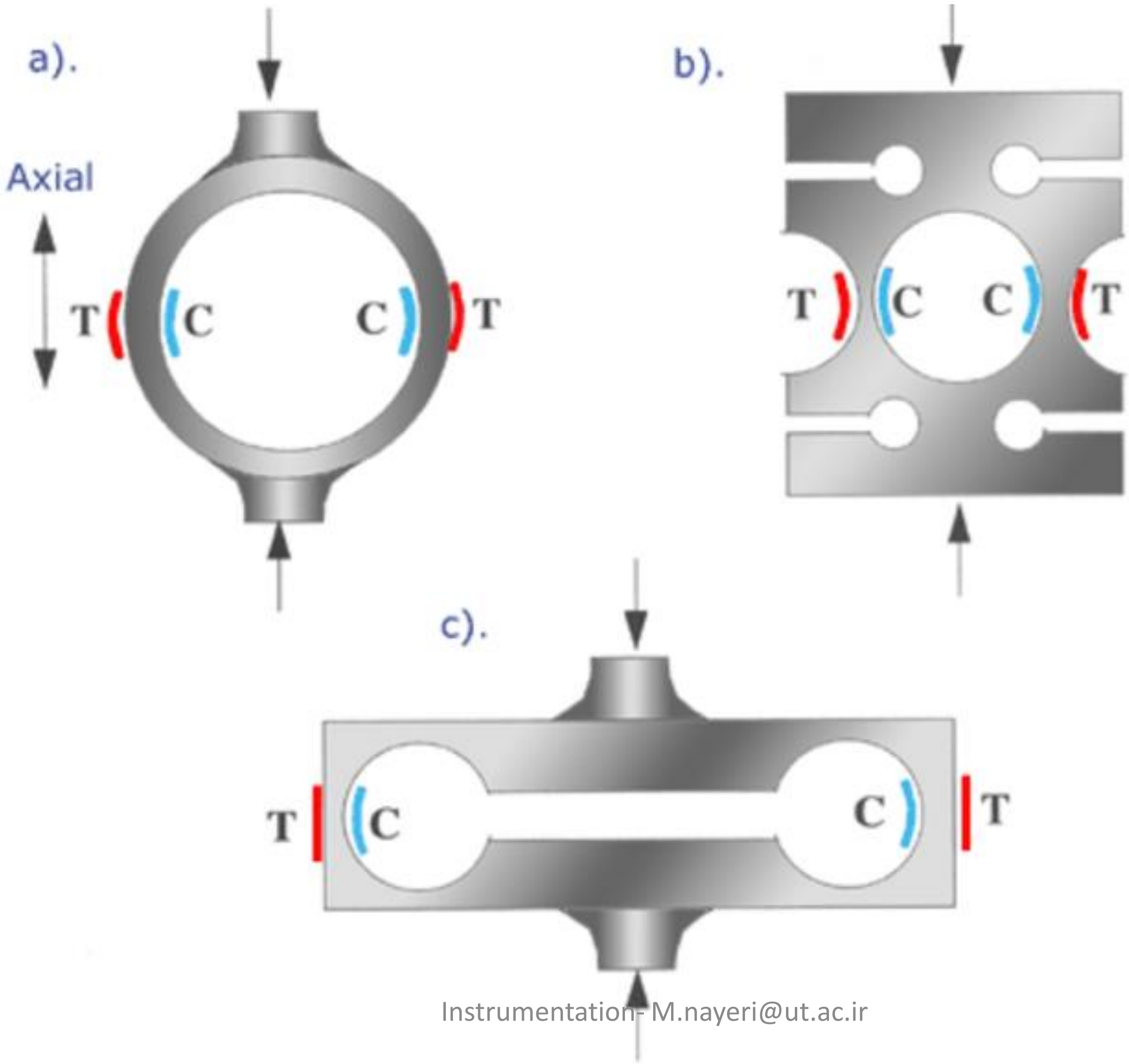

Tension
Compression
Compression
Tension
Wire: Thicker and Shorter
Wire: Thinner and Longer

Load cell

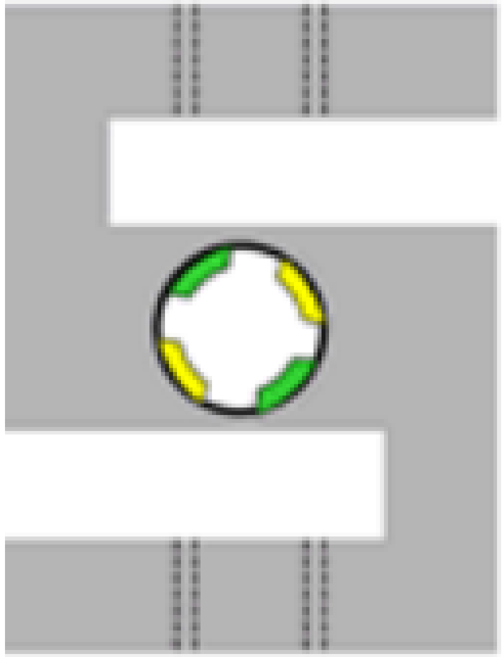


g element_1
y
x
z
Stress principal tensor symbol.
Max : 2.10103e+007 N_m2
Min : -2.03139e+007 N_m2
2.1e+007
1.8e+007
1.5e+007
1.2e+007
9e+006
6e+006
3e+006
169
-3e+006
-6e+006
-9e+006
-1.2e+007
-1.5e+007
-1.8e+007
-2.1e+007
On Boundary

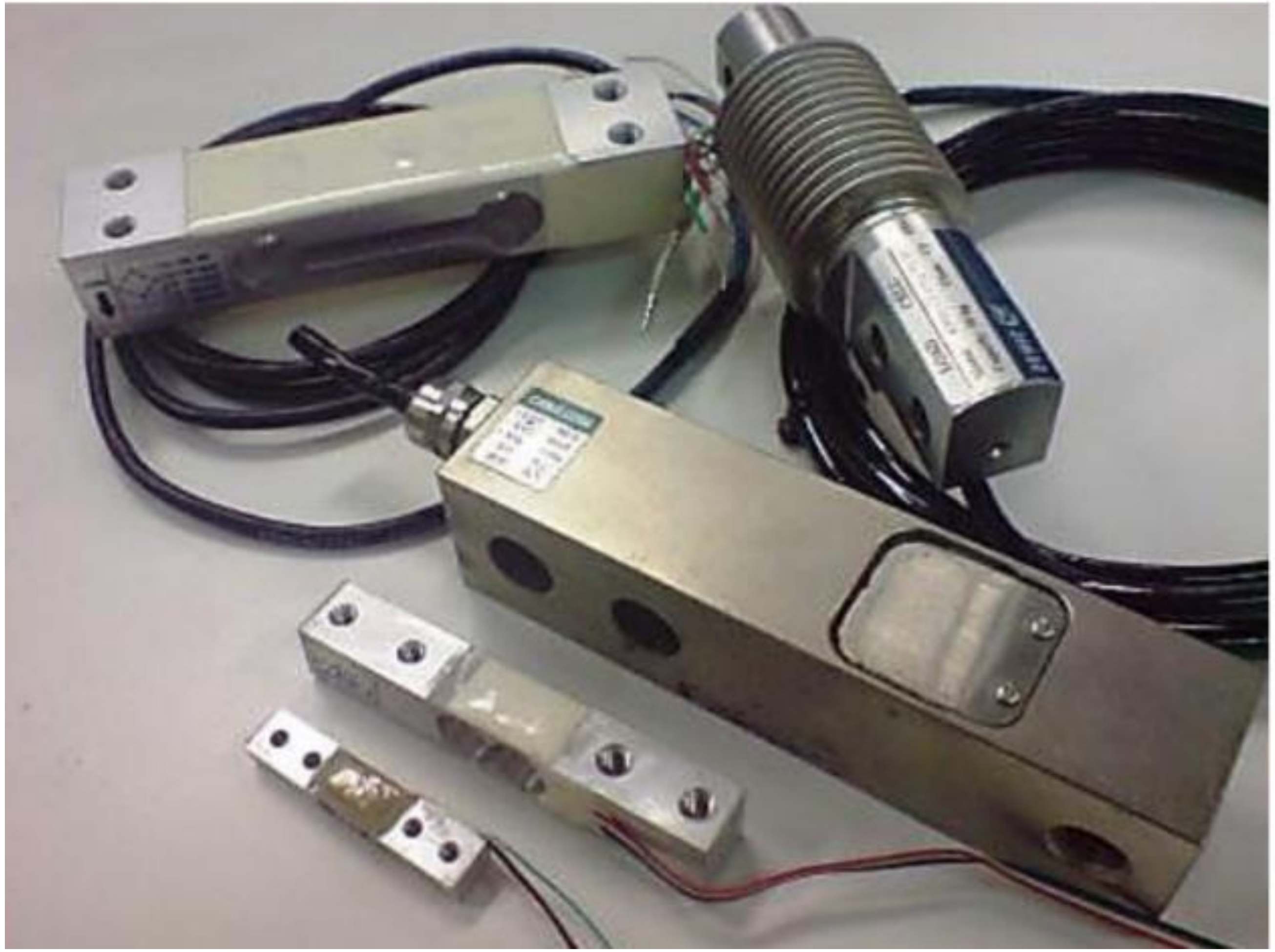
Load cell



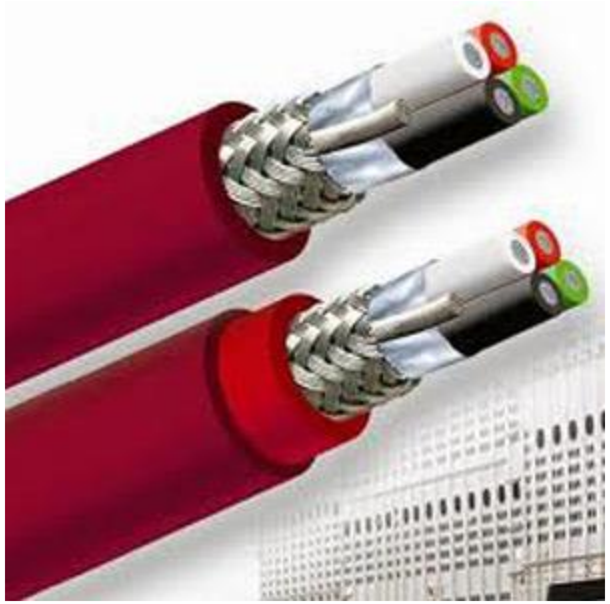
Load cell

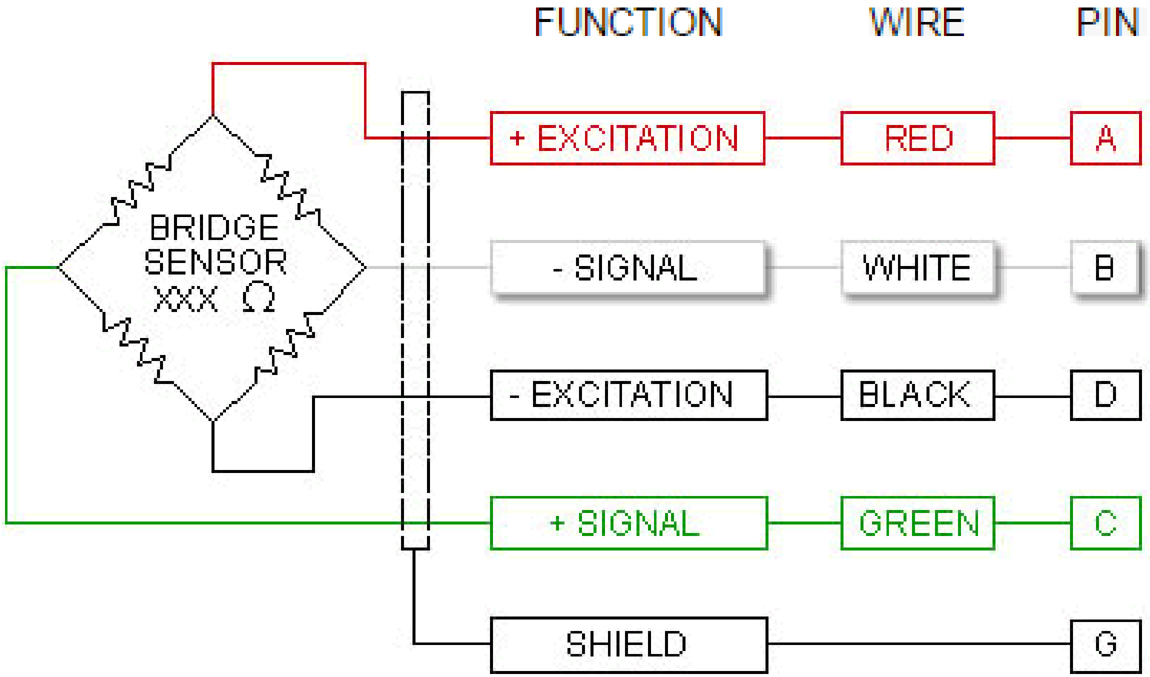


Load cell

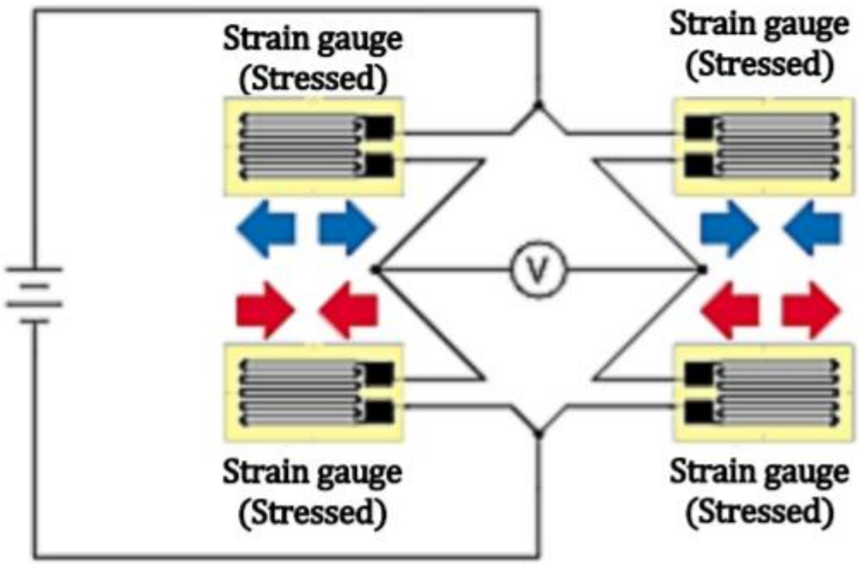


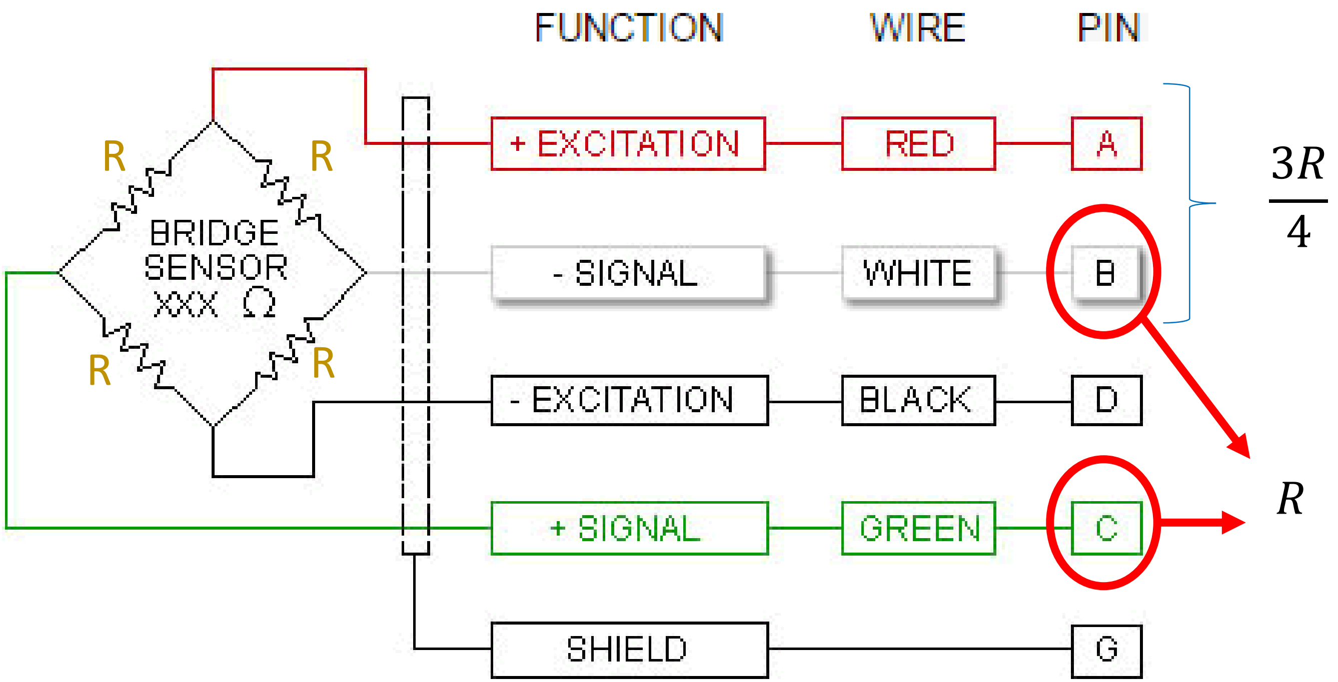
Load cell



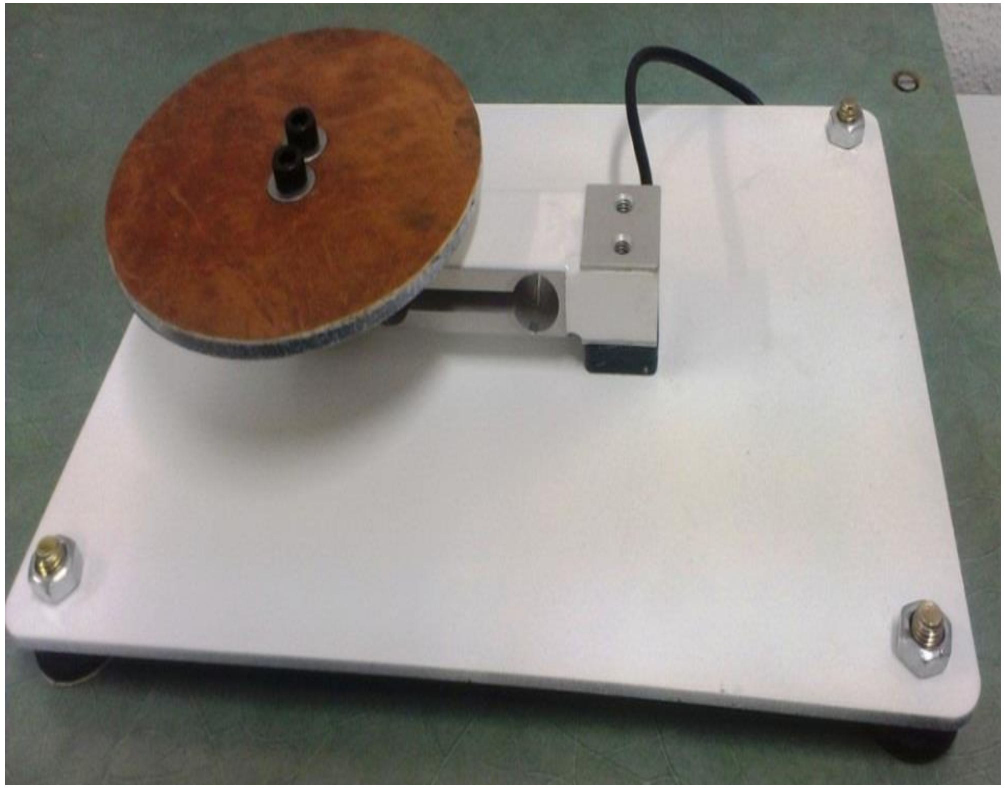

FUNCTION
WIRE
PIN
BRIDGE
SENSOR
XXX Ω
+ EXCITATION
RED
A
- SIGNAL
WHITE
B
- EXCITATION
BLACK
D
+ SIGNAL
GREEN
C
SHIELD
G

Load cell


Strain gauge
(Stressed)
Strain gauge
(Stressed)
V
Strain gauge
(Stressed)
Strain gauge
(Stressed)

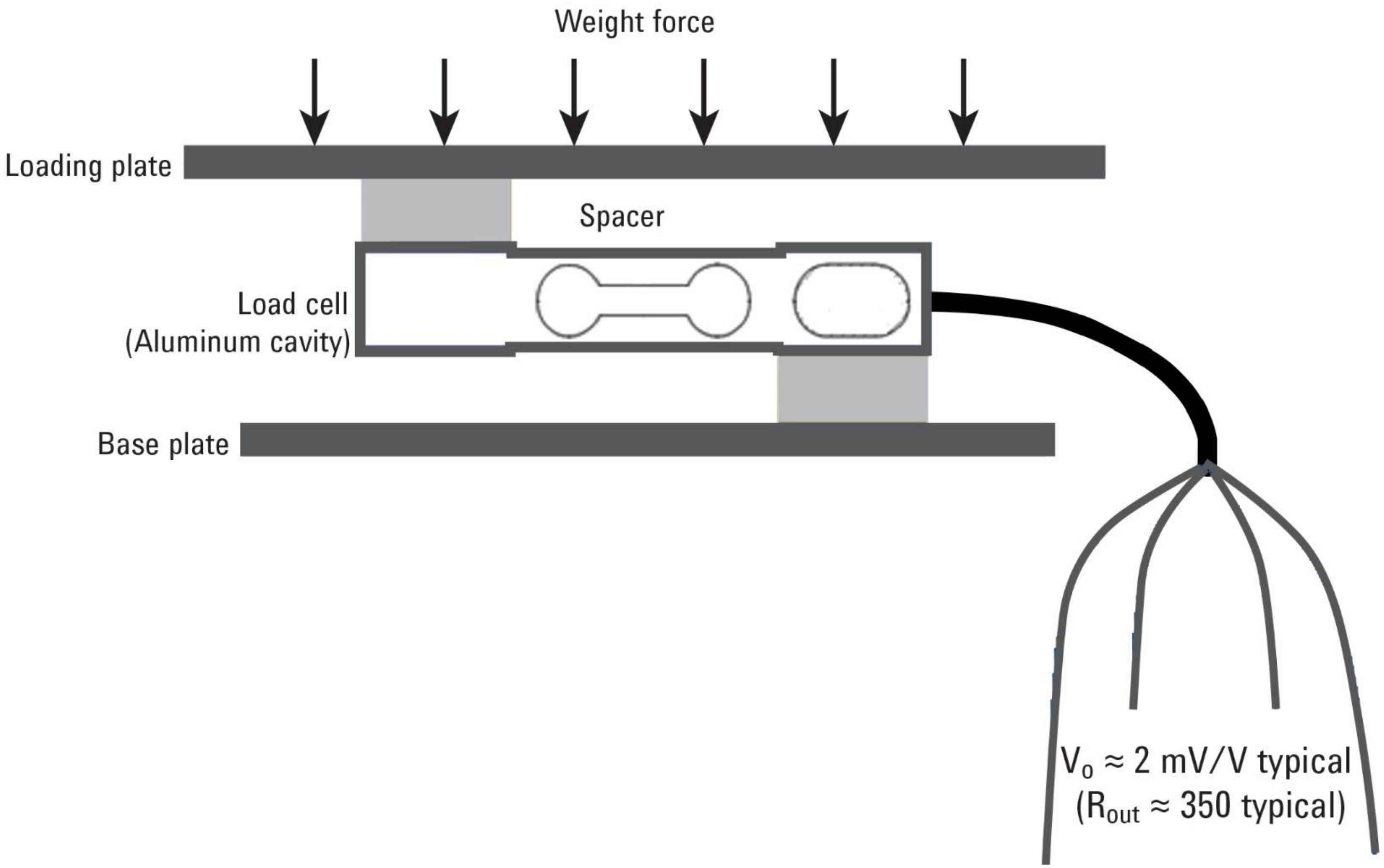

FUNCTION
WIRE
PIN
R
R
BRIDGE
SENSOR
XXX Ω
R
R
+ EXCITATION
RED
A
- SIGNAL
WHITE
B
- EXCITATION
BLACK
D
+ SIGNAL
GREEN
C
SHIELD
G
$\frac{3R}{4}$
R

Load cell

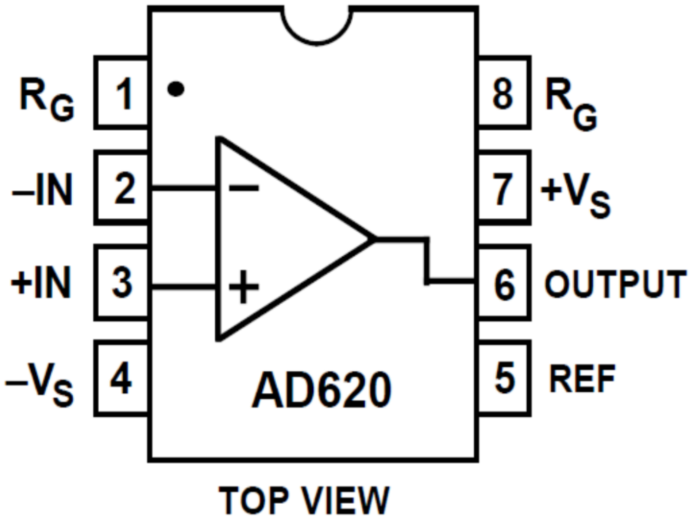


Output Sensitivity	1.8 ± 0.2 mV/V
Safe Overload	150%
Ultimate Overload	300%
Excitation Voltage	5 ~ 12 V
Excitation Maximum	18 V

Load cell

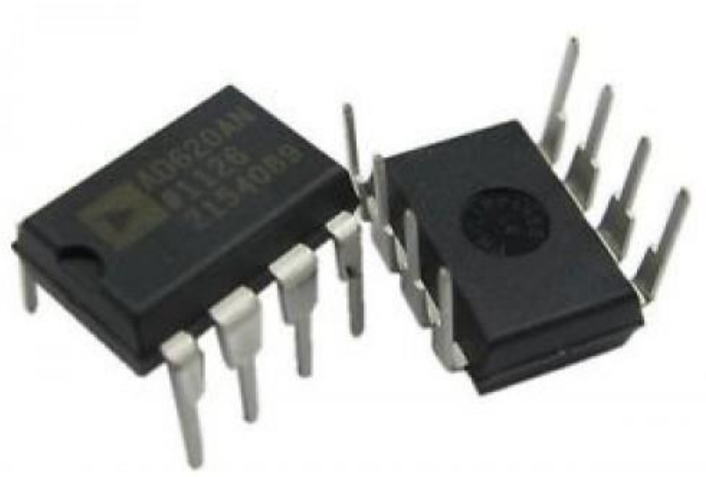


Weight force
Loading plate
Spacer
Load cell
(Aluminum cavity)
Base plate
$V_o \approx$ 2 mV/V typical
($R_{out} \approx$ 350 typical)

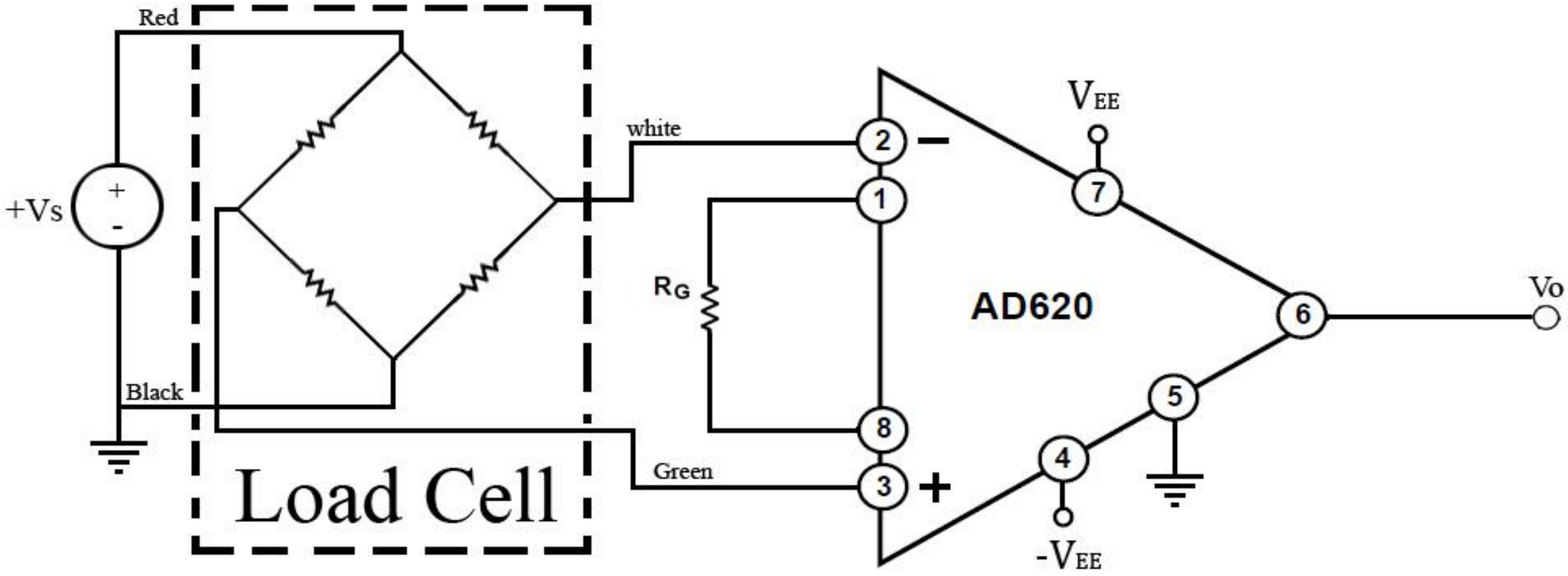


Load cell

مقاومت‌های استاندارد 1%	بهره	مقاومت‌های استاندارد 0.1%	بهره
49.9 KΩ	1.990	49.3 KΩ	2.002
12.4 KΩ	4.984	12.4 KΩ	4.984
5.49 KΩ	9.998	5.49 KΩ	9.998
2.61 KΩ	19.93	2.61 KΩ	19.93
1.00 KΩ	50.40	1.01 KΩ	49.91
499 Ω	100.0	499 Ω	100.0
249 Ω	199.4	249 Ω	199.4
100 Ω	495.0	98.8 Ω	501.0
49.9 Ω	991.0	49.3 Ω	1003

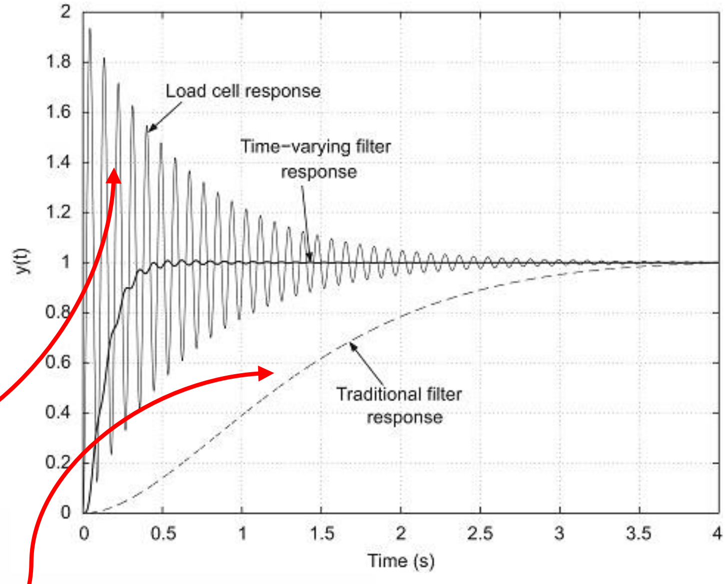


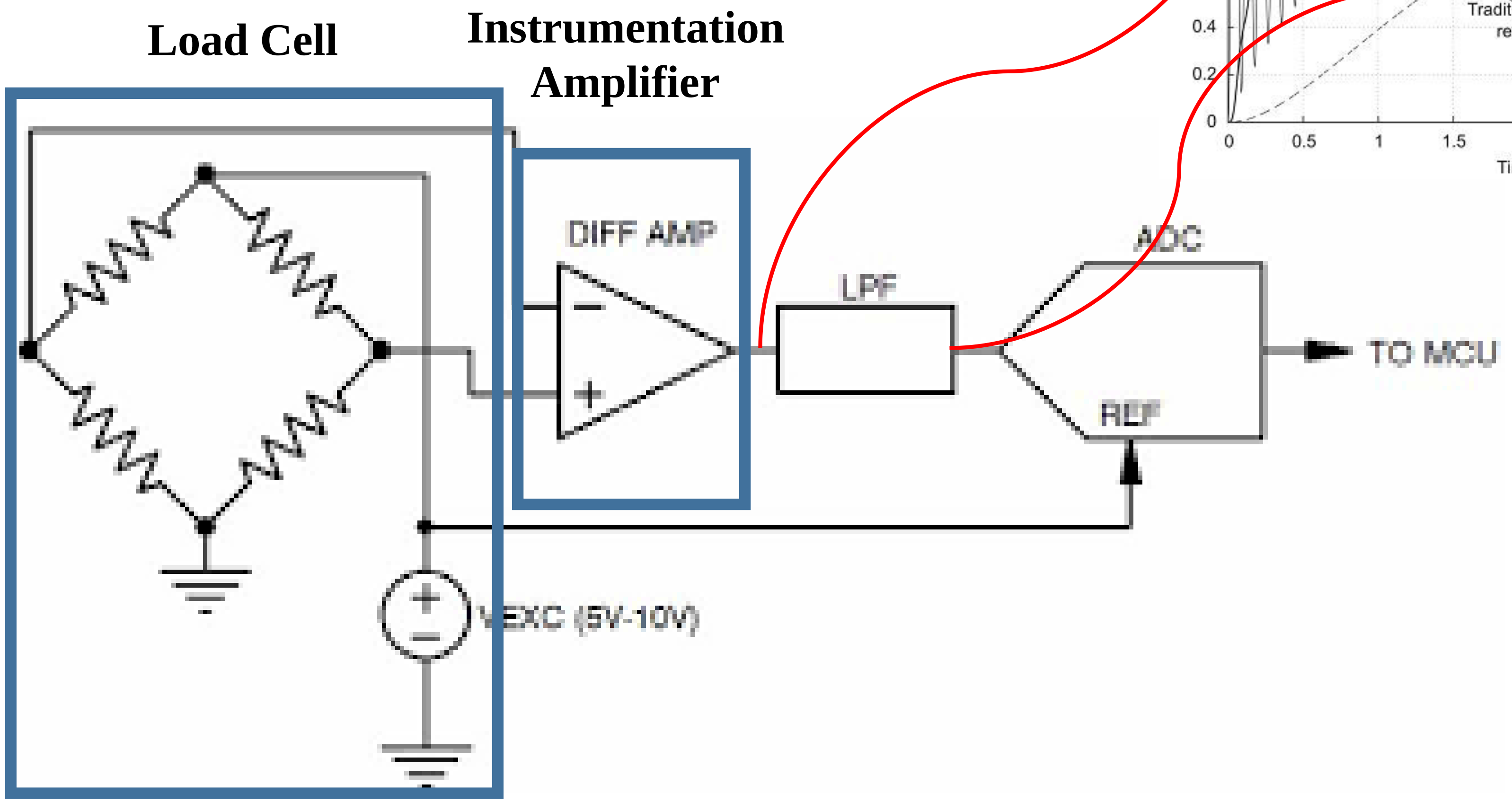
Load cell



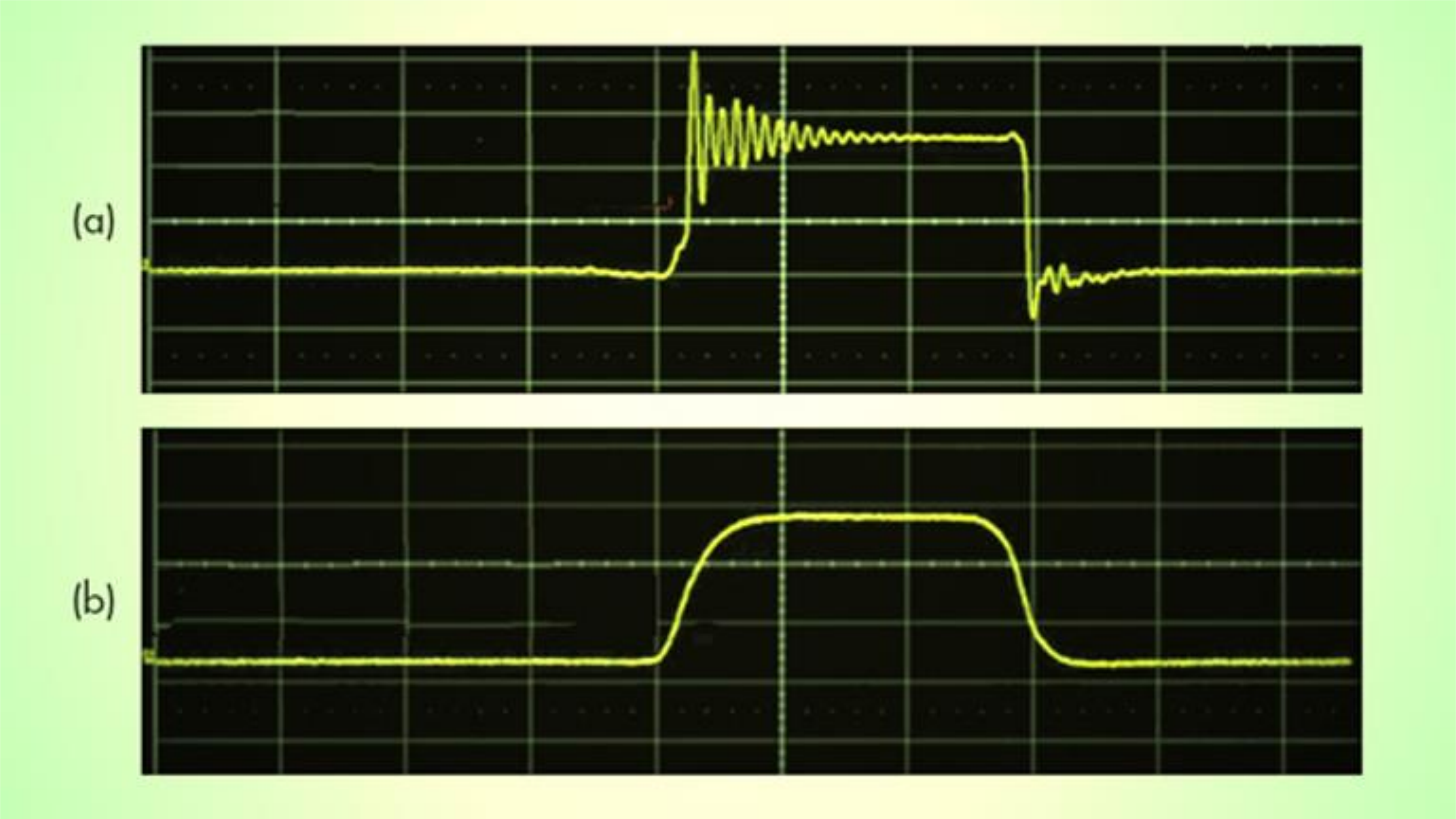
Red
+Vs
+
-
Black
Load Cell
white
Green
RG
2
1
8
3
AD620
VEE
7
6
Vo
5
4
-VEE

Load cell


2
1.8
1.6
1.4
1.2
1
0.8
0.6
0.4
0.2
0
y(t)
0
0.5
1
1.5
2
2.5
3
3.5
4
Time (s)
Load cell response
Time−varying filter
response
Traditional filter
response

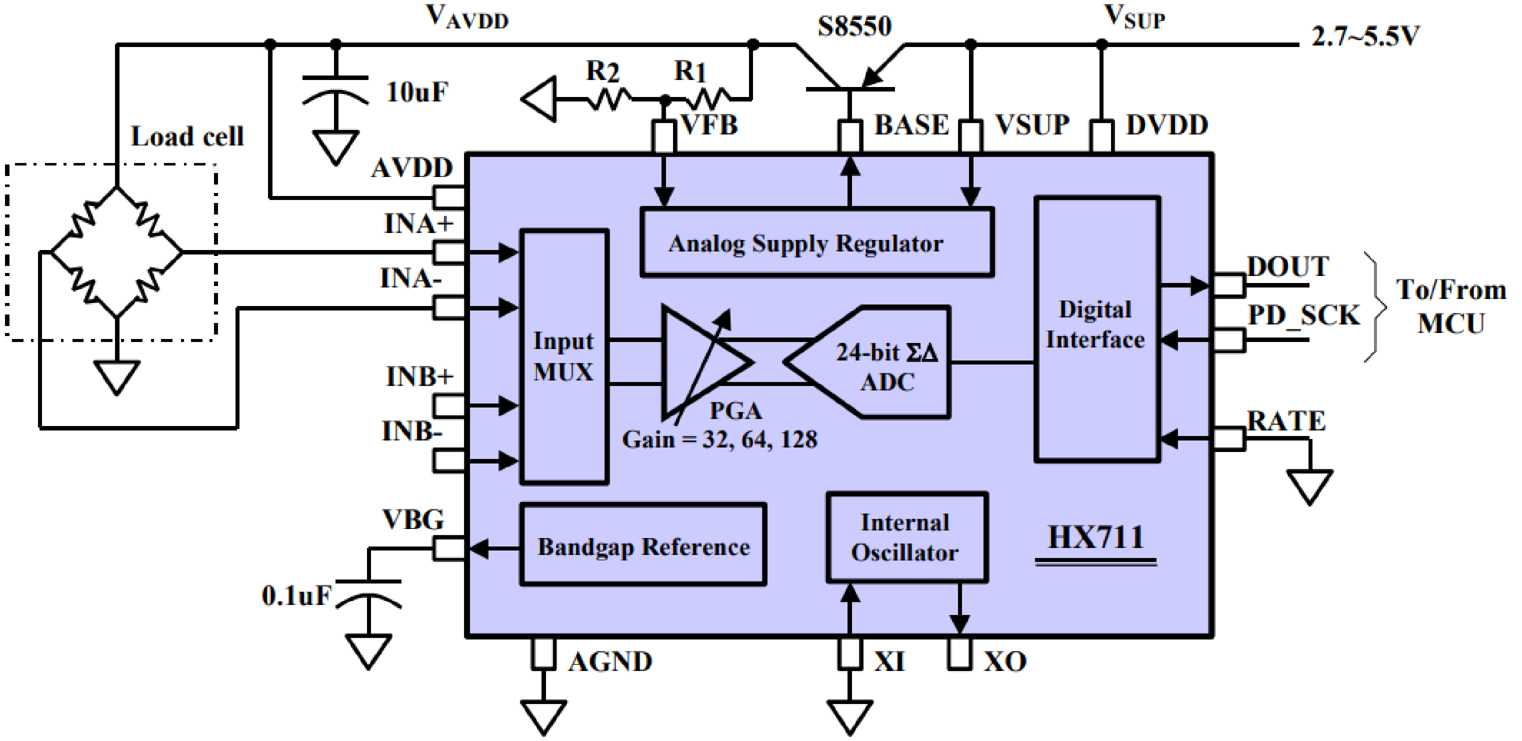

Load Cell
Instrumentation
Amplifier
DIFF AMP
LPF
ADC
REF
TO MCU
VEXC (5V-10V)

Load cell



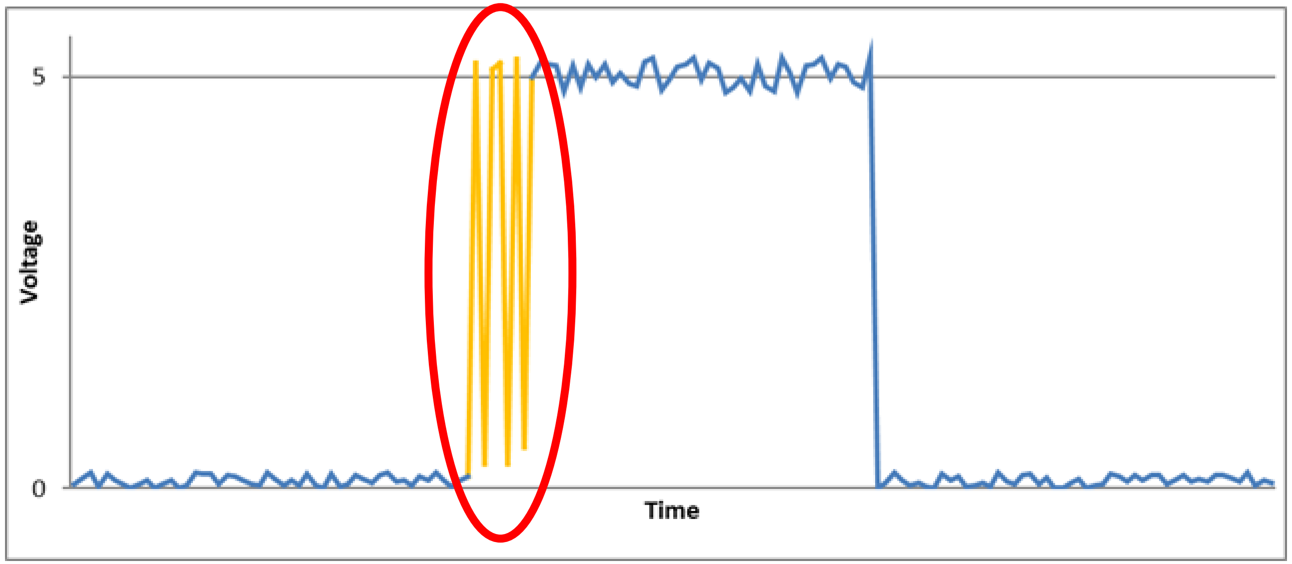
Load cell

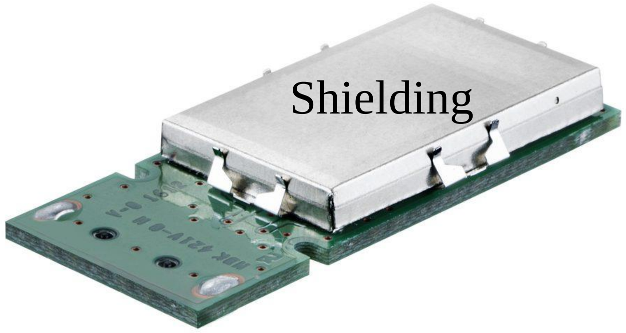
HX711 24-Bit Analog-to-Digital Converter (ADC) for Weigh Scales



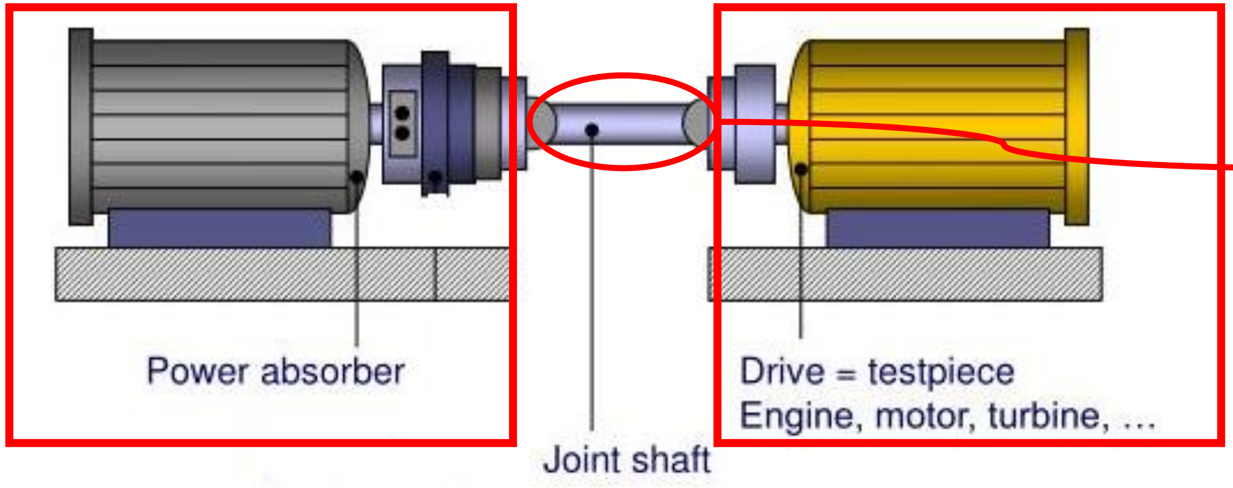
Load cell

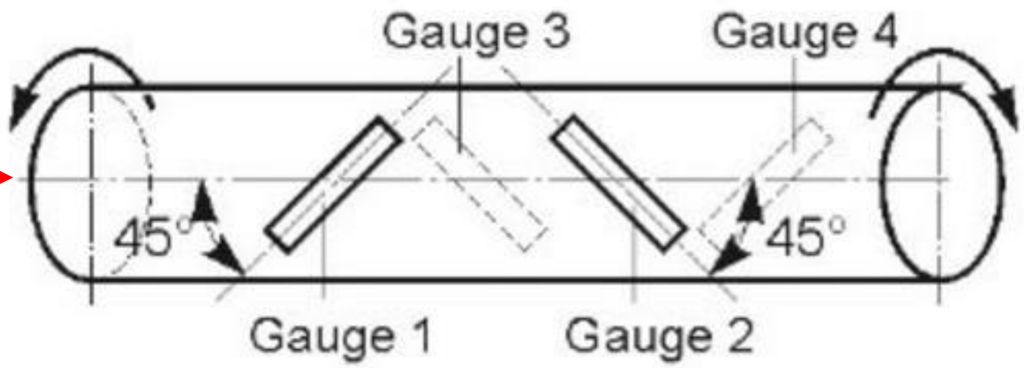
Ringing

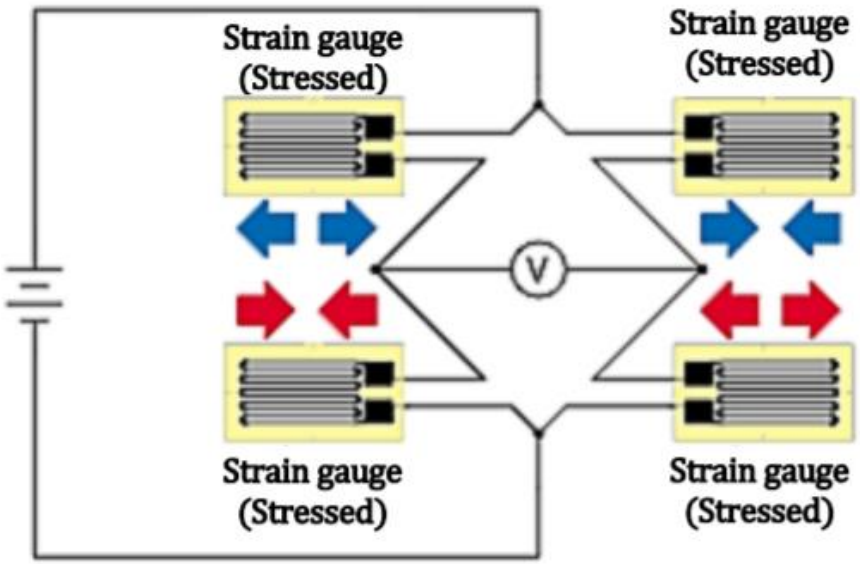


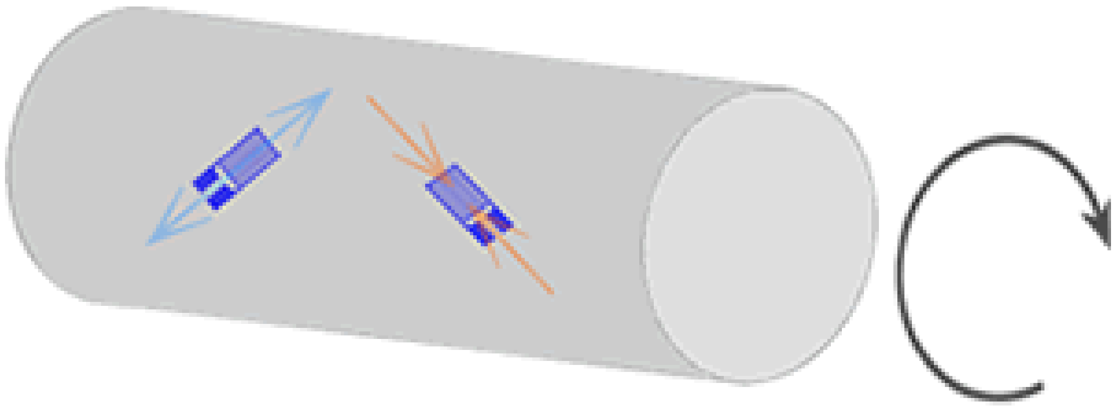


Torque Measurement



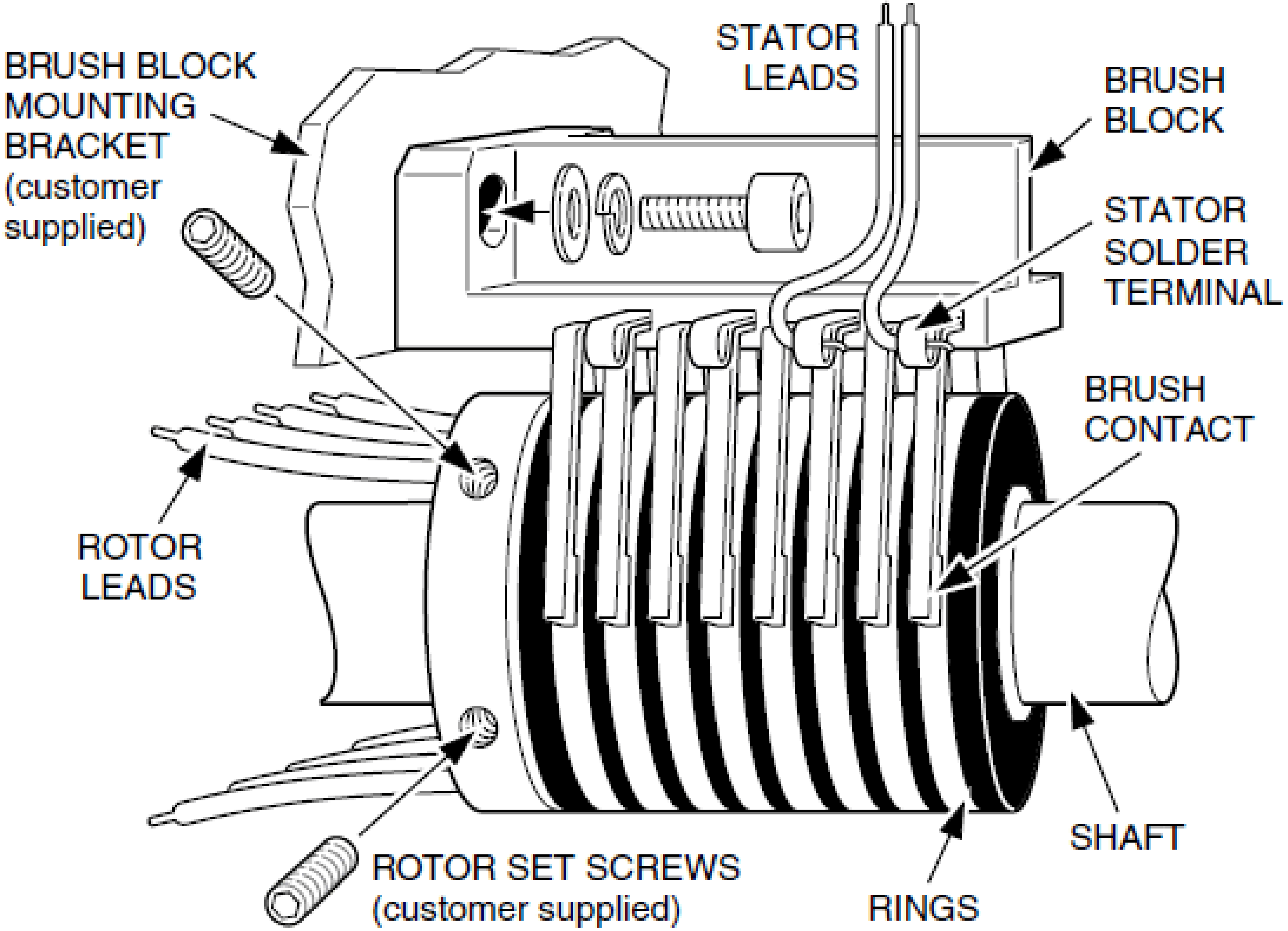






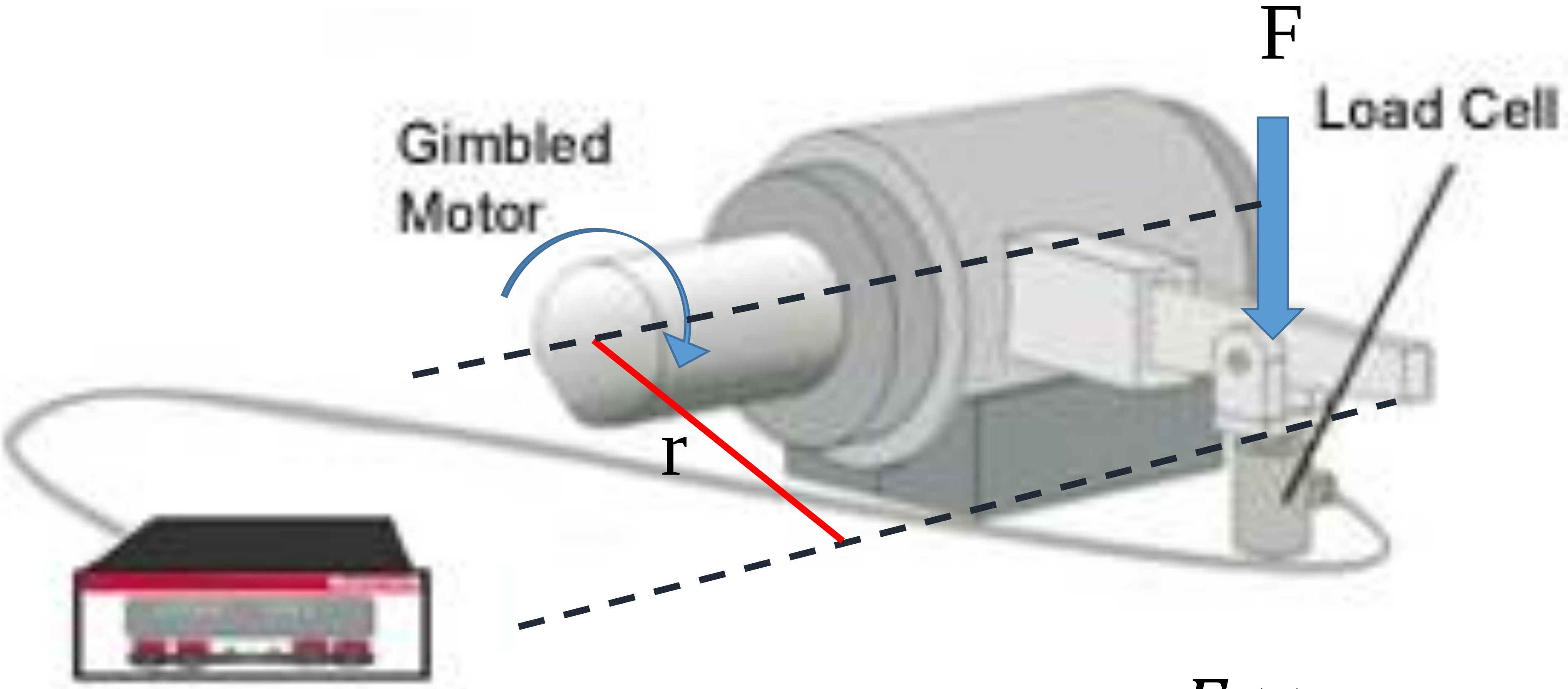
Strain Gauges Bonded to a Torque Shaft

Slip ring



BRUSH BLOCK MOUNTING BRACKET (customer supplied)
STATOR LEADS
BRUSH BLOCK
STATOR SOLDER TERMINAL
BRUSH CONTACT
ROTOR LEADS
SHAFT
ROTOR SET SCREWS (customer supplied)
RINGS

Torque Measurement



$$\tau = F \times r$$