LCS

Hw 10

محمد شرقی
810199492

$\frac{K}{s(s^2+s+4)} = G(s)$ $\rightarrow$ $s_1 = -\frac{1}{2} + \frac{i\sqrt{15}}{2}$, $s_2 = -\frac{1}{2} - \frac{i\sqrt{15}}{2}$

$\gamma = 50 \Rightarrow \angle G(j\omega_g) = -130$ & $|G(j\omega_g)| = 1$

$\angle(j\omega_g) = -90 - \tan^{-1}\left(\frac{\omega_g}{4-\omega_g^2}\right) = -130 \Rightarrow +40 = +\tan^{-1}\left(\frac{\omega_g}{4-\omega_g^2}\right)$

$\Rightarrow \tan(40^\circ) = \frac{\omega_g}{4-\omega_g^2} = 0.8391 \Rightarrow$ $\omega_g = -2.682$ ق ق ق ✗

$\omega_g = 1.491$ ق ق ق ✓

$|G(j\omega_g)| = 1 = \frac{K}{\omega_g\sqrt{(4-\omega^2)^2+\omega^2}} = 1 \Rightarrow$

$K = \omega_g\sqrt{(4-\omega^2)^2+\omega^2}$

$K = 3.45852$

$G(s) = \frac{3.4585}{s(s^2+s+4)}$

$\angle G(\omega_{pc}) = -180 \Rightarrow -90 - \tan^{-1}\left(\frac{\omega_0}{4-\omega_p^2}\right) = -180 \Rightarrow$

$\tan^{-1}\left(\frac{\omega_p}{4-\omega_p^2}\right) = 90 \Rightarrow 4 - \omega_p^2 = 0$ ($\tan(\pi/2) = \infty$)

$\omega_0 \begin{cases} +2 & \text{ق ق ق} \checkmark \\ -2 & \text{ق ق ق} \times \end{cases}$

$$G_M = K_g = \frac{1}{|G(j\omega_0)|} = \frac{1}{0.8645} = 1.1567$$

$$20\log(1.1567) = 1.2644$$

4) وقتی $w=1$ شد درنمودار GM را بدست می‌آوریم $GM \simeq 15$ دسی‌بل

$w_g = 2$ rad/s

حد فاز: درصورت $= 20\log|G(w)| = 0$ شد اختلاف زاویه با $(p.o. - 180)$

$\gamma \simeq 50 = 180 - 130$ درجه

فاز درجه: $\angle G(jw_o) = -180$

$w_o \simeq 7$ rad/s

اول ζ را بدست می‌آوریم

$$50 = \tan^{-1} \frac{2\zeta}{\sqrt{-2\zeta^2 + \sqrt{1+4\zeta^4}}}$$

$$\tan(50) = \frac{2\zeta}{\sqrt{-2\zeta^2 + \sqrt{1+4\zeta^4}}} \Rightarrow \boxed{\zeta \simeq 0.47774}$$

$$w_g = w_n \sqrt{-2\zeta^2 + \sqrt{1+4\zeta^4}} \Rightarrow w_n \simeq 1.6783$$

$$M_p = e^{\frac{-\zeta\pi}{\sqrt{1-\zeta^2}}} = 0.1811 \Rightarrow \boxed{\%18 \text{ درصد فراجهش}}$$

$$t_d = \frac{\pi}{w_d} = \frac{\pi}{w_n\sqrt{1-\zeta^2}} \cong \boxed{2.13072 = t_d}$$

$$t_s = \frac{4}{w_n \zeta} = 4.989 \simeq \boxed{5 = t_s}$$

محاسبه پهنای باند حلقه بسته سوال 4

از روی نمودار

$\frac{C(s)}{R(s)} = \left|\frac{G}{1+G}\right| = \frac{1}{\sqrt{2}}$

$\Rightarrow G(j\omega) = 2.41 \Rightarrow 20\log G(j\omega) = 20\log(2.41)$

$= 7.640$

$\omega_{bw} \simeq 4$

یا

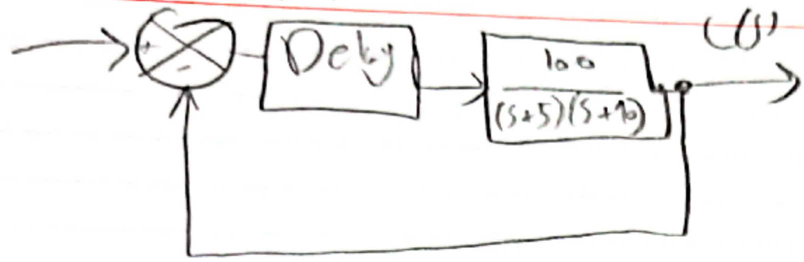
از فرمول

$\omega_{bw} = \omega_n\sqrt{1-2\zeta^2+\sqrt{1+(1-2\zeta^2)^2}} \simeq 2.63 \Rightarrow$

$20\log G(2.63) = 8.39$

در قسمت ب محاسبه شدند

5)



$$G(s) = \frac{e^{-sT}\,100}{(s+5)(s+10)}$$

↓ $-5, -10$

$$\frac{C(s)}{R(s)} = \frac{e^{-sT}100}{(s+5)(s+10)+e^{-sT}100}$$

$$1 + \frac{K e^{-sT} 100}{(s+5)(s+10)} = 0 \xrightarrow{t=0} -100 = (s+5)(s+10) \begin{cases} s_1 = -7.5 + j9.7 \\ s_2 = -7.5 - j9.7 \end{cases}$$

دو قطب موهومی — پایدار است

$$|G(j\omega)| = 1 \Rightarrow \frac{100}{\sqrt{\omega_g^2+25}\sqrt{\omega_g^2+100}} = 1 \Rightarrow \omega_g = 5\sqrt{\left(\frac{\sqrt{73}}{2} - 2.5\right)}$$

$$\omega_g = 6.6558$$

$$\gamma = 180 + \angle G(j\omega) = 180 - \tan^{-1}\left(\frac{6.65}{5}\right) - \tan^{-1}\left(\frac{6.65}{10}\right) - \frac{180}{\pi}T_d(6.65)$$

53.06   33.62 > 0

$\gamma = 93.28$ ↓ if $t = 0$ — پایدار

$$93.28 - \frac{180}{\pi}(6.65)T_d > 0$$

$$T_d < 0.244$$

برای پایداری

X $1, 0.5$ — ناپایدار

$0.2, 0.1, 0$ ✓ — پایدار

ملاحظات

5) ب)

حاشیه بهره: $\measuredangle G(j\omega_p) = -180 - \left(-t_d \omega_p \frac{180}{\pi}\right)$

$= -\left(\tan^{-1}\left(\frac{\omega_p}{5}\right) + \tan^{-1}\left(\frac{\omega_p}{10}\right)\right) = -180 + \left(t_d \omega_p \frac{180}{\pi}\right)$

$t_d = 0$:
- $\omega_g = 6.65 \Rightarrow |G(\omega_g)| = 1 \quad \gamma = 93.28$
- $\measuredangle G(j\omega_p) = -\pi \Rightarrow \tan^{-1}\left(\frac{\omega_p}{5}\right) + \tan^{-1}\left(\frac{\omega_p}{10}\right) = \pi$
- $\omega_p = \infty \Rightarrow$ حاشیه بهره $= \infty$ ✓

$t_d = 0.1$:
- $\omega_g = 6.65 \Rightarrow |G(\omega_g)| = 1 \Rightarrow \gamma = 55.178^\circ$
- $-\tan^{-1}\left(\frac{\omega_p}{5}\right) - \tan^{-1}\left(\frac{\omega_p}{10}\right) = \omega_p \frac{18}{\pi} - 180 \Rightarrow 11.3625 \Rightarrow \omega_p$!!
- $K_g = \frac{1}{|G(j\omega_p)|} = 1.87843$

$t_d = 0.2$:
- $\omega_g = 6.65 \Rightarrow |G(\omega_g)| = 1 \quad \gamma = 17.077$
- $\measuredangle(G(j\omega_p)) = -\pi + 0.2\omega_p \frac{180}{\pi} \Rightarrow \omega_p = 7.547$
- $K_g = \frac{1}{G(\omega_p)} = 1.133$

$t_d = 0.5$:
- $\omega_g = 6.65 \Rightarrow \gamma = -97.228$
- $G(\omega_p) = \pi + 0.5\,\omega_p \frac{180}{\pi} \Rightarrow \omega_p = 4.12224$
- $K_g = \frac{1}{G(\omega_p)} = 0.7607$

$t_d = 1$ {
- @ $w_g = 0.65 \Rightarrow \gamma = -287.73$
- $\angle G(jw_p) = -\pi + \frac{180}{\pi} w_p \Rightarrow \boxed{w_p = 2.44}$
- $K_p = \frac{1}{G(jw_p)} = 0.5728$

}

ج) $t_2 = 0.5 \Rightarrow 20 \log(0.7007) = -3.0899$

$t_2 = 1 \Rightarrow 20 \log(0.5728) = -4.8429$