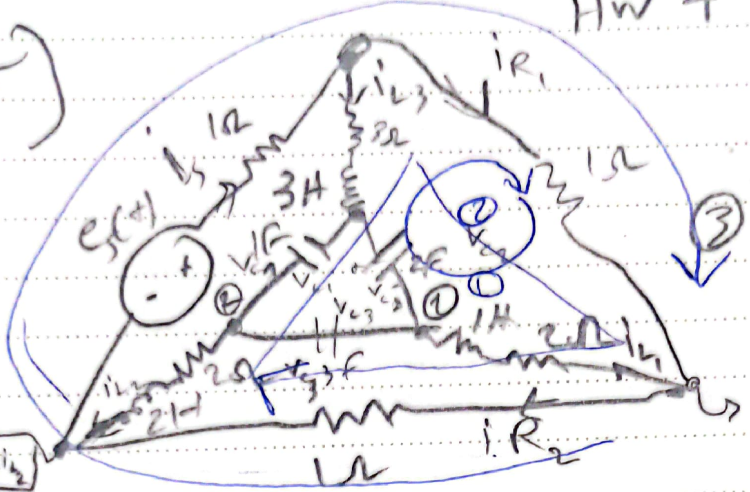
محمد عسکری
810199492

کنترل خطی استاد ...

HW 4

1) ب)



$i_{1L} + i_{2L} = i_{L3}$

$V_{c_1} + V_{c_2} = V_{c_3}$

$\boxed{i_{R_2} = i_s - i_2}$

$\boxed{i_{R_1} = i_{L_1} + i_s - i_{L_2}}$

$\boxed{+5\dot{V}_{c_2} + 3\dot{V}_{c_1} = -i_{L_1}}$

KCL ①: $-2\dot{V}_{c_2} - 3\dot{V}_{c_3} - i_{L_1} = 0 \Rightarrow -5\dot{V}_{c_2} - 3\dot{V}_{c_1} - i_{L_1} = 0 \Rightarrow$

$\dot{V}_{c_3} = \dot{V}_{c_1} + \dot{V}_{c_2}$

KCL ②: $\dot{V}_{c_1} + 3\dot{V}_{c_3} + i_2 = 0 \Rightarrow$ $\boxed{4\dot{V}_{c_1} + 3\dot{V}_{c_2} = -i_2}$

$\dot{V}_{c_3} = \dot{V}_{c_1} + \dot{V}_{c_2}$

$$\Rightarrow \begin{cases} \dot{V}_{c_1} = \frac{1}{11}(3i_{L_1} - 5i_{L_2}) \\ \dot{V}_{c_2} = \frac{1}{11}(-4i_{L_1} + 3i_{L_1}) \end{cases}$$

KVL ①: $-2i_{L_2} - 2\dot{i}_{L_2} + V_{c_1} - 3i_{L_3} - 3\dot{i}_{L_3} + i_{R_1} + i_{R_2} = 0$

KVL ②: $-V_{c_2} - 3i_{L_3} - 3\dot{i}_{L_3} + i_{R_1} + i_{L_1} + \dot{i}_{L_1} = 0$

KVL ③: $e_s(t) = i_s + i_{R_1} + i_{R_2} \Rightarrow e_s = i_s + i_s - i_{L_2} + i_{L_1} + i_s - i_{L_2}$

$e_s = 3i_s - 2i_{L_2} + i_{L_1} \Rightarrow$

$$\boxed{i_s = \frac{e_s - i_{L_1} + 2i_{L_2}}{3}}$$

$$\Rightarrow \begin{cases} i_{R_2} = \frac{e_s - i_{L_1} - i_{L_2}}{3} \\ i_{R_1} = \frac{e_s + 2i_{L_1} - i_{L_2}}{3} \end{cases} \quad \text{KVL } \boxed{1, 2}$$

$i_{R_1} + i_{R_2} = \frac{2e_s + i_{L_1} - 2i_{L_2}}{3}$

۲۲ فروردین جمعه

10 April 2020 Friday

KVL 1: $\boxed{-2i_{L_2} - 2\frac{di_{L_2}}{dt} + V_{c_1}} - 3i_{L_3} - \frac{3di_{L_3}}{dt} + \frac{2e_s + i_{L_1} - 2i_{L_2}}{3} = 0$

KVL 2: $\boxed{-V_{c_2} - 3i_{L_3}} - 3\frac{di_{L_3}}{dt} + \frac{e_s + 2i_{L_1} - i_{L_2}}{3} + i_{L_1} + \frac{di_{L_1}}{dt} = 0$

مرتب کنیم

$$\Rightarrow \begin{cases} -2\frac{di_{L_2}}{dt} - 3\frac{di_{L_3}}{dt} - \frac{8}{3}i_{L_2} - 3i_{L_3} + V_{c_1} + \frac{2e_s + i_{L_1}}{3} = 0 \\ -3\frac{di_{L_3}}{dt} + \frac{di_{L_1}}{dt} - V_{c_2} - 3i_{L_3} + \frac{e_s + 5i_{L_1} - i_{L_2}}{3} = 0 \end{cases}$$

$i_3 = i_L$

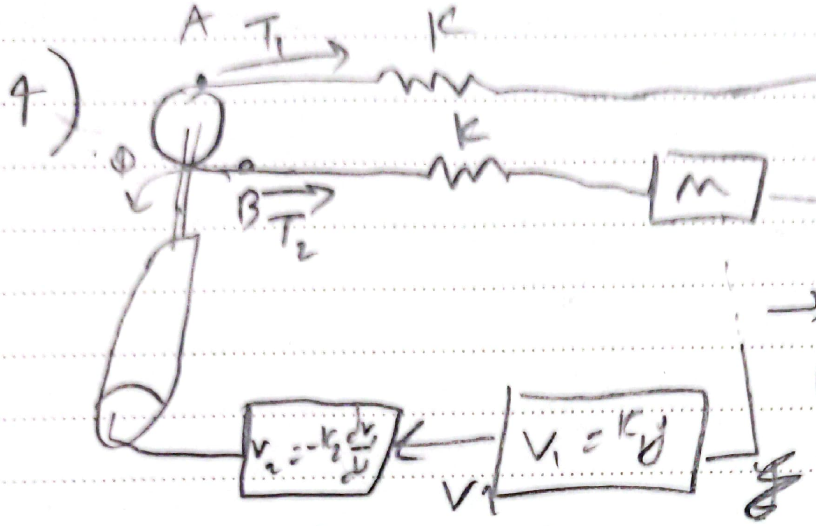
$$\Rightarrow \begin{cases} 5\frac{di_2}{dt} + 3\frac{di_1}{dt} = -\frac{17}{3}i_{L_2} - \frac{8i_{L_1}}{3} + \frac{2e_s}{3} + V_{c_1} \\ 2\frac{di_{L_1}}{dt} + 3\frac{di_{L_2}}{dt} = -V_{c_2} - \frac{4}{3}i_{L_1} - \frac{10}{3}i_{L_2} + \frac{e_s}{3} \end{cases}$$

$$\Rightarrow \begin{cases} \frac{di_{L_1}}{dt} = -\frac{e_s}{3} + \frac{1}{3}i_{L_2} + \frac{4}{3}i_{L_1} - 3V_{c_1} - 5V_{c_2} \\ \frac{di_{L_2}}{dt} = \frac{e_s}{3} - \frac{4}{3}i_2 - \frac{4}{3}i_{L_1} + 2V_{c_1} + 3V_{c_2} \end{cases}$$

$$\begin{bmatrix} \dot{V}_{c_1} \\ \dot{V}_{c_2} \\ \frac{di_{L_1}}{dt} \\ \frac{di_{L_2}}{dt} \end{bmatrix} = \begin{bmatrix} 0 & 0 & \frac{3}{11} & -\frac{5}{11} \\ 0 & 0 & -\frac{4}{11} & \frac{3}{11} \\ -3 & -5 & \frac{4}{3} & \frac{1}{3} \\ 2 & 3 & -\frac{4}{3} & -\frac{4}{3} \end{bmatrix} \begin{bmatrix} V_{c_1} \\ V_{c_2} \\ i_{L_1} \\ i_{L_2} \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ -\frac{1}{3} \\ \frac{1}{3} \end{bmatrix} e_s$$

$$i_s = \begin{bmatrix} 0 & 0 & -\frac{1}{3} & \frac{2}{3} \end{bmatrix} \begin{bmatrix} v_{c_1} \\ v_{c_2} \\ i_{L_1} \\ i_{L_2} \end{bmatrix} + \frac{1}{3} e_s$$

4)



$$\begin{cases} T = J\ddot{\theta} + b\dot{\theta} + r(T_1 - T_2) \\ T_m = \frac{K_m}{R} V_2 \end{cases}$$

A : $T_1 = K(r\theta - \cancel{r\theta_p}\, y)$, B : $T_2 = K(y - r\theta)$

$\cancel{T_1 - T_2 = m\frac{d^2 y}{dt^2}}$

$\cancel{A}$ m : $T_1 - T_2 = m\ddot{y}$ $\xrightarrow{A,B}$

$T_1 - T_2 = K(r\theta - y) - K(y - r\theta) = 2K(r\theta - y)$ $\Rightarrow$

$$2K(r\theta - y) = m\ddot{y} \quad \begin{cases} x_1 = r\theta - y \\ x_2 = \frac{dy}{dt} \\ x_3 = \frac{d\theta}{dt} \end{cases} \Rightarrow$$

$$2Kx_1 = m\dot{x}_2 \Rightarrow \boxed{\frac{2K}{m} x_1 = x^2}$$

$$x_1 = r\theta - y \xrightarrow{\frac{d}{dt}} \dot{x}_1 = r\dot{\theta} - \dot{y} \Rightarrow \frac{dx_1}{dt} = r x_3 - x_2$$

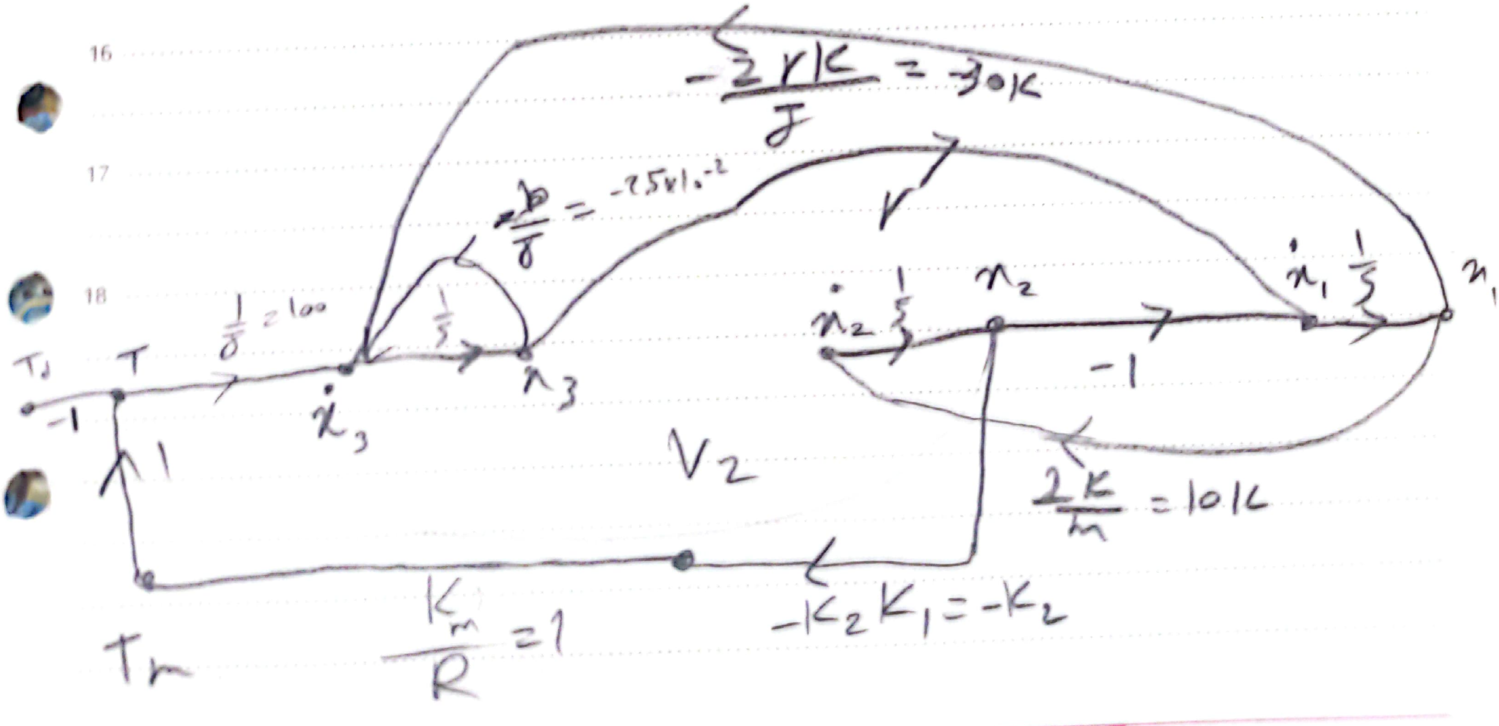
③ $\Rightarrow x_1 = r\theta - y \xRightarrow{\frac{d}{dt}} \dot{x}_1 = r\dot{\theta} - \dot{y} \Rightarrow \dot{x}_1 = r x_3 - x_2$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} & -1 & r \\ \frac{2K}{m} & & \\ \frac{-2r\cancel{?}K}{\cancel{?}J} & \frac{-K_m K_2 K_1}{JR} & \frac{-b}{J} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ -\frac{1}{J} \end{bmatrix} T_d$$

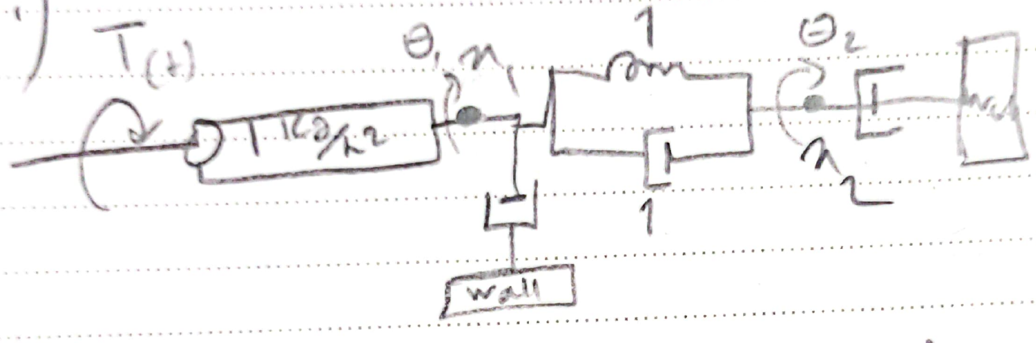
$$\dot{X} = \begin{bmatrix} 0 & -1 & 0.15 \\ 10K & 0 & 0 \\ -30K & \frac{-2K_2}{2\times10^{-2}} & -2.5\times10^{-2} \end{bmatrix} X + \begin{bmatrix} 0 \\ 0 \\ -100 \end{bmatrix} T_d$$

$\Downarrow$

$-100K_2$



5) ب)



$$n_1 : \ddot{\theta}_1 + (\theta_1 - \theta_2) + (\dot{\theta}_1 - \dot{\theta}_2) + \dot{\theta}_1 = T(t)$$

$$\xrightarrow{\mathcal{L}} (s^2 + 2s + 1)\Theta_1(s) \pm -(s+1)\Theta_2(s) = T(s) \quad (1)$$

$$n_2 : \quad + \dot{\theta}_2 + (\dot{\theta}_2 - \dot{\theta}_1) + (\theta_2 - \theta_1) = 0$$

$$\xrightarrow{\mathcal{L}} (2s+1)\Theta_2(s) \pm (s+1)\Theta_1(s) = 0 \Rightarrow \Theta_1(s) = \frac{2s+1}{s+1}\Theta_2 \quad (2)$$

$$(1),(2) \Longrightarrow \left((s+1)(2s+1) - (s+1)\right)\Theta_2 = T(s)$$

$$\left((s+1)(2s \cancel{+1-1})\right)\Theta_2 = T(s)$$

$$\left(2s(s+1)\right)\Theta_2 = T(s) \Rightarrow \boxed{\frac{\Theta_2}{T_s} = \frac{1}{2s(s+1)}}$$

✓