۳ فروردین یکشنبه ۲۷ رجب ۱۴۴۱ | 22 Mar./2020 Sunday

Hw6

محمد شرقی

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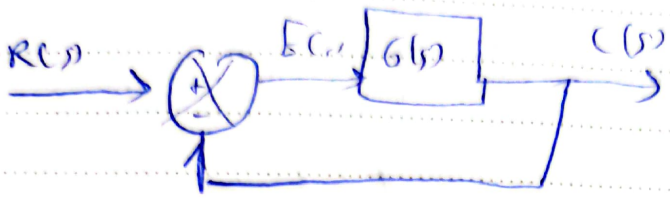
1)



2) $G(s) = \frac{K(s+3)(s+5)}{(s-2)(s-4)}$

$$\frac{\frac{K(s+3)(s+5)}{(s-2)(s-4)}}{1+\frac{K(s+3)(s+5)}{(s-2)(s-4)}} = \frac{K(s+3)(s+5)}{(s-2)(s-4)+K(s+3)(s+5)}$$

$$\frac{R(s)}{C(s)} = \frac{K(s^2+8s+15)}{(K+1)s^2+(8K-6)s+(15K+8)}$$

$$\frac{K(s^2+8s+15)}{(s^2-6s+8)+Ks^2+8Ks+15K}$$

s^2	$(K+1)$	$15K+8$	
s^1	$8K-6$	0	
s^0	$15K+8$	0	

$$\frac{(15K+8)(8K-6)}{(8K-5)} = 15K+8$$

حال دو حالت که داریم یا همه عبون اول مثبت یا همه منفی داریم:

همگی مثبت (+) :

$K+1>0 \Rightarrow K>-1$ ①

$8K-6>0 \Rightarrow K>\frac{3}{4}$ ②

$15K+8>0 \Rightarrow K>-\frac{8}{15}$ ③

① ∩ ② ∩ ③ $\Rightarrow K>\frac{3}{4}$

همگی منفی (−) :

$K+1<0 \Rightarrow K<-1$ ①

$8K-6<0 \Rightarrow K<\frac{3}{4}$ ②

$15K+8<0 \Rightarrow K<-\frac{8}{15}$ ③

① ∩ ② ∩ ③ $\Rightarrow K<-1$

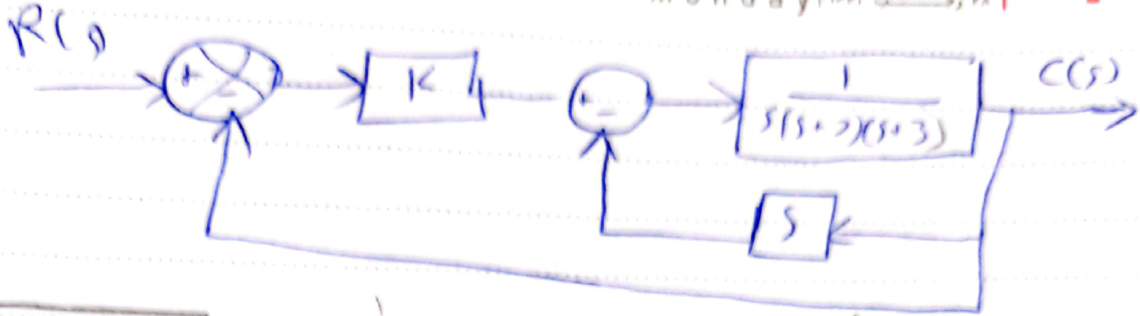
اجتماع ∪

K

$(K>\frac{3}{4}) \cup (K<-1)$

$K \in (-\infty,-1) \cup (\frac{3}{4},\infty)$

2)



$$\frac{\frac{1}{s(s+7)(s+3)}}{1+\frac{1}{(s+7)(s+3)}} = \frac{\frac{1}{s(s+7)(s+3)}}{\frac{(s+7)(s+3)+1}{(s+7)(s+3)}} = \frac{1}{s^3+10s^2+22s}$$

$$\frac{\frac{1}{s^3+10s^2+22s}}{1+\frac{K}{s^3+10s^2+22s}} = \frac{\frac{1}{s^3+10s^2+22s}}{\frac{s^3+10s^2+22s+K}{s^3+10s^2+22s}} = \frac{1}{s^3+10s^2+22s+K}$$

s^3	1	22	0
s^2	10	K	0
s^1	$-\frac{(K-220)}{10}$	0	$\Rightarrow K = 220$
s^0	K	0	$\Rightarrow K > 0$

الان دو حالت داریم **K = 0 یا 220** که باید بررسی کنیم

$10s^2 + K = 0 \Rightarrow s = \pm\sqrt{22}j$ $\Leftarrow K = 22$

$s^3+10s^2+22s+220 = (s-\sqrt{22}j)(s+\sqrt{22}j)(s-10)$

$f_1, f_2 = \pm\frac{\sqrt{22}}{2\pi}$

$(22 - \frac{15}{11})s = 0 \Rightarrow 225 = 0 \Rightarrow \boxed{s = 0}$ حالت دوم $K = 0$

$s^3 + 10s^2 + 225 = s(s^2 + 10s + 22) \Rightarrow s = \pm\sqrt{25 - 22} - 5$

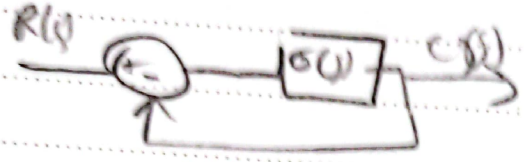
$= \pm\sqrt{3} - 5$

$= s(s + \sqrt{3} - 5)(s - \sqrt{3} - 5)$

چون ریشه حالت دوم حقیقی است پس نوسانی نیست ✗

K	220	✓
K	0	✗

4) $G(s) = \frac{5000}{s(s+75)}$



$$\Rightarrow \frac{C(s)}{R(s)} = \frac{\frac{5000}{s(s+75)}}{\frac{s(s+75)+5000}{s(s+75)}} = \frac{5000}{s(s+75)+5000}$$

$$\begin{cases} w_n^2 = 5000 \Rightarrow w_n = 50\sqrt{2} \\ 2\zeta w_n = 75 \Rightarrow \zeta = \frac{75}{2 \times 50\sqrt{2}} = \frac{3\sqrt{2}}{8} \end{cases}$$

سیستم زیرمیرا $\Rightarrow 0 < \zeta < 1$

$$M_p = e^{\left(\frac{-\zeta\pi}{\sqrt{1-\zeta^2}}\right)} = e^{\left(\frac{-\frac{3\sqrt{2}}{8}\pi}{\sqrt{\frac{64}{64}-\frac{18}{64}}}\right)} = e^{\left(\frac{-3\sqrt{2}\pi}{\sqrt{46}}\right)} \simeq \boxed{0.14 = M_p}$$

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$$t_s = \frac{4}{\zeta w_n} = \frac{4 \times 2}{75} = \frac{8}{75} \Rightarrow \boxed{t_s \simeq 0.107_s}$$

5) $R(t) = 5u(t) \Rightarrow R(s) = \frac{5}{s}$ $\qquad \frac{E(s)}{R(s)} = \frac{1}{1+G(s)}$

$$e_{ss} = \lim_{s \to 0} sE(s) = \lim_{s \to 0} s\left(\frac{5}{s}\right)\left(\frac{1}{1+\frac{5000}{s(s+75)}}\right) = \lim_{s \to 0} \frac{5s(s+75)}{s(s+75)+5000}$$

$\boxed{e_{ss} = 0}$ صحیح

ب) $5t\,u(t) = 5\left(\frac{1}{s^2}\right) = R(s)$

$$e_{ss} = \lim_{s \to 0} s\left(\frac{5}{s^2}\right)\left(\frac{1}{1+\frac{5000}{s(s+75)}}\right) = \lim_{s \to 0} \frac{5s(s+75)}{s\left(s(s+75)+5000\right)}$$

$$= \lim_{s \to 0} \frac{5 \times 75}{5000} = \boxed{0.075 = e_{ss}}$$ پاسخ

d) $\underbrace{5+^2 W(t)}_{a} = R(t) \Rightarrow R(s) = \frac{1}{s^3}$

$$\lim_{s\to 0} s\left(\frac{10}{s^2}\right)\left(\frac{1}{1+\frac{5000}{s(75+s)}}\right) = \lim_{s\to 0} \frac{10}{s}\left(\frac{s(75+s)}{s(75+s)+5000}\right) = \lim_{s\to 0} \frac{10(s+75)}{s(s(75+s)+5000)}$$

$$= \infty \Rightarrow \boxed{e_{ss} = \infty}$$