AI ML

AI = ML

AI $\subseteq$ ML

✓

AI $\supseteq$ ML

Data

$$x = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_d \end{bmatrix}$$

$$\begin{bmatrix} \text{قد} \\ \text{وزن} \end{bmatrix}$$

?

$$x \rightarrow \boxed{f} \rightarrow y$$

جنسیت $\left\{ \begin{array}{l} \text{مرد} \\ \text{زن} \end{array} \right.$

input
observation
feature vector

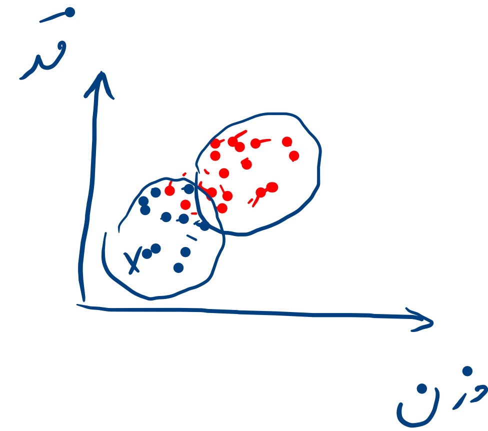
output
target value
label

x_1 y_1

x_2 y_2

$\vdots$

x_n y_n



$x \rightarrow \boxed{f} \rightarrow y$

$D = \{(x_1, y_1), \ldots, (x_n, y_n)\}$

Find $f = ?$

$y = f(x)$

y → پیوسته → Regression

y → گسسته → Classification

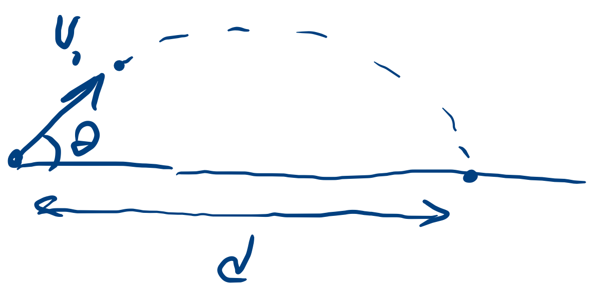
y ها موجود باشد → Supervised

y ها نباشد → unsupervised

یک بخشی از y ها موجود باشند → Semi-supervised

Bayes Classifier

1. we must have Data
2. There exists a pattern between input (x) and output (y), f
3. f is unknown.



$x = \begin{bmatrix} v_0 \\ \theta \end{bmatrix}$ $\quad y = d$

$x = \begin{bmatrix}170\\70\end{bmatrix}$ $\qquad$ $x = \begin{bmatrix}180\\90\end{bmatrix}$

$$x \rightarrow \boxed{g} \rightarrow \boxed{\sigma} \rightarrow \hat{y}$$

threshold

$$x \longrightarrow P(y|x) \begin{cases} P(y=1|x) = 0.7 \quad \checkmark \\ P(y=0|x) = 0.3 \end{cases}$$

$$P(y|x) = \frac{P(x,y)}{P(x)} = \frac{P(x|y)\,P(y)}{P(x)}$$

likelihood

پیشین

Prior

evidence

پسین

Posterior

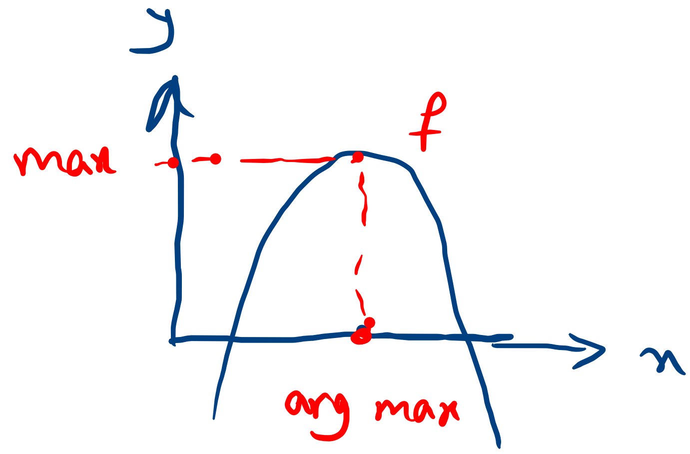
$$P(y = \text{مرد}) = 0.8$$

$$P(y = \text{زن}) = 0.2$$

$$\begin{bmatrix} ? \\ \cdot \\ ? \\ \cdot \end{bmatrix}$$

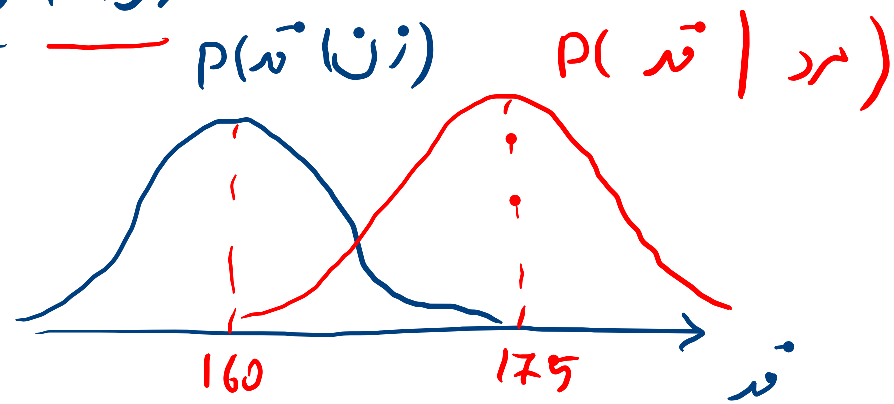
$$P\left(y = \text{مرد} \;\middle|\; \begin{bmatrix} ۱۷۰ \\ ۷۰ \end{bmatrix}\right)$$

$$P(y|x) = \frac{P(x|y)\,P(y)}{P(x)}$$



$$y^* = \arg\max_{y} \underline{P(y|x^*)}$$

$$\Rightarrow y^* = \arg\max_{y} \underline{P(x|y)}\,\underline{P(y)}$$



$$P(y=1 \mid x) \underset{0}{\overset{1}{\gtrless}} P(y=0 \mid x)$$

$$\underbrace{\frac{P(x \mid y=1)\,P(y=1)}{P(x \mid y=0)\,P(y=0)}}_{g(x)} \underset{0}{\overset{1}{\gtrless}} \underbrace{1}_{\theta}$$

$$\overset{g(x)}{\frac{P(x \mid y=1)}{P(x \mid y=0)}} \underset{0}{\overset{1}{\gtrless}} \overbrace{\frac{P(y=0)}{P(y=1)}}^{\theta}$$

likelihood Ratio

Bayes classifier is Optimal !

$$P(y|x) \propto P(x|y)\,P(y)$$

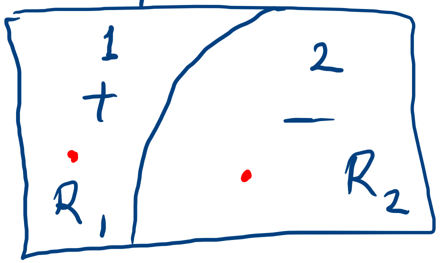
عکس

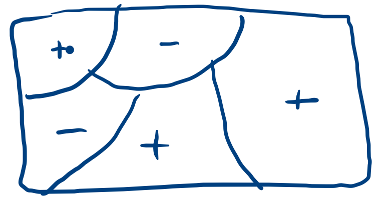
گربه

سگ

$$P(x \in R_2, y=1) = \int_{R_2} p(x,y)\, dx$$

input space





y

$\hat{y}$

$$\underline{\underline{P(x)}} = \int p(x,y)\, dy$$

$$P(\text{error}) = \underline{P(y \neq \hat{y})} = P(y=1, \hat{y}=2) + P(y=2, \hat{y}=1)$$

$$= P(y=1, x \in R_2) + P(y=2, x \in R_1)$$

$$+ P(y=1, x \in R_1) - P(y=1, x \in R_1)$$

$$= P(y=1) + \int_{R_1} p(x, y=2)\, dx - \int_{R_1} p(x, y=1)\, dx$$

$$P(\text{error}) = P(y=1) + \int_{R_1} \Big[\underbrace{P(y=2, x)}_{P(x)\,P(y=2|x)} - \underbrace{P(y=1, x)}_{P(x)\,P(y=1|x)}\Big]\,dx$$

$$= P(y=1) - \int_{R_1} P(x)\,\Big[P(y=1|x) \overset{>}{-} P(y=2|x)\Big]\,dx$$

$$\boxed{y \neq \hat{y}}$$

$y, \hat{y} \in \{0, 1\}$

Risk

λ_{11} λ_{12}

λ_{21} λ_{22}

$$R = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}_{k \times k}$$

-

$$\begin{bmatrix} 0 & 10 \\ 1 & 0 \end{bmatrix}$$

✓ Generative $\longrightarrow \boxed{P(x, y)}$ $\rightarrow$ $P(x)$, $P(y)$, $\boxed{P(y|x)}$ ✓, $P(x|y)$

discriminative $\longrightarrow \boxed{P(y|x)}$