Machine learning

Boosting

Can we make dumb learners smart?

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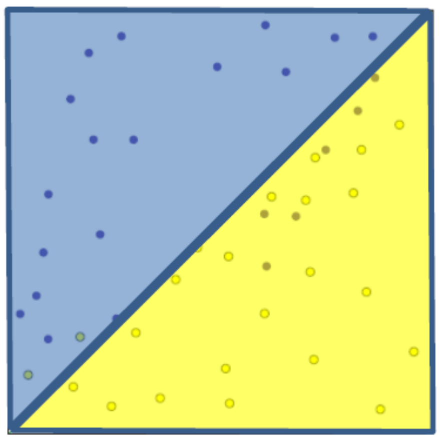
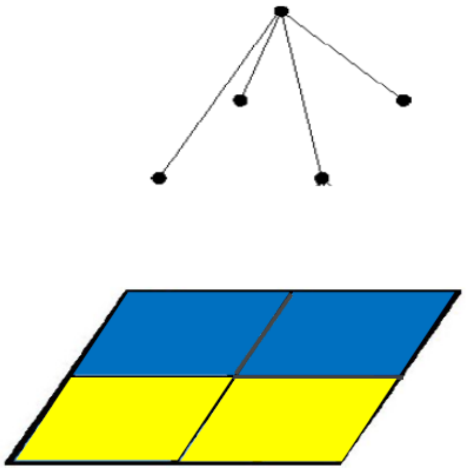
Slides are mainly adopted form cmu Aarti course

Fighting the bias-variance tradeoff

- **Simple (a.k.a. weak) learners** e.g., naïve Bayes, logistic regression, decision stumps (or shallow decision trees)





Are good ☺ - don't usually overfit

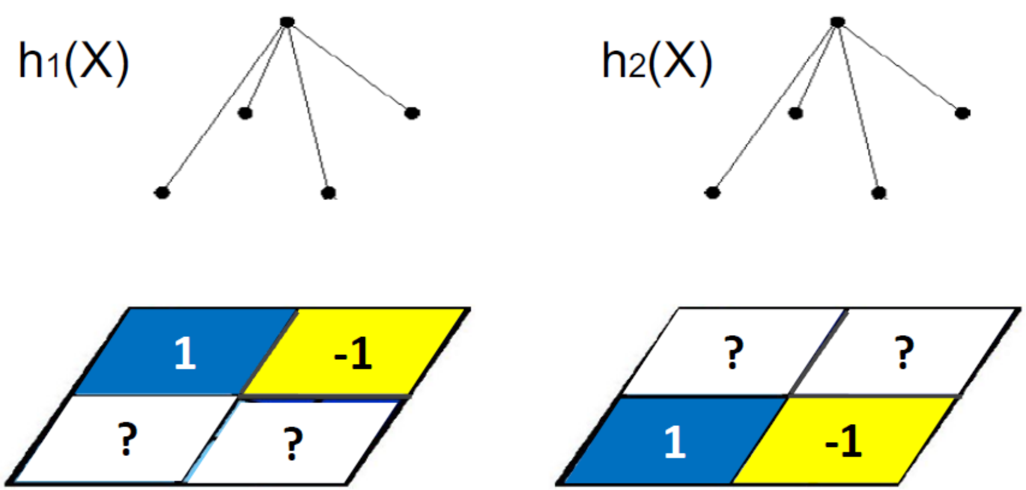
Are bad ☹ - can't solve hard learning problems

- **Can we make weak learners always good???**
  - **No!!!** **But often yes…**

Voting (Ensemble Methods)

- Instead of learning a single (weak) classifier, learn **many weak classifiers** that are **good at different parts of the input space**

- **Output class:** (Weighted) vote of each classifier
  - Classifiers that are most "sure" will vote with more conviction
  - Classifiers will be most "sure" about a particular part of the space
  - On average, do better than single classifier!



$H: X \rightarrow Y\ (-1,1)$

$H(X) = h_1(X)+h_2(X)$

$H(X) = \text{sign}(\sum_t \alpha_t h_t(X))$

weights

Voting (Ensemble Methods)

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  - On average, do better than single classifier!
- **But how do you ???**
  - force classifiers h_t to learn about different parts of the input space?
  - weigh the votes of different classifiers? α_t

Boosting [Schapire'89]

- **Idea:** given a weak learner, run it multiple times on (reweighted) training data, then let learned classifiers vote
- On each iteration *t*:
  - weight $D_t(i)$ for each training example i, based on how incorrectly it was classified
  - Learn a weak hypothesis – h_t
  - A weight for this hypothesis – α_t
- Final classifier: $\boxed{H(X) = \text{sign}(\sum \alpha_t \, h_t(X))}$
- **Practically useful**
- **Theoretically interesting**

Learning from weighted data

- **Consider a weighted dataset**
  - D(i) – weight of *i* th training example ($\mathbf{x}^i$, y^i)
  - Interpretations:
    - *i* th training example counts as D(i) examples
    - If I were to "resample" data, I would get more samples of "heavier" data points

- **Now, in all calculations, whenever used, *i* th training example counts as D(i) "examples"**
  - e.g., in MLE redefine *Count(Y=y)* to be weighted count

Unweighted data

$$Count(Y=y) = \sum_{i=1}^{m} \mathbf{1}(Y^i=y)$$

Weights D(i)

$$Count(Y=y) = \sum_{i=1}^{m} D(i)\mathbf{1}(Y^i=y)$$

AdaBoost [Freund & Schapire'95]

Given: $(x_1, y_1), \ldots, (x_m, y_m)$ where $x_i \in X$, $y_i \in Y = \{-1, +1\}$
Initialize $D_1(i) = 1/m$. **Initially equal weights**
For $t = 1, \ldots, T$:

- Train weak learner using distribution D_t. **Naïve bayes, decision stump**
- Get weak classifier $h_t : X \to \mathbb{R}$.
- Choose $\alpha_t \in \mathbb{R}$. **Magic (+ve)**
- Update:

$$D_{t+1}(i) = \frac{D_t(i)}{Z_t} \begin{cases} e^{-\alpha_t} & \text{if } y_i = h_t(x_i) \\ e^{\alpha_t} & \text{if } y_i \neq h_t(x_i) \end{cases}$$

$$= \frac{D_t(i) \exp(-\alpha_t y_i h_t(x_i))}{Z_t}$$

Increase weight if wrong on pt i
$y_i h_t(x_i) = -1 < 0$

where Z_t is a normalization factor

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where Z_t is a normalization factor

$$Z_t = \sum_{i=1}^{m} D_t(i) \exp(-\alpha_t y_i h_t(x_i))$$

Weights for all pts must sum to 1
$\sum_t D_{t+1}(i) = 1$

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Output the final classifier:

$$H(x) = \text{sign}\left(\sum_{t=1}^{T} \alpha_t h_t(x)\right).$$

What α_t to choose for hypothesis h_t?

Weight Update Rule: $$D_{t+1}(i) = \frac{D_t(i)\exp(-\alpha_t y_i h_t(x_i))}{Z_t}$$

$$\boxed{\alpha_t = \frac{1}{2}\ln\left(\frac{1-\epsilon_t}{\epsilon_t}\right)}$$

[Freund & Schapire'95]

Weighted training error

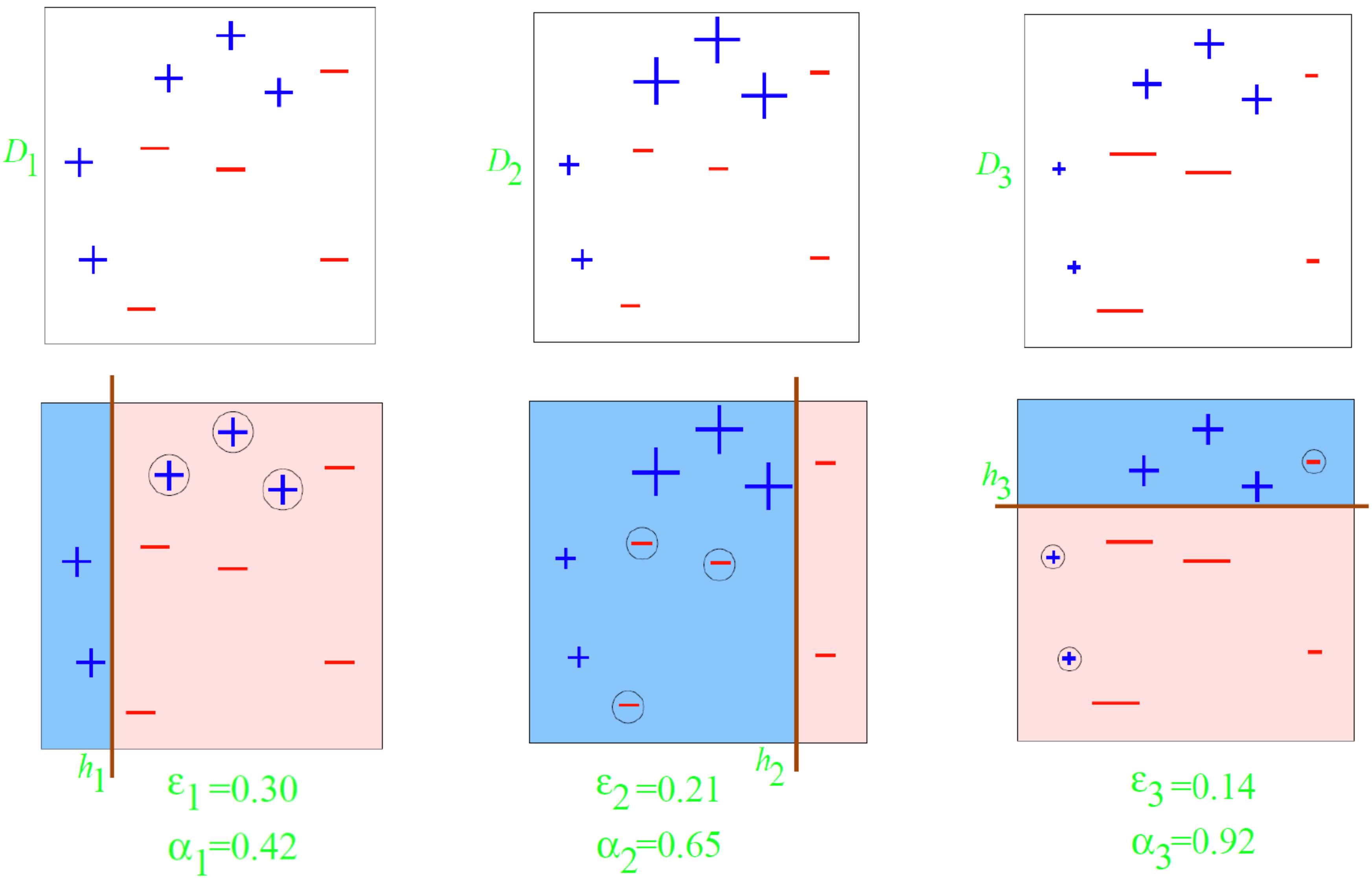
$$\epsilon_t = P_{i\sim D_t(i)}[h_t(\mathbf{x}^i) \neq y^i] = \sum_{i=1}^{m} D_t(i)\underbrace{\delta(h_t(x_i) \neq y_i)}_{\text{Does } h_t \text{ get } i^{th} \text{ point wrong}}$$

$\varepsilon_t = 0$ if h_t perfectly classifies all weighted data pts $\quad \alpha_t = \infty$

$\varepsilon_t = 1$ if h_t perfectly wrong => $-h_t$ perfectly right $\quad \alpha_t = -\infty$

$\varepsilon_t = 0.5 \quad \alpha_t = 0$

Boosting Example (Decision Stumps)



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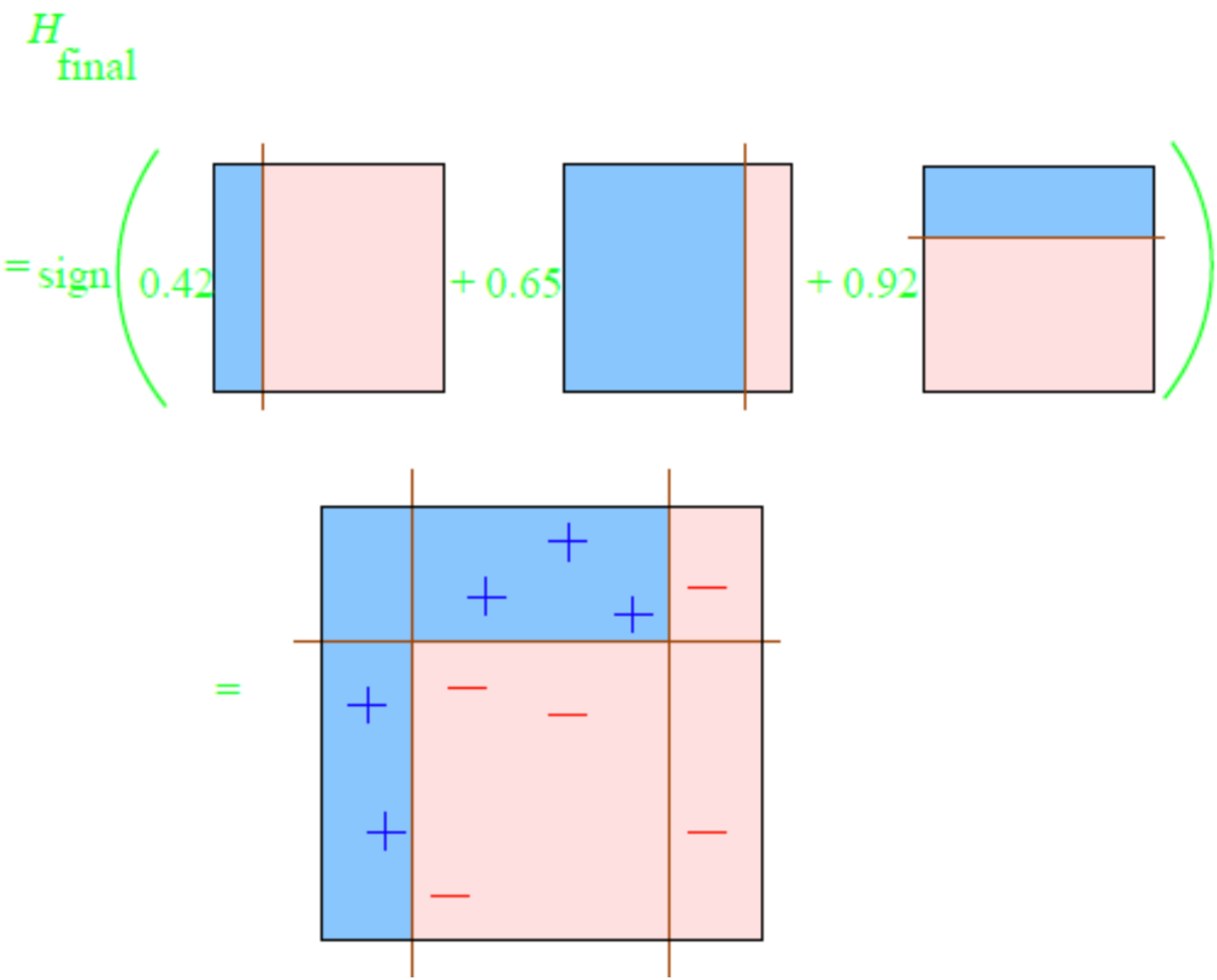
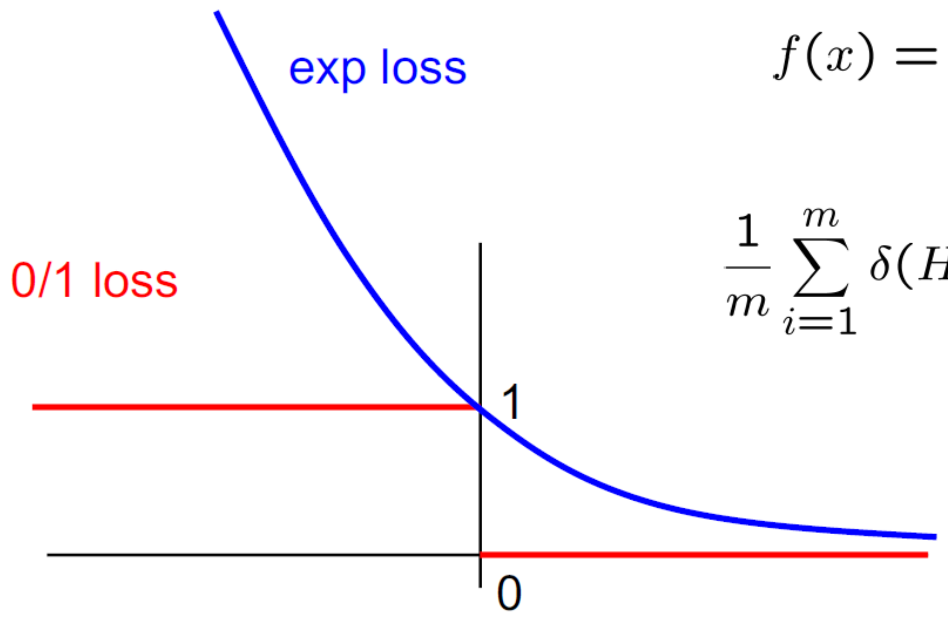


Analysis for Boosting

- Choice of α_t and hypothesis h_t obtained by coordinate descent on exp loss (convex upper bound on 0/1 loss)



$$f(x) = \sum_t \alpha_t h_t(x); H(x) = sign(f(x))$$

$$\frac{1}{m}\sum_{i=1}^{m} \delta(H(x_i) \neq y_i) \leq \frac{1}{m}\sum_{i=1}^{m} \exp(-y_i f(x_i))$$

0/1 loss exp loss

Analysis for Boosting

Analysis reveals:

- If each weak learner h_t is slightly better than random guessing ($\varepsilon_t < 0.5$), then training error of AdaBoost decays exponentially fast in number of rounds T.

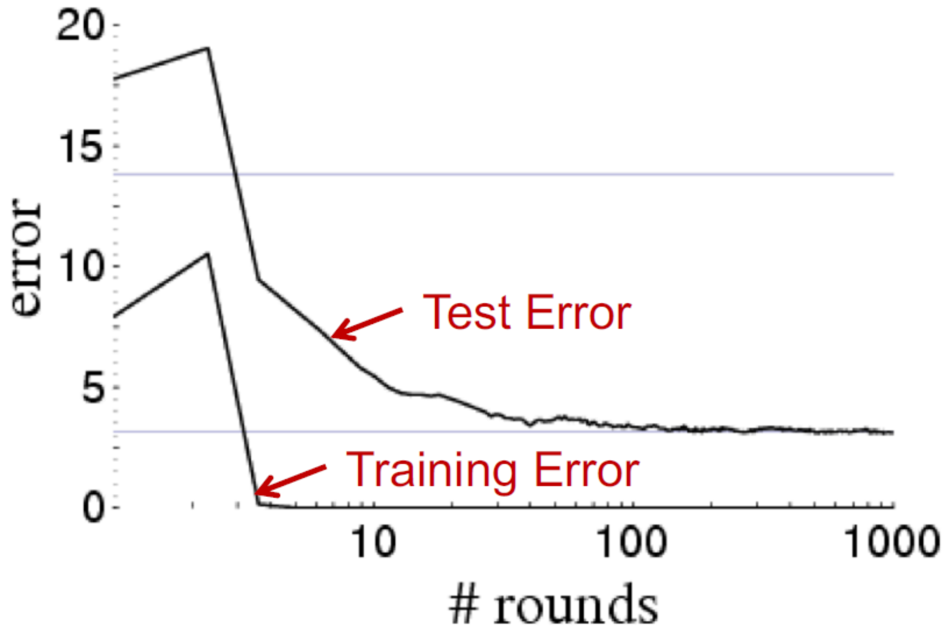
$$\frac{1}{m}\sum_{i=1}^{m}\delta(H(x_i) \neq y_i) \leq \exp\left(-2\sum_{t=1}^{T}(1/2-\epsilon_t)^2\right)$$

Training Error

What about test error?

Boosting results – Digit recognition

[Schapire, 1989]



- Boosting often,
  - Robust to overfitting
  - Test set error decreases even after training error is zero

- If margin between classes is large, subsequent weak learners agree and hence more rounds does not necessarily imply that final classifier is getting more complex.

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Boosting can overfit if margin between classes is too small (high label noise) or weak learners are too complex.

Boosting and Logistic Regression

Logistic regression assumes:

$$P(Y = 1|X) = \frac{1}{1 + \exp(f(x))} \qquad f(x) = w_0 + \sum_j w_j x_j$$

And tries to maximize data likelihood:

$$P(\mathcal{D}|f) \overset{\text{iid}}{=} \prod_{i=1}^{m} \frac{1}{1 + \exp(-y_i f(x_i))}$$

Equivalent to minimizing log loss

$$-\log P(\mathcal{D}|f) = \sum_{i=1}^{m} \ln(1 + \exp(-y_i f(x_i)))$$

Boosting and Logistic Regression

Logistic regression equivalent to minimizing log loss

$$\sum_{i=1}^{m} \ln(1 + \exp(-y_i f(x_i)))$$

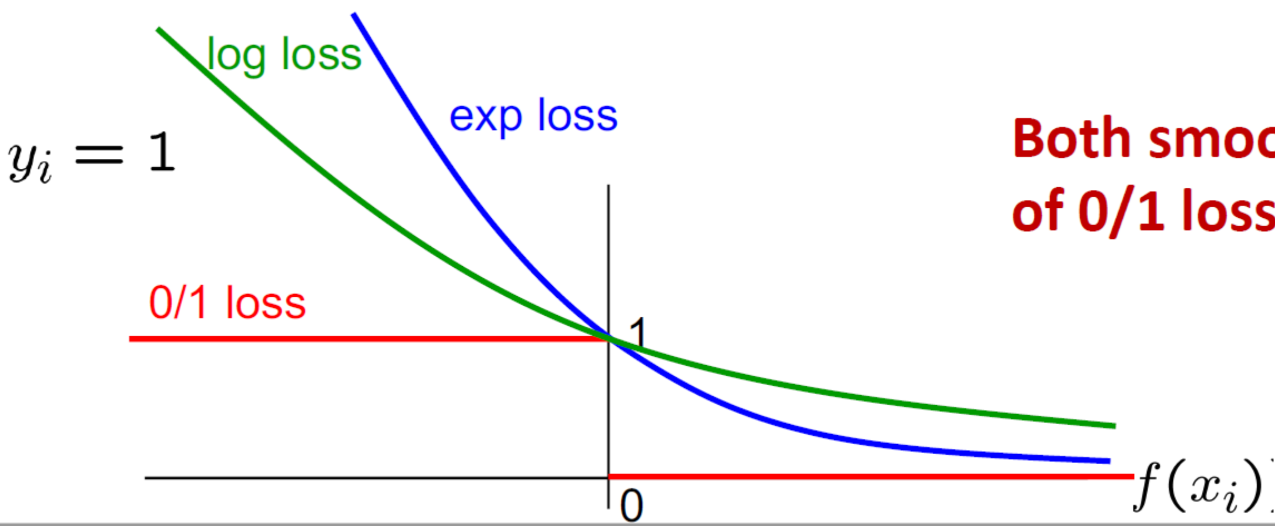
$$f(x) = w_0 + \sum_j w_j x_j$$

Boosting minimizes similar loss function!!

$$\frac{1}{m} \sum_{i=1}^{m} \exp(-y_i f(x_i))$$

$$f(x) = \sum_t \alpha_t h_t(x)$$

Weighted average of weak learners



Both smooth approximations of 0/1 loss!

Boosting and Logistic Regression

Logistic regression:

- Minimize log loss

$$\sum_{i=1}^{m} \ln(1 + \exp(-y_i f(x_i)))$$

- Define

$$f(x) = \sum_j w_j x_j$$

where x_j predefined features

(linear classifier)

- Jointly optimize over all weights $w_0, w_1, w_2 \ldots$

Boosting:

- Minimize exp loss

$$\sum_{i=1}^{m} \exp(-y_i f(x_i))$$

- Define

$$f(x) = \sum_t \alpha_t h_t(x)$$

where $h_t(x)$ defined dynamically to fit data

(not a linear classifier)

- Weights α_t learned per iteration t incrementally

Hard & Soft Decision

Weighted average of weak learners $$f(x) = \sum_t \alpha_t h_t(x)$$

Hard Decision/Predicted label: $$H(x) = sign(f(x))$$

Soft Decision:
(based on analogy with
logistic regression)

$$P(Y = 1|X) = \frac{1}{1 + \exp(f(x))}$$

Boosting Summary

- Combine weak classifiers to obtain very strong classifier
  - Weak classifier – slightly better than random on training data
  - Resulting very strong classifier – can eventually provide zero training error
- AdaBoost algorithm
- Boosting v. Logistic Regression
  - Similar loss functions
  - Single optimization (LR) v. Incrementally improving classification (B)
- Most popular application of Boosting:
  - Boosted decision stumps!
  - Very simple to implement, very effective classifier