

# Ensemble Learning

10  $\rightarrow$  55 %

$x$

$h_1 \rightarrow \pm 1$

$\vdots$

$h_k \rightarrow \pm 1$

$h_1 \rightarrow +$  0.99 %

$h_2 \rightarrow -$  0.51 %

$h_3 \rightarrow -$  0.51 %

$$F(x) = \sum_{i=1}^K h_i(x)$$

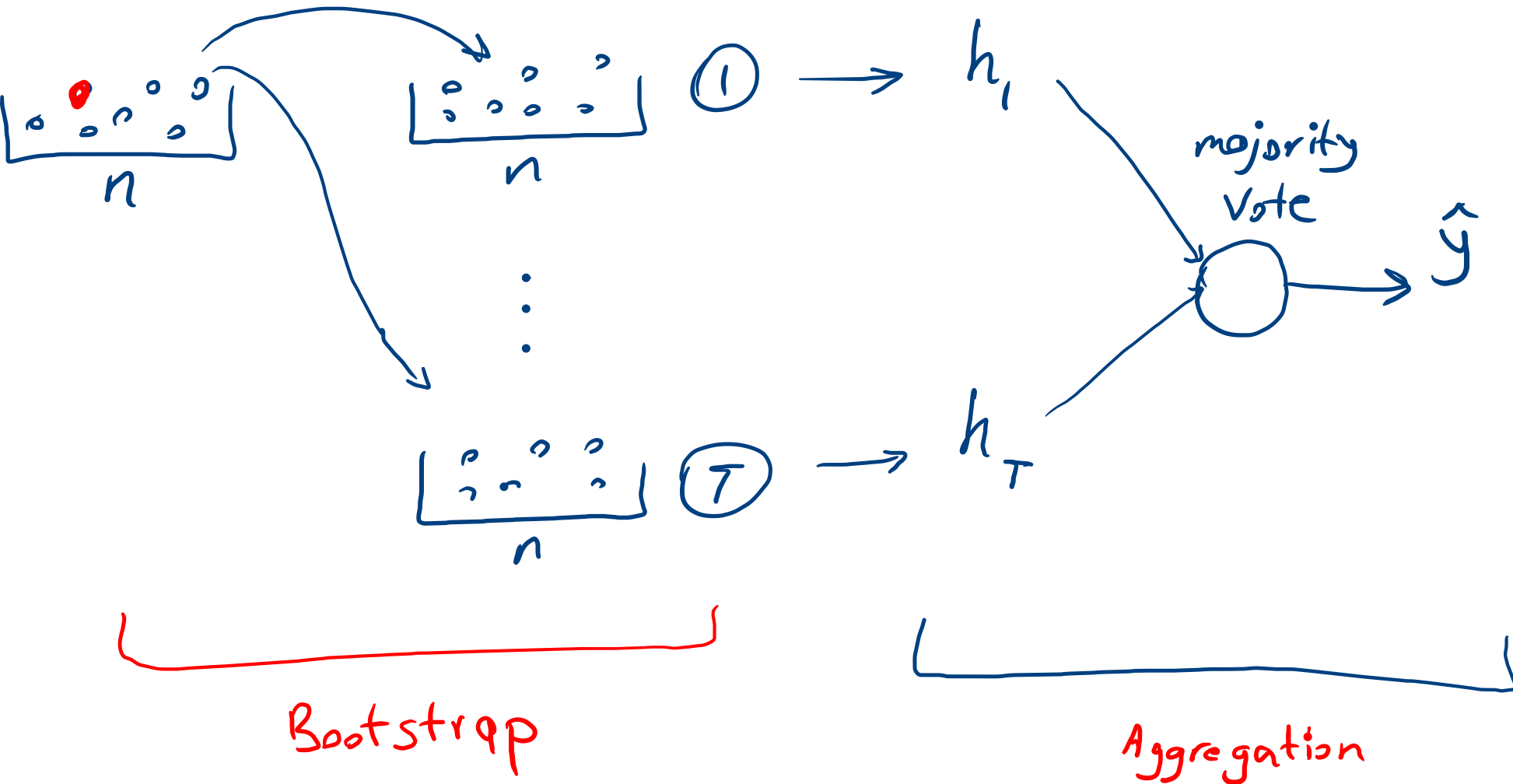
$$\text{sign}(F(x))$$

{ Bagging: Bootstrap + Aggregation

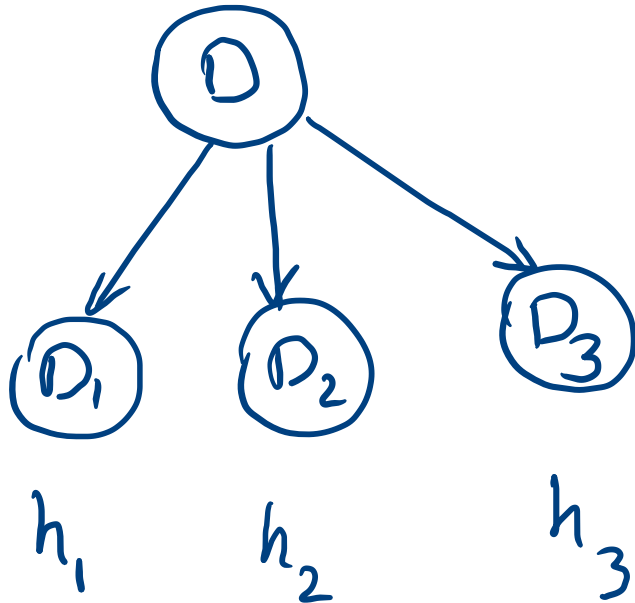
{ Boosting → Adaptive Boosting    AdaBoost

# Bagging

$$1 - \frac{1}{e} = 0.66$$



# Boosting



# AdaBoost

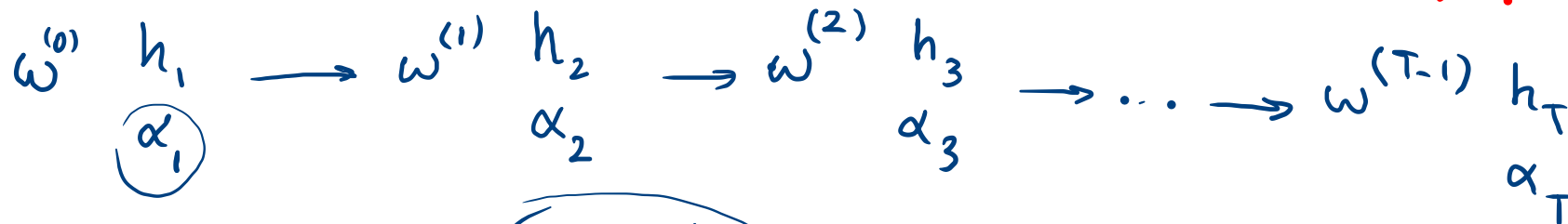
$h_1 \dots h_T$

$$F_T(x) = \sum_{t=1}^T \alpha_t h_t(x)$$

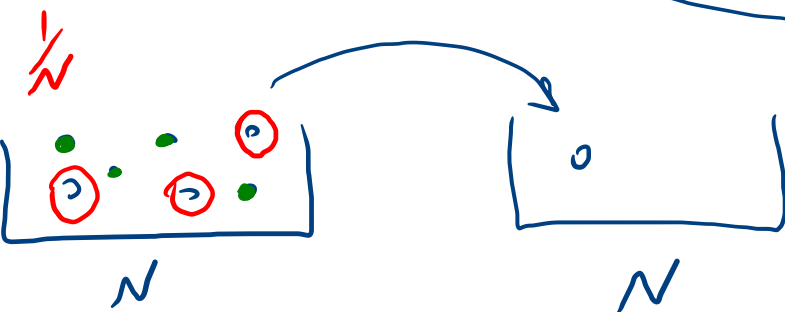
Base Classifier  
Weak Learner

$\alpha_t = ? \quad t=1, \dots, T$

$w_i = ?$



$$\alpha_1 h_1 + \alpha_2 h_2$$



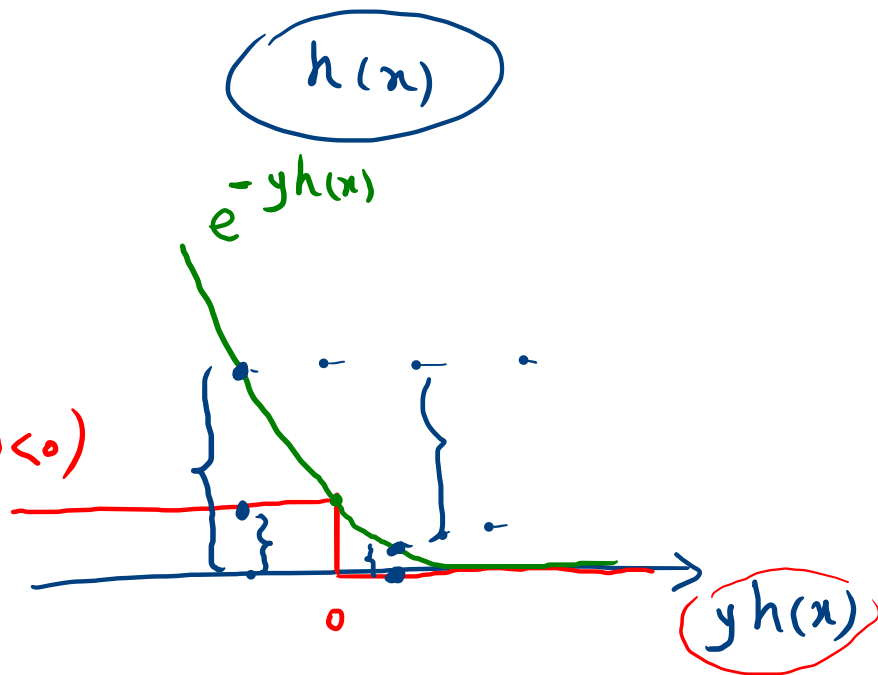
$\text{sign}(F_T(x))$

$$E = \sum_{i=1}^n I(\underline{y_i} \underline{h(x_i)} < 0)$$

$$\leftarrow (E) = \sum_{i=1}^n e^{-y_i h(x_i)}$$

$\alpha_t$

$I(yh(x) < 0)$



$h(x) \in \{-1, 1\}$

$$F_T(x) = \sum_{t=1}^T \alpha_t h_t(x) = F_{T-1}(x) + \alpha_T h_T(x)$$

$h_T$   
 $\alpha_T$

$$E = \sum_{i=1}^n e^{-y_i F_T(x_i)} = \sum_{i=1}^n \underbrace{e^{-y_i F_{T-1}(x_i)}}_{w_i^{(T-1)}} e^{-y_i \alpha_T h_T(x_i)}$$

$$= \sum_{i=1}^n w_i^{(T-1)} e^{-y_i \alpha_T h_T(x_i)} = \sum_{i: y_i \neq h_T(x_i)} w_i^{(T-1)} e^{\alpha_T}$$

$$+ \sum_{i: y_i = h_T(x_i)} w_i^{(T-1)} e^{-\alpha_T}$$

$$\frac{dE}{d\alpha_T} = \underbrace{e^{\alpha_T}}_{\text{C}} \sum_{i \in \mathcal{M}} w_i^{(T-1)} - e^{-\alpha_T} \sum_{i \in \mathcal{C}} w_i^{(T-1)} = 0$$

$$\Rightarrow \alpha_T = \frac{1}{2} \ln \frac{\sum_{i \in \mathcal{C}} w_i^{(T-1)}}{\sum_{i \in \mathcal{M}} w_i^{(T-1)}}$$

$$\epsilon_T = \frac{\sum_{i \in M} w_i^{(T-1)}}{\sum_{i=1}^n w_i^{(T-1)}}$$

$$h_T \quad 0 \leq \epsilon_T \leq 1$$

$$\alpha_T = \frac{1}{2} \ln \frac{1 - \epsilon_T}{\epsilon_T}$$

$$\epsilon_T \Rightarrow \frac{1}{2}$$

$$\epsilon_T = 0$$

$$\epsilon_T = 1$$



$$\epsilon_T = \frac{\sum_{i \in M} w_i^{(T-1)}}{\sum_{i=1}^n w_i^{(T-1)}}$$

$$h_T \quad 0 \leq \epsilon_T \leq 1$$

$$\alpha_T = \frac{1}{2} \ln \frac{1 - \epsilon_T}{\epsilon_T}$$

$$\begin{aligned} & \epsilon_T \Rightarrow \frac{1}{2} \\ & \epsilon_T = 0 \\ & \epsilon_T = 1 \end{aligned}$$