

Remember this?

ANOVA

Continuous (numerical)

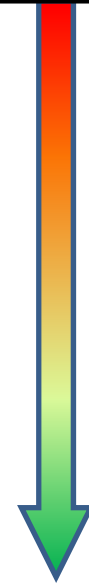
Samples are chosen randomly

Normal distribution for each group

Homoscedasticity

Degrees of freedom = $n-1 \geq 2$

Robustness

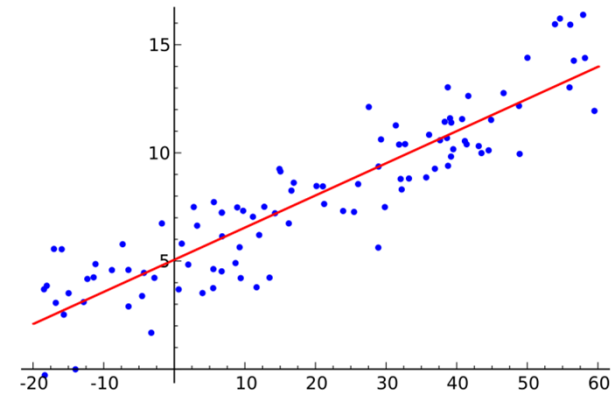


-
- Preferred because more powerful finding differences
 - Data needs to be Normal to fit the predicted distribution
 - One of many analysis based on the general linear model

GENERAL LINEAR MODELS

Simple analysis based on linear regression

- can **not** handle **not** continuous data
- can **not** handle **not** Normal data
- Can only handle a few variables at the same time



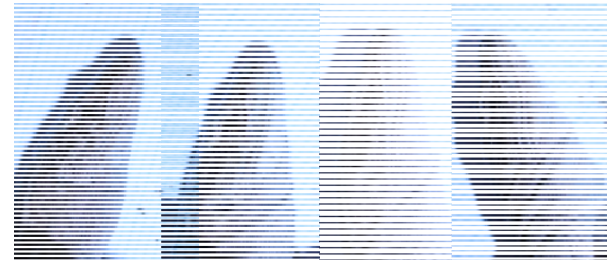
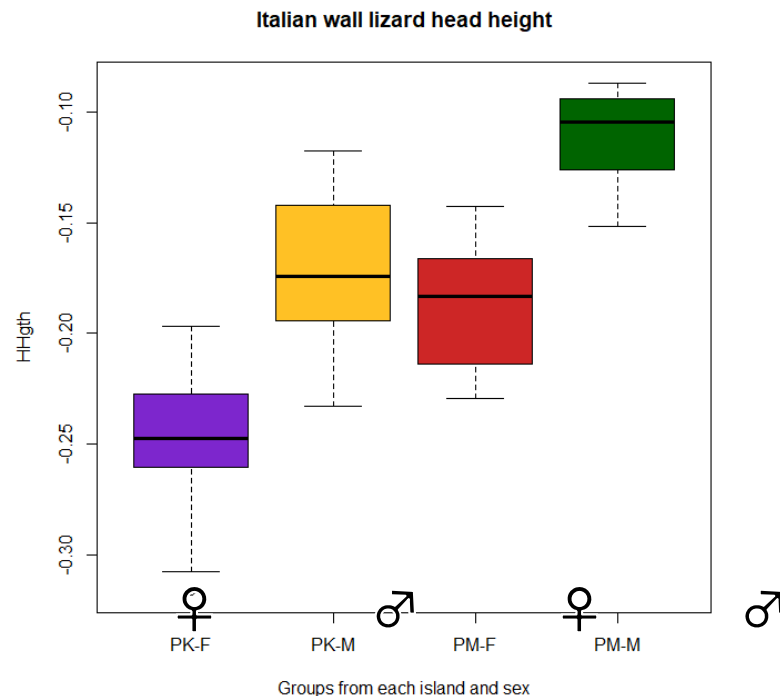
ANOVA, ANCOVA, MANOVA, MANCOVA, t-test, F-test...

MIND SETUP FOR LINEAR MODELS

Categorical variable = **explanatory variable**: Group (Sex+Population)

Continuous variable = **response variable**: Head Height

Are there significant differences in the response variable between groups?



MIND SETUP FOR LINEAR MODELS

Until now the question was:

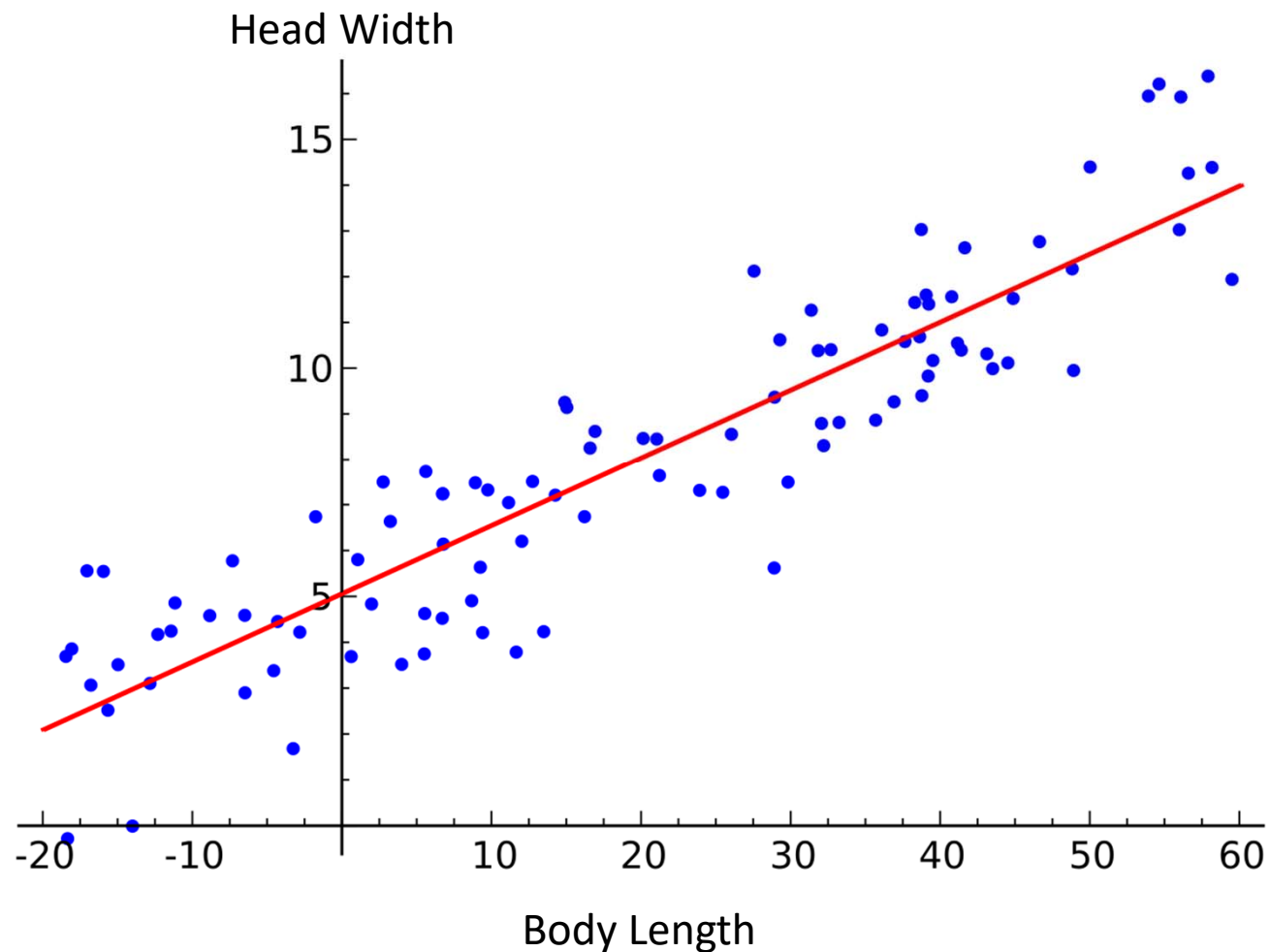
Are there significant differences in the response variable (Head Height) between groups?

How linear models work:

- 1.- Our data is adjusted to a linear model $y = ax + b$
- 2.- The model **PREDICTS** the expected values of Head Width according to another variable (Group, Body Length, etc.)
- 3.- Then **compares expected** values with real values

MIND SETUP FOR LINEAR MODELS

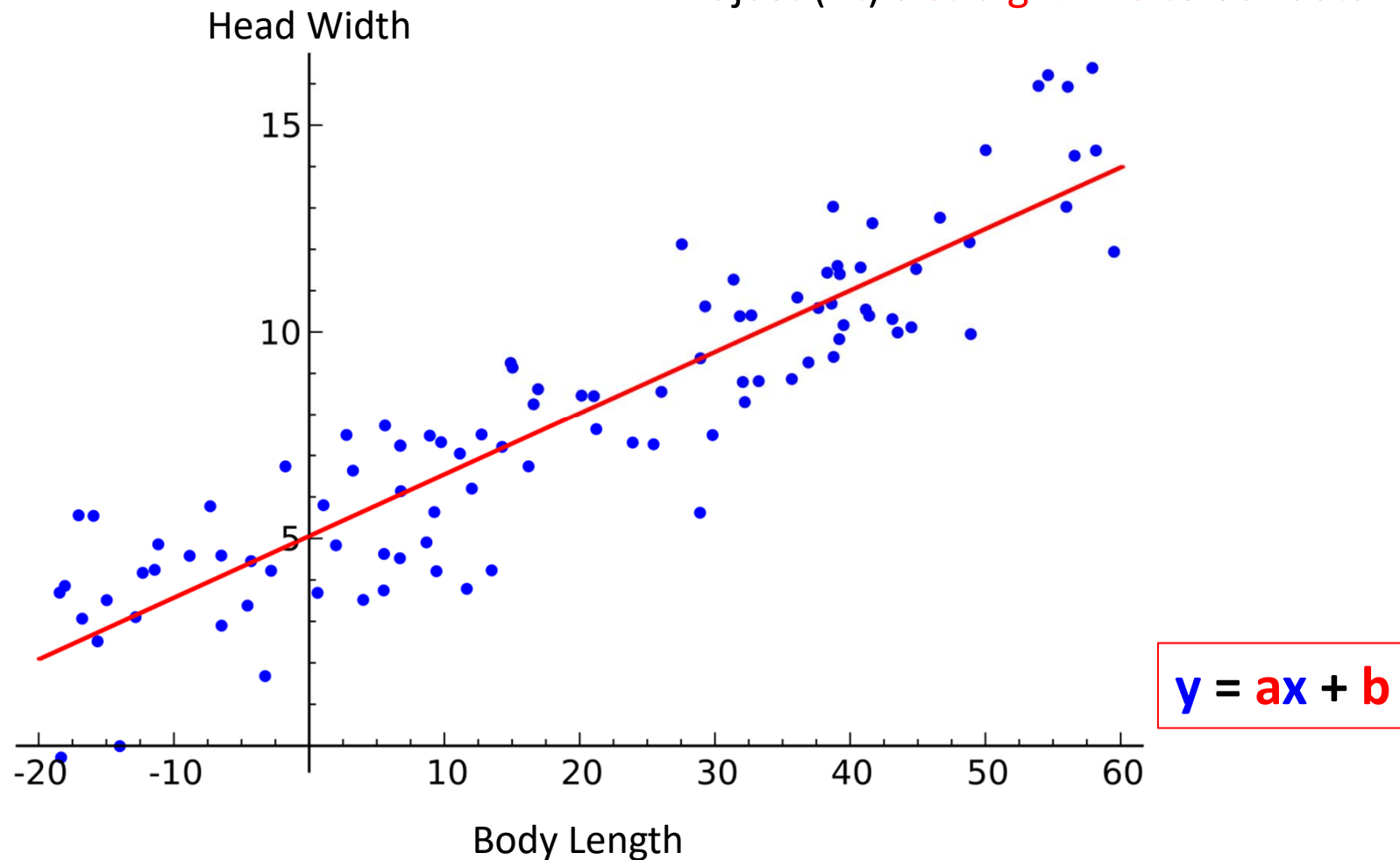
How do linear models make predictions?



LINEAR REGRESSION

1. Data is adjusted to linear regression model:

Adjust (fit) a **straight line** to our **data**

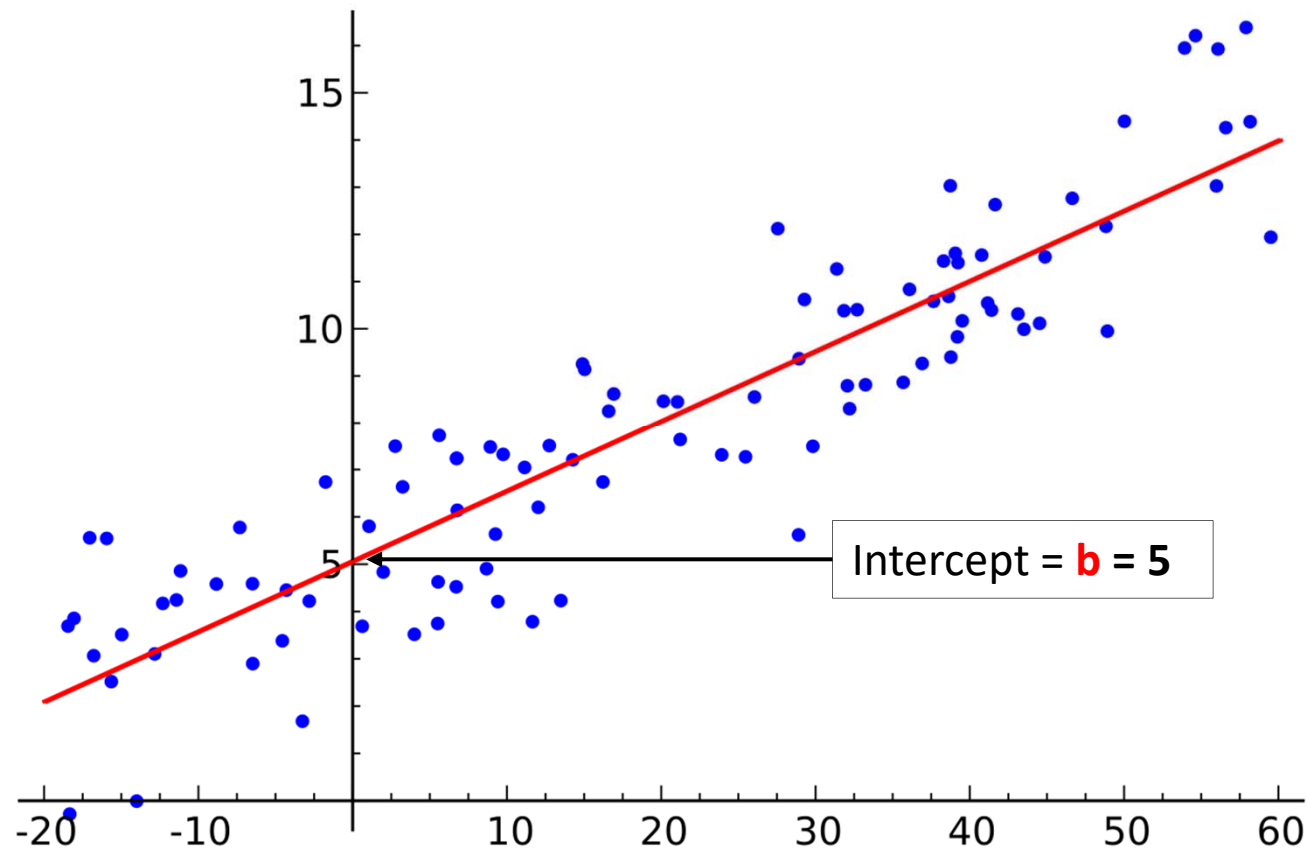


LINEAR REGRESSION

2. Calculate expected values:

Predict value of y from x (we need a and b)

$$y = ax + b$$



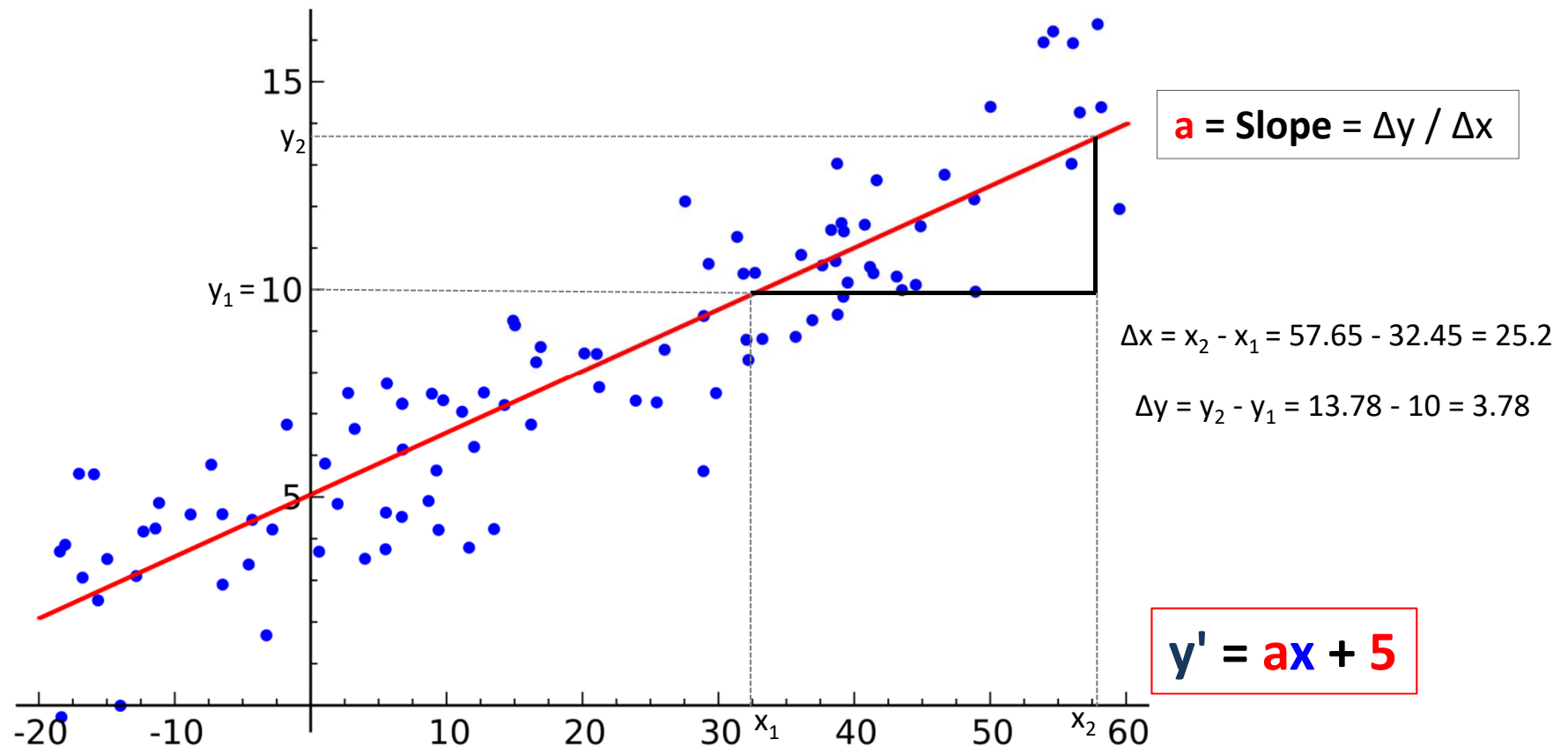
$$y' = ax + 5$$

LINEAR REGRESSION

2. Calculate expected values:

Predict value of y from x (we have b , still need a)

$$y = ax + b$$

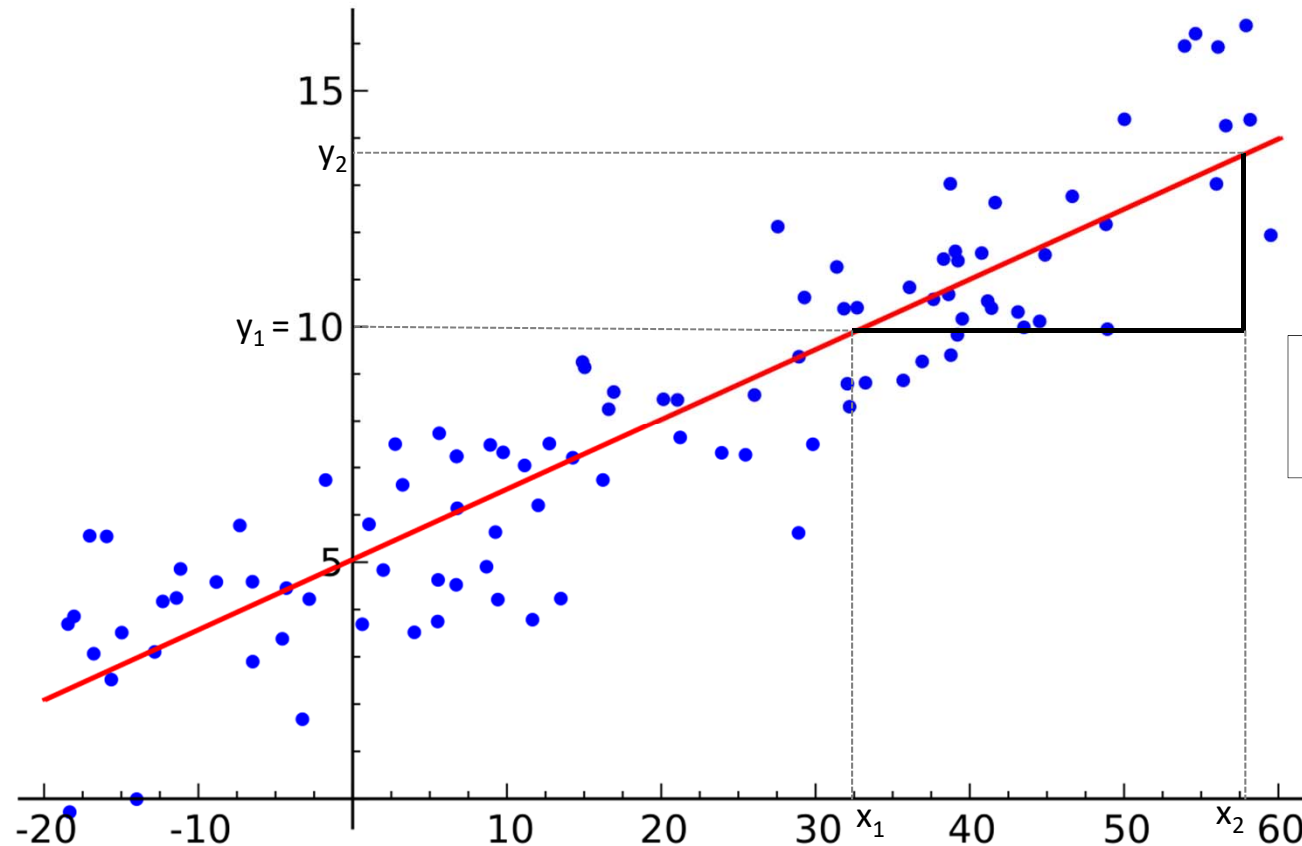


LINEAR REGRESSION

2. Calculate expected values:

Predict value of y from x (we have b , still need a)

$$y = ax + b$$



$$a = \text{Slope} = \Delta y / \Delta x$$

$$\text{Slope} = \Delta y / \Delta x = 3.78 / 25.2 = a = 0.15$$

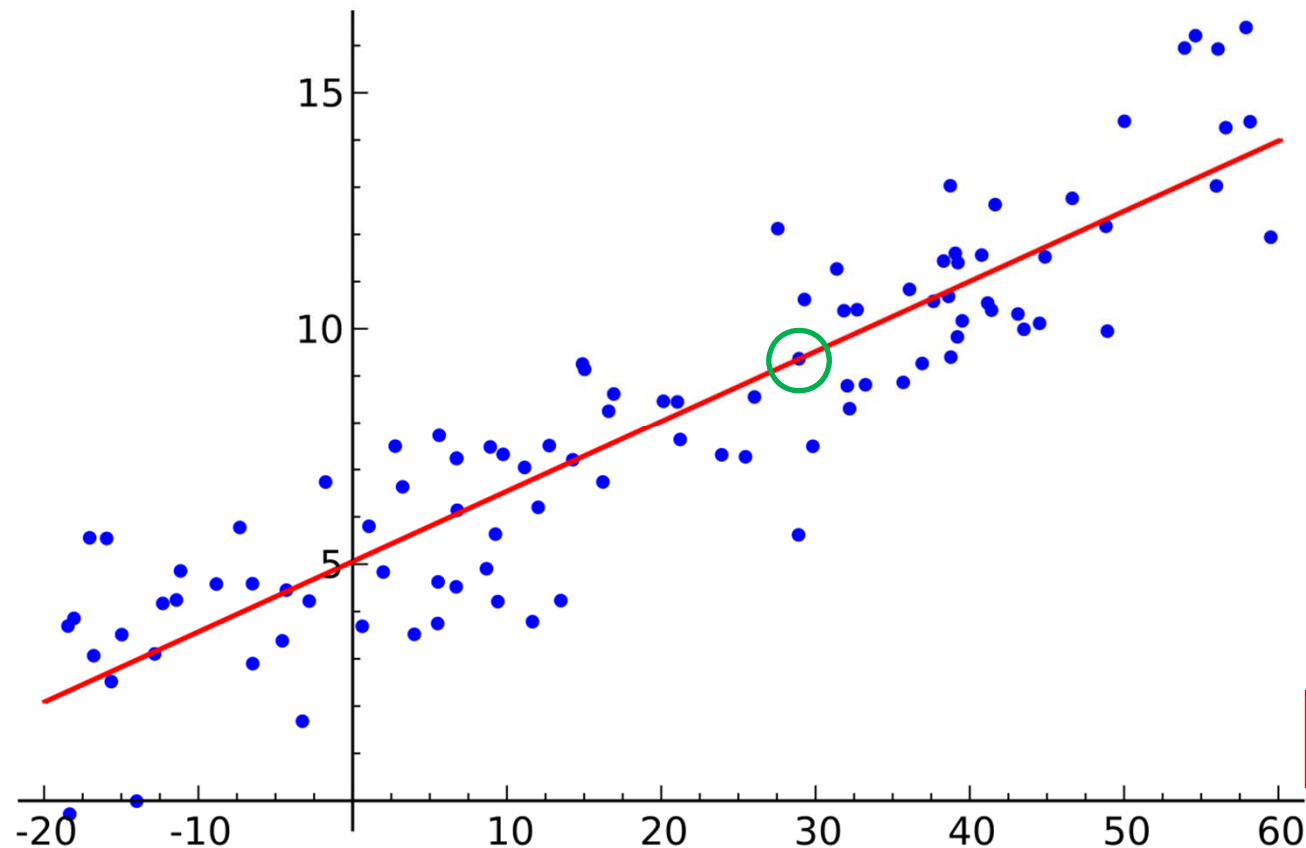
$$y' = 0.15x + 5$$

LINEAR REGRESSION

2. Calculate expected values:

Now we can predict value of y from x

$$y = 0.15x + 5$$

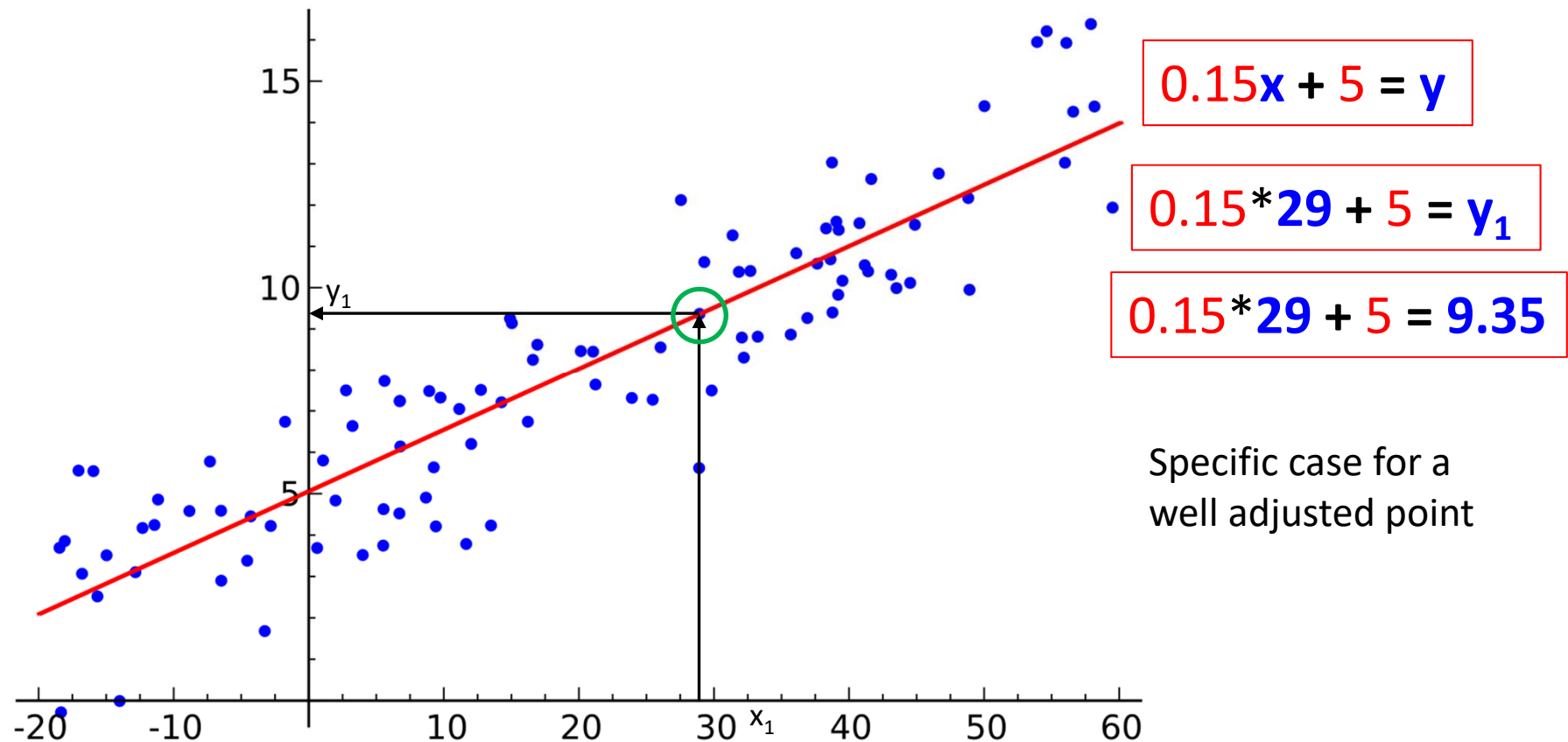


$$y' = 0.15x' + 5$$

LINEAR REGRESSION

2. Calculate expected values:

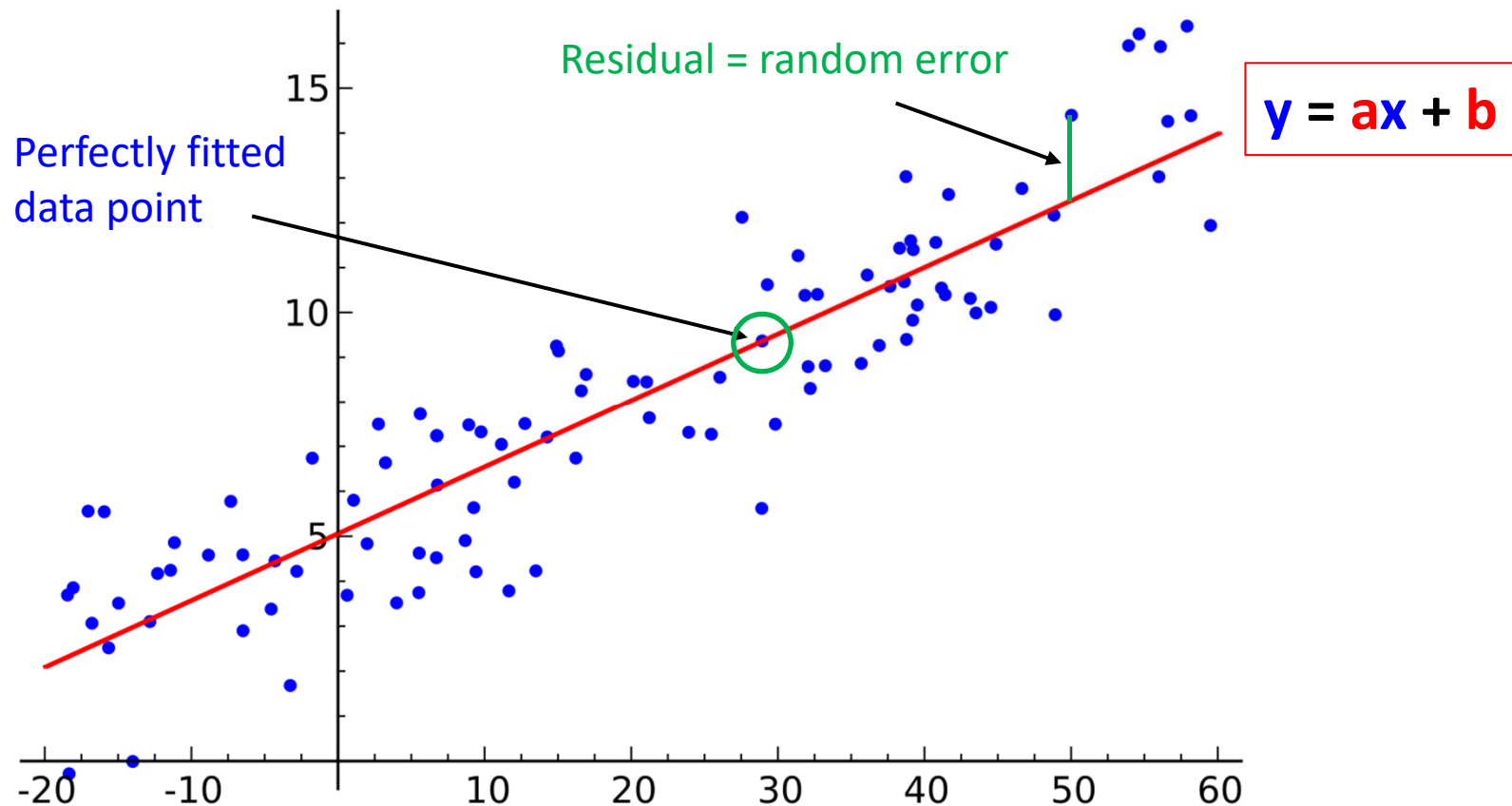
Now we can predict value of y from x



LINEAR REGRESSION

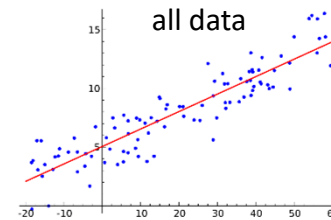
3. Compare expected values with real values:

difference (expected – observed) = **error**

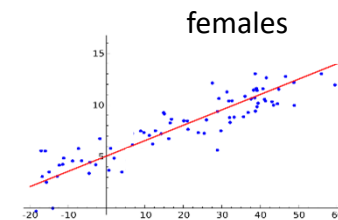
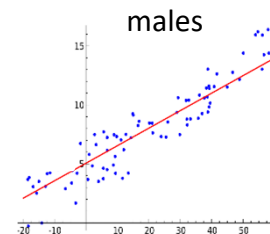


MIND SETUP FOR LINEAR MODELS

Linear models first adjust all data and predict values



Then adjust again the data but **for each group independently** and predict values again.



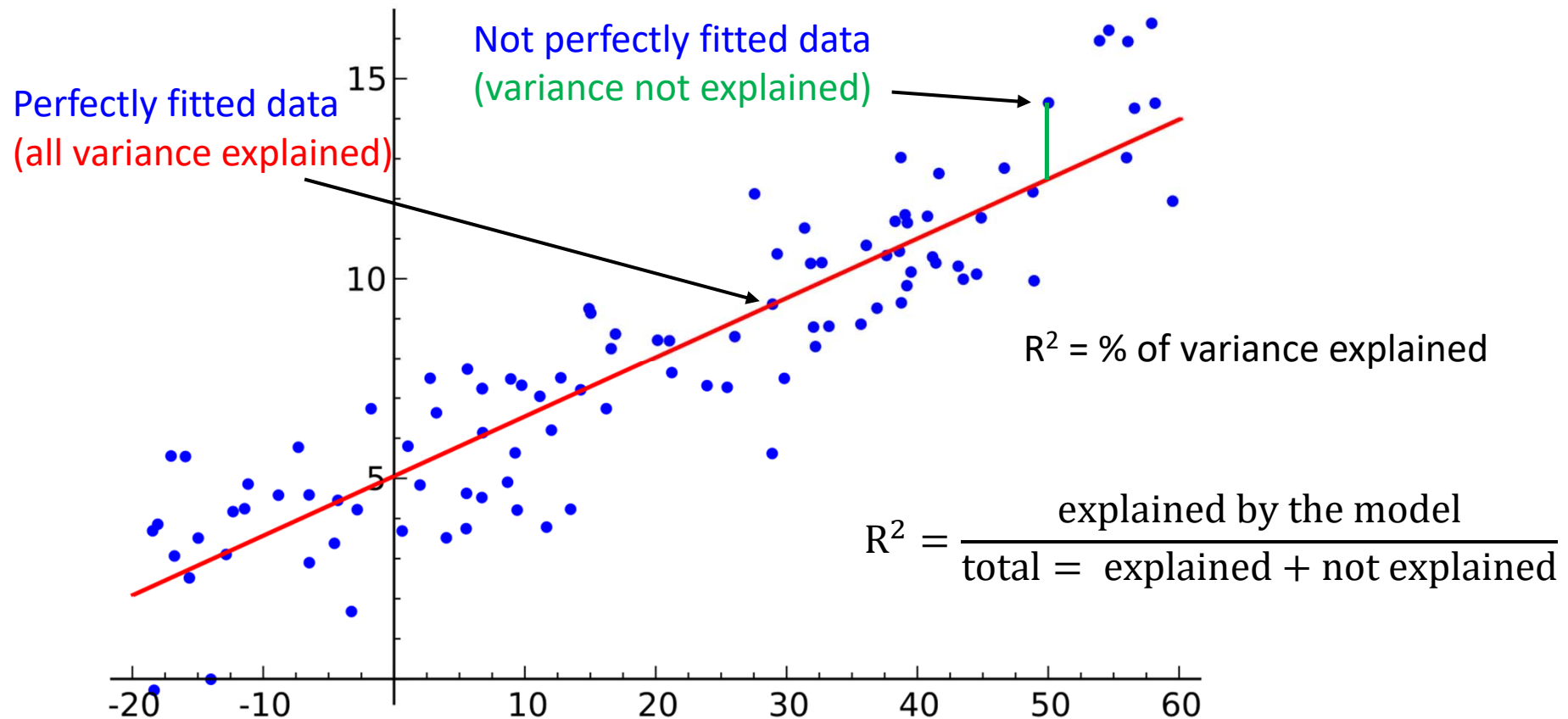
Then compares:

Which prediction is better the one from the model with all data or from the model adjusted separately for each group?

Better = smaller "error"

LINEAR REGRESSION

Linear regression model: Goodness of fit



General Linear Models

We already know how to run one **linear model** in **R**: ANOVA

aov (response~explanatory, data)

aov (Head Height ~ Group, data=dataset)

We could tell **R** to run a **general linear model**

Linear model -> **lm()**

- Will assume **normality**
- Will choose the analysis according to our data
 - Response is continuous; Explanatory is categorical/discrete -> **ANOVA**
 - Response is continuous; Explanatory is continuous -> **linear regression**

General Linear Models

lm ()

- Needs to be normal distributed
- Only one continuous response variable
- One or more (few) explanatory variables (categorical or continuous)

Data Distribution	Response variable	Explanatory variable	# Predictors	Test
<i>Normal</i>	<i>continuous</i>	<i>discrete</i>	<i>one</i>	<i>One-way ANOVA</i>
<i>Normal</i>	<i>continuous</i>	<i>discrete</i>	<i>multiple</i>	<i>Multi-way ANOVA</i>
<i>Normal</i>	<i>continuous</i>	<i>continuous</i>	<i>one</i>	<i>Linear regression</i>
<i>Normal</i>	<i>continuous</i>	<i>continuous</i>	<i>multiple</i>	<i>Multiple regression</i>
<i>Normal</i>	<i>continuous</i>	<i>discrete & continuous</i>	<i>multiple</i>	<i>ANCOVA</i>

General Linear Models

`lm (response ~ explanatory, data=dataset)`

<code>lm (Head Width~ Group)</code>	->	One-way ANOVA
<code>lm (Head Width ~ Pop*Sex)</code>	->	Multi-way ANOVA
<code>lm (Head Width ~ Body Length)</code>	->	Linear Regression
<code>lm (Head Width ~ Jaw Length*Body Length)</code>	->	Multiple Regression
<code>lm (Head Width ~ Body Length *Group)</code>	->	ANCOVA

Data Distribution	Response variable	Explanatory variable	# Predictors	Test
Normal	continuous	discrete	one	One-way ANOVA
Normal	continuous	discrete	multiple	Multi-way ANOVA
Normal	continuous	continuous	one	Linear regression
Normal	continuous	continuous	multiple	Multiple regression
Normal	continuous	discrete & continuous	multiple	ANCOVA

GENERAL LINEAR MODELS

Agrupation of Linear Models

- Handle only continuous response variables
- Handle only data with Normal distributions
- Only one type of regression (linear)
- Only one response variable
- Handle a few explanatory variables
- Significance checked by maximum likelihood (p-value)

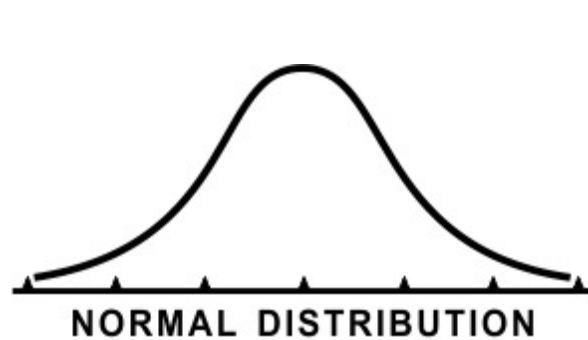
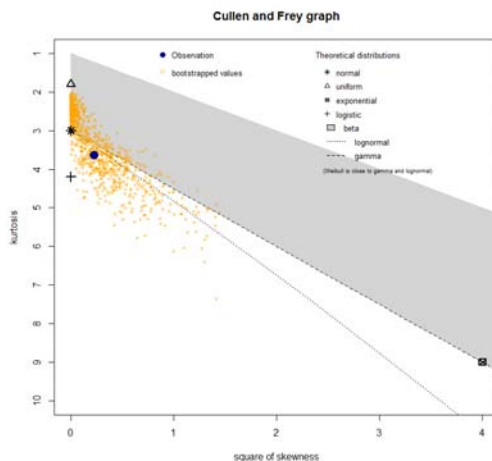
OTHER DISTRIBUTIONS?

If distribution does not fit the Normal distribution:

- transformation of dataset (log?)

OR

- use of low-power non parametric tests (Kruskal-Wallis)



OTHER REGRESSIONS?

- Logistic regression (1/0; yes/no; male/female)
- Poisson regression (discrete and ordinal data)



GENERAL LINEAR MODELS

Agrupation of Linear Models

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GENERALIZED LINEAR MODELS

Generalization of General Linear Models

- Response variable can be from continuous to categorical
- Handle data with Normal or **other** distributions
- Adjust model to linear and **other** regressions
- Handle many variables at the same time and their interactions (N/10)

GENERALIZED LINEAR MODELS

Regressions:

Linear, multivariate, logistic, Poisson's

Distributions:

Normal, Poisson, Binomial, Multinomial

Significance:

Maximum likelihood, Bayesian,
and least squares

Data:

Continuous, Count, Probability,
Frequency, Binary, etc.



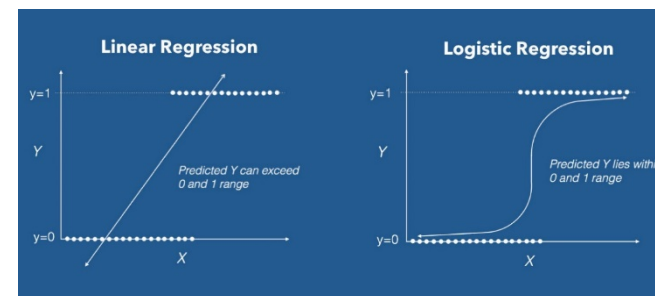
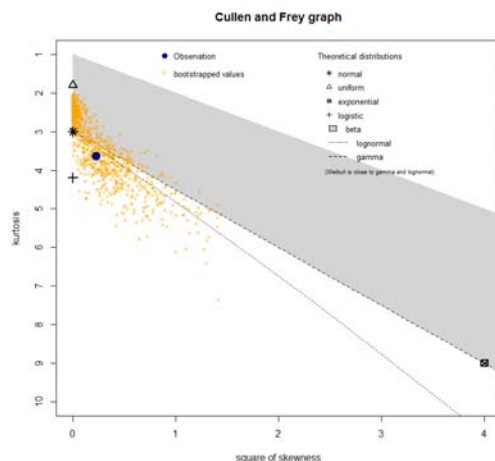
GLM LINK FUNCTIONS

Link function links the explanatory variables

Link function for linear regression

$$y = ax + b$$

GLM handles many types of data, regressions, and models
We need to tell R which kind of link function will need to apply



GLM LINK FUNCTIONS

Don't need to transform data to fit it in a distribution

You tell GLM to which **family** (type of data) belongs your data and it will choose the right link function to process it!

Which Link function?

Tell GLM to which FAMILY belongs our data:



Family is a combination of data characteristics and distributions:
Continuous / categorical, includes zeros (0), does not include zeros, has negative values, only positive, type of distribution, etc.

Rules of thumb to choose "family"

DATA

FAMILY

Normal Distribution

Positive continuous data, no zeros

Gamma distribution

Binary data (y/n)

Proportions (3:1)

Logistic distribution

Counts

Categorical ($SD = \bar{y}$)

Poisson distribution

Log Normal distribution

Other categorical

Gaussian

Gamma

Gamma

Binomial

Binomial

Binomial

Poisson

Poisson

Poisson

Poisson

Negative binomial

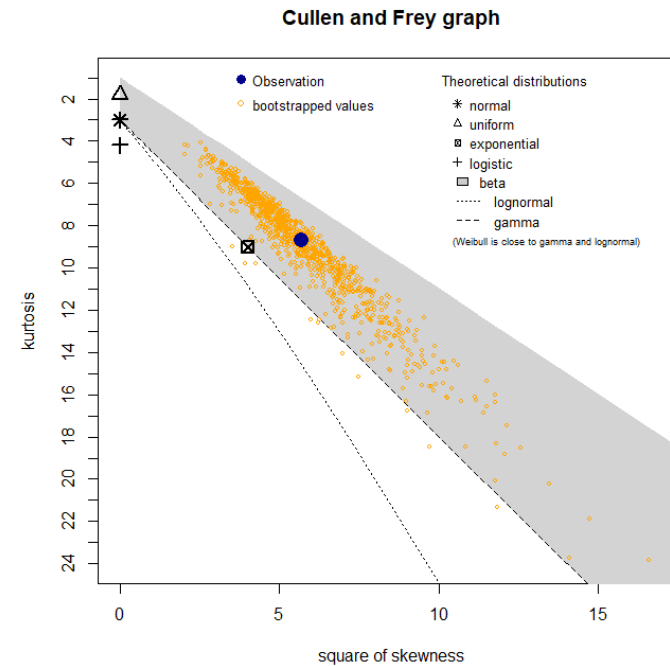
If we doubt which family fits better

fitdistrplus()

Check the fit of some typical distributions with **descdist()** plot

<u>Distribution</u>		<u>Family</u>
Normal	→	Gaussian
Logistic	→	Binomial
Gamma	→	Gamma
LogNormal	→	Poisson

Check other specific distributions with **fitdistr()**



Running GLMs

1. Choose variables, choose a family
2. If not sure, or want to be extra sure:
 `fitdistr()`
 `descdist()`
3. run the analysis and check if model is well fitted
 `glm (response(s)~predictor(s), family, data)`



Understanding

OUTPUTS

GENERALIZED LINEAR MODELS (open RStudio)

Understanding GLMs output

```
> summary(glm1)
Call:
glm(formula = Sex ~ Pop, family = "binomial", data = morelizards)
Deviance Residuals:
    Min       1Q   Median       3Q      Max
-1.317   -1.317    1.044    1.044    1.127

Coefficients:
              Estimate Std. Error z value Pr(>|z|)
(Intercept)  3.228e-01  2.026e-01   1.593   0.111
PK           7.837e-16  2.865e-01   0.000   1.000
PM          -2.026e-01  2.849e-01  -0.711   0.477

(Dispersion parameter for binomial family taken to be 1)

    Null deviance: 411.06  on 299  degrees of freedom
Residual deviance: 410.39  on 297  degrees of freedom
AIC: 416.39
Number of Fisher Scoring iterations: 4
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AIC can be used to compare with other models, but is not an absolute

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Coeff:

(Inter

PK

PM

Compare how well our data fits each distribution with:

Akaike Information Criterion (**AIC**)

The lower the value the better.

(Dispe

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HOW WELL ADJUSTED IS THE MODEL?
There is no R^2 we need to calculate ourselves

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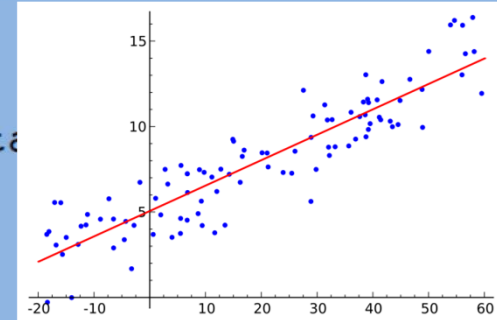
$$R^2 = \frac{\text{Null deviance} - \text{Residual deviance}}{\text{Null deviance}} = \frac{411.06 - 410.39}{411.06} = 0.0016$$

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HOW WELL ADJUSTED IS THE MODEL?

Rule of Thumb:

If: $\text{degrees of freedom} * 2 < \text{Residual deviance}$ → Data are **overdispersed**

$297 * 2 = 594 \gtrsim 410.39 \rightarrow$ Fit is not great, but data are **not overdispersed** ✓

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Each "free" group/covariable comparison against the intercept. Interesting, but not super useful

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TASKS

<https://tinyurl.com/evolecopract> --> **morelizards.csv** and **fifth_linear_models.R**

GLM

Libraries: `fitdistrplus`, `car`, `boot`

1. Choose variables (discrete or continuous, all allowed)
2. Plot the distribution: `descdist(variable)`
3. Fit your data to distributions that make sense for your type of data and compare how well they fit

`fit1<-fitdistr(na.omit(variable), "negative binomial")`

`AIC (fit1, fit2, ...)` #The lowest the better

4. Choose the families that better adjust to your data:
gaussian, poisson, poisson, Gamma, binomial, negative.binomial

5. Perform `glm()`

`glm(response~explanatory, family=negative.binomial, data=dataset)`

6. Check AIC, calculate R^2 and “degrees of freedom vs variance”
7. Perform an ANOVA with the output: `aov(glm_output)`
8. Are there significant effects for the chosen variables?
9. Plot

