Online Appendix for "Demand with Network Externalities: Identification and an Application to the Dating Websites Industry"*

Michael Sullivan

University of British Columbia

February 12, 2026

A Microfoundation for network externalities on dating websites

This appendix provides a model of search and matching on a dating website that justifies the inclusion of a network externality term in consumer payoffs from using dating websites. This model is a variant of that proposed by Smith (2006); the model features a mass L of users of a dating website, each of whom finds matches on the site at an exponential rate ρ. Matches are exogenously destroyed at rate δ. Each user's payoffs are discounted at the interest rate r. The flow value of a match between agents i and j is f_{ij} and the flow value of remaining unmatched is zero. I assume that f_{ij} are identically and independently distributed across pairs (i, j) from the distribution F. Let $\eta = \Pr(f_{ij} > 0)$ and let $\mu_f = \mathbb{E}[f_{ij}|f_{ij} > 0]$. Suppose that consumer i immediately forms any match that yields a positive flow value f_{ij} and does not account for the option value of waiting for a better match. The average present value[1] of being unmatched is

$$V_u = \frac{\rho\eta^2}{r} \int [V_m(f) - V_u] dF(f \mid f > 0), \tag{1}$$

where $u \leq L$ is the mass of unmatched users and $V_m(f)$ is the average present value of being matched with match value f:

$$V_m(f) = f + \frac{\delta}{r}[V_u - V_m(f)]. \tag{2}$$

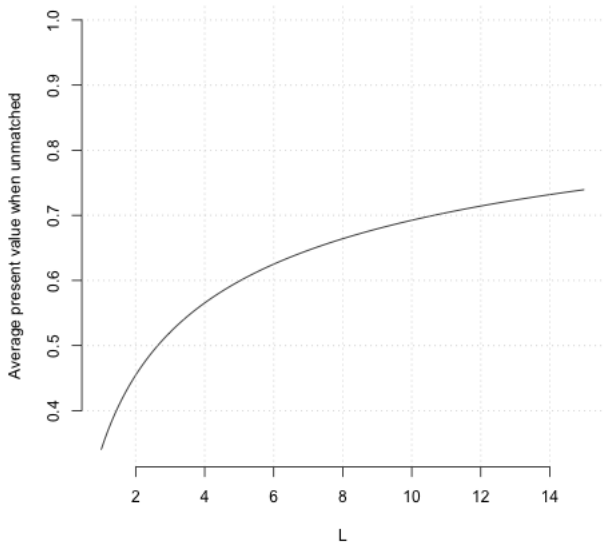
We can solve (2) to obtain

$$V_m(f) = \frac{rf + \delta V_u}{r + \delta}$$

*Email address: michael.sullivan@sauder.ubc.ca.

[1]The average present value is defined as the net present value times the interest rate r.

Figure 1: Relationship between platform usage and value



and then solve (1) to obtain

$$V_u = \frac{\eta^2 u \mu_f}{\psi + \eta^2 u}, \tag{3}$$

where $\psi = (r + \delta)/\rho$ is a measure of the search frictions in the market. The steady-state value of u is given by the condition

$$\delta(L - u) = \rho \eta^2 u^2,$$

which equates inflows to and outflows from the population of unmatched users. Solving for u yields

$$u = \sqrt{\frac{\delta^2}{4(\eta^2\rho)^2} + \frac{\delta}{\eta^2\rho} L} - \frac{\delta}{2\eta^2\rho}.$$

We can substitute this expression for u into (3) to obtain an expression for the value of being unmatched as a function of the mass of users L on the platform and of other model primitives; this expression is increasing in L. Figure 1 plots the relationship between V_u and L under the choice of parameters $\rho = 0.5$, $\delta = 0.5$, $\mu_f = 1$, $\eta = 1$, and $r = 0.10$. The positive relationship between L and the consumer's value of joining the platform as an unmatched user justifies the inclusion of a network externality term in the indirect utilities of the model considered in the main text.

B Multihoming

Consumers in my setting are able to use multiple websites, although few consumers multihome in practice. Among all panelists who use at least one site according to the criterion

Table 1: Multihoming patterns

Website	Share of panelists using	Share also using			
		`eharmony.com`	`match.com`	`okcupid.com`	`pof.com`
eharmony	0.35	1.00	0.38	0.02	0.09
match	0.67	0.20	1.00	0.02	0.07
okcupid	0.04	0.19	0.26	1.00	0.19
pof	0.15	0.21	0.32	0.05	1.00

established by the preceding section, 81% use only one website, 17% use two, and 2% use over two sites. Table 1 reports shares of each site's users who use other dating websites. Not only do few panelists multihome, panelists who multihome generally spend a large share of their time on a single dating website: the average number of minutes that a multihoming panelist spends on the panelist's primary site is 1023, whereas the average number of minutes spent on other sites is only 158. In addition, the average share of time that a panelist spends on the panelist's primary site among all dating websites that the panelist uses is 79%; the median across panelists is 81%. Although multihoming can play an important role in platform competition, the fact that panelists in my sample generally concentrate their online dating activity on a single site motivates my decision to use a model in which each consumer selects a single primary site.

C Asymptotic framework

Here, I provide notation and asymptotic assumptions under which I conduct inference. I write the data as

$$D^k = \{D_{it} : 1 \leq i \leq N_t^k, 1 \leq t \leq T^k\},$$

where k is an index that tends to infinity in my asymptotic analysis and D_{it} is a vector containing all observables pertaining to individual i in market t. I assume that, within each market t and for all k, $D_{it} \sim_{\text{iid}} \Psi_t$ for a distribution Ψ_t. My primary asymptotic assumptions are

$$T^k \xrightarrow{k\uparrow\infty} \infty$$
$$N_t^k / T^k \xrightarrow{k\uparrow\infty} \infty.$$

That is, I assume that the number of markets T^k tends to infinity and the number of observations within each market, N_t^k, also tends to infinity. This latter assumption is reasonable in settings such as my own in which the number of individuals in the microdata within each market is large. Last, I assume that the number of platforms J is fixed (i.e., it does not depend on k).

D Microstep results

Table 2 provides results of the microstep estimation for the baseline specification. Table 4 provides results of the microstep estimation for the "Age" demographic group specification. For each specification, standard errors are computed from the inverse Hessian estimate of the estimator's asymptotic variance.

E Quantity-type model results

Bibliography

Smith, Lones. 2006. "The Marriage Model with Search Frictions." *Journal of Political Economy* 114 (6): 1124–1144.

Table 2: First-stage parameter estimates – "Overall" demographic group specification, demographic variables

	eharmony.com	match.com	okcupid.com	pof.com
Education: High school or less (Omit.)	0.000	0.000	0.000	0.000
	-	-	-	-
Education: Some college	0.072	0.106	-0.071	0.369
	(0.150)	(0.103)	(0.326)	(0.228)
Education: College degree	-0.076	-0.063	-0.293	-0.206
	(0.161)	(0.109)	(0.355)	(0.273)
Education: Advanced degree	-0.028	0.048	-1.329	-0.139
	(0.182)	(0.121)	(0.627)	(0.304)
Education: Unknown	0.230	0.020	-0.356	0.462
	(0.112)	(0.078)	(0.238)	(0.179)
Age: Under 25yo (Omit.)	0.000	0.000	0.000	0.000
	-	-	-	-
Age: 25-29yo	0.141	0.285	-0.110	0.344
	(0.178)	(0.135)	(0.411)	(0.253)
Age: 30-34yo	0.063	0.294	-0.460	0.424
	(0.167)	(0.126)	(0.402)	(0.238)
Age: 35-39yo	0.026	0.169	-0.539	0.266
	(0.163)	(0.124)	(0.382)	(0.236)
Age: 40-49yo	0.108	0.215	-0.216	0.013
	(0.154)	(0.118)	(0.343)	(0.226)
Age: 50+yo	0.097	0.249	-0.326	0.187
	(0.153)	(0.117)	(0.343)	(0.223)
Children in HH: No (Omit.)	0.000	0.000	0.000	0.000
	-	-	-	-
Children in HH: Yes	0.094	0.055	0.028	-0.045
	(0.067)	(0.050)	(0.180)	(0.097)
Race: White (Omit.)	0.000	0.000	0.000	0.000
	-	-	-	-
Race: Black	-0.082	-0.496	-0.587	-0.369
	(0.089)	(0.077)	(0.322)	(0.150)
Race: Asian	-0.287	-0.185	0.252	-0.993
	(0.216)	(0.142)	(0.388)	(0.510)
Race: Other	-0.293	-0.280	0.228	0.170
	(0.192)	(0.136)	(0.392)	(0.247)
Broadband: No (Omit.)	0.000	0.000	0.000	0.000
	-	-	-	-
Broadband: Yes	-0.496	-0.116	-0.131	-0.570
	(0.106)	(0.093)	(0.372)	(0.142)
Hispanic: No (Omit.)	0.000	0.000	0.000	0.000
	-	-	-	-
Hispanic: Yes	-0.107	0.069	-0.085	-0.208
	(0.059)	(0.043)	(0.168)	(0.091)
Income: Under 25k (Omit.)	0.000	0.000	0.000	0.000
	-	-	-	-
Income: 25-75k	0.089	-0.024	0.048	0.077
	(0.064)	(0.047)	(0.180)	(0.090)
Income: 75-100k	0.068	-0.054	0.061	-0.048
	(0.080)	(0.059)	(0.222)	(0.117)
Income: Over 100k	0.001	-0.052	0.306	-0.297
	(0.079)	(0.057)	(0.204)	(0.120)
HH size: 1 (Omit.)	0.000	0.000	0.000	0.000
	-	-	-	-
HH size: 2	-0.136	-0.206	-0.228	-0.008
	(0.098)	(0.072)	(0.255)	(0.146)
HH size: 3	-0.323	-0.295	-0.318	-0.037
	(0.117)	(0.086)	(0.306)	(0.172)
HH size: Over 3	-0.233	-0.357	-0.438	-0.056
	(0.114)	(0.084)	(0.300)	(0.169)
Log local population	0.042	-0.023	-0.097	-0.061
	(0.021)	(0.015)	(0.054)	(0.029)

Table 3: First-stage parameter estimates – "Overall" demographic group specification, web usage variables

	eharmony.com	match.com	okcupid.com	pof.com
Log Pages Viewed	0.141	0.237	0.548	0.100
	(0.073)	(0.057)	(0.201)	(0.104)
Log Browsing Duration	-0.136	-0.094	0.016	-0.022
	(0.060)	(0.044)	(0.154)	(0.085)
Pages Viewed: Adult	0.062	0.042	-0.022	0.078
	(0.012)	(0.011)	(0.050)	(0.012)
Pages Viewed: Advert	-0.011	-0.001	-0.001	-0.003
	(0.011)	(0.005)	(0.013)	(0.012)
Pages Viewed: Finance	0.030	0.017	-0.089	-0.126
	(0.027)	(0.019)	(0.084)	(0.050)
Pages Viewed: Gaming	-0.002	-0.033	-0.035	-0.012
	(0.011)	(0.011)	(0.032)	(0.020)
Pages Viewed: Government	0.018	0.010	-0.676	-0.108
	(0.028)	(0.021)	(0.346)	(0.088)
Pages Viewed: Info	-0.061	0.021	0.140	-0.087
	(0.053)	(0.030)	(0.046)	(0.087)
Pages Viewed: Malware	0.009	-0.017	-0.041	-0.013
	(0.007)	(0.006)	(0.023)	(0.014)
Pages Viewed: Media	-0.054	0.008	0.043	-0.174
	(0.021)	(0.011)	(0.021)	(0.044)
Pages Viewed: Other	0.000	-0.003	0.001	0.001
	(0.001)	(0.001)	(0.001)	(0.001)
Pages Viewed: Portal	0.016	0.033	0.011	0.024
	(0.004)	(0.003)	(0.009)	(0.005)
Pages Viewed: Retail	0.002	0.002	-0.000	0.004
	(0.004)	(0.003)	(0.010)	(0.006)
Pages Viewed: Social Media	-0.004	-0.003	-0.004	0.002
	(0.001)	(0.001)	(0.003)	(0.002)
Pages Viewed: Video	-0.021	-0.070	0.025	-0.076
	(0.012)	(0.011)	(0.013)	(0.025)
Pages Viewed: Weather	-0.102	-0.053	-0.135	-0.193
	(0.074)	(0.043)	(0.179)	(0.119)
Pages Viewed: Webservice	0.007	-0.004	0.010	-0.048
	(0.008)	(0.007)	(0.014)	(0.023)
Pages Viewed: Internet/Wireless	-0.006	-0.001	-0.021	-0.043
	(0.025)	(0.018)	(0.065)	(0.050)
Pages Viewed: News	-0.185	0.063	-0.026	0.047
	(0.076)	(0.031)	(0.123)	(0.064)
Pages Viewed: Sports	-0.123	0.011	-0.123	-0.043
	(0.049)	(0.022)	(0.103)	(0.056)
Pages Viewed: Travel	0.133	0.205	-0.992	-0.108
	(0.079)	(0.055)	(0.417)	(0.159)
Pages Viewed: Career	0.205	0.102	-0.127	0.166
	(0.062)	(0.054)	(0.275)	(0.095)
Pages Viewed: Downloads	0.110	-0.102	-0.388	0.160
	(0.108)	(0.109)	(0.495)	(0.146)
Pages Viewed: Directory	0.898	0.739	1.033	1.084
	(0.431)	(0.372)	(1.120)	(0.593)

Table 4: First-stage parameter estimates ("Age" demographic group specification), demographic variables

	eharmony.com	match.com	okcupid.com	pof.com
Education: High school or less (Omit.)	0.000	0.000	0.000	0.000
	-	-	-	-
Education: Some college	0.071	0.112	-0.068	0.355
	(0.148)	(0.102)	(0.323)	(0.224)
Education: College degree	-0.078	-0.060	-0.295	-0.214
	(0.159)	(0.109)	(0.353)	(0.270)
Education: Advanced degree	-0.027	0.056	-1.313	-0.146
	(0.181)	(0.120)	(0.625)	(0.301)
Education: Unknown	0.228	0.024	-0.350	0.453
	(0.110)	(0.076)	(0.235)	(0.173)
Age: Under 25yo (Omit.)	0.000	0.000	0.000	0.000
	-	-	-	-
Age: 25-29yo	0.149	0.319	-0.094	0.357
	(0.171)	(0.131)	(0.407)	(0.244)
Age: 30-34yo	0.059	0.321	-0.451	0.428
	(0.161)	(0.122)	(0.397)	(0.228)
Age: 35-39yo	-2.006	-2.042	-7.552	-2.154
	(0.156)	(0.120)	(0.376)	(0.226)
Age: 40-49yo	-1.923	-1.995	-7.236	-2.409
	(0.146)	(0.113)	(0.335)	(0.214)
Age: 50+yo	-1.933	-1.962	-7.343	-2.231
	(0.145)	(0.112)	(0.334)	(0.210)
Children in HH: No (Omit.)	0.000	0.000	0.000	0.000
	-	-	-	-
Children in HH: Yes	0.093	0.056	0.017	-0.041
	(0.067)	(0.050)	(0.180)	(0.097)
Race: White (Omit.)	0.000	0.000	0.000	0.000
	-	-	-	-
Race: Black	-0.085	-0.497	-0.584	-0.369
	(0.087)	(0.076)	(0.318)	(0.148)
Race: Asian	-0.286	-0.187	0.281	-0.996
	(0.213)	(0.140)	(0.381)	(0.508)
Race: Other	-0.300	-0.282	0.248	0.172
	(0.189)	(0.135)	(0.383)	(0.242)
Broadband: No (Omit.)	0.000	0.000	0.000	0.000
	-	-	-	-
Broadband: Yes	-0.498	-0.115	-0.131	-0.572
	(0.106)	(0.093)	(0.368)	(0.142)
Hispanic: No (Omit.)	0.000	0.000	0.000	0.000
	-	-	-	-
Hispanic: Yes	-0.108	0.069	-0.084	-0.206
	(0.059)	(0.042)	(0.166)	(0.090)
Income: Under 25k (Omit.)	0.000	0.000	0.000	0.000
	-	-	-	-
Income: 25-75k	0.081	-0.026	0.052	0.082
	(0.064)	(0.047)	(0.178)	(0.090)
Income: 75-100k	0.063	-0.057	0.066	-0.046
	(0.079)	(0.059)	(0.221)	(0.116)
Income: Over 100k	-0.004	-0.053	0.316	-0.294
	(0.078)	(0.056)	(0.201)	(0.118)
HH size: 1 (Omit.)	0.000	0.000	0.000	0.000
	-	-	-	-
HH size: 2	-0.142	-0.212	-0.228	-0.014
	(0.097)	(0.072)	(0.252)	(0.143)
HH size: 3	-0.328	-0.300	-0.306	-0.040
	(0.116)	(0.085)	(0.303)	(0.170)
HH size: Over 3	-0.239	-0.362	-0.439	-0.064
	(0.113)	(0.084)	(0.298)	(0.167)
Log local population	0.043	-0.023	-0.095	-0.062
	(0.016)	(0.012)	(0.044)	(0.023)

Table 5: First-stage parameter estimates ("Age" demographic group specification), web usage variables

	eharmony.com	match.com	okcupid.com	pof.com
Log Pages Viewed	0.142	0.241	0.538	0.094
	(0.067)	(0.051)	(0.174)	(0.096)
Log Browsing Duration	-0.136	-0.097	0.014	-0.020
	(0.060)	(0.044)	(0.152)	(0.085)
Pages Viewed: Adult	0.063	0.042	-0.023	0.077
	(0.012)	(0.011)	(0.049)	(0.012)
Pages Viewed: Advert	-0.011	-0.001	-0.001	-0.003
	(0.011)	(0.006)	(0.013)	(0.012)
Pages Viewed: Finance	0.031	0.018	-0.093	-0.126
	(0.026)	(0.019)	(0.083)	(0.050)
Pages Viewed: Gaming	-0.002	-0.033	-0.034	-0.010
	(0.011)	(0.011)	(0.032)	(0.019)
Pages Viewed: Government	0.016	0.007	-0.742	-0.106
	(0.028)	(0.021)	(0.345)	(0.086)
Pages Viewed: Info	-0.057	0.021	0.140	-0.090
	(0.053)	(0.030)	(0.046)	(0.086)
Pages Viewed: Malware	0.009	-0.017	-0.042	-0.013
	(0.007)	(0.006)	(0.023)	(0.014)
Pages Viewed: Media	-0.055	0.008	0.042	-0.174
	(0.021)	(0.011)	(0.020)	(0.044)
Pages Viewed: Other	0.000	-0.004	0.001	0.001
	(0.001)	(0.001)	(0.001)	(0.001)
Pages Viewed: Portal	0.016	0.033	0.012	0.023
	(0.004)	(0.003)	(0.008)	(0.005)
Pages Viewed: Retail	0.002	0.002	0.000	0.005
	(0.004)	(0.003)	(0.010)	(0.006)
Pages Viewed: Social Media	-0.004	-0.003	-0.004	0.002
	(0.001)	(0.001)	(0.002)	(0.002)
Pages Viewed: Video	-0.022	-0.070	0.025	-0.076
	(0.012)	(0.011)	(0.012)	(0.025)
Pages Viewed: Weather	-0.105	-0.057	-0.130	-0.193
	(0.074)	(0.043)	(0.179)	(0.119)
Pages Viewed: Webservice	0.007	-0.004	0.011	-0.049
	(0.008)	(0.007)	(0.014)	(0.023)
Pages Viewed: Internet/Wireless	-0.006	-0.002	-0.025	-0.042
	(0.025)	(0.018)	(0.066)	(0.049)
Pages Viewed: News	-0.186	0.064	-0.037	0.052
	(0.076)	(0.031)	(0.124)	(0.064)
Pages Viewed: Sports	-0.123	0.014	-0.122	-0.041
	(0.049)	(0.022)	(0.103)	(0.056)
Pages Viewed: Travel	0.135	0.205	-0.998	-0.110
	(0.079)	(0.055)	(0.417)	(0.158)
Pages Viewed: Career	0.204	0.101	-0.123	0.168
	(0.062)	(0.054)	(0.272)	(0.094)
Pages Viewed: Downloads	0.106	-0.105	-0.387	0.151
	(0.107)	(0.109)	(0.492)	(0.146)
Pages Viewed: Directory	0.878	0.714	0.940	1.091
	(0.428)	(0.369)	(1.172)	(0.587)

Table 6: Market step parameter estimates – "Overall" demographic group specification, quantity-type model

Panel A: Parameter estimates

	OLS	IV
$\log(M_t s_{jt})$	0.98	0.69
	(0.02)	(0.14)
$\log(M_t)$	-0.98	-0.72
	(0.02)	(0.12)
eharmony	0.07	-0.23
	(0.16)	(0.28)
match	-0.84	-0.92
	(0.16)	(0.24)
okcupid	-3.41	-4.31
	(0.18)	(0.49)
pof	0.41	-0.13
	(0.17)	(0.35)
p_j $(\hat{\alpha})$		0.0099
		(0.0043)

Panel B: First stage of IV regression

	$\widetilde{\log(s_{jt})}$
$\tilde{z}_{jt}$	0.97
	(0.32)
F	9.12