بنام خدا

تمرین 8 : محاسبه مارکف بر سیستم stand by spare

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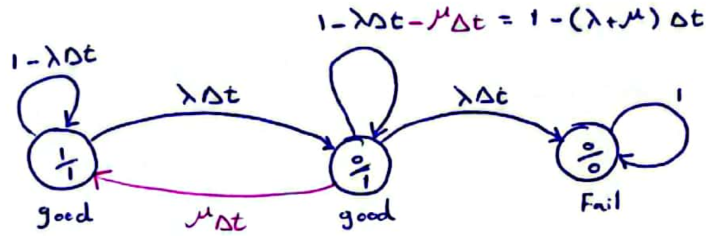
Cold stand By Spare :

∴ Assume $\begin{cases} \text{Failure rate} : \lambda \\ \text{Repair rate} : \mu \end{cases}$ and assume we have many Repairman exist.

we can write models as bellow :



$$\frac{P_A(t+\Delta t)-P_A(t)}{\Delta t} = \frac{1}{\Delta t}\left[P_A(t)\cdot(\cancel{1}-\lambda\Delta t)+P_B(t)\cdot\mu\Delta t - \cancel{P_A(t)}\right]$$

$$\frac{P_B(t+\Delta t)-P_B(t)}{\Delta t} = \frac{1}{\Delta t}\left[P_A(t)\lambda\Delta t+\cancel{P_B(t)}\cdot(1-(\lambda+\mu)\Delta t)-\cancel{P_B(t)}\right]$$

$$\frac{P_F(t+\Delta t)-P_F(t)}{\Delta t} = \frac{1}{\Delta t}\left[P_B(t)\lambda\Delta t+\cancel{P_F(t)}-\cancel{P_F(t)}\right]$$

$$\Rightarrow \begin{cases} P_A'(t) = -\lambda P_A(t)+\mu P_B(t) \\ P_B'(t) = \lambda P_A(t)-(\lambda+\mu)P_B(t) \\ P_F'(t) = \lambda P_B(t) \end{cases} \xrightarrow{\mathcal{L}} \begin{cases} S P_A(s)-\cancel{P_A(0)}^{1} = -\lambda P_A(s)+\mu P_B(s) \\ S P_B(s)-\cancel{P_B(0)}^{0} = \lambda P_A(s)-(\lambda+\mu)P_B(s) \\ S P_F(s)-\cancel{P_F(0)}^{0} = \lambda P_B(s) \end{cases}$$

$$\Rightarrow \begin{array}{l} (s+\lambda)P_A(s)-\mu P_B(s)=1 \longrightarrow \frac{1}{\lambda}\Big((s+\lambda)(s+\lambda+\mu)\Big)P_B(s)-\mu P_B(s)=1 \\ (s+\lambda+\mu)P_B(s)=\lambda P_A(s) \longrightarrow P_A(s)=\frac{s+\lambda+\mu}{\lambda}P_B(s) \end{array}$$

$$\Rightarrow \frac{1}{\lambda}\left[(s+\lambda)(s+\lambda+\mu)-\lambda\mu\right]P_B(s)=1 \Rightarrow P_B(s)=\frac{\lambda}{(s+\lambda)(s+\lambda+\mu)-\lambda\mu}$$

$$P_A(s) = \frac{s+\lambda+\mu}{\cancel{\lambda}} \times \frac{\cancel{\lambda}}{(s+\lambda)(s+\lambda+\mu)-\lambda\mu} = \frac{s+\lambda+\mu}{s^2+\lambda s+\mu s+\lambda s+\lambda^2} = \frac{s+\lambda+\mu}{(s+\lambda)^2+\mu s}$$

$$\begin{cases} P_A(s) = \dfrac{s+\lambda+\mu}{(s+\lambda)^2+\mu s} & (I) \\[2ex] P_B(s) = \dfrac{\lambda}{(s+\lambda)(s+\lambda+\mu)-\mu\lambda} & (II) \end{cases}$$

(I) $L^{-1}\{P_A(s)\}$: if Assume $\lambda = 1$ & $\mu = 2$ $\Rightarrow P_A(s) = \dfrac{s+3}{(s+1)^2+2s} = \dfrac{N(s)}{D(s)}$

$$= \frac{s+3}{s^2+2s+1+2s} = \frac{s+3}{s^2+4s+1}$$

$s_1 = -2+\sqrt{3}$

$s_2 = -2-\sqrt{3}$

$\Rightarrow D(s) = (s+2-\sqrt{3})(s+2+\sqrt{3})$

$$\Rightarrow \frac{s+3}{(s+2-\sqrt{3})(s+2+\sqrt{3})} \xrightarrow{L^{-1}} P_A(t) = \frac{1}{6}\left[\sqrt{3}\left((1+\sqrt{3})e^{2\sqrt{3}t} - (1-\sqrt{3})\right)e^{-t(2+\sqrt{3})}\right]$$

$$\boxed{= \frac{\sqrt{3}}{6}\left[(1+\sqrt{3})e^{2\sqrt{3}t} - (1-\sqrt{3})\right]e^{-t(2+\sqrt{3})t}}$$

(II) $L^{-1}\{P_B(s)\}$: Assume again $\lambda = 1$ & $\mu = 2$ $\Rightarrow P_B(s) = \dfrac{1}{(s-1)(s+3)-2} = \dfrac{N(s)}{D(s)}$

$D(s) = (s-1)(s+3)-2 = s^2+3s-3-3-2 = s^2+2s-5 \Rightarrow P_B(s) = \dfrac{1}{s^2+2s-5}$

$s_1 = -1+\sqrt{6}$

$s_2 = -1-\sqrt{6}$

$$\Rightarrow P_B(s) = \frac{1}{(s+1-\sqrt{6})(s+1-\sqrt{6})} = \frac{A}{s+1-\sqrt{6}} + \frac{B}{s+1+\sqrt{6}}$$

$\Rightarrow 1 = A(s+1+\sqrt{6}) + B(s+1-\sqrt{6})$

$A = \dfrac{1}{2\sqrt{6}}$

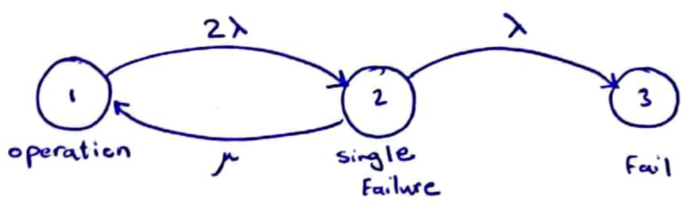
$B = \dfrac{-1}{2\sqrt{6}}$

$\Rightarrow P_B(s) = \dfrac{\frac{1}{2\sqrt{6}}}{s+1-\sqrt{6}} + \dfrac{-\frac{1}{2\sqrt{6}}}{s+1+\sqrt{6}}$

$$\xRightarrow{L^{-1}} \frac{1}{2\sqrt{6}}e^{-(1-\sqrt{6})t} - \frac{1}{2\sqrt{6}}e^{-(1+\sqrt{6})t} = \frac{1}{2\sqrt{6}}\left[e^{-(1-\sqrt{6})t} - e^{-(1+\sqrt{6})t}\right]$$

$$\Rightarrow \boxed{P_B(t) = \frac{\sqrt{6}}{12}\left[e^{2\sqrt{6}t} - 1\right]e^{-t(1+\sqrt{6})}}$$

Hot Stand By Spare:



Transition Rate Matrix is: $Q = \begin{bmatrix} -2\lambda & 2\lambda & 0 \\ \mu & -(\lambda+\mu) & \lambda \\ 0 & 0 & 0 \end{bmatrix}$

in state 1: $P_1'(t+\Delta t) = -2\lambda P_1(t) + \mu P_2(t)$

in state 2: $P_2'(t+\Delta t) = 2\lambda P_1(t) - (\lambda+\mu) P_2(t)$ $\xrightarrow{L}$

in state 3: $P_3'(t+\Delta t) = \lambda P_2(t)$

$$\begin{cases} sP_1(s) - \cancel{P_1(0)}^{\,0} = -2\lambda P_1(s) + \mu P_2(s) \\ sP_2(s) - \cancel{P_2(0)}^{\,0} = 2\lambda P_1(s) - (\lambda+\mu) P_2(s) \\ sP_3(s) - \cancel{P_3(0)}^{\,0} = \lambda P_2(s) \end{cases} \Rightarrow \begin{cases} (I)\ sP_1(s) = -2\lambda P_1(s) + \mu P_2(s) \\ (II)\ sP_2(s) = 2\lambda P_1(s) - (\lambda+\mu) P_2(s) \\ (III)\ sP_3(s) = \lambda P_2(s) \end{cases}$$

$$\xrightarrow{(III)} P_2(s) = \frac{sP_3(s)}{\lambda} \xrightarrow{\text{Put in (II) \& (I)}} \begin{cases} sP_1(s) = -2\lambda P_1(s) + \frac{\mu s P_3(s)}{\lambda} \\ \frac{s^2 P_3(s)}{\lambda} = 2\lambda P_1(s) - (\lambda+\mu) \cdot \frac{sP_3(s)}{\lambda} \end{cases}$$

$$\Rightarrow \begin{cases} P_1(s) = 0 \\ P_2(s) = 0 \\ P_3(s) = 0 \end{cases} \quad \underline{\underline{???}}$$