Dependable System Design

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RBD Calculator

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1 Introduction

The Reliability Block Diagram (RBD) method is a graphical and analytical approach used to model and evaluate the reliability of systems. It represents a system as a combination of interconnected blocks, each corresponding to a system component or subsystem, with the connections illustrating their functional relationships. By analyzing the configuration of these blocks—whether in series, parallel, or complex hybrid structures—engineers can predict the overall system reliability, identify weak points, and optimize designs to enhance performance. The RBD method is widely used in industries like aerospace, automotive, electronics, and manufacturing, where reliability is critical. Its flexibility allows for the modeling of simple systems as well as complex, interdependent systems with varying failure modes and repair strategies.

2 Definition

The Reliability Block Diagram (RBD) method involves calculating the reliability of a system based on the arrangement of its components. The total reliability depends on the configuration of these components—whether in series, parallel, or a combination. Below are the key formulas used in RBD analysis:

2.1 Series Configuration

In a series configuration, the failure of any single component leads to the failure of the entire system. The total reliability of a system with n components in series is given by:

$$R_{\text{total}}(t) = \prod_{i=1}^{n} R_i(t)$$

2.2 Parallel Configuration

In a parallel configuration, the system functions as long as at least one component remains operational. The total reliability of a system with n components in parallel is expressed as:

$$R_{\text{total}}(t) = 1 - \prod_{i=1}^{n} \big(1 - R_i(t)\big)$$

2.3 $m - of - n$ Configuration

An m-of-n configuration means the system operates if at least m out of n components are functioning. The total reliability of the system is calculated using the binomial distribution:

$$R_{\text{total}}(t) = R_{m\text{-of-}n}(t) = \sum_{i=m}^{n} \binom{n}{i} R^i(t) \big(1 - R(t)\big)^{n-i}$$

3 Implementation

We designed this project using two models:

1. Standard Design: A conventional approach.
2. GUI Design: A visual and interactive approach.

The details of both models will be explained below.

3.1 Standard Design

In this section, by running the `main.py` code, you can input your model in the following format:

```
Name of model-Start node-End node-Reliability
```

For example, if you want to describe a system consisting of two subsystems in series and one parallel subsystem connected to the previous two (as shown in "Fig. 1"),

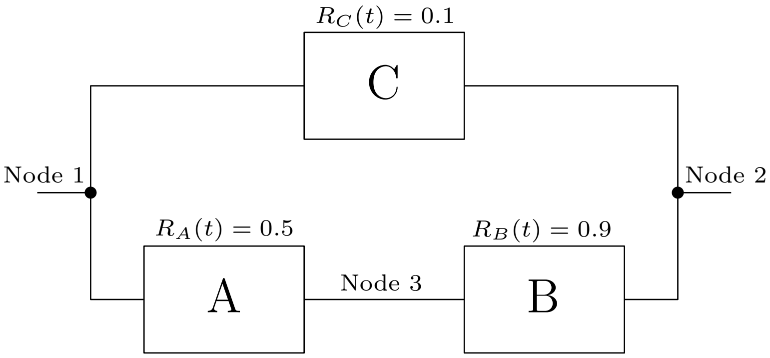

Figure 1: Example System

you can proceed as follows:

```
A-1-3-0.5, B-3-2-0.9, C-1-2-0.1
```

3.2 GUI Design

In the web application developed using `NodeJS`, the server can be run as follows:

3.2.1 Install Dependencies

First, install the required dependencies as follows:

```
# For Ubuntu/Debian:

$ curl -fsSL https://deb.nodesource.com/setup_lts.x | sudo -E bash -
$ sudo apt-get install -y nodejs

```

```
# For macOS (using Homebrew):
$ brew install node

# For Windows:
# Download and run the installer from https://nodejs.org/en/download/
```

3.2.2 Verify Installation

```
$ node --version
$ npm --version
```

3.3 Run Application

After installing all the dependencies, you can prepare the server to run using the following commands:

```
$ npm install
$ npm run dev
```

The application should now be running as bellow! You can access it at `http://localhost:5173` (or whatever port Vite assigns).

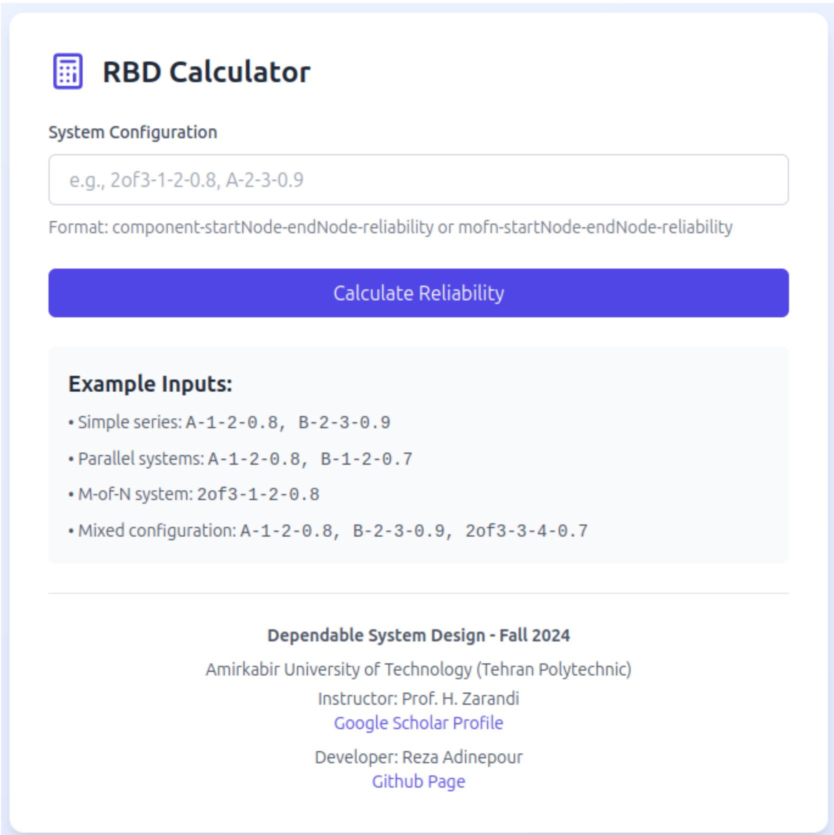

Figure 2: Web Application Overview

In this App, the inputs are entered in the same way as in the previous case.

4 Evaluation

To test the functionality of the developed program, several models are calculated theoretically, and the results are then compared with the outputs of the program.

4.1 model 1

We solve the model presented in Figure 1 theoretically.
The system configuration is as follows:

- Subsystems A and B form a series path from node 1 to node 3 to node 2.
- Subsystem C is in parallel with the series combination of A and B, connecting node 1 directly to node 2.

Step 1: Series Combination of A and B For the series combination of A and B, the reliability is calculated as:

$$R_{\text{series}} = R_A \cdot R_B$$

Substituting $R_A = 0.5$ and $R_B = 0.9$:

$$R_{\text{series}} = 0.5 \cdot 0.9 = 0.45$$

This represents the reliability of the path $1 \to 3 \to 2$.

Step 2: Parallel Combination of $R_{\textbf{series}}$ and C The reliability of the parallel system, consisting of R_{series} and R_C, is given by:

$$R_{\text{parallel}} = 1 - [(1 - R_{\text{series}}) \cdot (1 - R_C)]$$

Substituting $R_{\text{series}} = 0.45$ and $R_C = 0.1$:

$$R_{\text{parallel}} = 1 - [(1 - 0.45) \cdot (1 - 0.1)]$$

$$R_{\text{parallel}} = 1 - [0.55 \cdot 0.9]$$

$$R_{\text{parallel}} = 1 - 0.495 = 0.505$$

Step 3: Final Answer The total reliability of the system is approximately:

$$\boxed{0.505}$$

If we provide the same input to the application, the output will match the results of our analysis.

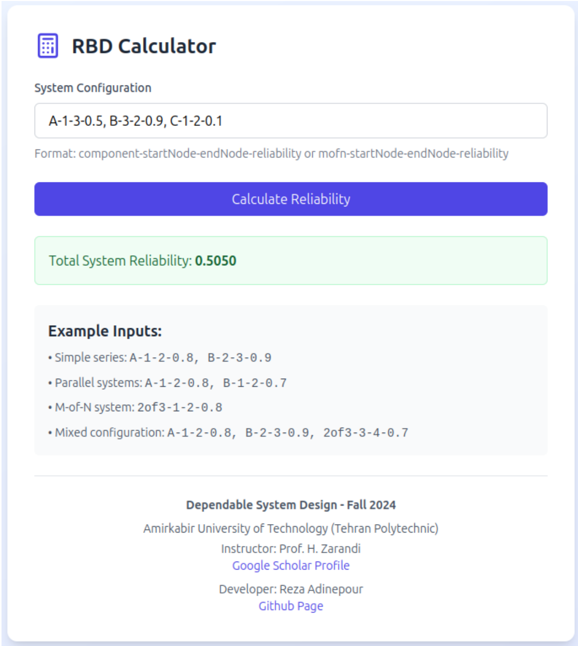

Figure 3: Test of Model 1

4.2 model 2

If our model is as shown in the following figure:

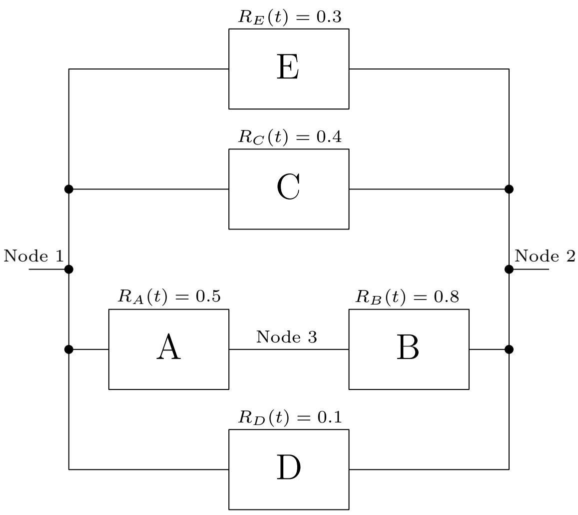

Figure 4: Model 2

It can be theoretically solved as follows:

Problem Setup We are given the following subsystems:

- $A - 1 - 3 - 0.5$: Subsystem A connects node 1 to node 3 with reliability 0.5.
- $B - 3 - 2 - 0.8$: Subsystem B connects node 3 to node 2 with reliability 0.8.

- $C - 1 - 2 - 0.4$: Subsystem C connects node 1 to node 2 with reliability 0.4.
- $D - 1 - 2 - 0.1$: Subsystem D connects node 1 to node 2 with reliability 0.1.
- $E - 1 - 2 - 0.3$: Subsystem E connects node 1 to node 2 with reliability 0.3.

The system configuration is as follows:

- Subsystems A and B form a **series** path from node $1 \to 3 \to 2$.
- Subsystems C, D, and E are in **parallel** with the series combination of $A + B$.

Step 1: Series Combination of A and B For the series combination of A and B, the reliability is:

$$R_{\text{series}} = R_A \cdot R_B$$

Substituting $R_A = 0.5$ and $R_B = 0.8$:

$$R_{\text{series}} = 0.5 \cdot 0.8 = 0.4$$

Step 2: Parallel Combination of $A + B$, C, D, and E The reliability of parallel components is calculated as:

$$R_{\text{parallel}} = 1 - \prod_{i=1}^{n} (1 - R_i)$$

Here, the components are $R_{\text{series}} = 0.4$, $R_C = 0.4$, $R_D = 0.1$, and $R_E = 0.3$. Substituting the values:

$$R_{\text{parallel}} = 1 - [(1 - 0.4) \cdot (1 - 0.4) \cdot (1 - 0.1) \cdot (1 - 0.3)]$$

Calculate each term:

$$(1 - 0.4) = 0.6, \quad (1 - 0.4) = 0.6, \quad (1 - 0.1) = 0.9, \quad (1 - 0.3) = 0.7$$

$$R_{\text{parallel}} = 1 - [0.6 \cdot 0.6 \cdot 0.9 \cdot 0.7]$$

$$R_{\text{parallel}} = 1 - (0.6 \cdot 0.6 = 0.36, \quad 0.36 \cdot 0.9 = 0.324, \quad 0.324 \cdot 0.7 = 0.2268)$$

$$R_{\text{parallel}} = 1 - 0.2268 = 0.7732$$

Step 3: Final Answer The total reliability of the system is approximately:

$$\boxed{0.773}$$

If we provide the same input to the application, the output will match the results of our analysis.

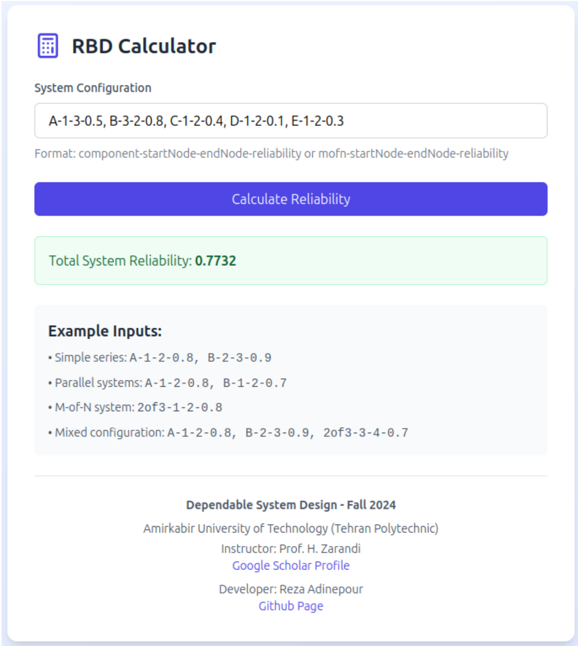

Figure 5: Test of Model 2

4.3 Model 3

If our model is as shown in the following figure:

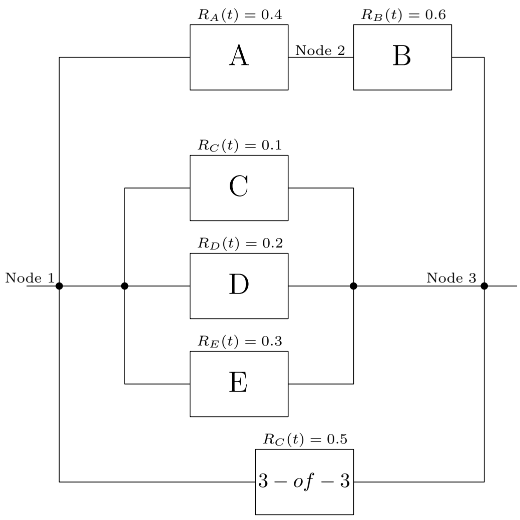

Figure 6: Model 3

It can be theoretically solved as follows:

Problem Setup We are given the following subsystems:

- $A - 1 - 2 - 0.4$: Subsystem A connects node 1 to node 2 with reliability 0.4.

- $B - 2 - 3 - 0.6$: Subsystem B connects node 2 to node 3 with reliability 0.6.
- $C - 1 - 3 - 0.1$: Subsystem C connects node 1 to node 3 with reliability 0.1.
- $D - 1 - 3 - 0.2$: Subsystem D connects node 1 to node 3 with reliability 0.2.
- $E - 1 - 3 - 0.3$: Subsystem E connects node 1 to node 3 with reliability 0.3.
- $3\text{of}3 - 1 - 3 - 0.5$: A 3-of-3 subsystem connects node 1 to node 3 with reliability 0.5.

The system configuration is as follows:

- Subsystems A and B form a **series** path from node $1 \to 2 \to 3$.
- Subsystems C, D, E, and the 3-of-3 subsystem are in **parallel** with the series combination of $A + B$.

Step 1: Series Combination of A and B For the series combination of A and B, the reliability is:

$$R_{\text{series}} = R_A \cdot R_B$$

Substituting $R_A = 0.4$ and $R_B = 0.6$:

$$R_{\text{series}} = 0.4 \cdot 0.6 = 0.24$$

Step 2: Reliability of the 3-of-3 Subsystem The reliability of a 3-of-3 subsystem is calculated as:

$$R_{\text{3-of-3}} = \sum_{i=3}^{3} \binom{3}{i} R^i (1-R)^{3-i}$$

Substituting $R = 0.5$:

$$R_{\text{3-of-3}} = \binom{3}{3} R^3 (1-R)^0$$

$$R_{\text{3-of-3}} = 1 \cdot (0.5)^3 \cdot 1 = 0.125$$

Step 3: Parallel Combination of All Paths The components in parallel are:

- $R_{\text{series}} = 0.24$ (from $A + B$),
- $R_C = 0.1$,
- $R_D = 0.2$,
- $R_E = 0.3$,
- $R_{\text{3-of-3}} = 0.125$.

The reliability of the parallel combination is given by:

$$R_{\text{parallel}} = 1 - \prod_{i=1}^{n} (1 - R_i)$$

Substituting the values:

$$R_{\text{parallel}} = 1 - [(1 - 0.24) \cdot (1 - 0.1) \cdot (1 - 0.2) \cdot (1 - 0.3) \cdot (1 - 0.125)]$$

Calculate each term:

$$(1 - 0.24) = 0.76, \quad (1 - 0.1) = 0.9, \quad (1 - 0.2) = 0.8, \quad (1 - 0.3) = 0.7, \quad (1 - 0.125) = 0.875$$

$$R_{\text{parallel}} = 1 - [0.76 \cdot 0.9 \cdot 0.8 \cdot 0.7 \cdot 0.875]$$

Compute step by step:

$$0.76 \cdot 0.9 = 0.684, \quad 0.684 \cdot 0.8 = 0.5472, \quad 0.5472 \cdot 0.7 = 0.38304, \quad 0.38304 \cdot 0.875 = 0.33516$$

$$R_{\text{parallel}} = 1 - 0.33516 = 0.66484$$

Final Answer The total reliability of the system is approximately:

$$\boxed{0.665}$$

If we provide the same input to the application, the output will match the results of our analysis.

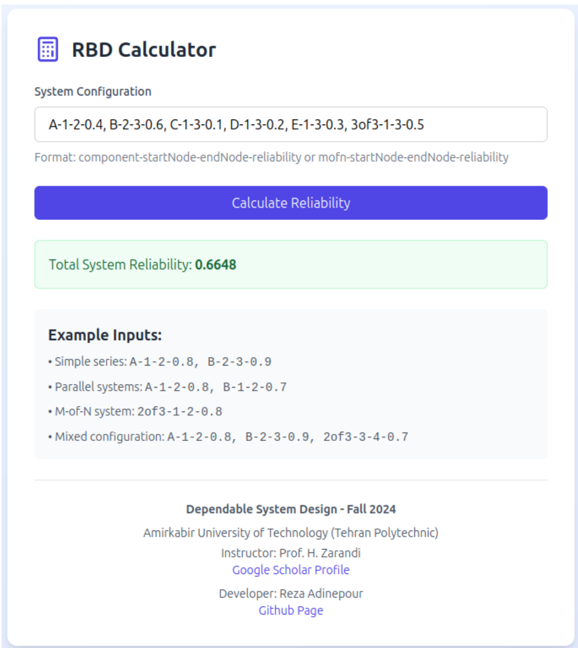

Figure 7: Test of Model 3

4.4 Model 4

If our model is as shown in the following figure:

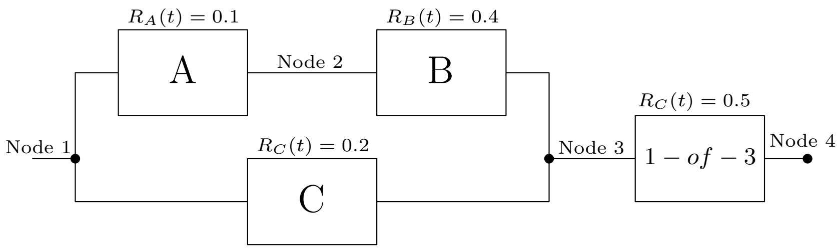

Figure 8: Model 4

It can be theoretically solved as follows:

Problem Setup We are given the following subsystems:

- $A - 1 - 2 - 0.1$: Subsystem A connects node 1 to node 2 with reliability 0.1.
- $B - 2 - 3 - 0.4$: Subsystem B connects node 2 to node 3 with reliability 0.4.
- $C - 1 - 3 - 0.2$: Subsystem C connects node 1 to node 3 with reliability 0.2.
- $1\text{of}3 - 3 - 4 - 0.5$: A 1-of-3 subsystem connects node 3 to node 4 with reliability 0.5.

The system configuration is as follows:

- Subsystems A and B are in **series**, forming a path from node $1 \to 2 \to 3$.
- Subsystem C is in **parallel** with the $A + B$ path, connecting nodes $1 \to 3$.
- The 1-of-3 subsystem connects node 3 to node 4.

Step 1: Series Combination of A and B For the series combination of A and B, the reliability is:

$$R_{\text{series}} = R_A \cdot R_B$$

Substituting $R_A = 0.1$ and $R_B = 0.4$:

$$R_{\text{series}} = 0.1 \cdot 0.4 = 0.04$$

Step 2: Parallel Combination of R_{series} and C For the parallel combination of the $A + B$ series ($R_{\text{series}} = 0.04$) and C ($R_C = 0.2$), the reliability is:

$$R_{\text{parallel}} = 1 - [(1 - R_{\text{series}}) \cdot (1 - R_C)]$$

Substituting:

$$R_{\text{parallel}} = 1 - [(1 - 0.04) \cdot (1 - 0.2)]$$
$$R_{\text{parallel}} = 1 - [0.96 \cdot 0.8]$$
$$R_{\text{parallel}} = 1 - 0.768 = 0.232$$

Step 3: 1-of-3 Subsystem The reliability of a 1-of-3 subsystem is calculated as:

$$R_{\text{1-of-3}} = \sum_{i=1}^{3} \binom{3}{i} R^i (1-R)^{3-i}$$

Substituting $R = 0.5$:

$$R_{\text{1-of-3}} = \binom{3}{1} R^1 (1-R)^2 + \binom{3}{2} R^2 (1-R)^1 + \binom{3}{3} R^3 (1-R)^0$$

Calculate each term:

- For $i = 1$:
$$\binom{3}{1} R^1 (1-R)^2 = 3 \cdot (0.5)^1 \cdot (0.5)^2 = 3 \cdot 0.5 \cdot 0.25 = 0.375$$
- For $i = 2$:
$$\binom{3}{2} R^2 (1-R)^1 = 3 \cdot (0.5)^2 \cdot (0.5)^1 = 3 \cdot 0.25 \cdot 0.5 = 0.375$$
- For $i = 3$:
$$\binom{3}{3} R^3 (1-R)^0 = 1 \cdot (0.5)^3 \cdot 1 = 0.125$$

Summing these:

$$R_{\text{1-of-3}} = 0.375 + 0.375 + 0.125 = 0.875$$

Step 4: Series Combination of R_{parallel} and $R_{\text{1-of-3}}$ The combined reliability of R_{parallel} and $R_{\text{1-of-3}}$ in series is:

$$R_{\text{total}} = R_{\text{parallel}} \cdot R_{\text{1-of-3}}$$

Substituting $R_{\text{parallel}} = 0.232$ and $R_{\text{1-of-3}} = 0.875$:

$$R_{\text{total}} = 0.232 \cdot 0.875 = 0.203$$

Step 5: Final Answer The total reliability of the system is approximately:

$$\boxed{0.203}$$

If we provide the same input to the application, the output will match the results of our analysis.

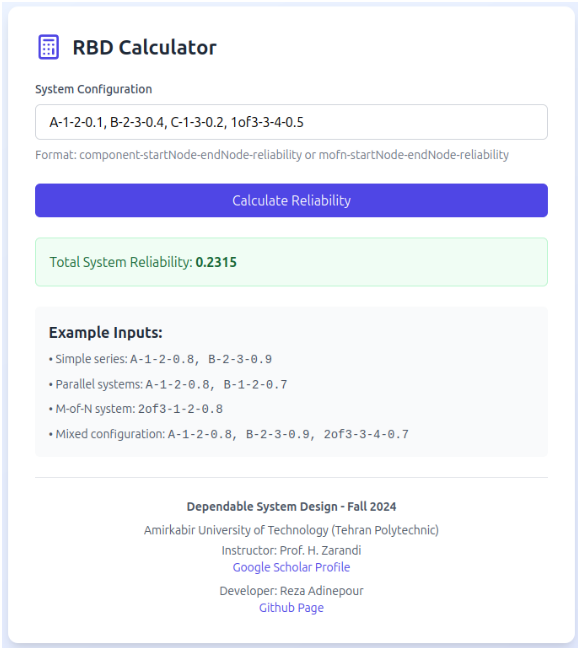

Figure 9: Test of Model 4

4.5 Model 5

If our model is as shown in the following figure:

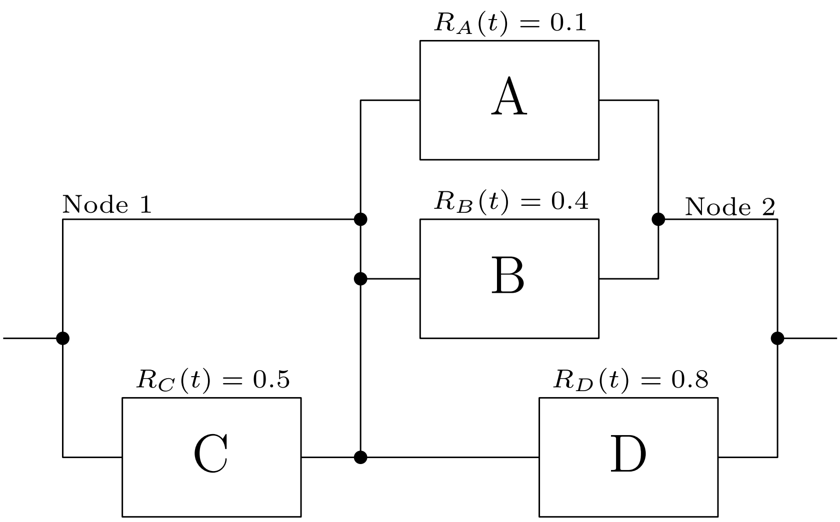

Figure 10: Model 5

It can be theoretically solved as follows:

Problem Setup We are given the following subsystems:

- $A - 1 - 2 - 0.1$: Subsystem A connects node 1 to node 2 with reliability 0.1.

- $B - 1 - 2 - 0.4$: Subsystem B connects node 1 to node 2 with reliability 0.4.
- $C - 1 - 1 - 0.5$: Subsystem C forms a self-loop at node 1 with reliability 0.5.
- $D - 1 - 2 - 0.8$: Subsystem D connects node 1 to node 2 with reliability 0.8.

The system configuration is as follows:

- Subsystems A, B, and D are in **parallel** as they all connect node 1 to node 2.
- Subsystem C is a **self-loop** at node 1, which does not contribute to the reliability between nodes 1 and 2.

Step 1: Parallel Combination of A, B, and D The reliability of parallel subsystems is calculated as:

$$R_{\text{parallel}} = 1 - \prod_{i=1}^{n} (1 - R_i)$$

Here, the components are $R_A = 0.1$, $R_B = 0.4$, and $R_D = 0.8$. Substituting these values:

$$R_{\text{parallel}} = 1 - [(1 - 0.1) \cdot (1 - 0.4) \cdot (1 - 0.8)]$$

Calculate each term:

$$(1 - 0.1) = 0.9, \quad (1 - 0.4) = 0.6, \quad (1 - 0.8) = 0.2$$

$$R_{\text{parallel}} = 1 - [0.9 \cdot 0.6 \cdot 0.2]$$

$$R_{\text{parallel}} = 1 - (0.9 \cdot 0.6 = 0.54, \quad 0.54 \cdot 0.2 = 0.108)$$

$$R_{\text{parallel}} = 1 - 0.108 = 0.892$$

Step 2: Contribution of Subsystem C (Self-Loop) Subsystem C is a self-loop at node 1. Since it does not contribute to the path between nodes 1 and 2, it does not affect the total reliability.

Step 3: Final Answer The total reliability of the system is approximately:

$$\boxed{0.892}$$

If we provide the same input to the application, the output will match the results of our analysis.

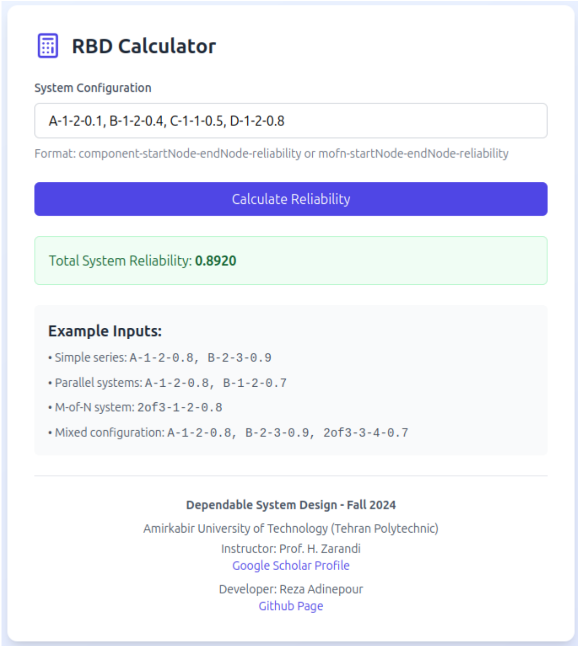

Figure 11: Test of Model 5

5 Codes

You can find all of codes Here.

References

[1] Elena Dubrova. *Fault-Tolerant Design.* Springer, Boston, MA, 2008.