Embedded Systems Design and Modeling

Chapter 4
Hybrid Systems

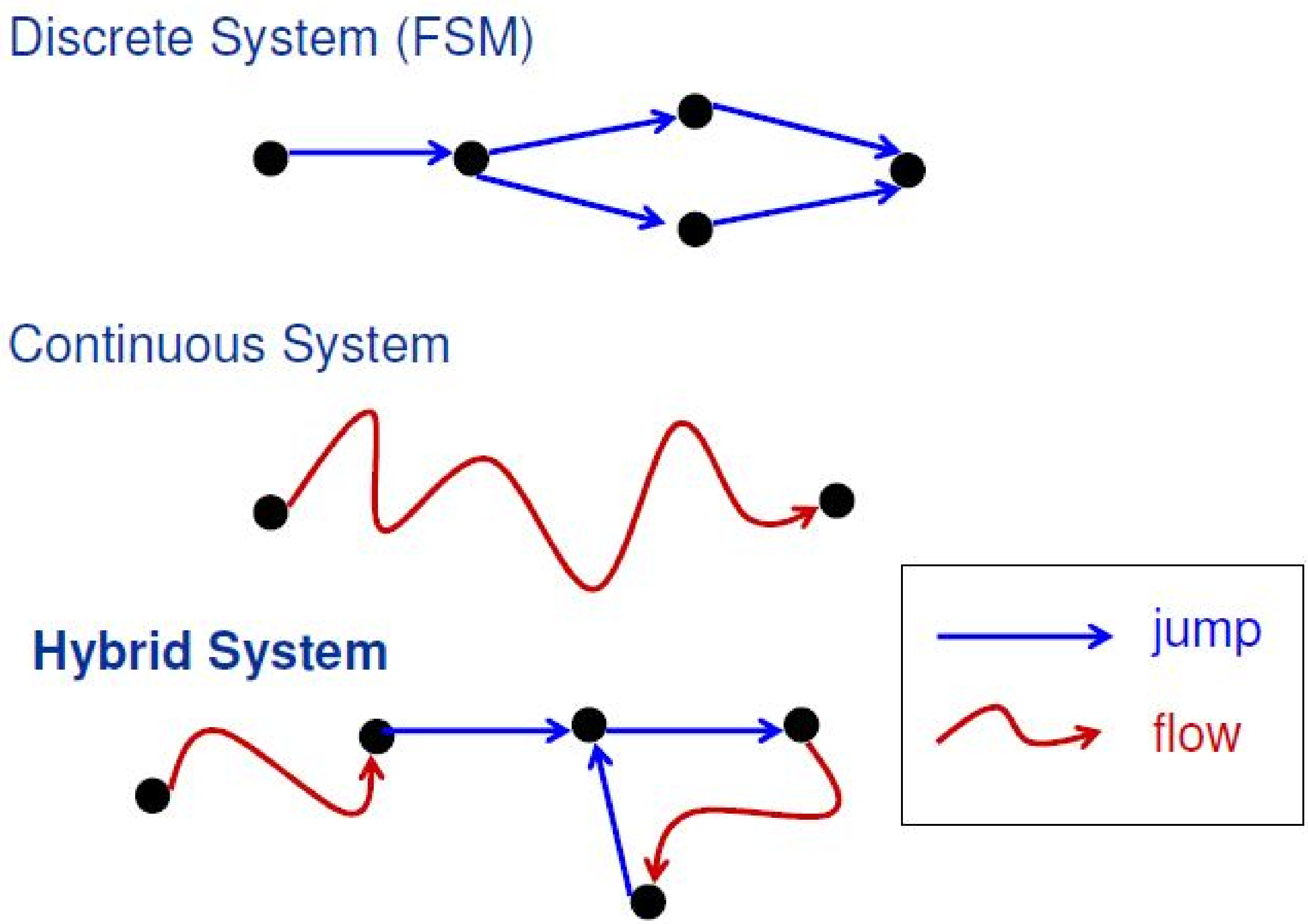
Outline

- Introduction
- Modeling hybrid systems
- Finite state machines as actors
- General form of hybrid systems
- Hybrid systems classes
- Conclusions

Introduction

- Hybrid systems are a bridge between the time-based and state-based models
- Possible cases of hybrid systems:
  - Digital controllers for physical systems such as thermostats, traffic light controllers
  - Phased operation of a natural phenomenon such as a bouncing ball
  - Multi-agent systems where independent systems have to interact with each other such as transportation systems

Visual Example of Hybrid Systems

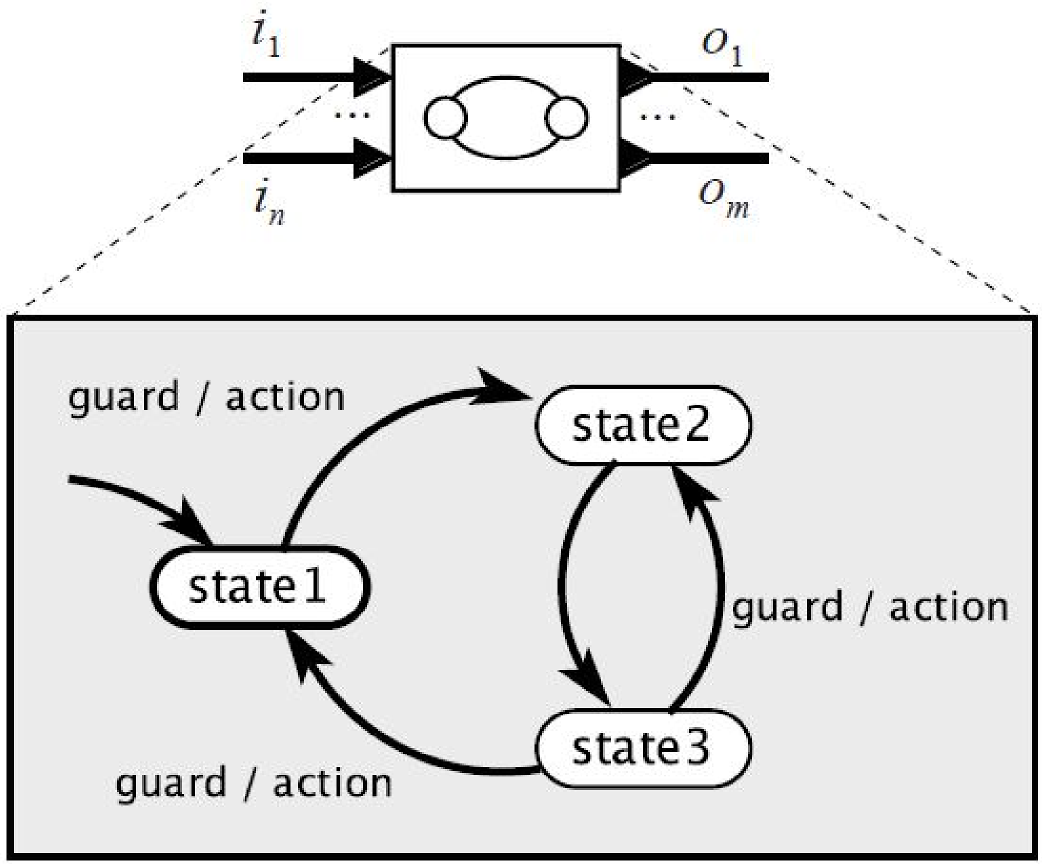


Modeling Hybrid Systems

- One way to model a hybrid system is to use an FSM which describes an overall discrete system but there may be continuous elements
- Example: a thermostat
  - The input is continuous
  - Each state has both continuous and discrete components:
    - The guards are continuous
    - The actions are discrete
    - Transitions to/from states happen <u>instantaneously</u>

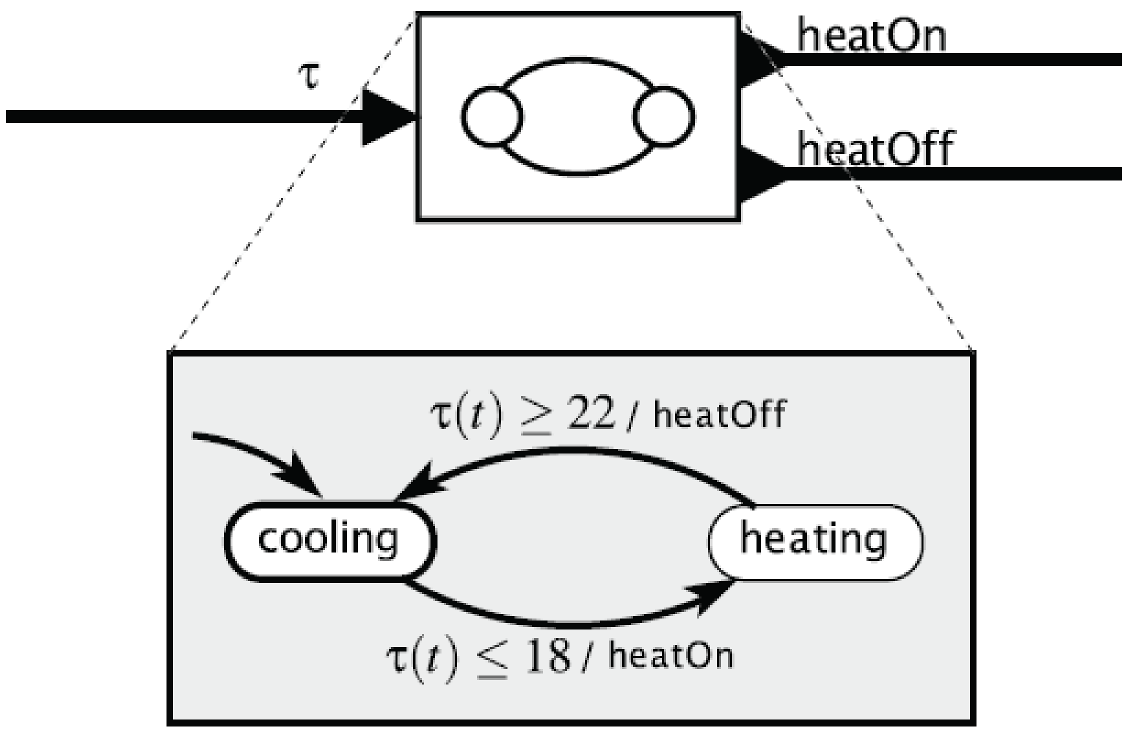
FSM as an Actor

- FSMs can accept continuous inputs and generate continuous outputs
- So they can be viewed as actors
- Between reactions the FSM stutters at the current state
- No need for the inputs and outputs to be absent then

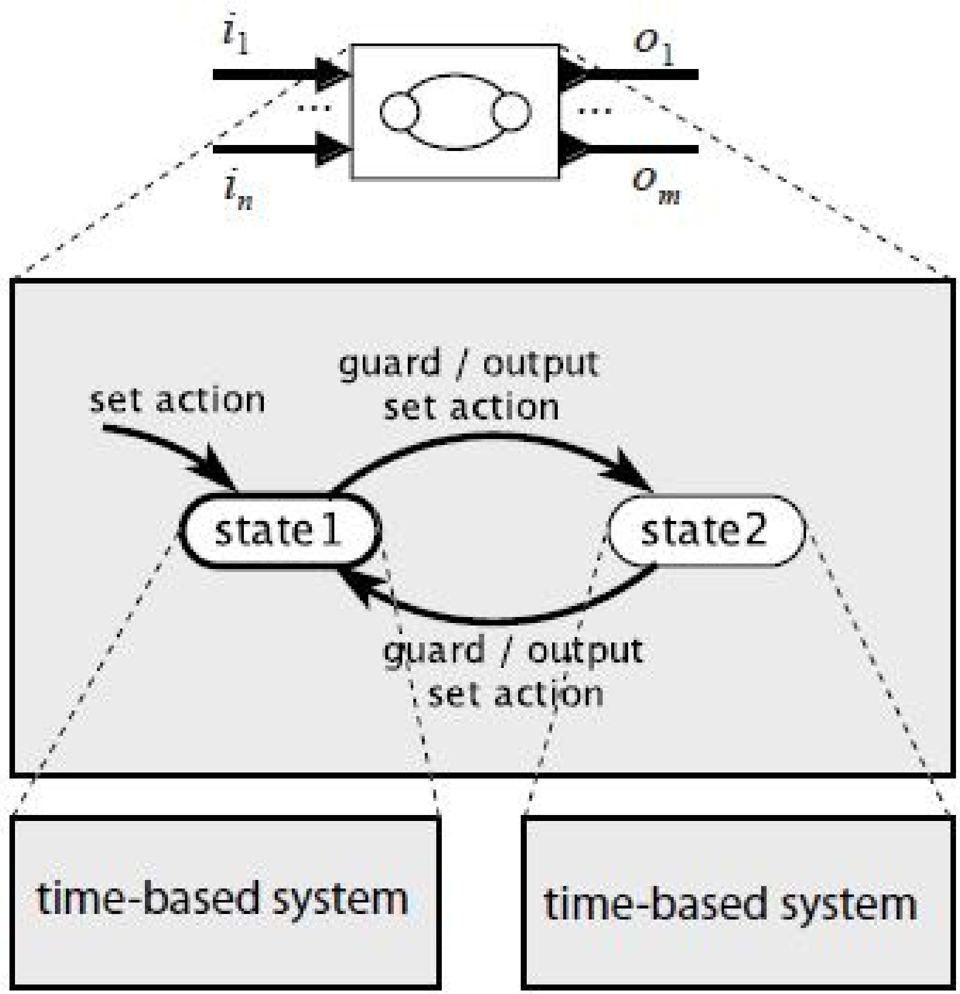


Thermostat Example

- Thermostat described by an FSM:



General Form of A Hybrid System



Hybrid Systems Classes

- Hybrid systems are sometimes called “modal models” because they have several modes of operation
- Classes of hybrid systems:

1. Timed automata
2. Higher order dynamics systems
3. Supervisory control

Automata

1. Timed automata:

- Time-based changes are implemented simply as the passage of time
- A clock with a period "*a*" is modeled by a 1st order differential equation:

$$\forall\, t \in T_m, \quad \dot{s}(t) = a,$$

$s: \mathbb{R} \to \mathbb{R}$ is a continuous-time signal

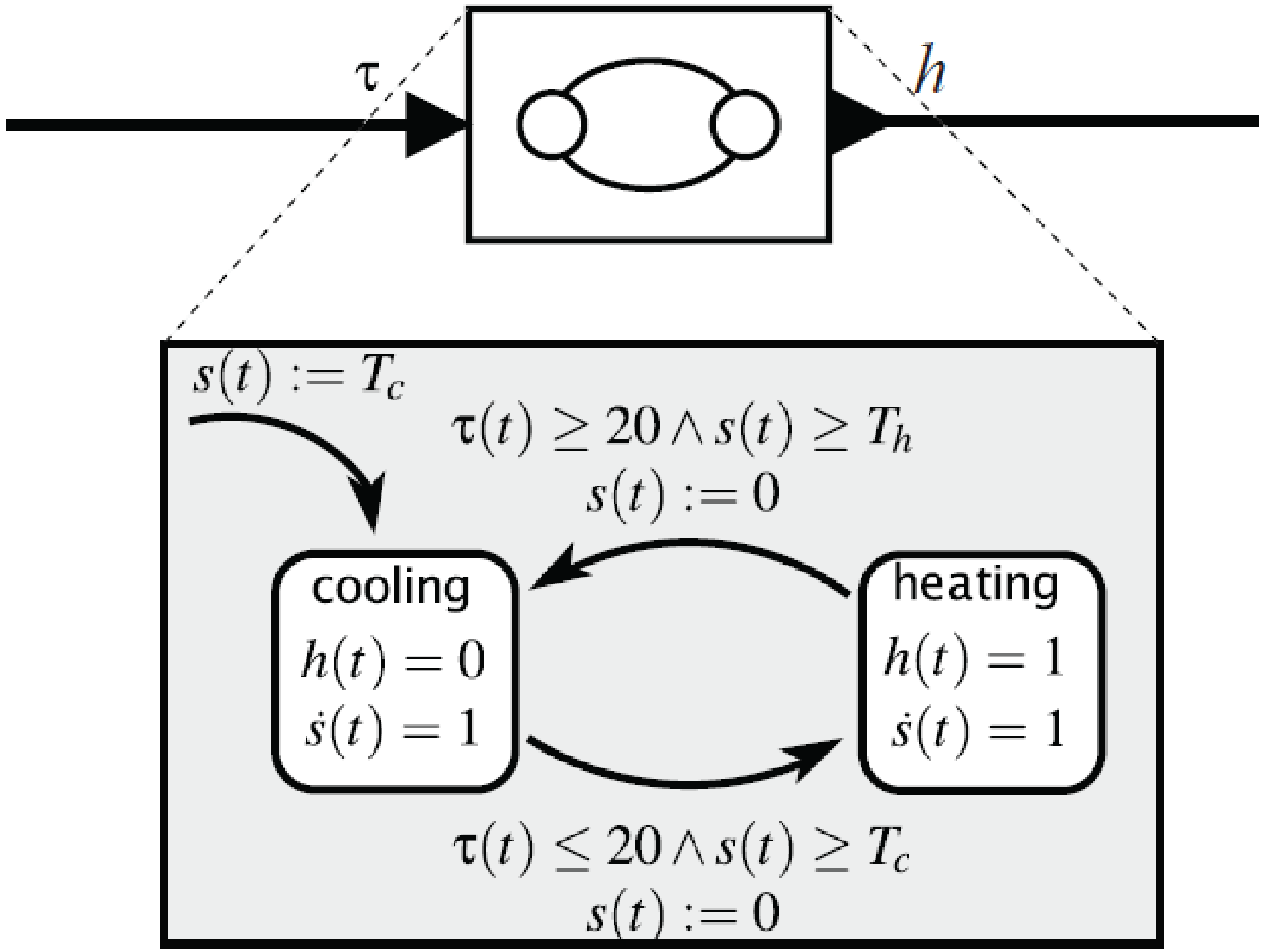
$s(t)$ is the value of the clock at time t.

$T_m \subset \mathbb{R}$ is the subset of time during which the hybrid system is in mode m

Timed Automaton Model of Thermostat

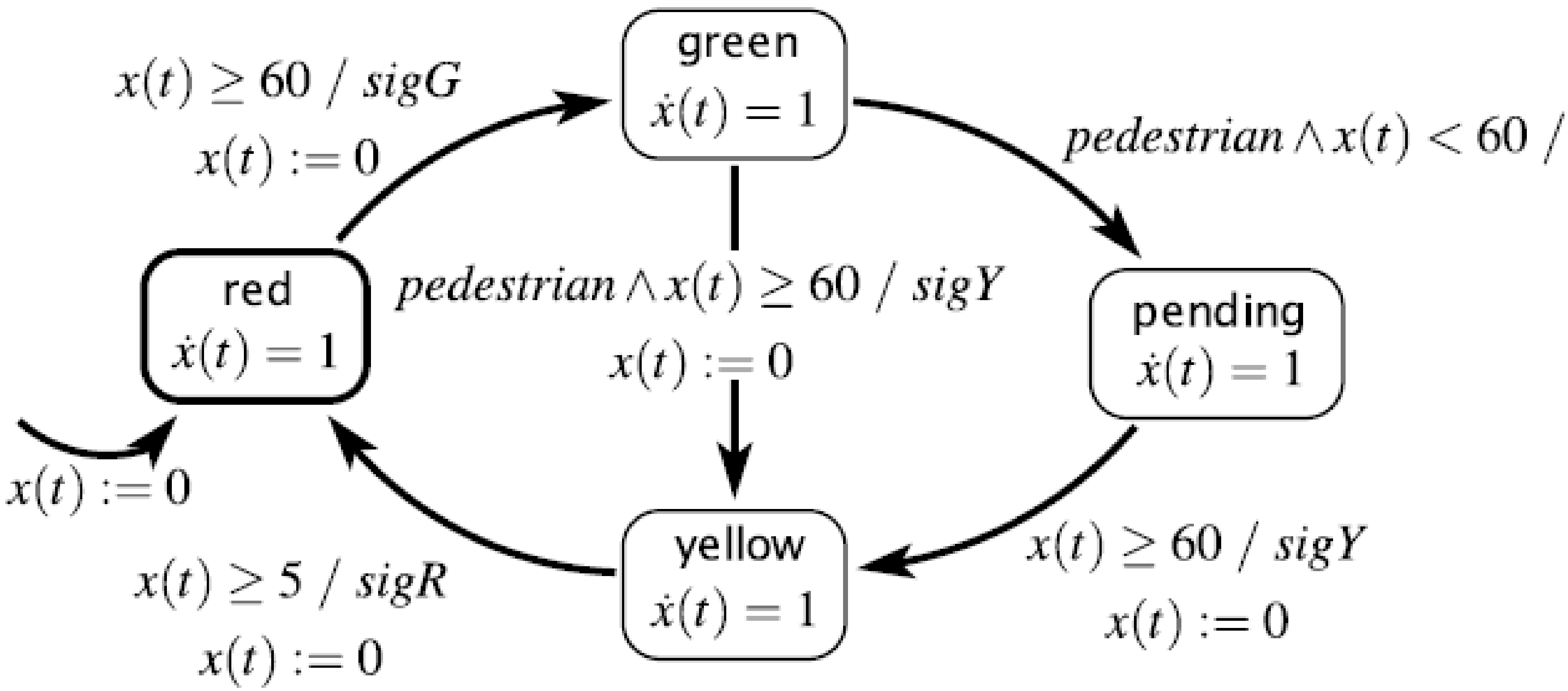
- The continuous-time states are implemented using timed automaton
- Hysteresis implemented using time (compared to the previous example that was done by temperature)
- Both h(t) and s(t) are continuous functions of time
- Transitions occur instantaneously, i.e., they take no time
- Clock rates the same, not always the case

Thermostat by Automata



Traffic Light by Timed Automata

continuous variable: $x(t)$: $\mathbb{R}$
inputs: $pedestrian$: pure
outputs: $sigR, sigG, sigY$: pure



Higher Order Systems

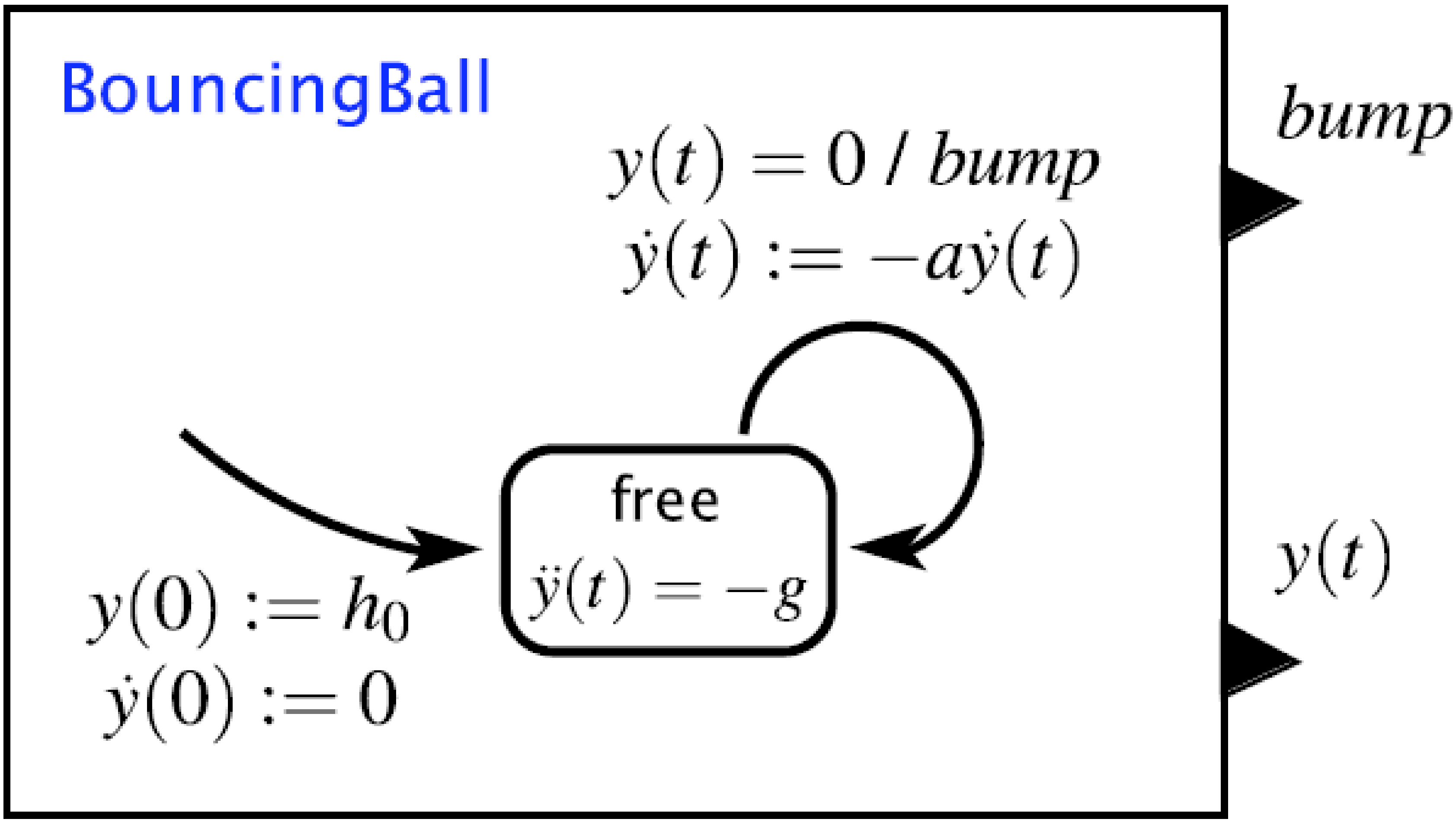
2. Modeling systems with higher order dynamics:

- In timed automata, the continuous states can only represent the passage of time
- Not capable of handling cases that the continuous states are more complex
- Solution: have more complex behavior in the mode(s)

Bouncing Ball Example

- Example: a bouncing ball modeled with hybrid automaton
- At time 0: height = h0, velocity = 0
- Free fall means the acceleration = -g
- When hitting the ground a signal "bump" is produced
- The ball bounces back up to a certain height
- Bouncing factor is "*a*"

Bouncing Ball Actor Model

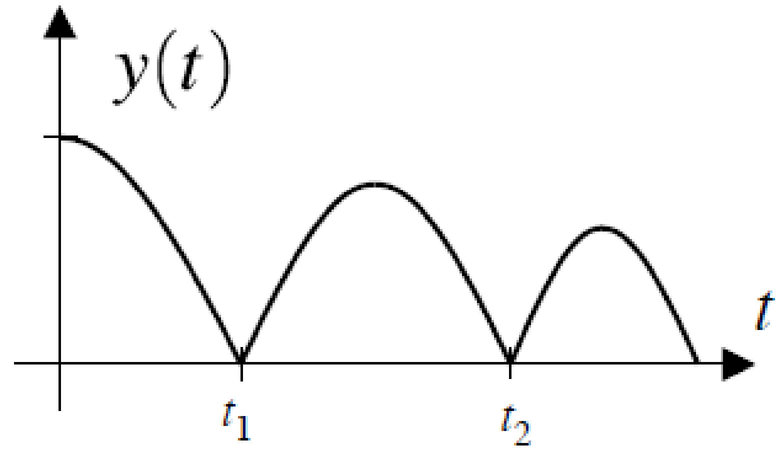


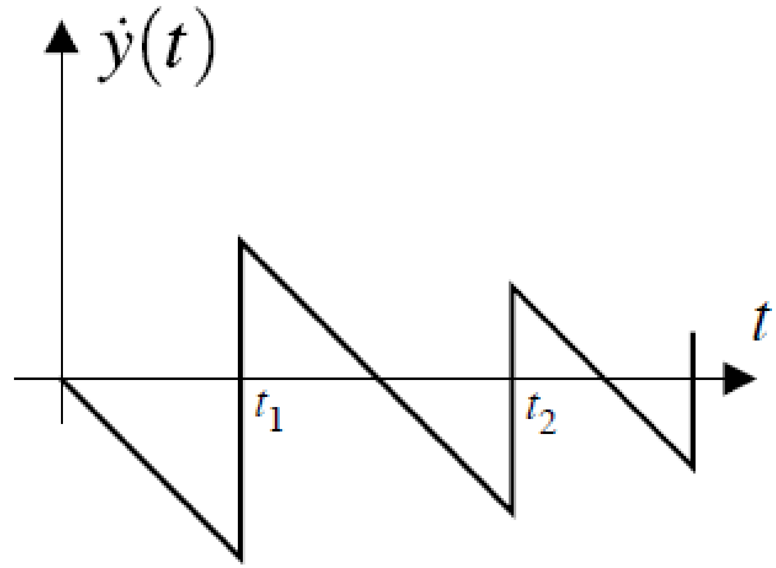
Example Continued

- The actor includes only one state or mode called “free”
- But this mode includes a 2nd order differential equation (not just the passage of time)
- There are two outputs: a signal and a continuous value (height)
- The plot is in the next slide

Height, Velocity Curves

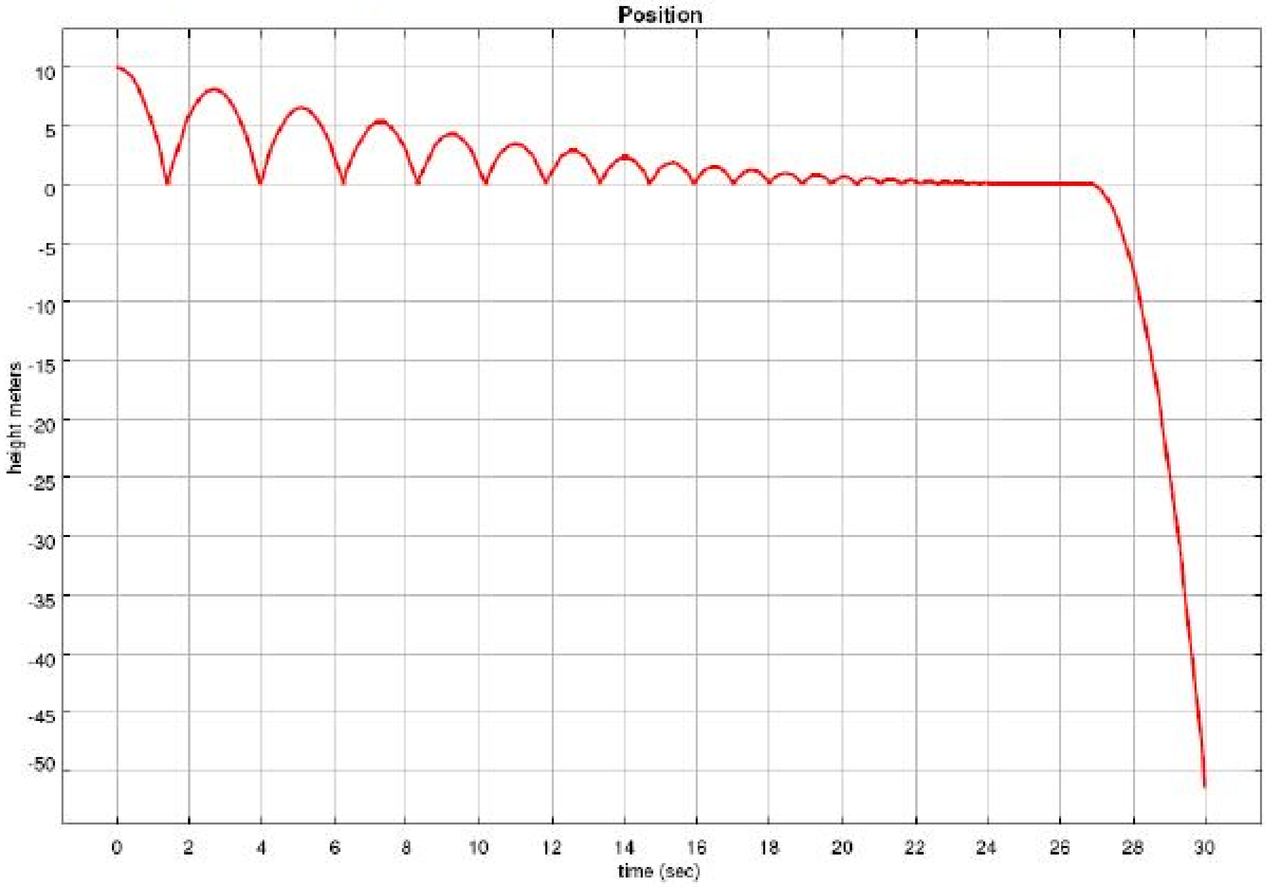
- The height and velocity are drawn over time
- t1 is the first time the ball bounces
- t2 is the second time, so on …





Simulation Results

Simulation of Bouncing Ball Automaton in Ptolemy II / HyVisual

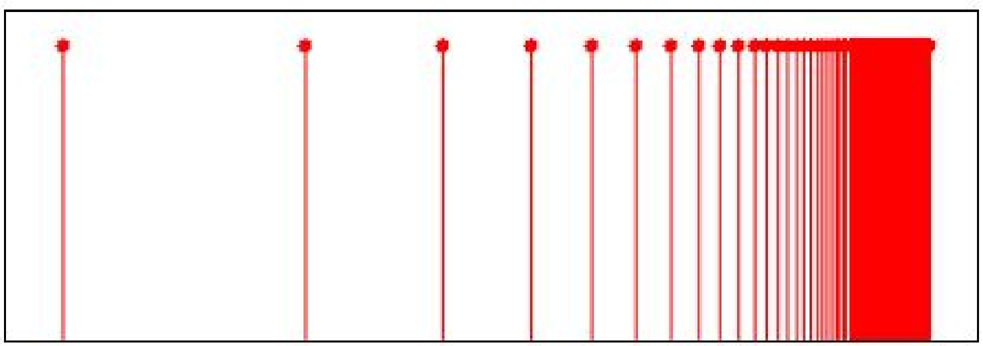


Modeling Artifacts

Zeno Behavior

Informally:

The system makes an infinite number of jumps in finite time



Causes of Artifact(s)

Why does Zeno Behavior Arise?

Our model is a mathematical artifact

Zeno behavior is mathematically possible, but it is infeasible in the real, physical world

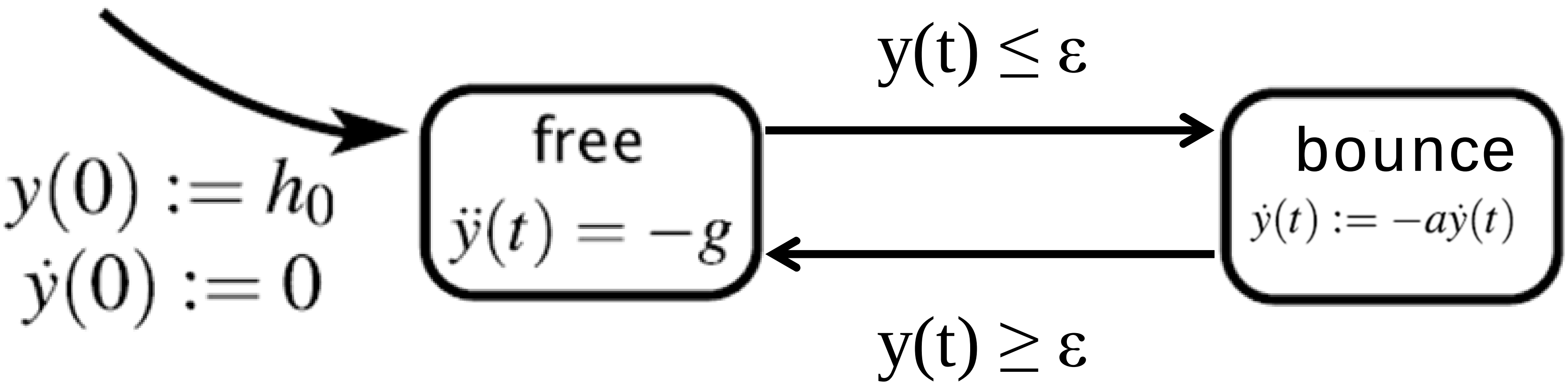
Points to some unrealistic assumption in the model

Modeling Artifacts

- Question: what is unrealistic in this model?
- Answer: that the mode changes back to itself instantaneously
- So to eliminate the Zeno behavior we introduce a second mode (bounce)
- Now the transitions occur from one mode to another and then back to the first one

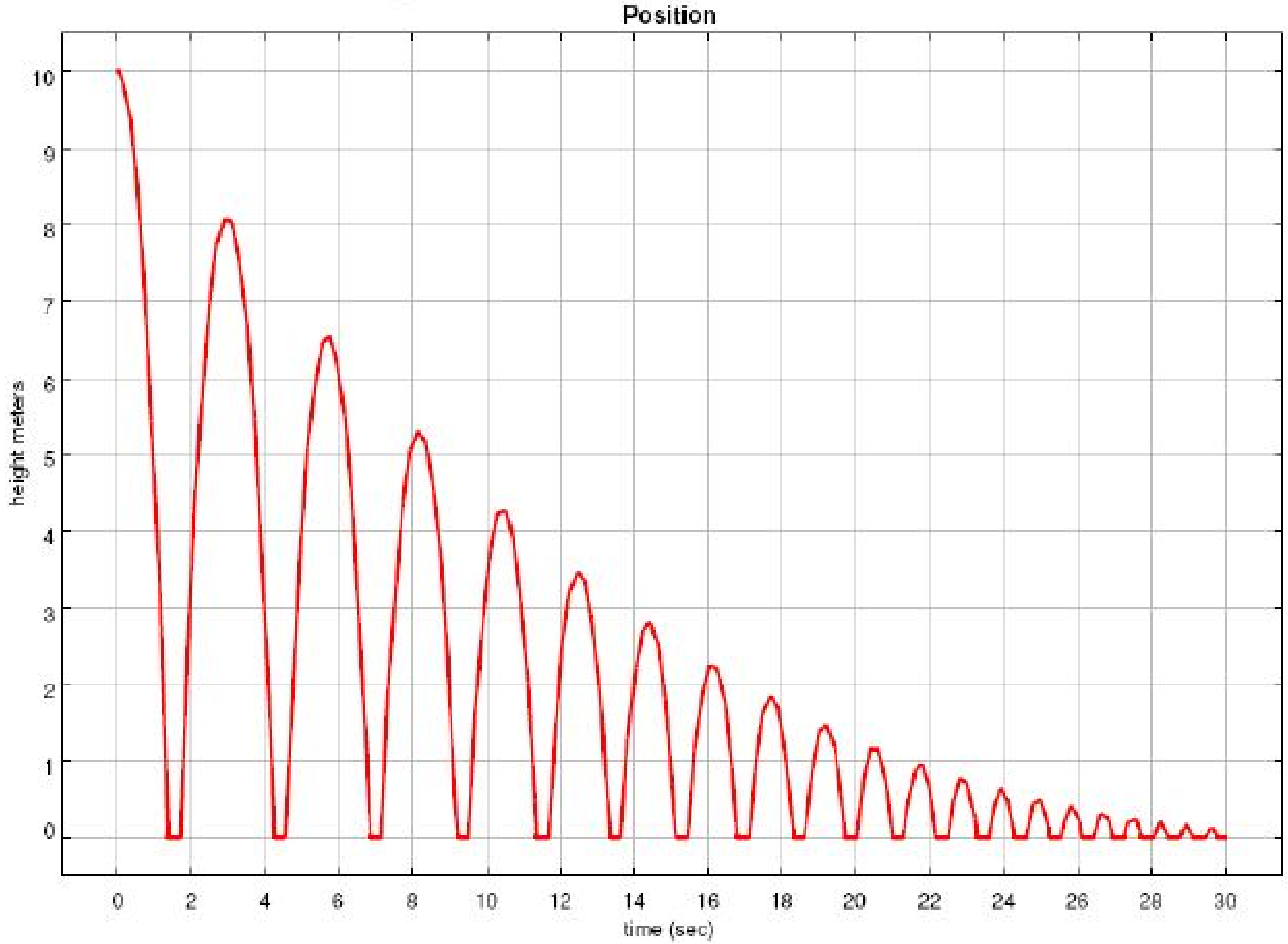
Preventing Zeno Behavior

- Parameter ε is needed to set the guard for the new mode
- What is the effect of ε?



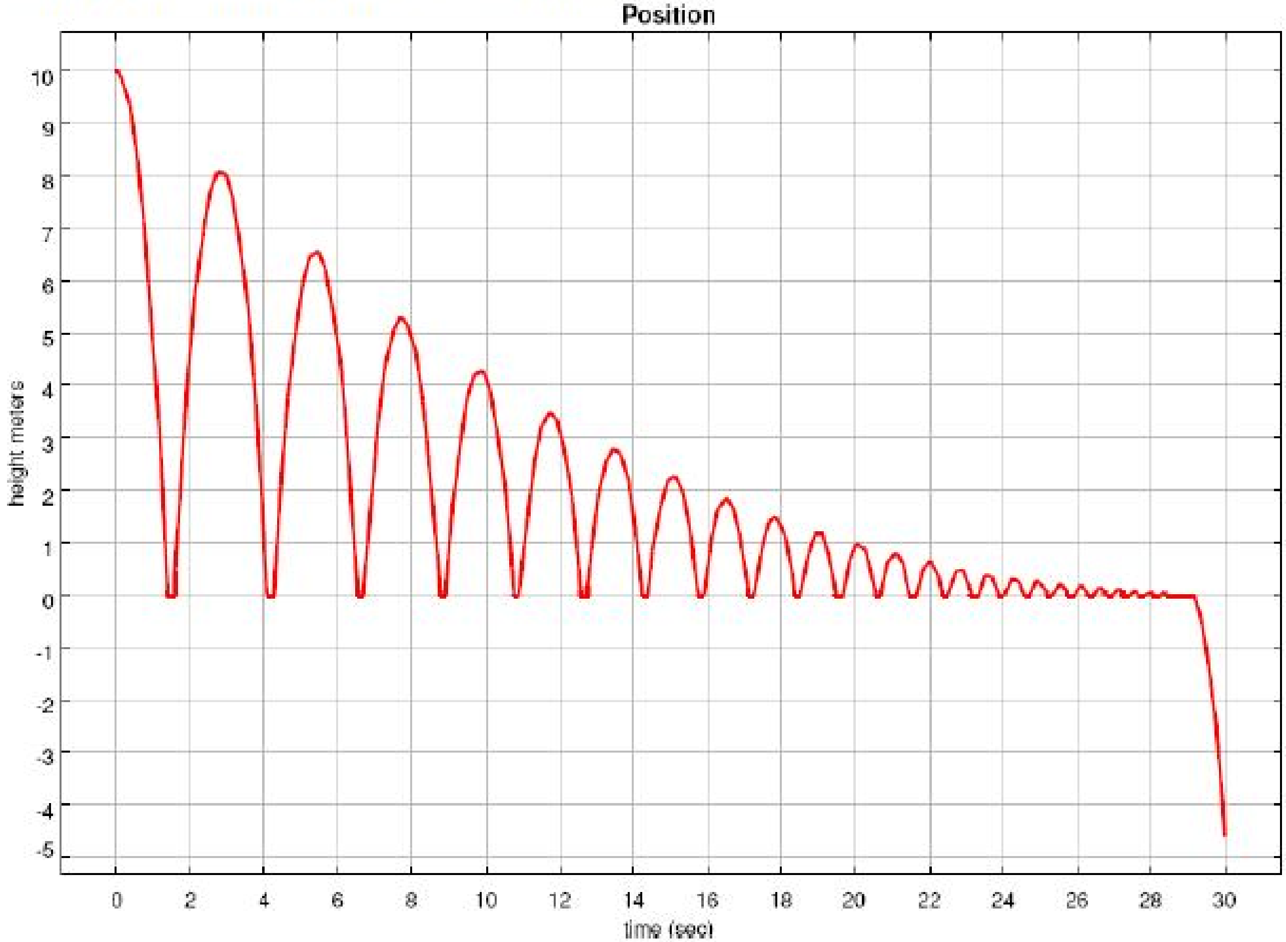
Effect of ε

Simulation for ε = 0.3



Effect of ε

Simulation for ε = 0.15



Thoughts on Artifacts

- Adding the new mode didn't <u>completely</u> solve the problem of Zeno behavior
- But with a reasonable value for ε, the effect can be minimized
- This is usually the case that the models have some unwanted artifacts
- The unwanted artifact usually makes the model correct or accurate under special conditions and not accurate without those conditions

Supervisory Control

3. Modeling systems with supervisory control:

- A control system has 4 components:
  - The physical system to be controlled
  - The environment
  - The sensors
  - The controller which itself has two levels:
    - Supervisory control level that determines the mode transitions
    - Low level control that determines the time-based inputs

Supervisory Control (Cont’d)

- It is ideal to use the hybrid systems to model these cases:
  - The higher level is used to model the overall control strategy by showing each main event in form of a mode
  - The detailed implementation of the strategies are modeled in each mode
- In these systems, the modes usually contain continuous-time descriptions
- The guards and actions are discrete-time

Conclusions

- Cyber-physical systems require modeling of the continuous time along with the discrete computational aspects
- This makes the hybrid systems an integral part of embedded systems modeling
- FSMs are one of the main modeling methods
- There are three classes of hybrid systems depending on the complexity of the modes

Homework Assignments

- Chapter 4:
  - 1, 2, 5, 7, 9, 10, 11, 12
  - The rest: optional
  - For Sunday 1403/1/19