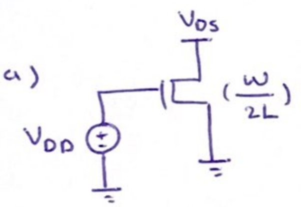
In the name of god

2.2:

Reza Adinepour
402131055

$$I_{DS}: \begin{cases} 0 & ; \quad V_{gs} < V_{th} \quad (\text{cutoff}) \\ \beta\left(V_{gs}-V_{th}-\frac{V_{ds}}{2}\right)V_{ds} & : \quad V_{ds} < V_{ds,sat} \quad (\text{Linear}) \\ \frac{\beta}{2}\left(V_{gs}-V_{th}\right)^2 & ; \quad V_{ds} > V_{ds,sat} \quad (\text{Saturation}) \end{cases}$$

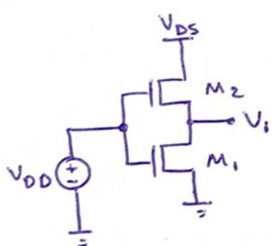
a)



$\Rightarrow$ in This MOS I have $\begin{cases} V_{gs} = V_{DD} \\ V_{ds} = V_{DS} \end{cases}$

So, $I_{DS(a)} = K\frac{W}{2L}\left(V_{DD}-V_{th}-\frac{V_{DS}}{2}\right)V_{DS}$

b)



$\Rightarrow$ for M_1 MOSFET : $\begin{cases} V_{gs(M_1)} = V_{DD} \\ V_{ds(M_1)} = V_1 \end{cases}$

for M_2 MOSFET : $\begin{cases} V_{gs(M_2)} = V_{DD} - V_1 \\ V_{ds(M_2)} = V_{DS} - V_1 \end{cases}$

So, I know $I_{DS(M_1)} = I_{DS(M_2)}$:

$$\cancel{\frac{KW}{L}}\left[V_{DD}-V_{th}-\frac{V_1}{2}\right]V_1 = \cancel{\frac{KW}{L}}\left[(V_{DD}-V_1)-V_{th}-\frac{1}{2}(V_{DS}-V_1)\right](V_{DS}-V_1)$$

$$\Rightarrow V_1 = (V_{DD}-V_{th}) - \left[(V_{DD}-V_{th})^2 - \left(V_{DD}-\frac{1}{2}V_{DS}-V_{th}\right)V_{DS}\right]^{\frac{1}{2}} \qquad (I)$$

Thus: $I_{DS(M_1)} = K\frac{W}{L}\left(V_{DD}-V_{th}-\frac{V_1}{2}\right)V_1 \xrightarrow{(I)}$

$$I_{DS(M_1)} = K\frac{W}{L}\left[V_{DD}-V_{th}-\frac{1}{2}\left((V_{DD}-V_{th})-\left(V_{DD}-\frac{1}{2}V_{DS}-V_{th}\right)V_{DS}\right)^{\frac{1}{2}}\right] \times V_1$$

$$\Rightarrow I_{DS(M_1)} = k\frac{W}{L}\Big[V_{DD}-V_{th}\ldots$$

$$-\frac{1}{2}\left((V_{DD}-V_{th})-\left(V_{DD}-\frac{1}{2}V_{DS}-V_{th}\right)V_{DS}\right)^{\frac{1}{2}}\times\left((V_{DD}-V_{th})-\left[(V_{DD}-V_{th})^2-\left(V_{DD}-\frac{1}{2}V_{DS}-V_{th}\right)\times V_{DS}\right]^{\frac{1}{2}}\right)$$

$$\Rightarrow I_{DS(M_2)} = k\frac{W}{L}\Big[V_{DD}-\left((V_{DD}-V_{th})-\left((V_{DD}-V_{th})^2-\left(V_{DD}-\frac{1}{2}V_{DS}-V_{th}\right)V_{DS}\right)^{\frac{1}{2}}\right)$$

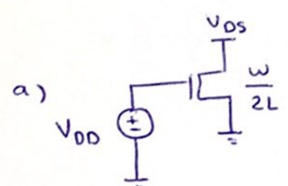
$$-V_{th}-\frac{1}{2}\left(V_{DS}-\left((V_{DD}-V_{th})-\left((V_{DD}-V_{th})^2-\left(V_{DD}-\frac{1}{2}V_{DS}-V_{th}\right)V_{DS}\right)^{\frac{1}{2}}\right)\right)$$

$$\times\left(V_{DS}-\left((V_{DD}-V_{th})-\left((V_{DD}-V_{th})^2-\left(V_{DD}-\frac{1}{2}V_{DS}-V_{th}\right)V_{DS}\right)^{\frac{1}{2}}\right)\right)$$

after simplifing $\Rightarrow I_{DS(M_1)} = I_{DS(M_2)} = 2\,I_{DS(a)}$

2.3 : Consider body effect and calculate $I_{DS_1} = I_{DS_2}$ or $I_{DS_1} > I_{DS_2}$ or $I_{DS_1} < I_{DS_2}$

a)



in this section I know : $\begin{cases} V_{gs} = V_{DD} \\ V_{ds} = V_{DS} \\ V_{sb} = V_s - V_b = -V_b \end{cases}$

$\longrightarrow$ if body and source are short circuit from the inside of transistor, we have : $\underline{V_{sb} = 0}$

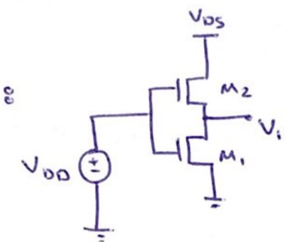
** <u>Assumption</u> ** in this question we assume that the source and the body of the transistor are short circuit.

so : $V_t = V_{to} + \gamma\left(\sqrt{\varphi_s + V_{sb}} - \sqrt{\varphi_s}\right) \xrightarrow{V_{sb(M_1)}=0} V_t = V_{to} + \gamma\left(\sqrt{\varphi_s + 0} - \sqrt{\varphi_s}\right)$

$\Rightarrow V_t = V_{to} = V_{th} \longrightarrow$ no changed threshold Voltage for M_1 transistor

so the current no changing.

but for section <u>b</u> :



$M_1 \begin{cases} V_{gs} = V_{DD} \\ V_{ds} = V_1 \\ V_{sb} = V_s - V_b = 0 \end{cases}$ $\rightarrow$ no changed I_s

$M_2 \begin{cases} V_{gs} = V_{DD} - V_{DS} \\ V_{ds} = V_1 - V_{DS} \\ V_{sb} = V_{DS} - V_b > 0 \end{cases}$

in M_2 transistor V_{sb} voltag is changed and the current changed as follows :

$$I_{DS_2} = \frac{KW}{L}\left[(V_{DD} - V_{DS}) - V_{to} - \gamma\left(\sqrt{\varphi_s + V_{DS} - V_b} - \sqrt{\varphi_s}\right) - \frac{1}{2}(V_1 - V_{DS})\right](V_1 - V_{DS})$$

from the previous part I know : $I_{DS(a)} = K\frac{W}{2L}\left(V_{DD} - V_{th} - \frac{V_{DS}}{2}\right)V_{DS}$

$\Rightarrow$ So, Considering the body effect, the threshold Voltage has Increase for M_2 transistor and the I_{DS} decreases $\Rightarrow I_{DS_2} < I_{DS_1}$

<u>2.6</u> : prove this equation : $V_{dsat} = \dfrac{V_{GT} V_c}{V_{GT} + V_c}$

I know : $I_{ds} = \begin{cases} \dfrac{\mu_{eff}}{1 + \frac{V_{ds}}{V_c}} C_{ox}\dfrac{W}{L}\left(V_{GT} - \dfrac{V_{ds}}{2}\right)V_{ds} & ; \quad V_{ds} < V_{d,sat} \quad \text{(Linear)} \\ C_{ox}W\left(V_{GT} - V_{d,sat}\right)\upsilon_{sat} & ; \quad V_{ds} > V_{d,sat} \quad \text{(Saturation)} \end{cases}$

in $V_{ds} = V_{ds,sat}$ two part of equation is equal. So I replace V_{ds} with $V_{ds,sat}$ in both and I make them equal :

$$I_{ds} = \begin{cases} \dfrac{\mu_{eff}}{1+\dfrac{V_{ds,sat}}{V_c}} C_{ox}\dfrac{W}{L}\left(V_{GT}-\dfrac{V_{ds,sat}}{2}\right)V_{ds,sat} \\ \\ C_{ox}W\left(V_{GT}-V_{d,sat}\right)\upsilon_{sat} \end{cases}$$

$$\Rightarrow \cancel{C_{ox}}\cancel{W}\left(V_{GT}-V_{d,sat}\right)\upsilon_{sat} = \frac{\mu_{eff}}{1+\frac{V_{d,sat}}{V_c}}\cancel{C_{ox}}\frac{\cancel{W}}{L}\left(V_{GT}-\frac{V_{d,sat}}{2}\right)V_{d,sat}$$

$$\Rightarrow \left(V_{GT}-V_{d,sat}\right)\upsilon_{sat} = \frac{\mu_{eff}}{L\left(1+\frac{V_{d,sat}}{V_c}\right)}\left(V_{GT}-\frac{V_{d,sat}}{2}\right)V_{d,sat}$$

$$\Rightarrow -V_{sat}V_{d,sat} = -V_{sat}V_{GT} + \frac{\mu_{eff}}{L\left(1+\frac{V_{d,sat}}{V_c}\right)}\left(V_{GT}-\frac{V_{d,sat}}{2}\right)V_{d,sat}$$

$$\Rightarrow \left(V_{GT}-V_{d,sat}\right)\upsilon_{sat} = V_{GT}V_{dsat}\times\frac{V_c\,\mu_{eff}}{L\left(V_c+V_{d,sat}\right)} - \frac{V_{d,sat}^2}{2}\times\frac{V_c\,\mu_{eff}}{L\left(V_c+V_{d,sat}\right)}$$

after solve this second order equation we can find : $V_{dsat} = \dfrac{V_{GT}V_c}{V_{GT}+V_c}$

2.7 :

$$\begin{cases} \text{Process} = 0.6\,\mu m \\ \text{gate oxide thickness} = 100\,\mathring{A} = 100\times10^{-8}\,m \\ N_A = 2\times10^{17}\,cm^{-3} \\ V_{th_0} = 0.7^V,\ V_{sb} = 4 \end{cases} \longrightarrow \text{if } V_{sb}=0 \longrightarrow V_{th} = 0.7 \quad \underline{\text{No changed}}$$

in first I should calculate surface potential at threshold :

$$\varphi_s = 2U_{thermal}\ln\left(\frac{N_A}{n_i}\right) \xrightarrow[n_i=1.45\times10^{10}]{V_{thermal}=0.026^V} \varphi_s = 2\times0.026\times\ln\left(\frac{2\times10^{17}}{1.45\times10^{10}}\right) = \boxed{0.854^V}$$

in this part I want calculate body effect coefficient :

$$\gamma = \frac{t_{ox}}{\varepsilon_{ox}}\sqrt{2q\,\varepsilon_{si}N_A} = \frac{\sqrt{2q\,\varepsilon_{si}N_A}}{C_{ox}} = \frac{100\times10^{-8}}{3.9\times8.85\times10^{-14}}\times\sqrt{2\times1.6\times10^{-19}\times11.7\times8.85\times10^{-14}\times2\times10^{17}}$$

$= 0.745$

$$\Rightarrow V_{th} = V_{th_0} + \gamma\left(\sqrt{\varphi_s+V_{sb}} - \sqrt{\varphi_s}\right) = 0.7 + 0.745\left(\sqrt{0.854+4} - \sqrt{0.854}\right) = \boxed{1.65^V}$$

2.8 : No, any number of transistors may be ~~rep~~ placed in series Although the delay increases with the square of the number of series transistor.

2.9 : The threshold is increased by applying a negative body voltage so $V_{sb} > 0$

2.10 : $\begin{cases} V_{th} = 0.4^{v} \\ V_{DD} = 1.2^{v} \end{cases}$

a) $\dfrac{(V_{DD} - 0.3)^2}{(V_{DD} - 0.4)^2} = \dfrac{(1.2 - 0.3)^2}{(1.2 - 0.4)^2} = 1.26$

b) $e^{\frac{-V_{th}}{n V_{thermal}}} = \dfrac{e^{\frac{-0.3}{1.4 \times 0.026}}}{e^{\frac{-0.4}{1.4 \times 0.026}}} = 15.6$

c) $V_{Thermal} = \dfrac{K_B T}{q} = \dfrac{1.38 \times 10^{-23} \times 393}{1.6 \times 10^{-19}} = \boxed{34^{mv}}$ $\Rightarrow \dfrac{e^{\frac{-0.3}{1.4 \times 0.034}}}{e^{\frac{-0.4}{1.4 \times 0.034}}} = 8.2$

$120^{\circ}C \longrightarrow 393\ K$

$K_B = 1.381 \times 10^{-23} \frac{J}{K} = 8.617 \times 10^{-5} \frac{ev}{K}$

* The total leakage will normaly be higher for both threshold voltages at high temperature