Embedded Systems

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Homework 5

Chapter 6 - Concurrent Models of Computation

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Question 1

Show how each of the following actor models can be transformed into a feedback system by using a reorganization similar to that in Figure 6.1(b). That is, the actors should be aggregated into a single side-by-side composite actor.

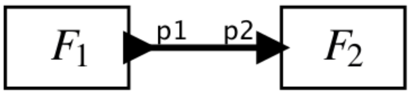
1. 

Figure 1: Actor models of question 1 (a)

Solution

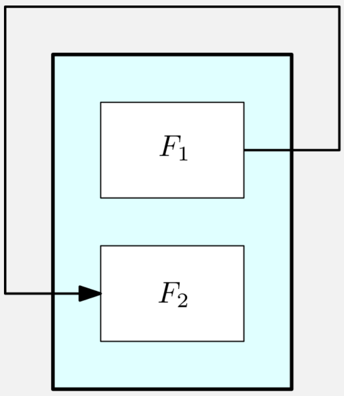


Figure 2: Equivalent system with feedback

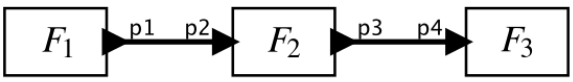
2. 

Figure 3: Actor models of question 1 (b)

Solution

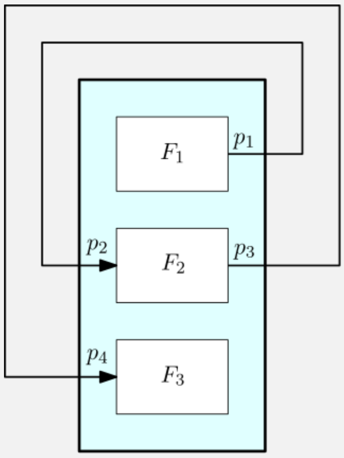


Figure 4: Equivalent system with feedback

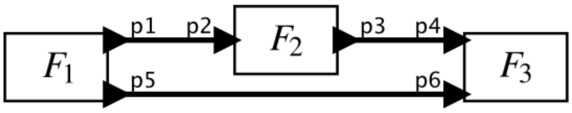
3. 

Figure 5: Actor models of question 1 (c)

Solution

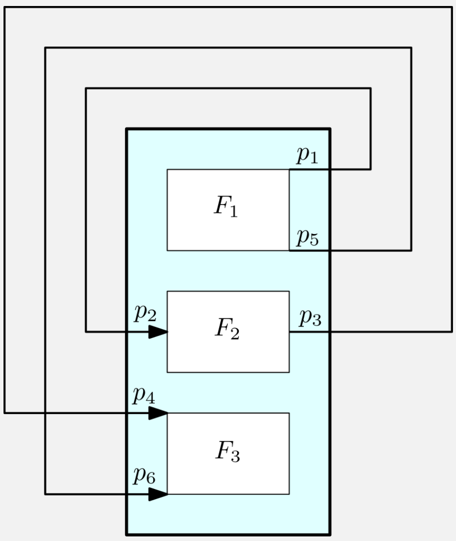


Figure 6: Equivalent system with feedback

Question 2

Consider the following state machine in a synchronous feedback composition:

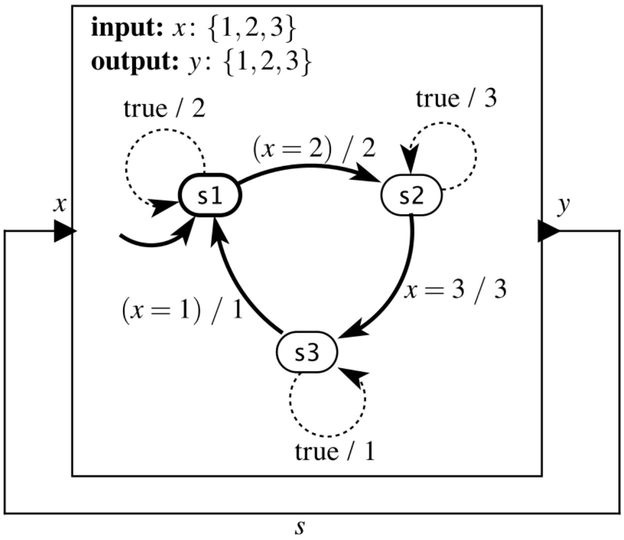


Figure 7: State machine of question 2

1. Is it well-formed? Is it constructive?

Solution

Yes, It is well-formed and constructive because the output can be determined in each state, even when the input is unknown.

2. If it is well formed and constructive, then find the output symbols for the first 10 reactions. If not, explain where the problem is.

Solution

The output sequence for the first 10 reactions is:

$$(2, 3, 1, 2, 3, 1, 2, 3, 1, 2)$$

3. Show the composition machine, assuming that the composition has no input and that the only output is y.

Solution

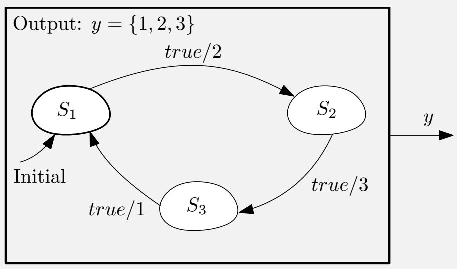


Figure 8: Composition machine of (c)

Question 3

For the following synchronous model, determine whether it is well formed and constructive, and if so, determine the sequence of values of the signals S_1 and S_2 .

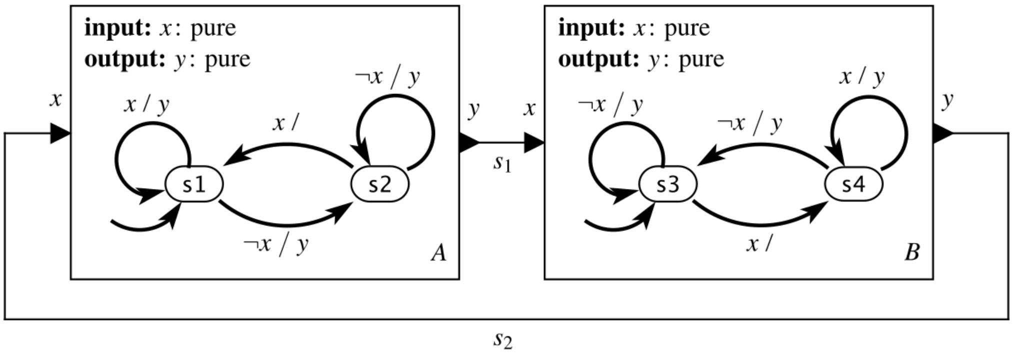


Figure 9: Model of question 3

Solution

It obviously seen that this system is well-formed because each states have one fixed point. In the case of constructivists, we have:

State (s1, s3):
with X = *present* → s1 = Y *present* → s2 = Y *absent*
with X = *absent* → s1 = Y *present* → s2 = Y *absent*

State (s1, s4):
with X = *present* → s1 = Y *present* → s2 = Y *present*
with X = *absent* → s1 = Y *present* → s2 = Y *present*

State (s2, s3):
with X = *present* → s1 = Y *absent* → s2 = Y *present*
with X = *absent* → s1 = Y *present* → s2 = Y *absent*

State (s2, s4):
with X = *present* → s1 = Y *absent* → s2 = Y *present*
with X = *absent* → s1 = Y *present* → s2 = Y *present*

Question 4

For the following synchronous model, determine whether it is well formed and constructive, and if so, determine the possible sequences of values of the signals s_1 and s_2 . Note that machine A is nondeterministic.

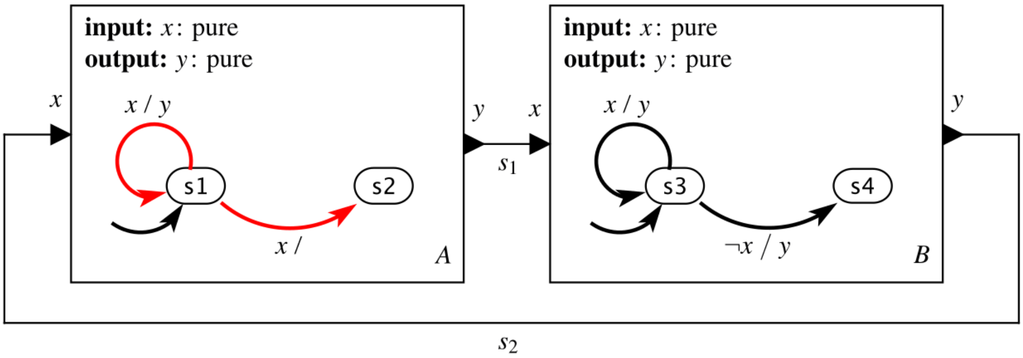


Figure 10: Model of question 4

Solution

Suppose that we are in state S_2 then with each input we don't have any clear output and this leads to this model be unconstructive and subsequently it is ill-formed.
Also, it can be seen that, with nondeterministic behavior this model will not be wel formed.

Question 8

Consider the following SDF model:

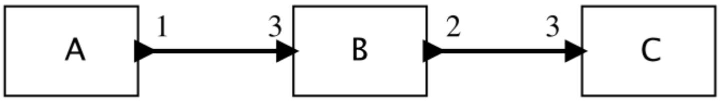


Figure 11: SDF of question 8

The numbers adjacent to the ports indicate the number of tokens produced or consumed by the actor when it fires. Answer the following questions about this model.

1. Let q_A , q_B , and q_c denote the number of firings of actors **A**, **B**, and **C**, respectively. Write down the balance equations and find the least positive integer solution.

Solution

$$\begin{aligned} 1q_A &= 3q_B \\ 2q_B &= 3q_C \\ \rightarrow \begin{cases} 2q_A = 6q_B \\ 6q_B = 9q_C \end{cases} \rightarrow 2q_A &= 9q_C \rightarrow \begin{cases} q_A = 9 \\ q_B = 3 \\ q_C = 2 \end{cases} \end{aligned}$$

2. Find a schedule for an unbounded execution that minimizes the buffer sizes on the two communication channels. What is the resulting size of the buffers?

Solution

The sequence of actions is as like as below:

$$(A, A, A, B, A, A, A, B, C, A, A, A, B, C)$$

Size of buffer between A and B is 3 and the buffer size of between B and C is 4.

Question 9

For each of the following dataflow models, determine whether there is an unbounded execution with bounded buffers. If there is, determine the minimum buffer size.

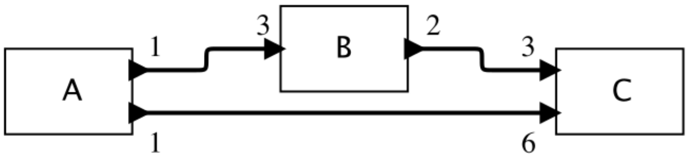
1. 

Figure 12: Dataflow models of question 9 (a)

Solution

$$\begin{cases} q_A = 3q_B \\ 2q_B = 3q_C \\ q_A = 6q_C \end{cases} \rightarrow \begin{cases} q_A = 3q_B \\ 18q_A = 2q_B \end{cases} \rightarrow 10q_A = 4q_B \rightarrow \begin{cases} q_A = 4 \\ q_B = 10 \end{cases} \rightarrow 4 = 6q_C$$

This dataflow model has not unbounded execution with bounded buffers. because of unusual result.

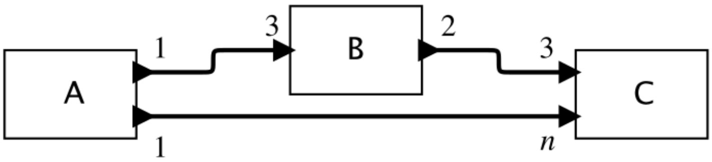
2. 

Figure 13: Dataflow models of question 9 (b)

where n is some integer.

Solution

$$\begin{cases} q_A = 3q_B \\ 2q_B = 3q_C \\ q_A = nq_C \end{cases} \rightarrow \frac{2}{3}q_A = 3q_C \rightarrow 2q_A = q_C \rightarrow \begin{cases} q_A = 1 \\ q_C = 2 \end{cases} \rightarrow q_A = 3q_B \rightarrow q_B = \frac{1}{3}$$

This dataflow model has not an unbounded execution because of fraction result

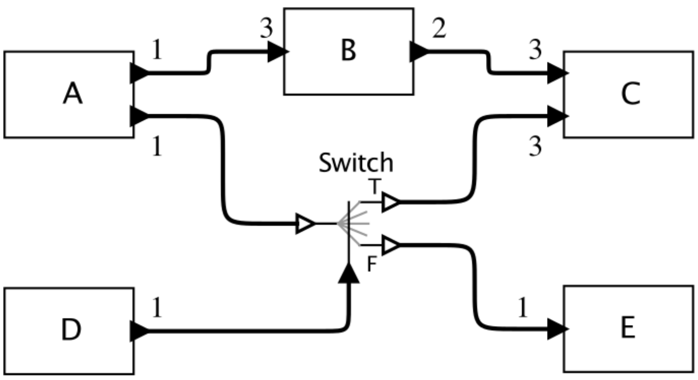
3. 

Figure 14: Dataflow models of question 9 (c)

where **D** produces an arbitrary boolean sequence.

Solution

$$\begin{cases} q_A = 3q_B \\ 2q_B = 3q_C \\ q_A = 3q_C \\ q_A = q_E \end{cases} \rightarrow \begin{cases} q_A = 3q_B \\ 2q_B = q_A \end{cases} \rightarrow 2q_B = 3q_B \rightarrow q_B = 0$$

This system has not unbounded execution because of 0 result

4. For the same dataflow model as in part (c), assume you can specify a periodic boolean output sequence produced by **D**. Find such a sequence that yields bounded buffers, give a schedule that minimizes buffer sizes, and give the buffer sizes.

Solution

Let D produce following sequence:

$$(true, true, true, false, false, false, true, true, true)$$

Let the schedule be:

$$A, D, S, A, D, S, A, D, S, B,$$

$$A, D, S, E, A, D, S, E, A, D, S, E, B, C,$$

$$A, D, S, A, D, S, A, D, S, B, C$$

where **S** is short for Switch. The buffer sizes are as follows:

$$A \rightarrow B : 3$$
$$A \rightarrow S : 1$$
$$D \rightarrow S : 1$$
$$S \rightarrow E : 1$$
$$S \rightarrow C : 3$$
$$B \rightarrow C : 4$$

Question 10

Consider the SDF graph shown below:

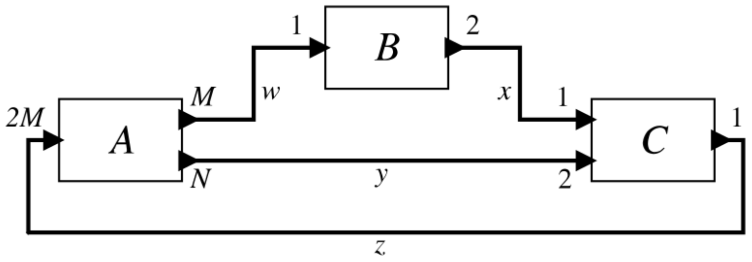


Figure 15: SDF graph of question 10

In this figure, A, B, and C are actors. Adjacent to each port is the number of tokens consumed or produced by a firing of the actor on that port, where N and M are variables with positive integer values. Assume the variables w, x, y, and z represent the number of initial tokens on the connection where these variables appear in the diagram. These variables have non-negative integer values.

1. Derive a simple relationship between N and M such that the model is consistent, or show that no positive integer values of N and M yield a consistent model.

Solution

$$\begin{cases} Mq_A = q_B \\ Nq_A = 2q_C \\ 2q_B = q_C \\ q_C = 2Mq_A \end{cases} \rightarrow \begin{cases} 2Mq_A = q_C \\ Nq_A = 2q_C \end{cases} \rightarrow 4Mq_A = Nq_A \rightarrow 4M = N$$

2. Assume that $w = x = y = 0$ and that the model is consistent and find the minimum value of z (as a function N and M) such that the model does not deadlock.

Solution

$$\begin{cases} Mq_A = q_B \\ Nq_A = 2q_C \\ 2q_B = q_C \\ q_C + z = 2Mq_A \end{cases} \rightarrow \begin{cases} 2Mq_A = q_C \\ Nq_A = 2q_C \end{cases} \rightarrow 4Mq_A = Nq_A \rightarrow 4M = N$$

$$\begin{cases} 2Mq_A = q_C \\ Nq_A = 2q_C \end{cases} \rightarrow Nq_A - 2Mq_A = q_C \rightarrow Nq_A - 2Mq_A = 2Mq_A - z$$

$$\rightarrow z = 4Mq_A - Nq_A$$

$$\rightarrow \begin{cases} 2q_B = q_C \\ 2Mq_A = 2q_C \end{cases} \rightarrow \begin{cases} q_A = 1 \\ q_B = 1 \\ q_C = 2 \end{cases}$$

3. Assume that $z = 0$ and that the model is consistent. Find values for w, x, and y such that the model does not deadlock and $w + x + y$ is minimized.

Solution

$$\begin{cases} 1) Mq_A + w = q_B \\ 2) Nq_A + y = 2q_C \\ 3) 2q_B + x = q_C \\ 4) q_C = 2Mq_A \end{cases} \xrightarrow{3,4} 5) 2q_B + x = 2Mq_A \xrightarrow{5,1} 2Mq_A + 2w = 2Mq_A - x$$

$$\rightarrow x + 2w = 0$$

$$\xrightarrow{2, N=4M} 6) 4Mq_A + y = 2q_C \xrightarrow{6,4} 4Mq_A + y = 4Mq_A \rightarrow y = 0$$

$$\rightarrow \begin{cases} x + 2w = 0 \\ y = 0 \end{cases} \rightarrow \begin{cases} x = 0 \\ y = 0 \\ w = 0 \end{cases}$$

4. Assume that $w = x = y = 0$ and z is whatever value you found in part (b). Let b_w , b_x , b_y , and b_z be the buffer sizes for connections w, x, y, and z, respectively. What is the minimum for these buffer sizes?

Solution

$$\begin{cases} Mq_A = q_B \\ Nq_A = 2q_C \\ 2q_B = q_C \\ q_C = 2Mq_A \end{cases} \rightarrow \begin{cases} 2Mq_A = q_C \\ Nq_A = 2q_C \end{cases} \rightarrow 4Mq_A = Nq_A \rightarrow 4M = N$$

$$\rightarrow \begin{cases} Mq_A = q_B \\ 4Mq_A = 2q_C \\ 2q_B = q_C \\ q_C = 2Mq_A \end{cases} \rightarrow \begin{cases} Mq_A = q_B \\ 2q_A = 2q_C \\ 2q_B = q_C \end{cases} \rightarrow \begin{cases} M = 1 \\ N = 4 \\ q_A = 1 \\ q_B = 1 \\ q_C = 2 \end{cases}$$

End of Homework 5

All figure of this HW are draw with Ipe.
You can download it from here: https://ipe.otfried.org/