

Embedded Systems Design and Modeling



Chapter 2 Modeling with an emphasis on Continuous Dynamics Part 2

System Properties: Causality

- ❑ Causal: output(s) depend only on current and past inputs (including that point)
- ❑ Need a mathematical representation:
 - A function “restriction in time” represents the current and past inputs
 - It is defined as: for times $t \leq \tau$
$$x_1|_{t \leq \tau} = x_2|_{t \leq \tau} \Rightarrow S(x_1)|_{t \leq \tau} = S(x_2)|_{t \leq \tau}$$
- ❑ Strictly causal: excluding that particular point in time (e.g., integrator actor)
$$x_1|_{t < \tau} = x_2|_{t < \tau} \Rightarrow S(x_1)|_{t \leq \tau} = S(x_2)|_{t \leq \tau}$$

System Properties: Memorylessness

- Memoryless: output(s) depend only on the current inputs (not the past inputs)
- Consider continuous time system S :

$$S: X \rightarrow Y, \text{ where } X = A^{\mathbb{R}} \text{ and } Y = B^{\mathbb{R}}$$

if there is a function f that for all x :

$$(S(x))(t) = f(x(t))$$

then the system is memoryless

- Integrator actor is NOT memoryless
- Adder actor is memoryless

System Properties: Linearity

- Linearity: a system S is linear if the superposition property holds:

$$\forall x_1, x_2 \in X \text{ and } \forall a, b \in \mathbb{R}, \quad S(ax_1 + bx_2) = aS(x_1) + bS(x_2)$$

- Scale actor is always linear
 - Integrator actor is linear if initial value=0
- Linearity is a very important property in the study of systems because:
 - Combination of linear actors or systems (e.g., cascaded) will form another linear system

System Properties: Time Invariance

- Time invariant: when a system's behavior is independent of when the input is applied
- In other words, the system is repeatable
- Formal mathematical representation:

- Define the delay actor:

$$\forall x \in X \text{ and } \forall t \in \mathbb{R}, \quad (D_{\tau}(x))(t) = x(t - \tau)$$

- Time invariance:

$$\forall x \in X \text{ and } \forall \tau \in \mathbb{R}, \quad S(D_{\tau}(x)) = D_{\tau}(S(x))$$

- LTI: both linear and time invariant

System Properties: Stability

- Physical phenomena are bounded:
 - All inputs are bounded \Rightarrow
 - All outputs have to be bounded in physical world too
- Unbounded behavior is perceived as instability
 - Example: natural frequency oscillations
- Stable system definition: for all bounded input signals output signals have to remain bounded too

System Properties: Stability (Cont'd)

- Formal mathematical definition:
 - Input, $w(t)$, is bounded if there is a real number $A < \infty$ such that $|w(t)| \leq A$ for all $t \in \mathbb{R}$
 - Output, $v(t)$, is bounded if there is a real number $B < \infty$ such that $|v(t)| \leq B$ for all $t \in \mathbb{R}$
 - A system is stable if for an arbitrary bound A , a bound B can be found for the output
- Integrator is NOT a stable system

Summary

- ❑ ODEs and actor models are examples of good MoCs
- ❑ Systems with continuous dynamics can be nicely described by them
- ❑ System properties are important parts of modeling embedded systems
- ❑ There are many challenges in the proper modeling, design, and analysis of well-known MoCs

Homework Assignments

- ▣ Chapter 2: 2, 3, 4, 5, 6, 7 for the next Tuesday 1402/12/15