Embedded Systems

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Homework 7
Chapter 12 - Scheduling

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Question 1

This problem studies fixed-priority scheduling. Consider two tasks to be executed periodically on a single processor, where task 1 has period $p_1 = 4$ and task 2 has period $p_2 = 6$.

1. Let the execution time of task 1 be $e_1 = 1$. Find the maximum value for the execution time e_2 of task 2 such that the RM schedule is feasible.

Soloution

According to the proposed schedule:

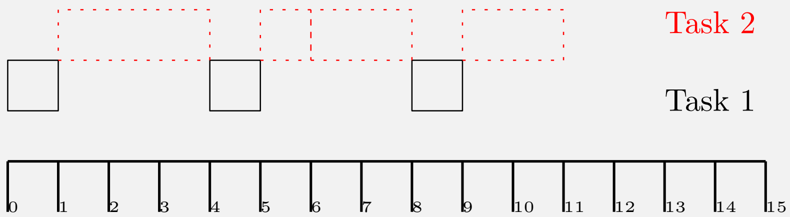

Figure 1: Time schedule of question 1-a

The answer is 4.

2. Again let the execution time of task 1 be $e_1 = 1$. Let non-RMS be a fixed priority schedule that is not an RM schedule. Find the maximum value for the execution time $e_2$of task 2 such that non-RMS is feasible.

Soloution

According to the following formula:

$$e_1 + e_2 \leq p_1$$

The answer is 3.

3. For both your solutions to (a) and (b) above, find the processor utilization. Which is better?

Soloution

(a) For RM mode, the idle rate of the system is equal to 1 cycle in 12 cycles.

(b) For the non-RM mode, the idle rate of the system is equal to 3 cycles in 12 cycles. Therefore, in the RM mode, the number can be $\frac{11}{12}$ and in the non-RM mode reached of $\frac{9}{12}$ So, in RM mode, the amount of processor usage is higher.

4. For RM scheduling, are there any values for e_1 and e_2 that yield 100% utiliza tion? If so, give an example.

Soloution

If there is a need to run both, it seems it is not shared for 100% of processor usage. but if possible, do not execute a task, it means e is equal to zero and by equalizing e and other p tasks, we will reach 100%.

Question 2

This problem studies dynamic-priority scheduling. Consider two tasks to be exe cuted periodically on a single processor, where task 1 has period $p_1 = 4$ and task 2 has period $p_2 = 6$. Let the deadlines for each invocation of the tasks be the end of their period. That is, the first invocation of task 1 has deadline 4, the second invocation of task 1 has deadline 8, and so on.

1. Let the execution time of task 1 be $e_1 = 1$. Find the maximum value for the execution time e_2 of task 2 such that EDF is feasible.

Soloution

In the EDF (Earliest Deadline First) mode, an attempt is made to minimize the lateness as much as possible by executing each task with the nearest deadline. According to the text of the book, this is calculated using the following formula:

$$L_{\max} = \max_{i \in T}(f_i - d_i)$$

If the value of L can be kept negative or zero, the relationship is feasible.

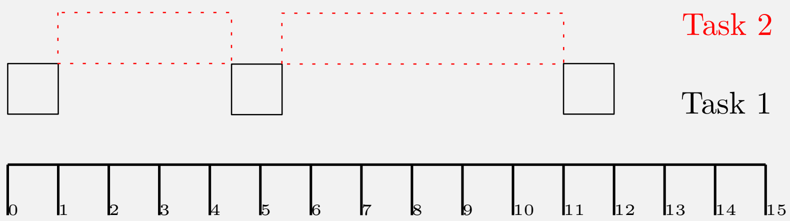

Figure 2: Time schedule of question 2-a

For this purpose, e_2 must be selected in such a way that the conditions for this are met and no gaps exist, as shown in the figure above. Therefore, considering the problem statement, we have:

$$K = \frac{(4(K+1)) - 1}{1 + e_2}$$

We test this equation for different values of K:
For $K = 1$, we have $e_2 = 6$ which is equal to p_2.
For $K = 2$, we have $e_2 = 4.5$ which correctly satisfies the equation. Therefore, the value $e_2 = 4.5$ is the answer to the question.

2. For the value of e_2 that you found in part (a), compare the EDF schedule against the RM schedule from Exercise 1 (a). Which schedule has less pre emption? Which schedule has better utilization?

Soloution

Considering the scenario given in the previous question and the obtained value, there should be no pre-emption in EDF. However, pre-emption does occur in RM. Additionally, in EDF, we will achieve 100% utilization, which is higher than in RM.

Question 3

This problem compares RM and EDF schedules. Consider two tasks with periods $p_1 = 2$ and $p_2 = 3$ and execution times $e_1 = e_2 = 1$. Assume that the deadline for each execution is the end of the period.

1. Give the RM schedule for this task set and find the processor utilization. How does this utilization compare to the Liu and Layland utilization bound of (12.2)?

Soloution

n every 6 time cycles, the first task must be executed 3 times and the second task 2 times. Since $e_1 = e_2 = 1e_1 = e_2 = 1$, the total utilized time is 5. Therefore, we have:

$$\mu \leq n(2^{\frac{1}{n}} - 1) = \mu = \frac{5}{6} \approx 0.833$$

Therefore, for $n = 2$, this number is approximately 0.82842, which is less than the calculated value for μ. Hence, RM is probably not feasible in this case.

2. Show that any increase in e_1 or e_2 makes the RM schedule infeasible. If you hold $e_1 = e_2 = 1$ and $p_2 = 3$ constant, is it possible to reduce p_1 below 2 and still get a feasible schedule? By how much? If you hold $e_1 = e_2 = 1$ and $p_1 = 2$ constant, is it possible to reduce p_2 below 3 and still get a feasible schedule? By how much?

Soloution

Intuitively, it can be said that given the value of p_1, this task is executed alternately. Therefore, if the execution time is changed, there will be no time left for the proper execution of the other task (unless the execution time of the other task is reduced by the same amount). For this reason, the execution time of e_2 cannot be increased either, because e_1 is executed alternately, leaving no room to increase the execution time of e_2.
Mathematically, assuming 100% processor utilization, p_1 can be considered equivalent to 1.5, meaning it is executed 4 times in 6 cycles, which with two executions of the other task equals 6. Similarly, for task2, assuming 3 executions, p_2 can be considered equal to 2, which with 3 executions of task1 equals 6.

3. Increase the execution time of task 2 to be $e_2 = 1.5$, and give an EDF sched- ule. Is it feasible? What is the processor utilization?

Soloution

Yes, this is possible using EDF. According to the problem definition, we have:

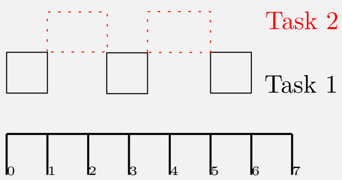

Figure 3: Time schedule of question 3-c

And the processor utilization is also 100%.

Question 4

This problem, formulated by Hokeun Kim, also compares RM and EDF schedules. Consider two tasks to be executed periodically on a single processor, where task 1 has period $p_1 = 4$ and task 2 has period $p_2 = 10$. Assume task 1 has execution time $e_1 = 1$, and task 2 has execution time $e_2 = 7$.

1. Sketch a rate-monotonic schedule (for 20 time units, the least common multiple of 4 and 10). Is the schedule feasible?

Soloution

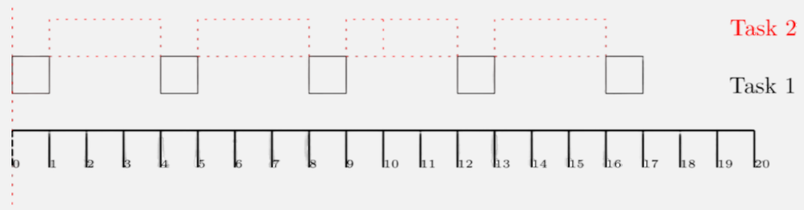

Figure 4: Time schedule of question 4-a

Yes, it seems feasible.

2. Now suppose task 1 and 2 contend for a mutex lock, assuming that the lock is acquired at the beginning of each execution and released at the end of each execution. Also, suppose that acquiring or releasing locks takes zero time and the priority inheritance protocol is used. Is the rate-monotonic schedule feasible?

Soloution

No, specifically after the mutex is locked by task2, due to its long execution time, task1 misses its deadline.

3. Assume still that tasks 1 and 2 contend for a mutex lock, as in part (b). Sup pose that task 2 is running an **anytime algorithm**, which is an algorithm that can be terminated early and still deliver useful results. For example, it might be an image processing algorithm that will deliver a lower quality image when terminated early. Find the maximum value for the execution time e_2 of task 2 such that the rate-monotonic schedule is feasible. Construct the resulting schedule, with the reduced execution time for task 2, and sketch the schedule for 20 time units. You may assume that execution times are always positive integers.

Soloution

We decrease the value of e_2 one by one. For $e_2 = 6$, we have:

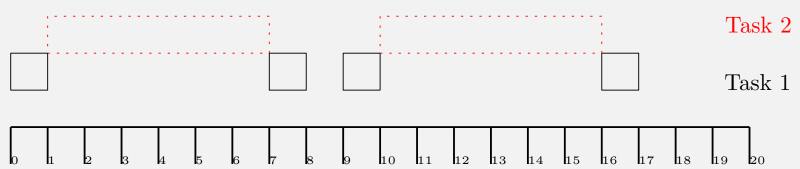

Figure 5: Time schedule of question 4-c

Soloution

Which clearly shows that task1 misses its deadline in the 16^{th} time cycle.
Since it is mentioned that the execution time is of integer type, for $e_2 = 5$, we have:

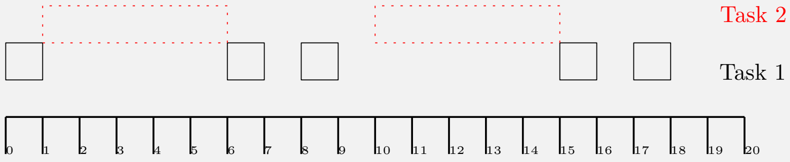

Figure 6: Time schedule of question 4-c

Which seems correct. Therefore, with $e_2 = 5$, RM can be feasible.

4. For the original problem, where $e_1 = 1$ and $e_2 = 7$, and there is no mutex lock, sketch an EDF schedule for 20 time units. For tie-breaking among task executions with the same deadline, assume the execution of task 1 has higher priority than the execution of task 2. Is the schedule feasible?

Soloution

To solve this problem, we need to calculate the difference between the execution time of each task and its deadline, and execute the task with the smaller value. Additionally, if the values are equal, task1 has the priority.

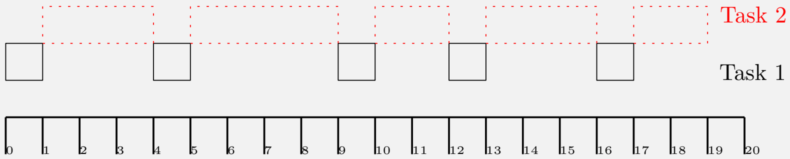

Figure 7: Time schedule of question 4-d

5. Now consider adding a third task, task 3, which has period $p_3 = 5$ and ex ecution time $e_3 = 2$. In addition, assume as in part (c) that we can adjust execution time of task 2. Find the maximum value for the execution time e_2 of task 2 such that the EDF schedule is feasible and sketch the schedule for 20 time units. Again, you may assume that the execution times are always positive integers. For tie-breaking among task executions with the same deadline, assume task i has higher priority than task j if $i < j$.)

Soloution

For this case, within a span of 20 time cycles, task1 is executed 5 times and task3 is executed 4 times. Therefore, we have:

$$20 - (5 \times 1 + 4 \times 2) = 7$$

This amount is the total available time for task2, which, given $p_2 = 10$ and the need to execute task2 twice, and also considering the condition that the execution time must be an integer, we have:

$$e_2 = 3$$

Solution

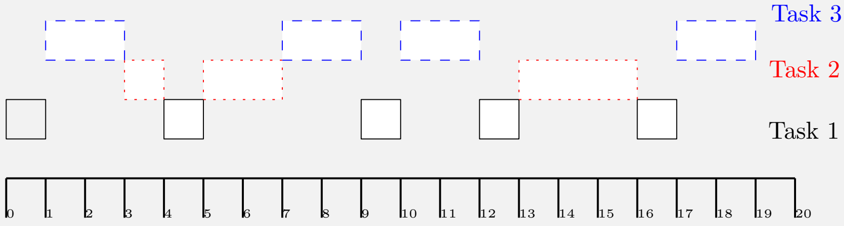

Figure 8: Time schedule of question 4-e

Question 5

This problem compares fixed vs. dynamic priorities, and is based on an example by Burns and Baruah (2008). Consider two periodic tasks, where task τ_1 has period $p_1 = 2$, and task τ_2 has period $p_2 = 3$. Assume that the execution times are $e_1 = 1$ and $e_2 = 1.5$. Suppose that the release time of execution i of task τ_1 is given by

$$r_{1,i} = 0.5 + 2(i-1)$$

for $i = 1, 2, \ldots$. Suppose that the deadline of execution i of task τ_1 is given by

$$d_{1,i} = 2i$$

Correspondingly, assume that the release times and deadlines for task τ_2 are

$$r_{2,i} = 3(i-1)$$

and

$$d_{2,i} = 3i$$

1. Give a feasible fixed-priority schedule.

Soloution

Given the ambiguity in the problem statement, considering the previous questions, the task with higher priority is assumed. If this assumption is not made, each question must be solved twice: once for task1 with higher priority and once for task2 with higher priority.

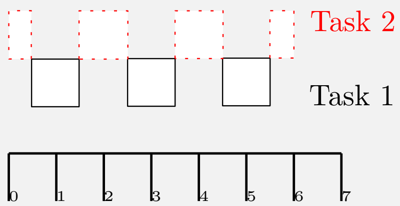

Figure 9: Time schedule of question 5-a

2. Show that if the release times of all executions of task τ_1 are reduced by 0.5, then no fixed-priority schedule is feasible.

Soloution

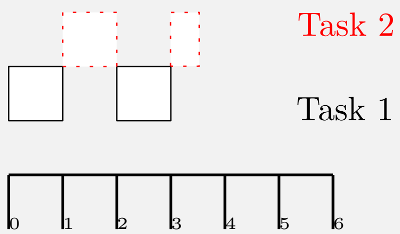

Figure 10: Time schedule of question 5-b

It is clear that in this case, task2 misses its deadline.

3. Give a feasible dynamic-priority schedule with the release times of task τ_1 reduced to

$$r_{1,i} = 2(i-1)$$

Soloution

In this case we have:

$$D_{2,2} = 6 \;\; D_{2,1} = 3 \;\; D_{1,3} = 6 \;\; D_{1,2} = 4 \;\; D_{1,1} = 2$$

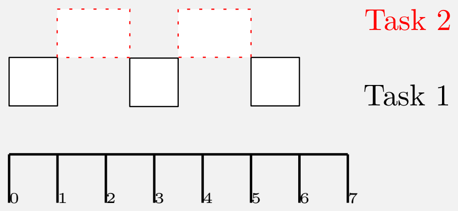

Figure 11: Time schedule of question 5-c

End of Homework 7

All of this figures, draw with `ipe`. You can download this software here:
`ipe.otfried.org`