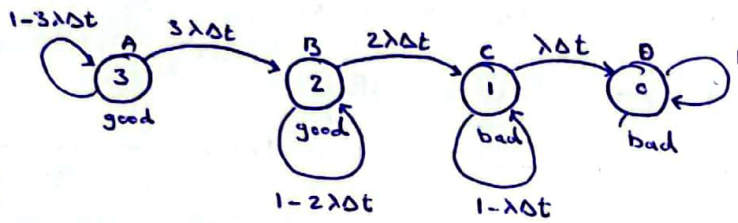سوال ①

$$P_A(t+\Delta t) = P_A(t) \cdot (1 - 3\lambda\Delta t)$$
$$P_B(t+\Delta t) = P_B(t) \cdot (1 - 2\lambda\Delta t) + P_A(t) \cdot 3\lambda\Delta t$$
$$P_C(t+\Delta t) = P_C(t) \cdot (1 - \lambda\Delta t) + P_B(t) \cdot 2\lambda\Delta t$$
$$P_D(t+\Delta t) = P_D(t) + P_C(t) \cdot \lambda\Delta t$$

$$\longrightarrow$$

$$P_A'(t) = -3\lambda P_A(t)$$
$$P_B'(t) = -2\lambda P_B(t) + 3\lambda P_A(t)$$
$$P_C'(t) = -\lambda P_C(t) + 2\lambda P_B(t)$$
$$P_D'(t) = \lambda P_C(t)$$

$$\begin{cases} SP_A(s) - P_A(0) = -3\lambda P_A(s) \\ SP_B(s) - P_B(0) = -2\lambda P_B(s) + 3\lambda P_A(s) \\ SP_C(s) - P_C(0) = -\lambda P_C(s) + 2\lambda P_B(s) \\ SP_D(s) - P_D(0) = \lambda P_C(s) \end{cases}$$

$$\Rightarrow SP_A(s) + 3\lambda P_A(s) = 1 \longrightarrow \boxed{P_A = \dfrac{1}{s+3\lambda}} \quad (I)$$

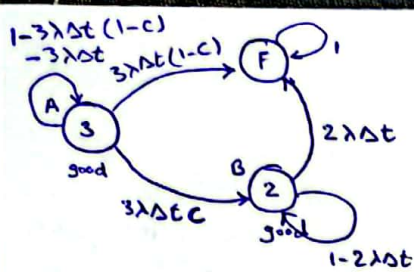$$SP_B(s) + 2\lambda P_B(s) = 3\lambda \cdot \dfrac{1}{s+3\lambda}$$

$$\Rightarrow \boxed{P_B(s) = \dfrac{3\lambda}{(s+3\lambda)(s+2\lambda)}} \quad (II)$$

$$\begin{cases} P_A(t) = e^{-3\lambda t} \\[2mm] P_B(t): \quad P_B(s) = \dfrac{3\lambda}{(s+3\lambda)(s+2\lambda)} = \dfrac{A}{s+3\lambda} + \dfrac{B}{s+2\lambda} \end{cases}$$

$$\begin{cases} A = (s+3\lambda) P_{(s)} \Big|_{s=-3\lambda} = -3 \\[3mm] B = (s+2\lambda) P_{(s)} \Big|_{s=-2\lambda} = 3 \end{cases}$$

$$\Rightarrow P_B(s) = \dfrac{-3}{s+3\lambda} + \dfrac{3}{s+2\lambda} \xrightarrow{\mathcal{L}^{-1}} \boxed{P_B(t) = -3e^{-3\lambda t} + 3e^{-2\lambda t}}$$

$$\Rightarrow R(t) = P_A(t) + P_B(t) = e^{-3\lambda t} - 3\left(e^{-3\lambda t} - e^{-2\lambda t}\right)$$

State diagram labels: $1-3\lambda\Delta t(1-c)$, $-3\lambda\Delta t$, $3\lambda\Delta t(1-c)$, F, 1, A, 3, good, $2\lambda\Delta t$, B, 2, $3\lambda\Delta t c$, good, $1-2\lambda\Delta t$

$$P_A(t+\Delta t) = P_A(t)\left(1 - (3\lambda\Delta t(1-c) + 3\lambda\Delta t)\right)$$

$$P_B(t+\Delta t) = P_B(t)(1-2\lambda\Delta t) + P_A(t)(3\lambda\Delta t \cdot c)$$

$$P_F(t+\Delta t) = P_F(t) + P_B(t)(2\lambda\Delta t)$$

② جلیه

$$\Rightarrow \quad \begin{cases} P_A'(t) = -6\lambda P_A(t) + 3c\lambda P_A(t) \\ P_B'(t) = -2\lambda P_B(t) + 3\lambda c P_A(t) \\ P_F'(t) = 2\lambda P_B(t) \end{cases} \xrightarrow{\mathcal{L}}$$

$$SP_A(S) - P_A'(0) = -6\lambda P_A(S) + 3c\lambda P_A(S) \quad (I)$$

$$SP_B(S) = -2\lambda P_B(S) + 3\lambda c P_A(S) \quad (II)$$

$$SP_F(S) = 2\lambda P_B(S) \quad (III)$$

$(I):\ SP_A(S) + 6\lambda P_A(S) - 3c\lambda P_A(S) = 1 \quad \Rightarrow P_A(S)(S+6\lambda - 3c\lambda) = 1 \Rightarrow P_A(S) = \dfrac{1}{S+(6-3c)\lambda}$

$(II):\ SP_B(S) + 2\lambda P_B(S) = \dfrac{3\lambda c}{S+(6-3c)\lambda} \quad \Rightarrow P_B(S) = \dfrac{3\lambda c}{(S+2\lambda)(S+(6-3c)\lambda)} = \dfrac{A}{S+2\lambda} + \dfrac{B}{S+(6-3c)\lambda}$
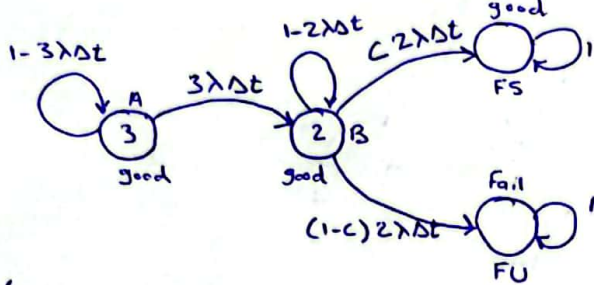
$$\begin{cases} A = (S+2\lambda)P_B(S)\Big|_{S=-2\lambda} = \dfrac{3c}{4-3c} \\ B = (S+(6-3c)\lambda)P_B(S)\Big|_{S=-(6-3c)\lambda} = \dfrac{3c}{-4+3c} \end{cases}$$

$$\Rightarrow \dfrac{\frac{3c}{4-3c}}{S+2\lambda} + \dfrac{\frac{3c}{-4+3c}}{S+(6-3c)\lambda} \xleftarrow{\mathcal{L}^{-1}} P_B(t) = \dfrac{3c}{4-3c}e^{-2\lambda t} + \dfrac{3c}{-4+3c}e^{-(6-3c)\lambda t}$$

$$P_A(t) = e^{-(6-3c)\lambda t} \quad \Rightarrow R(t) = P_A(t) + P_B(t) = e^{-(6-3c)\lambda t} + \dfrac{3c}{4-3c}e^{-2\lambda t} + \dfrac{3c}{-4+3c}e^{-(6-3c)\lambda t}$$

$$MTTF = \int_0^\infty R(t)\,dt = \int_0^\infty e^{-(6-3c)\lambda t}\,dt + \int_0^\infty \dfrac{3c}{4-3c}e^{-2\lambda t}\,dt + \dfrac{3c}{-4+3c}\int_0^\infty e^{-(6-3c)\lambda t}\,dt$$

$$= \dfrac{1}{\lambda(-3c+6)} + \left[\dfrac{3c}{-4+3c}\times\dfrac{1}{\lambda(-3c+6)}\right] + \left[\dfrac{3c}{4-3c}\times\dfrac{1}{2\lambda}\right]$$

$$P_A(t+\Delta t) = P_A(t) \cdot (1 - 3\lambda\Delta t)$$

$$P_B(t+\Delta t) = P_B(t) \cdot (1 - 2\lambda\Delta t) + P_A(t) \cdot (3\lambda\Delta t)$$

$$P_{FS}(t+\Delta t) = P_{FS}(t) + P_B(t) \cdot (2c\lambda\Delta t)$$

$$P_{FU}(t+\Delta t) = P_{FU}(t) + P_B(t) \cdot (2\lambda(1-c)\Delta t)$$

$$P'_A(t) = -3\lambda P_A(t)$$

$$P'_B(t) = -2\lambda P_B(t) + 3\lambda P_A(t)$$

$$P'_{FS}(t) = 2c\lambda P_B(t)$$

$$P'_{FU}(t) = 2\lambda(1-c) P_B(t)$$

$\xrightarrow{L}$

$$S P_A(s) - P_A(0) = -3\lambda P_A(s) \qquad (I)$$

$$S P_B(s) - P_B(0) = -2\lambda P_B(s) + 3\lambda P_A(s) \qquad (II)$$

$$S P_{FS}(s) - P_{FS}(0) = 2c\lambda P_B(s) \qquad (III)$$

$$S P_{FU}(s) - P_{FU}(0) = 2\lambda(1-c) P_B(s) \qquad (IV)$$

$(I): \quad S P_A(s) + 3\lambda P_A(s) = 1 \longrightarrow P_A(s)(S+3\lambda) = 1 \longrightarrow P_A(s) = \dfrac{1}{S+3\lambda} \xrightarrow{L^{-1}} \boxed{P_A(t) = e^{-3\lambda t}}$

$(II): \quad S P_B(s) + 2\lambda P_B(s) = 3\lambda P_A(s) \xrightarrow{(I)} P_B(s)(S+2\lambda) = \dfrac{3\lambda}{S+3\lambda}$

$$\Rightarrow P_B(s) = \frac{\frac{3\lambda}{S+3\lambda}}{S+2\lambda} = \frac{3\lambda}{(S+3\lambda)(S+2\lambda)} = \frac{A}{S+3\lambda} + \frac{B}{S+2\lambda}$$

$$\begin{cases} A = (S+3\lambda) P_B(s)\Big|_{s=-3\lambda} = \dfrac{3\lambda}{-3\lambda+2\lambda} = -3 \\[2mm] B = (S+2\lambda) P_B(s)\Big|_{s=-2\lambda} = \dfrac{3\lambda}{-2\lambda+3\lambda} = 3 \end{cases}$$

$$\Rightarrow P_B(s) = \frac{-3}{S+3\lambda} + \frac{3}{S+2\lambda} \xrightarrow{L^{-1}} \boxed{P_B(t) = -3e^{-3\lambda t} + 3e^{-2\lambda t}}$$

$(III): \quad S P_{FS}(s) = 2c\lambda \cdot \dfrac{3\lambda}{(S+3\lambda)(S+2\lambda)}$

$$\Rightarrow P_{FS}(s) = \frac{6c\lambda^2}{S(S+3\lambda)(S+2\lambda)} = \frac{A}{S} + \frac{B}{S+3\lambda} + \frac{C}{S+2\lambda}$$

$$A = S P_{FS}(s)\Big|_{s=0} = \frac{6c\lambda^2}{3\lambda \cdot 2\lambda} = \frac{6c\lambda^2}{6\lambda^2} = C$$

$$B = (S+3\lambda) P_{FS}(s)\Big|_{s=-3\lambda} = \frac{6c\lambda^2}{-3\lambda \cdot (-3\lambda+2\lambda)} = \frac{6c\lambda^2}{3\lambda^2} = 2C$$
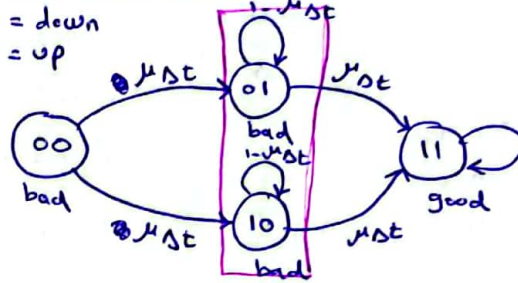
$$C = (S+2\lambda) P_{FS}(s)\Big|_{s=-2\lambda} = \frac{6c\lambda^2}{-2\lambda(-2\lambda+3\lambda)} = \frac{6c\lambda^2}{-2\lambda^2} = -3C$$

$$\Rightarrow P_{FS}(s) = \frac{C}{S} + \frac{2C}{S+3\lambda} + \frac{-3C}{S+2\lambda} \xrightarrow{L^{-1}} \boxed{P_{FS}(t) = C + 2Ce^{-3\lambda t} - 3Ce^{-2\lambda t}}$$

$$\Rightarrow S(t) = P_A(t) + P_B(t) + P_{FS}(t) = e^{-3\lambda t} - 3e^{-3\lambda t} + 3e^{-2\lambda t} + C + 2Ce^{-3\lambda t} - 3Ce^{-2\lambda t}$$

$$= (1-2+2c)e^{-3\lambda t} + (3-3c)e^{-2\lambda t} + C$$

output

M₁ → output

= → error

M₂ →

0 = down
1 = up

M(t) = P_c(t)

$$P_A(t+\Delta t) = P_A(t) \cdot (1 - 2\mu\Delta t)$$

$$P_B(t+\Delta t) = P_B(t) \cdot (1 - \mu\Delta t) + P_A(t) \cdot (2\mu\Delta t) \longrightarrow$$

$$P_C(t+\Delta t) = P_C(t) + P_B(t) \cdot (1 - \mu\Delta t)$$

$$P_A'(t) = -2\mu P_A(t)$$

$$P_B'(t) = \mu P_B(t) + 2\mu P_A(t)$$

$$P_C'(t) = P_B(t) - \mu P_B(t)$$

$$\begin{cases} S P_A(s) - P_A(0) = -2\mu P_A(s) & (I) \\ S P_B(s) = \mu P_B(s) + 2\mu P_A(s) & (II) \\ S P_C(s) = P_B(s) - \mu P_B(s) & (III) \end{cases}$$

(I): $P_A(s)(S+2\mu) = 1 \longrightarrow P_A(s) = \dfrac{1}{S+2\mu} \xrightarrow{L^{-1}} \boxed{P_A(t) = e^{-2\mu t}}$

(II): $P_B(s)(S - \mu) = \dfrac{2\mu}{S+2\mu} \longrightarrow P_B(s) = \dfrac{\frac{2\mu}{S+2\mu}}{S+\mu} = \dfrac{2\mu}{(S+2\mu)(S+\mu)}$

$$= \dfrac{A}{S+2\mu} + \dfrac{B}{S+\mu}$$

$$\begin{cases} A = (S+2\mu)P_B(s)\Big|_{S=-2\mu} = \dfrac{2\mu}{-2\mu+\mu} = -2 \\[4mm] B = (S+\mu)P_B(s)\Big|_{S=-\mu} = \dfrac{2\mu}{-\mu+2\mu} = 2 \end{cases}$$

$$\Rightarrow P_B(s) = \dfrac{-2}{S+2\mu} + \dfrac{2}{S+\mu} \xrightarrow{L^{-1}} \boxed{P_B(t) = -2e^{-2\mu t} + 2e^{-\mu t}}$$

(III) $P_C(s) = \dfrac{1}{S}\left[(1-\mu)P_B(s)\right] = \dfrac{1}{S}\left[(1-\mu) \cdot \dfrac{2\mu}{(S+2\mu)(S+\mu)}\right]$

$$= \dfrac{2\mu(1-\mu)}{S(S+2\mu)(S+\mu)} = \dfrac{A}{S} + \dfrac{B}{S+2\mu} + \dfrac{C}{S+\mu}$$

$$\begin{cases} A = S P_C(s)\Big|_{S=0} = \dfrac{2\mu(1-\mu)}{2\mu(\mu)} = \dfrac{2\mu - 2\mu^2}{2\mu^2} \\[3mm] B = (S+2\mu)P_C(s)\Big|_{S=-2\mu} = \dfrac{2\mu(1-\mu)}{-2\mu(-\mu)} = \dfrac{2\mu - 2\mu^2}{2\mu^2} \\[3mm] C = (S+\mu)P_C(s)\Big|_{S=-\mu} = \dfrac{2\mu(1-\mu)}{-\mu(\mu)} = \dfrac{2\mu - 2\mu^2}{-\mu^2} \end{cases}$$

$$\rightarrow P_C(s) = \dfrac{\frac{2\mu(1-\mu)}{2\mu^2}}{S} + \dfrac{\frac{2\mu(1-\mu)}{2\mu^2}}{S+2\mu} + \dfrac{\frac{-2\mu(1-\mu)}{\mu^2}}{S+\mu}$$

$$\xrightarrow{L^{-1}} P_C(t) = \dfrac{2\mu(1-\mu)}{2\mu^2} + \dfrac{2\mu(1-\mu)}{2\mu^2}e^{-2\mu t} - \dfrac{2\mu(1-\mu)}{\mu^2}e^{-\mu t} = M(t)$$

1-2μΔt   1-μΔt

A ⟲ 00 → 2μΔt → B ⟲ μ¹ → μΔt → C ⟲ μ² → 1

bad   bad   good